Collusion in Auctions with Constrained Bids:

Theory and Evidence from Public Procurement[*]

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Abstract

We study the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. Under collusion, bidding constraints weaken cartels by limiting the scope for punishment. This yields a test of repeated collusive behavior exploiting the counter-intuitive prediction that introducing minimum prices can lower the distribution of winning bids. The model's predictions are borne out in procurement data from Japan, where we find evidence that collusion is weakened by the introduction of minimum prices. A robust design insight is that setting a minimum price at the bottom of the observed distribution of winning bids necessarily improves over a minimum price of zero.

KEYWORDS: collusion, cartel enforcement, minimum prices, entry deterrence, procurement.

1 Introduction

This paper studies the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. Minimum prices, which place a lower bound on the price at which procurement contracts can be awarded, are frequently used in public procurement. Because minimum prices make price wars less effective, they can also make cartel enforcement more difficult. This leads to the counter-intuitive prediction that the introduction of minimum prices may lead to a first-order stochastic dominance drop in the right tail of winning bids. Because this prediction does not arise in competitive environments, it provides a joint test of collusion and of the specific channel we outline: enforcement constraints are binding, and they can be affected by institution design. The model's predictions are borne out in procurement data from Japan, showing that binding enforcement constraints are an empirically relevant determinant of cartel behavior.

From a policy perspective, our findings show that in the presence of colluding bidders, attempts at surplus extraction may foster collusion and reduce the auctioneer's surplus. Inversely, providing minimum surplus guarantees can limit collusion and improve the auctioneer's surplus. A robust take-away from our analysis is that introducing a minimum price at the bottom of the distribution of observed bids always dominates setting no minimum price. If there is no collusion, it does not affect the distribution of bids, and if there is collusion it can only reduce the distribution of bids.

We model firms as repeatedly playing a first-price procurement auction with i.i.d. production costs. We assume that costs are commonly observed among cartel members, and that firms are able to make transfers. In this environment, cartel behavior is limited by self-enforcement constraints: firms must be willing to follow bidding recommendations, as well as make equilibrium transfers. We provide an explicit characterization of optimal cartel behavior: first, contract allocation is efficient, provided that price constraints are not binding; second, cartel members implement the highest possible winning bid for which the

sum of deviation temptations is less than the cartel's total pledgeable surplus. This simple characterization lets us delineate distinctive predictions of the model in a transparent manner.

Our main predictions relate the introduction of minimum prices and changes in the distribution of winning bids. In our repeated game environment, minimum prices may weaken cartel discipline by limiting the impact of price wars. When this is the case, sustaining collusive bids above the minimum price becomes more difficult, causing a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price. A key observation is that minimum prices have either no impact, or the opposite impact in environments without collusion. Under competition, regardless of whether firms have complete or asymmetric information about costs, minimum prices lead to a weak first-order stochastic dominance increase in the right tail of winning bids. This provides a joint test of collusion and of the mechanics of cartel enforcement.

Allowing for entry lets us extend this test and generate new predictions. Under our model, minimum prices reduce the right tail of winning bids conditional on a cartel member winning. However, minimum prices have no effect on the right tail of winning bids conditional on an entrant winning. The reason for this is that cartel members seek to dissuade entry by pinning entrants' winning bids to their production costs. As a result the right tail of entrant winning bids does not depend on minimum prices. In contrast, minimum prices still affect the highest sustainable winning bid among potential cartel winners. This differential impact of minimum prices on cartel and entrant winners allows us to distinguish our model from a competitive model in which the introduction of minimum prices also increases entry. In such a model, the winning bids of both cartel and entrant winners should be affected by minimum prices.

We explore the impact of enforcement constraints on cartel behavior by using data from public procurement auctions taking place in Japanese cities of the Ibaraki prefecture between 2007 and 2016. The introduction of minimum prices in several cities lets us use difference-

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in-differences and change-in-changes frameworks (Athey and Imbens, 2006) to recover the counterfactual distribution of winning bids after the policy change. The data consistently exhibit significant drops in the distribution of winning bids to the right of the minimum price. Using frequent participation as a proxy for cartel membership, we also show that possible cartel members are disproportionately affected by the policy change. These findings imply that: (i) there is collusion; (ii) enforcement constraints limit the scope of collusion; (iii) minimum prices successfully weaken cartel discipline.

Our paper lies at the intersection of different strands of the literature on collusion in auctions. The seminal work of Graham and Marshall (1987) and McAfee and McMillan (1992) studies static collusion in environments where bidders are able to contract. A key take-away from their analysis is that the optimal response from the auctioneer should involve setting more constraining reserve prices. In a procurement setting this means reducing the maximum price that the auctioneer is willing to pay. We argue, theoretically and empirically, that when bidders cannot contract and must enforce collusion through repeated game play, minimum price guarantees can weaken cartel enforcement.

An important observation from McAfee and McMillan (1992) is that in the absence of cash transfers, the cartel's ability to collude is severly limited even when commitment is available. A recent strand of work takes seriously the idea that in repeated games, continuation values may successfully replace transfers. Aoyagi (2003) studies bid rotation schemes and allows for communication. Skrzypacz and Hopenhayn (2004) (see also Blume and Heidhues, 2008) study collusion in environments without communication and show that while cartel members may still be able to collude, they will remain bounded away from efficient collusion. Athey et al. (2004) study collusion in a model of repeated Bertrand competition and emphasize that information revelation costs will push cartel members towards rigid pricing schemes. Because we focus on obedience rather than information revelation constraints, our model simplifies away the strategic issues emphasized in this body of work: we assume complete information

among cartel members and transferable utility.[1] This yields a simple characterization of optimal collusion closely related to that obtained in the relational contracting literature (Bull, 1987, Baker et al., 1994, 2002, Levin, 2003), and provides a transparent framework in which to study the effect of price constraints on winning bids.

Several recent papers study the impact of the auction format on collusion. Fabra (2003) compares the scope for tacit collusion in uniform and discriminatory auctions. Marshall and Marx (2007) study the role of bidder registration and information revelation procedures in facilitating collusion. Pavlov (2008) and Che and Kim (2009) consider settings in which cartel members can commit to mechanisms and argue that appropriate auction design can successfully limit collusion provided participants have deep pockets and can make ex ante payments. Abdulkadiroglu and Chung (2003) make a similar point when bidders are patient.

More closely related to our work, Lee and Sabourian (2011) as well as Mezzetti and Renou (2012) study full implementation in repeated environments using dynamic mechanisms. They show that implementation in all equilibria can be achieved by restricting the set of continuation values available to players to support repeated game strategies. The incomplete contracts literature (see for instance Bernheim and Whinston, 1998, Baker et al., 2002) has suggested that the same mechanism, used in the opposite direction, provides foundations for optimally incomplete contracts. Specifically, it may be optimal to keep contracts more incomplete than needed, in order to maintain the range of continuation equilibria needed to enforce efficient behavior. We provide empirical evidence that this theoretical mechanism plays a significant role in practice, and can be meaningfully used to affect collusion between firms.

On the empirical side, an important set of papers develops empirical methods to detect collusion (see Harrington (2008) for a detailed survey of prominent empirical strategies and their theoretical underpinnings). Porter and Zona (1993, 1999) contrast the behavior of sus-

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[1]Note that we allow for incomplete information when we study the impact of minimum prices under competition. This ensures that our test of collusion is not driven by this stark modeling assumption.

pected cartel members with that of non-cartel members, controlling for observables. Bajari and Ye (2003) use excess correlation in bids as a marker of collusion. Porter (1983), along with Ellison (1994) (see also Ishii, 2008) use patterns of price wars of the sort predicted by repeated game models of oligopoly behavior (Green and Porter, 1984, Rotemberg and Saloner, 1986) to identify collusion. In a multi-stage auction context, Kawai and Nakabayashi (2014) argue that excess switching of second and third bidder across bidding rounds, compared to first and second bidders, is a smoking gun for collusion. We propose a test of collusion exploiting changes in the cartel's ability to implement effective punishments.

The paper is structured as follows. Section 2 sets up our benchmark model of cartels and characterizes optimal cartel behavior. Section 3 derives empirical predictions from this model that distinguish it from competitive behavior. Section 4 briefly extends these results in a setting with entry. Section 5 takes the model to data. Section 6 discusses endogenous participation by cartel members, non-performing bidders, and robustness tests for our empirical analysis. Appendix A provides theoretical and empirical extensions. Key proofs are collected in Appendix B. An Online Appendix further develops endogenous participation by cartel members, provides a calibration assessing the magnitude of our findings, and collects remaining proofs.

2   Self-Enforcing Cartels

Modeling strategy.   McAfee and McMillan (1992)'s classic model of cartel behavior focuses on the constraints imposed by information revelation among asymmetrically informed cartel members. Instead, we are interested in the enforcement of cartel recommendations through repeated play. Viewed from the mechanism design perspective of Myerson (1986), McAfee and McMillan (1992) focus on truthful revelation, while we focus on obedience. The implications of the two frictions turn out to be different: interpreted in a procurement context, McAfee and McMillan (1992) show that collusion makes lower maximum prices

desirable; we argue that higher minimum prices may help weaken cartels.

This different emphasis is reflected in our modeling choices. We have three main goals:

(i)   we want to provide transparent intuition on how bidding constraints, here mini-mum prices, can affect cartel behavior and the distribution of bids;

(ii) we want to assess empirically whether enforcement constraints are a significant determinant of cartel behavior;

(iii) we want to exploit this understanding of cartel behavior to derive a test of col-lusion.

Given those goals, we use a tractable complete information model of collusion when fleshing out implications of our H_1 hypothesis ("there is collusion and enforcement constraints are binding"). To ensure that our test is not dependent on this simplification, we allow for more general informational environments when we characterize behavior under our H_0 hypothesis ("there is no collusion"). This results in a transparent but powerful test.

2.1   The model

Players and payoffs.   Each period $t \in \mathbb{N}$, a buyer procures a single unit of a good through a first-price auction described below. A set $N = \{1, ..., n\}$ of long-lived firms is present in the market. In each period t, a subset $\widehat{N}_t \subset N$ of firms is able to participate in the auction. Participant set $\widehat{N}_t$ is exogenous, i.i.d. over time, and cartel members are exchangeable.[2] In other terms, for all subsets $J \subset N$ of cartel members, and all permutations $\alpha : N \to N$ of cartel member identities, we have that

$$\text{prob}(\widehat{N}_t = J) = \text{prob}(\widehat{N}_t = \alpha(J)).$$

We think of this set of participating firms as those potentially able to produce in the current period.[3] In period t, each participating firm $i \in \widehat{N}_t$ can deliver the good at a cost

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[2]Appendix OA extends the model to allow for endogenous participation.
[3]We consider the endogenous participation of entrants in Section 4.

$c_{i,t}$. Cost $c_{i,t}$ is drawn i.i.d. across participants and time periods from a c.d.f. F with support $[\underline{c}, \overline{c}]$ and density f with $f(c) > 0$ for all $c \in [\underline{c}, \overline{c}]$.

Firms are able to send transfers to each other, regardless of whether or not they participate in the auction. We denote by $T_{i,t}$ the net transfer received or sent by firm i. Let $x_{i,t} \in \{0, 1\}$ denote whether firm i wins the procurement contract in period t. Let $b_{i,t}$ denote her bid. We assume that firms have quasi-linear preferences, so that firm i's overall stage game payoff is

$$\pi_{i,t} = x_{i,t}(b_{i,t} - c_{i,t}) + T_{i,t}.$$

Firms value future payoffs using a common discount factor $\delta < 1$.

The stage game. The procurement contract is allocated according to a first price auction with constrained bids. Specifically, each participant must submit a bid b_i in the range $[p, r]$ where r is a maximum (or reserve) price, and $p < r$ is a minimum price. Bids outside of this range are discarded. The winner is the lowest bidder, with ties broken randomly. The winner then delivers the good at the price she bid. For simplicity, we assume that $r \geq \overline{c}$.[4]

To keep the model tractable and to focus on how enforcement constraints affect bidding behavior, we assume that all firms belong to the cartel, and firms in the cartel observe one another's production costs. In addition, we assume that payoffs are transferable.[5] The timing of information and decisions within period t is as follows.

1. The set of participating firms $\widehat{N}_t$ is drawn and observed by all cartel members.

2. The production costs $\mathbf{c}_t = (c_{i,t})_{i \in \widehat{N}_t}$ of participating firms are publicly observed by cartel members.

3. Participating firms $i \in \widehat{N}_t$ submit public bids $\mathbf{b}_t = (b_{i,t})_{i \in \widehat{N}_t}$. This yields allocation

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[4]This assumption is largely verified in our data. Indeed, 99.02% of auctions in our data have a winner.

[5]The assumption that firms can transfer money is not unrealistic. Indeed, many known cartels used monetary transfers; see for instance Pesendorfer (2000), Asker (2010) and Harrington and Skrzypacz (2011). In practice these transfers can be made in ways that make it difficult for authorities to detect them, like sub-contracting between cartel members or, in the case of cartels for intermediate goods, between-firms sales.

$\mathbf{x}_t = (x_{i,t})_{i \in \widehat{N}_t} \in [0,1]^{\widehat{N}_t}$ such that: if $b_{j,t} > b_{i,t}$ for all $j \in \widehat{N}_t \backslash \{i\}$ then $x_{i,t} = 1$; if there exists $j \in \widehat{N}_t \backslash \{i\}$ with $b_{j,t} < b_{i,t}$ then $x_{i,t} = 0$.

In the case of ties, we follow Athey and Bagwell (2001) and let the bidders jointly determine the allocation. This simplifies the analysis but requires some formalism (which can be skipped at moderate cost to understanding). We allow bidders to simultaneously pick numbers $\gamma_t = (\gamma_{i,t})_{i \in \widehat{N}_t}$ with $\gamma_{i,t} \in [0,1]$ for all i, t. When lowest bids are tied, the allocation to a lowest bidder i is

$$x_{i,t} = \frac{\gamma_{i,t}}{\sum_{\{j \in \widehat{N}_t \text{ s.t. } b_{j,t} = \min_k b_{k,t}\}} \gamma_{j,t}}.$$

4. Firms make transfers $T_{i,t}$.

   Positive transfers are always accepted and only negative transfers will be subject to an incentive compatibility condition. We require exact budget balance within each period at the overall cartel level, i.e. $\sum_{i \in N} T_i = 0$.

Our model is intended to capture commonly observed features of public construction procurement (see McMillan (1991) for a reference). Governments need to procure construction services on an ongoing basis. They face a limited and stable set of firms that can potentially perform the work, a subset of which participates regularly. Legislation frequently requires participants to register, and governments make bids and outcomes public after each auction is completed. The repeated and public nature of the interaction makes collusion a realistic concern.

Note that procurement auctions with minimum acceptable bids are frequently used in practice. For instance, auctions with minimum bids are used for procurement of public works in several countries in the European Union and by local governments in Japan. The common rationale for introducing minimum bids in the auction is to limit the incidence of strategic default by non-performing contractors (Calveras et al., 2004).[6]

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[6]Appendix OB extends our model to allow for non-performing contractors. Such firms can be viewed as entrants with zero costs, producing a worthless good. Since our model and predictions focus exclusively on the bidders' side of the market, our predictions regarding bid distributions hold regardless of whether such

The repeated game. Interaction is repeated and firms can use the promise of continued collusion to enforce obedient bidding and transfers. Formally, bids and transfers need to be part of a subgame perfect equilibrium of the repeated game among firms.

The history among cartel members at the beginning of time t is

$$h_t = \{\mathbf{c}_s, \mathbf{b}_s, \gamma_s, \mathbf{x}_s, \mathbf{T}_s\}_{s=0}^{t-1}.$$

Let $\mathcal{H}^t$ denote the set of period t public histories and $\mathcal{H} = \bigcup_{t \geq 0} \mathcal{H}^t$ denote the set of all histories. Our solution concept is subgame perfect equilibrium (SPE), with strategies

$$\sigma_i : h_t \mapsto (b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t), T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t))$$

such that bids $(b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t))$ and transfers $T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t)$ can depend on all public data available at the time of decision-making.

Denote by Σ the set of SPE in the repeated stage game. Let

$$V(\sigma, h_t) = \mathbb{E}_\sigma \left[\sum_{s \geq 0} \delta^s \sum_{i \in \widehat{N}_{t+s}} x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) \middle| h_t \right]$$

denote the total surplus generated under equilibrium σ conditional on history h_t. We denote by

$$\overline{V}_p \equiv \max_{\sigma \in \Sigma} V(\sigma, h_0)$$

the highest equilibrium surplus sustainable in equilibrium.[7] We emphasize that this highest equilibrium value depends on minimum price p.

We say that a strategy σ_i is non-collusive whenever bids at history h_t depend only on the

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non-performing firms are included in the model. However, the presence of non-performing firms would affect aggregate welfare assessments.

[7]The existence of surplus maximizing and surplus minimizing equilibria follows from Proposition 2.5.2 in Mailath and Samuelson (2006).

costs of participating bidders at history h_t, but not their identities: $\sigma_i(h_t) = \hat{\sigma}_i\left(c_{i,t}, \{c_{j,t}\}_{j \in \widehat{N}_t \setminus i}\right)$ for all histories h_t. Since there is no persistent state in this game, non-collusive strategies coincide with Markov perfect strategies.

Definition 1 (collusive and competitive environments). *We say that we are in a collusive environment if firms play a Pareto efficient SPE; i.e., an SPE that attains $\overline{V}_p$.*

We say that we are in a competitive environment if firms play a SPE in non-collusive strategies that is Pareto efficient among non-collusive equilibria.

Under complete information, the unique competitive equilibrium outcome is such that the winning bid is equal to the maximum between the second lowest cost and the minimum price. The contract is allocated to the bidder with the lowest cost whenever the winning bid is above the minimum price, and is allocated randomly among all bidders with cost below the minimum price when the winning bid is equal to the minimum price.

2.2 Optimal collusion

Given a history h_t and a strategy profile σ, we denote by $(\beta(\mathbf{c}_t|h_t, \sigma), \gamma(\mathbf{c}_t|h_t, \sigma))$ the bidding profile induced by strategy profile σ at history h_t as a function of realized costs $\mathbf{c}_t$.

Lemma 1 (stationarity). *Consider a subgame perfect equilibrium σ that attains $\overline{V}_p$. Equilibrium σ delivers surplus $V(\sigma, h_t) = \overline{V}_p$ after all on-path histories h_t.*

There exists a fixed bidding profile (β^*, γ^*) such that, in a Pareto efficient equilibrium, firms bid $(\beta(\mathbf{c}_t|h_t, \sigma), \gamma(\mathbf{c}_t|h_t, \sigma)) = (\beta^*(\mathbf{c}_t), \gamma^*(\mathbf{c}_t))$ after all on-path histories h_t.

For any $i \in N$ and any $\sigma \in \Sigma$, let

$$V_i(\sigma, h_t) = \mathbb{E}_\sigma\left[\sum_{s \geq 0} \delta^s(x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) + T_{i,t+s})\,\Big|\,h_t\right]$$

denote the expected discounted payoff that firm i gets in equilibrium σ conditional on history

h_t. Let

$$\underline{V}_p \equiv \min_{\sigma \in \Sigma} V_i(\sigma, h_0)$$

denote the lowest possible equilibrium payoff for a given firm.

Given a bidding profile (β, γ), let us denote by $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ the induced winning bid and allocation profile for realized costs $\mathbf{c}$. For each firm i, we define

$$\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) \equiv \mathbf{1}_{\beta^W(\mathbf{c})>p} + \frac{\mathbf{1}_{\beta^W(\mathbf{c})=p}}{1 + \sum_{j \in \widehat{N} \setminus \{i\}: x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})}.$$

Term $\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$ corresponds to a deviator's highest possible chance of winning the contract by attempting to undercut the equilibrium winning bid.

Lemma 2 (enforceable bidding). *A winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ are sustainable in SPE if and only if for all $\mathbf{c}$,[8]*

$$\sum_{i \in \widehat{N}} (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \leq \delta(\overline{V}_p - n\underline{V}_p). \quad (1)$$

As in Levin (2003), a bidding profile can be implemented in SPE if and only if the sum of deviation temptations (both from bidders abstaining to bid above their cost, and bidders having to bid below their cost) is less than or equal to the total pledgeable surplus $\delta(\overline{V}_p - n\underline{V}_p)$, i.e. the difference between the highest possible continuation surplus, and the sum of minimal continuation surpluses guaranteed to each player in equilibrium.

For each cost realization $\mathbf{c}$, let $\mathbf{x}^*(\mathbf{c})$ denote the efficient allocation. It allocates the procurement contract to the participating firm with the lowest cost (ties are broken randomly). We define

$$b_p^*(\mathbf{c}) \equiv \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c})) \left[b - c_i\right]^+ \leq \delta(\overline{V}_p - n\underline{V}_p) \right\}.$$

For values of $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$, this value is the highest enforceable winning bid when

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[8]We note that cost vector $\mathbf{c} = (c_i)_{i \in \widehat{N}}$ uniquely determines the set $\widehat{N}$ of participating bidders.

the cartel allocates the good efficiently. Note that $b_p^*(\mathbf{c})$ is always weakly greater than the second lowest cost.

Proposition 1. *On the equilibrium path, the bidding strategy in any SPE that attains $\overline{V}_p$ sets winning bid $\beta_p^*(\mathbf{c}) = \max\{b_p^*(\mathbf{c}), p\}$ in every period. Moreover, the allocation is conditionally efficient: whenever $\beta_p^*(\mathbf{c}) > p$, the contract is allocated to the bidder with the lowest procurement cost.*

This result follows from obedience constraint (1). Bid $\beta_p^*(\mathbf{c})$ is the highest enforceable bid. Furthermore, allocating the good efficiently increases the surplus accruing to the cartel while also relaxing (1). Indeed, the lowest cost bidder has the largest incentives to undercut other bidders.

The following bidding profile implements the optimal collusive scheme when $\beta_p^*(\mathbf{c}) > p$. The firm with the lowest cost bids $\beta_p^*(\mathbf{c})$ and wins the contract at this price. At least one other firm bids immediately above $\beta_p^*(\mathbf{c})$.[9]

The following comparative statics hold. First, winning bid $\beta_p^*(\mathbf{c})$ is weakly decreasing in the procurement cost of each participating firm. Indeed, lowering the cost of any firm $i \in \widehat{N}$ tightens the obedience constraint (1). Second, winning bid $\beta_p^*(\mathbf{c})$ is weakly decreasing in the number of participating firms: holding other firms' costs constant, adding a new participant also tightens the obedience constraint.

The firm's behavior in a competitive environment with complete information is an immediate corollary: it coincides with collusive behavior in a game with discount factor $\delta = 0$. For any profile of cost realizations $\mathbf{c}$, let $c_{(2)}$ denote the second lowest cost.

Corollary 1 (behavior under competition). *In a competitive environment, the winning bid is $\beta_p^{comp}(\mathbf{c}) = \max\{p, c_{(2)}\}$.*

We now clarify how minimum prices affect the set of payoffs that firms can sustain in SPE. We denote by $\beta_0^*(\underline{c})$ the lowest bid in a Pareto efficient SPE when there is no minimum

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[9]Tie-breaking profile γ is needed to make this statement precise.

price. We note that $\beta_0^*(\underline{c})$ is observable from data: it is the lowest equilibrium winning bid.

Proposition 2 (worst case punishment). *(i) $\underline{V}_0 = 0$, and $\underline{V}_p > 0$ whenever $p > \underline{c}$;*

(ii) there exists $\eta > 0$ such that for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\overline{V}_p - n\underline{V}_p < \overline{V}_0 - n\underline{V}_0$.

Proposition 2(i) shows that with no minimum price, the cartel can force a firm's payoff down to a minmax value of 0, but that minmax values are bounded away from zero when the minimum price is within the support of procurement costs. Proposition 2(ii) establishes that the pledgeable surplus $\overline{V}_p - n\underline{V}_p$ that the cartel can use to provide incentives decreases after introducing a low minimum price. The reason for this is that a minimum price p in the neighborhood of $\beta_0^*(\underline{c})$ increases the lowest equilibrium value $\underline{V}_p$ by an amount bounded away from 0, even for $\eta > 0$ small. This tightens enforcement constraint (1) and reduces the bids that the cartel can sustain in equilibrium.[10]

We end this section by noting that the results above extend to environments with asymmetric firms. When firms are asymmetric, the cartel's pledgeable surplus becomes $\overline{V}_p - \sum_{i \in N} \underline{V}_{i,p}$, where $\underline{V}_{i,p}$ is firm i's lowest equilibrium value. With this change, the results in Lemma 2 and Propositions 1 and 2 continue to hold when firms are asymmetric.

3   Empirical implications

The effect of minimum prices on the distribution of bids.   We now delineate several empirical implications of our model. Specifically, we contrast the effect that a minimum price has on the distribution of winning bids under competition and under collusion.

Proposition 3 (the effect of minimum prices on bids). *Under collusion, minimum prices can induce a first-order stochastic dominance drop in the right tail of winning bids. Under competition, minimum prices don't affect the right tail of winning bids. Formally:*

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[10]In contrast, a minimum price significantly larger than $\beta_0^*(\underline{c})$ may increase the cartel's pledgeable surplus.

(i) there exists $\eta > 0$ such that, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$,

$$prob(\beta_p^* \geq q | \beta_p^* \geq p) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p),$$

the inequality being strict for some $q > p$ whenever $prob(\beta_0^* < r) > 0$.

(ii) for all $p > 0$ and all $q > p$,

$$prob(\beta_p^{comp} \geq q | \beta_p^{comp} > p) = prob(\beta_0^{comp} \geq q | \beta_0^{comp} > p).^{11}$$

Consider now equilibrium bidding data from auctions without minimum price. Bidders may be either collusive or competitive. Let $\underline{\beta}_0^{obs}$ denote the lowest observed winning bid. Since competitive bids are not affected when the minimum price is below the observed distribution of winning bids, we obtain the following corollary.

Corollary 2 (robust policy take-away). *Regardless of whether there is collusion, setting a minimum price $p \leq \underline{\beta}_0^{obs}$ causes a weak first-order dominance drop in procurement costs.*

This corollary is a robust policy take-away. Setting a minimum price at the bottom of the distribution of observed winning bids weakly dominates setting no minimum price. Setting a minimum price strictly within the distribution of observed winning bids may increase procurement costs if there little or no collusion.[12]

One design subtlety worth emphasizing is that the minimum prices studied in this paper are not indexed on bids. In some settings (e.g. Italy) minimum prices are set as an increasing function of submitted bids, e.g. a quantile of submitted bids (Conley and Decarolis,

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[11]Conditioning on a strict inequality is meaningful because the distribution of winning bids may have mass points at the minimum price, which we need to correctly take care of. When the mass of bids at the minimum price is small, the conditioning events in Proposition 3 (i) and (ii) coincide. In data from our lead example city, Tsuchiura, 1.2% of auctions with a minimum price have a winning bid equal to the minimum price.

[12]Note that the rule-of-thumb described in Corollary 2 can be extended if the distribution of costs changes over time. Minimum prices should be adjusted to be as high as possible without being binding. If a mass of bids is concentrated at the minimum price, the minimum price should be lowered.

2011, Decarolis, 2013). We expect such minimum price policies to be less effective than fixed minimum prices in deterring collusion: by coordinating on low bids, cartel bidders can still bring minimum prices down, limiting the effect that the policy has on punishments.

Proposition 3 provides a joint test of collusion and of the fact that cartel enforcement constraints are binding. Consider the introduction of a minimum price close to the minimum observed winning bid. Under collusion, the introduction of such a minimum price will lead to a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price. Under competition, the introduction of minimum prices will lead to a (weak) first order stochastic dominance increase in the distribution of winning bids.

We emphasize that the predictions under competition in Proposition 3(ii) do not rely on the assumption that firms can make monetary transfers: indeed, no transfers are used in competitive equilibrium. Moreover, the results under collusion in Proposition 3(i) also continue to hold in the absence of monetary transfers: minimum prices still reduce the cartel's ability to punish deviators, thereby lowering the highest sustainable bid.

We now strengthen this test by showing that Proposition 3(ii) extends to asymmetric information settings.

Competitive comparative statics under asymmetric information. We assume now that firms are privately informed about their own procurement cost. Let $b_0^{AI} : [\underline{c}, \overline{c}] \to \mathbb{R}_+$ denote the equilibrium bidding function in the unique symmetric equilibrium of the first-price procurement auction with reserve price r and *no minimum price*.

Proposition 4. *Under private information, a first-price auction with reserve price r and minimum price $p < \min\{r, \overline{c}\}$ has a unique symmetric equilibrium with bidding function b_p^{AI}.*

If $b_0^{AI}(\underline{c}) \geq p$, then $b_p^{AI}(c) = b_0^{AI}(c)$ for all $c \in [\underline{c}, \overline{c}]$;

If $b_0^{AI}(\underline{c}) < p$, there exists a cutoff $\hat{c} \in (\underline{c}, \bar{c})$ with $b_0^{AI}(\hat{c}) > p$ such that

$$b_p^{AI}(c) = \begin{cases} b_0^{AI}(c) & \text{if } c \geq \hat{c}, \\ p & \text{if } c < \hat{c}. \end{cases}$$

An immediate corollary of Proposition 4 is that minimum prices can only yield a first order stochastic dominance increase in the right tail of winning bids. Let $\beta_p^{AI}(\mathbf{c}) \equiv \min_i b_p^{AI}(c_i)$ denote winning bids.

Corollary 3. *For all $p > 0$ and all $q > p$,*

$$prob(\beta_p^{AI} \geq q \mid \beta_p^{AI} > p) \geq prob(\beta_0^{AI} \geq q \mid \beta_0^{AI} > p).$$

This strengthens the test of collusion provided in Proposition 3. A first-order stochastic dominance drop in the right tail of winning bids cannot be explained away by a competitive model with incomplete information.

Similar results continue to hold when bidders are asymmetric and face interdependent costs. Under competition, setting a binding minimum price creates a mass of bids at the minimum price, and a gap in the support of the winning bid distribution just above the minimum price. As a result, in these competitive environments a minimum price cannot generate a first-order stochastic dominance drop in the right tail of winning bids.

4   Entry

We now extend the model of Section 2 to allow for entry. The goal of this extension is twofold. First, we want to show that the testable predictions in Proposition 3 continue to hold when non-cartel members can participate. Second, this extension allows us to derive additional predictions on the differential effect of minimum prices on cartel members and entrants. These additional predictions are important since they let us distinguish our model

from a competitive one in which minimum prices somehow increases entry.

We assume that in each period t, a short-lived firm may bid in the auction along with participating cartel members $\widehat{N}_t$. To participate, the short-lived firm has to pay an entry cost k_t drawn i.i.d. over time from a distribution F_k with support $[0, \overline{k}]$. The distribution of entry costs may have a point mass at 0. We let $E_t \in \{0, 1\}$ denote the entry decision of the short-lived firm in period t, with $E_t = 1$ denoting entry.

Upon paying the entry cost, the short-lived firm learns its cost $c_{e,t}$ for delivering the good, which is drawn i.i.d. from a c.d.f. F_e with support $[\underline{c}, \overline{c}]$ and density f_e. We assume that the short-lived firm's entry decision and her procurement cost upon entry $c_{e,t}$ are publicly observed.

The timing of information and decisions within each period t is as follows:

1. The short-lived firm's entry cost k_t is drawn and privately observed. The short-lived firm makes entry decision E_t, which is observed by cartel members.

2. The set of participating cartel members $\widehat{N}_t$ is drawn and observed by both cartel members and the short-lived firm.

3. The production costs $\mathbf{c}_t$ of participating firms are drawn and publicly observed by all firms.

4. Participating firms submit public bids $\mathbf{b}_t = (b_{i,t})$ and numbers $\gamma = (\gamma_{i,t})$ with $\gamma_{i,t} \in [0, 1]$, resulting in allocation $\mathbf{x}_t = (x_{i,t})$.[13]

5. Cartel members make transfers $T_{i,t}$ to one another.

The public history at the beginning of time t is now $h_t = \{E_s, \mathbf{c}_s, \mathbf{b}_s, \gamma_s, \mathbf{x}_s, \mathbf{T}_s\}_{s=0}^{t-1}$, and is observed by both cartel members and entrants. Let $\mathcal{H}^t$ denote the set of period t public histories and $\mathcal{H} = \bigcup_{t \geq 0} \mathcal{H}^t$ denote the set of all histories. Our solution concept is subgame perfect public equilibrium, with strategies

$$\sigma_i : h_t \mapsto (b_{i,t}(E_t, \mathbf{c}_t), \gamma_{i,t}(E_t, \mathbf{c}_t), T_{i,t}(E_t, \mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t))$$

---

[13]The allocation is determined in the same way as in Section 2.

for cartel members and strategies

$$\sigma_e : h_t \mapsto (E_t(k_t), b_{e,t}(k_t, \mathbf{c}_t), \gamma_{e,t}(k_t, \mathbf{c}_t))$$

for the short-lived firms.

We note that the cartel in this model is not all-inclusive. In each period participating cartel members compete against short-lived entrants.[14]

The analysis of this model is essentially identical to that of the model of Section 2 except that now the cartel will deter entry in addition to enforcing collusive bidding. Given that procurement costs are observed after entry, entry depends only on cost k_t and takes a threshold-form. Entrants with entry costs above a certain level are deterred from entering, while entrants with an entry cost below this threshold participate in the auction.

For concision, we focus on extending the main empirical predictions of our model. Appendix B provides further details on optimal cartel behavior.

Proposition 5 (the effect of minimum prices on bids). *(i) Under collusion, there exists $\eta > 0$ such that for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $q > p$, and $E \in \{0, 1\}$,*

$$prob(\beta_p^* \geq q | \beta_p^* \geq p, E) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p, E).$$

(ii) Under competition, for all $p > 0$, $q > p$, and $E \in \{0, 1\}$,

$$prob(\beta_p^{comp} \geq q | \beta_p^{comp} > p, E) = prob(\beta_0^{comp} \geq q | \beta_0^{comp} > p, E).$$

In other words, the contrasting comparative statics of Proposition 3 continue to hold conditional on the entrant's entry decision.[15]

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[14]See Hendricks et al. (2008) or Decarolis et al. (2016) for recent analyses of cartels that are not all-inclusive.

[15]We note that a competitive model with incomplete information and entry cannot explain the predictions in Proposition 5(i). Indeed, in such a model the introduction of a binding minimum price generates a mass

A notable new prediction is that under collusion minimum prices have different impacts on cartel and entrant winners.

Proposition 6 (differential effect of minimum prices on bids). *Under collusion, there exists $\eta > 0$ such that, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$:*

(i) $prob(\beta_p^* \geq q | \beta_p^* \geq p, E, cartel\ wins) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p, E, cartel\ wins)$ for $E = 0, 1$;

(ii) $prob(\beta_p^* \geq q | \beta_p^* > p, entrant\ wins) = prob(\beta_0^* \geq q | \beta_0^* > p, entrant\ wins)$.

In words, minimum prices should only affect the right tail of winning bids when the winners are cartel members. The intuition behind this stark prediction is straightforward. Since costs are complete information, under optimal entry deterrence, entrants either win at the minimum price, or at their production cost. As a result, the right tail of winning bids conditional on an entrant being the winner is independent of the cartel's pledgeable surplus, and independent of the minimum price. The prediction holds approximately if cartel members only get a noisy but precise signal of the entrant's production cost.[16]

Alternative models of entry. Proposition 6 is important because it lets us distinguish our model with models in which minimum prices are associated with greater (potentially unobserved) entry, but for reasons unrelated to collusion. For instance, because of media coverage of the policy change. Under such a model, minimum prices could reduce the distribution of winning bids even in a competitive environment. However, entry would affect the winning bids of both entrants and cartel members. Under this alternative model, unlike ours, minimum prices should have a similar impact on cartel and entrant winners.

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point of bids at the minimum price and a gap in the support of the winning bid distribution just above the minimum price. As a result, such a model cannot generate a first-order stochastic dominance drop in the right-tail of the winning bid distribution.

[16]The prediction continues to hold if the cartel knows the entrant's cost, but the entrant has imperfect information about the costs of cartel members. Indeed, in such a setting, under optimal entry deterrence entrants would still either win at the minimum price or at their production cost.

5   Empirical Analysis

Sections 2, 3 and 4 lay out a theoretical mechanism through which minimum prices can affect the distribution of winning bids, and clarify its implications for data. This empirical section aims to assess the relevance of this mechanism in a real life context and answer the following questions: are enforcement constraints binding? are they affected by minimum prices? what is the impact on cartel members? what is the impact on entrants?

5.1   Data and Empirical Strategy

We provide empirical answers to the questions above using auction data from Japanese cities located in the Ibaraki prefecture.

Context.   Local procurement in Japan is an appropriate context for us to test the model developed in Sections 2, 3 and 4. McMillan (1991)'s account of collusive practices in Japan's construction industry vindicates many of our assumptions. It confirms the role of transfers in sustaining collusion, as well as the importance of selective tendering and observed participation in limiting entry, especially at the local level.[17] More, recently, Ishii (2008) and Kawai and Nakabayashi (2014) provide evidence of widespread collusion in Japanese procurement auctions. This suggests that local procurement in Japan is an environment where minimum price constraints could plausibly have an effect.

Sample selection.   We collected our data as follows. In a study of paving auctions, Ishii (2008) notes the use of minimum prices in Japanese procurement auctions. The author was able to point us to one of our treatment cities. We then proceeded to search for all publicly available data from the 30 most populous cities in the prefecture. We kept all cities that had public data available covering the relevant period. This left us with the fourteen cities

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[17]We further refer to McMillan (1991) for details on real world collusion, including organizational aspects of allowing managers to maintain deniability.

included in the study. We treat these fourteen cities as distinct markets.[18] The data covers public work projects auctioned off between May 2007 and March 2016, corresponding to 10533 auctions.

city	population	density	min price increases	#auctions	data time range
Hitachiomiya	42,438	122	2011-04, 2012-12	1371	2007-07 — 2016-03
Hokota	51,519	253	—	597	2011-04 — 2016-04
Inashiki	43,178	210	2010-07, 2012-04	1003	2007-04 — 2016-03
Kamisu	94,551	642	—	671	2012-05 — 2016-02
Kasumigaura	45,373	382	—	487	2013-07 — 2014-10
Moriya	64,644	1810	—	312	2012-04 — 2016-03
Sakuragawa	49,387	275	—	472	2010-05 — 2016-01
Shimotsuma	45,289	560	—	335	2007-07 — 2016-02
Toride	109,926	1571	2012-03, 2013-04	423	2010-05 — 2016-09
Tsuchiura	142,931	1162	2009-10, 2014-04	1748	2007-05 — 2016-03
Tsukuba	215,306	758	2014-04	1855	2009-04 — 2016-03
Tsukubamirai	48,807	617	2014-04	290	2008-05 — 2015-12
Ushiku	81,532	1385	—	672	2008-04 — 2016-03
Yuki	51,429	782	—	242	2014-04 — 2016-01

Table 1: City characteristics.

Throughout the period, all cities use first-price auctions. Six cities — Hitachiomiya, Inashiki, Toride, Tsuchiura, Tsukuba, and Tsukubamirai — experience at least one policy change going from a zero minimum price to a positive minimum price. Within this set, Tsuchiura provides us with the richest data, including bidder names, non-winning bids, and minimum prices.[19] Cities other than the six mentioned above use first-price auctions with no minimum price throughout the period.

Policy documents available from municipal websites clarify that minimum prices are chosen by a formal rule and contain no more information than reserve prices. Reserve prices are computed by adding up engineering estimates of material, labor, administrative, and financing costs. Minimum prices are obtained by mutliplying each expense category by a

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[18]We discuss this assumption in Section 6.

[19]Notable trivia: Tsuchiura is a sister city of Palo Alto, CA.

pre-determined coefficient.

Publicly available policy documents, as well as exchanges with city officials confirm that minimum prices were introduced to avoid excessively low bids that could only be executed at the expense of quality.[20] We found no evidence that policy changes were triggered by city specific factors also affecting the distribution of bids.

Descriptive statistics. Some facts about our sample of auctions are worth noting. The first is that although all auctions include a reserve price, these reserve prices are not set to extract greater surplus for the city along the lines of Myerson (1981) or Riley and Samuelson (1981). Rather, consistent with recorded practice, reserve prices are engineering estimates (Ohashi, 2009, Tanno and Hirai, 2012, Kawai and Nakabayashi, 2014) that provide an upper-bound to the range of possible costs for the project. This is largely verified in our data, since 99.02% of auctions have a winner. This lets us treat reserve prices as an exogenous scaling parameter and use it to normalize the distribution of bids to $[0, 1]$. Normalized winning bids are defined as follows:

$$norm\_winning\_bid = \frac{winning\_bid}{reserve\_price}.$$

This normalization lets us take the comparative statics of Propositions 3, 4 and 5 to the data, even though there is heterogeneity in minimum prices. As a robustness test, we also study the distribution of log-winning-bids using reserve prices as a control variable (see Table A.7). Our findings are unchanged.

The distribution of winning bids is closely concentrated near reserve prices. Throughout all of our data, the aggregate cost savings from running an auction rather than using reserve prices as a take-it-or-leave-it offer are equal to 4.9%. This could be because reserve prices are obtained through very precise engineering estimates, but this provides justifiable concern that collusion may be present. In the city of Tsuchiura, for which we observe minimum prices,

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[20]Our model can capture non-performing bidders by treating them as entrants with zero costs. We elaborate on this point in Section 6 and Online Appendix OB.

the median minimum price is in the first decile of the distribution of normalized winning bids. This matches the premise of Propositions 3, 5 and 6. Minimum prices in Tsuchiura range from .75 to .85 of the reserve.

Empirical strategy. The data lets us evaluate the prediction of Propositions 3 and 6 directly. If there is no collusion the introduction of a low minimum price should not change the right tail of wining bids. In fact, in a competitive environment, introducing such a low minimum price should have a very limited effect on bidding behavior. In contrast, if there is collusion, we anticipate a drop in the right tail of winning bids.

We measure the impact of a policy change on the distribution of winning bids at the city level by forming either change-in-changes (Athey and Imbens (2006)) or difference-in-differences estimates for the sample of normalized winning bids above a threshold of .8.[21] Given the heterogeneity in city characteristics reported in Table 1, we match each treatment city to two cities that are most suitable as controls according to the following criteria:

- the control city has data before and after the treatment city's policy change; during that period the control city does not itself experience a policy change;

- the control city minimizes the distance between the treatment city t and potential control city c according to distance

$$d_{t,c} = \left| \frac{population_t - population_c}{population_t} \right| + \left| \frac{density_t - density_c}{density_t} \right|.$$

When two minimum price increases occur in the treatment city we keep only data corresponding to the first policy change. It corresponds to going from no minimum price to a positive minimum price, and matches the premise of our theoretical results. We let cities experiencing a policy change serve as a control city when they do not experience a policy change. Table 2 shows how treatment and control cities are matched.

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[21]This is the mid-range point for minimum prices in Tsuchiura. The results are unchanged if we consider the distribution of normalized winning bids conditional on prices being above .78 or .82 of the reserve price.

Treatment	Control 1	Control 2
Hitachiomiya	Inashiki	Sakuragawa
Inashiki	Hitachiomiya	Sakuragawa
Toride	Ushiku	Tsuchiura
Tsuchiura	Ushiku	Tsukuba
Tsukuba	Kamisu	Shimotsuma
Tsukubamirai	Shimotsuma	Kasumigaura

Table 2: Treatment and control cities, matched according to population and density.

We report our findings in three steps. We first provide a detailed description of our approach using Tsuchiura as a treatment city. We use Tsuchiura as a benchmark because it is the city for which we have the richest and most abundant data. We observe minimum prices, non-winning bids, and the identity of all bidders. We then present aggregated results clustering standard errors at the (city, year) level, performing wild bootstrap to obtain p-values (Cameron et al. (2008)). For completeness we report individual treatment-city regressions in Appendix A.

5.2 Findings for Tsuchiura

5.2.1 The impact of minimum prices on the distribution of winning bids

Figure 1 plots distributions of normalized winning bids for Tsuchiura and the corresponding control cities Tsukuba and Ushiku, using data two years before and two years after the policy change (October 28th 2009). The three cities are broadly comparable: their populations range from 82K to 215K, with Tsuchiura at 143K. They are located within 15km of one another, and within 75km of Tokyo. The data appears well suited to a difference-in-differences approach. Remarkably, the distribution of normalized winning bids in the control cities seems essentially unchanged. Figure 1 also suggests a first-order stochastic-dominance drop in normalized winning bids of the treatment city above .8.
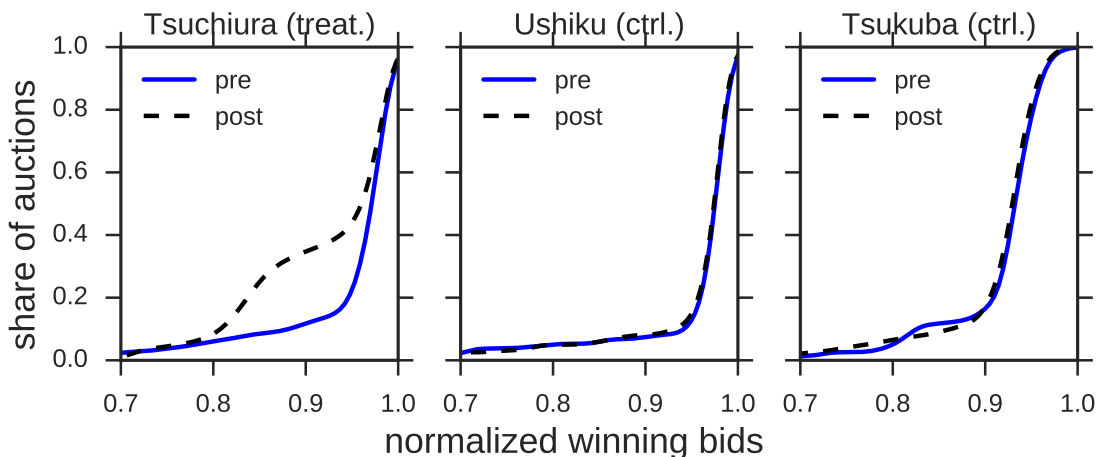

See Appendix A for details.

Figure 1: Distribution of winning bids, before and after treatment: 2007-2009, 2009-2011.

Change-in-changes. The framework of Athey and Imbens (2006) allows us to formalize this observation by estimating the counterfactual distribution of normalized winning bids in our treatment city, absent minimum prices. The actual and counterfactual quantiles of normalized winning bids, conditional on prices being above 80% of the reserve price are given in Table 3. We use both Tsukuba and Ushiku as a controls.[22]

quantile of conditional dist	0.1	0.25	0.5	0.75	0.9
actual – counterfactual	-0.051***	-0.054***	0.003	0.004	0.004*
std error	(0.018)	(0.02)	(0.005)	(0.003)	(0.003)
actual	0.833	0.881	0.959	0.977	0.984
counterfactual	0.884	0.935	0.956	0.973	0.98

Table 3: Change-in-changes estimates: quantiles of the actual and counterfactual conditional distributions of normalized winning bids ($> .8$)

Difference-in-differences. The findings displayed in Table 3 are confirmed by a difference-in-differences approach including additional controls.[23] We define variable

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[22]We do not merge the control data. This would bias results since the relative sample size of the pre and post period is different across control cities. Instead we separately run the algorithm of Athey and Imbens (2006) for each control city, and then average the corresponding counterfactual estimates. We report bootstrapped standard errors for our aggregated estimates.

[23]Throughout the empirical analysis, we winsorize the normalized winning bids at 1% and 99%.

$$policy\_change = \mathbf{1}_{date \geq \text{October } 28^{th} \text{ 2009 \& } city=Tsuchiura}$$

and perform both OLS and quantile regressions of the linear model with city fixed-effects, month fixed-effects, year fixed-effects and city-specific trends:

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 \log GDP$$
$$+ city\_fe + month\_fe + year\_fe + city\_trends. \qquad (2)$$

We continue to use the cities of Ushiku and Tsukuba as controls. To match the theoretical predictions of Proposition 3, we perform regressions on the subsample of auctions whose normalized winning bid is above .8, corresponding to the sample of auctions whose winning bids are (or would have been) above the minimum price. For completeness, we also report mean effects for the unconditional sample of auctions. Throughout this section we refer to the sample as *conditional*, when normalized winning bids are constrained to be above .8, and as *unconditional* when normalized winning bids are unconstrained.

For now, we present standard errors for our estimates assuming that shocks are independent at the auction level. In Section 5.3 we deal with possible city-level shocks by aggregating the data from all cities in our sample and clustering errors at the (city, year) level.

The outcome of regression (2), summarized in Table 4, vindicates the mechanism we explore in Sections 2, 3 and 4. The introduction of minimum prices is associated with a first-order stochastic dominance drop in the right tail of winning bids. The implication is not only that there is collusion, but that cartel enforcement constraints are binding, and that the sustainability of collusion is limited by price constraints.

	unconditional sample	sample s.t. $norm\_winning\_bid > .8$				
norm_winning_bid	mean effect	mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.008	-0.026***	-0.084***	-0.021***	-0.006*	0.003
	(0.007)	(0.005)	(0.010)	(0.005)	(0.003)	(0.002)
lngdp	0.065	0.027	0.010	-0.047	-0.022	-0.065*
	(0.103)	(0.070)	(0.148)	(0.068)	(0.049)	(0.036)
N	3705	3459	3459	3459	3459	3459

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table 4: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; regressions include city fixed-effects, month fixed-effects, year fixed-effects and city specific time-trends.

5.2.2 Who does the policy change affect?

Proposition 6 offers another test of the mechanism analyzed in Sections 2, 3 and 4. Under collusion, our theory predicts that the price paid by winning cartel members should go down, but not the price paid by winning entrants. Under competition, winning cartel members and winning entrants should be similarly affected.

Defining cartel membership. The key step consists in deciding which firms are potential cartel members, and which firm are likely entrants. We observe that Proposition 6 remains true when using a proxy for cartel membership that is a superset of cartel bidders. For this reason we err on the side of inclusiveness when proxying for potential cartel membership. If a proxy misclassifies entrants as cartel members, the measured effect of the policy change on entrants remains equal to zero. If cartel members are misclassified as entrants, the measured effect of the policy on entrants may become non-zero.

For Tsuchiura, participation data allows us to form a measure directly consistent with the theory. We can classify firms as cartel members and entrants according to the frequency with which they participate in auctions. Tsuchiura exhibits considerable heterogeneity in the degree of bidder activity. The median number of auctions a bidder participates in is 4, whereas the mean is at 20. The 25% most active bidders make up 83% of the auction×bidder

data. Accordingly, we define cartel measure $\widehat{cartel}_i$, which takes a value of one if bidder i belongs to the 25% most active bidders (82 out of 330 bidders), and is equal to zero otherwise.

This measure cannot be computed for cities other than Tsuchiura: we observe winners but not participants. For this reason, we use winning an auction as a proxy for participation. Accordingly, we define cartel membership measure $\widetilde{cartel}_i$ which takes a value of one whenever, bidder i belongs in the set of bidders who belong to the 35% of bidders that *win* auctions most often out of those who win at least once (71 firms; 77% of the auction×bidder data in Tsuchiura), and is equal to zero otherwise.

The threshold 35% is the round number threshold that generates the best overlap between $\widehat{cartel}$ and $\widetilde{cartel}$. All but 5 of the firms in our data that belong to cartel measure $\widetilde{cartel}_i$ also belong to cartel measure $\widehat{cartel}_i$. It is plausible that $\widetilde{cartel}$ may be less precise than $\widehat{cartel}$ because winning events are approximately 4 times rarer than participation events.[24]

Findings. We estimate the differential impact of minimum prices on cartel members and entrants using both $\widehat{cartel}$ and $\widetilde{cartel}$ measures. Since $\widetilde{cartel}$ is available for all cities, it can be used in a a difference-in-differences approach estimating the linear model

$$
norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 \widetilde{cartel} + \beta_3 \widetilde{cartel}\_Tsuchiura
$$
$$
+ \beta_4 \widetilde{cartel}\_Tsuchiura \times policy\_change + \beta controls + \gamma fixed\_effects
$$
$$
\tag{3}
$$

on the sample of auctions with normalized winning bids above .8.[25] The controls include $\log GDP$ of Japan. Fixed-effects include city specific time-trends, as well as city, month, and year fixed-effects.

To confirm the findings obtained for cartel measure $\widetilde{cartel}$, we replicate them with mea-

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[24]On average, 3.8 bidders participate in each auction in the city of Tsuchiura.

[25]The variable $\widetilde{cartel}$ is a proxy indicating whether a firm in either city belongs to a potential cartel or not. The variable $\widetilde{cartel}\_Tsuchiura$ interacts $\widetilde{cartel}$ with a Tsuchiura indicator. Our specification allows cartels to behave differently in the treatment and control cities.

sure $\widehat{cartel}$, which is potentially more accurate, but is only available for Tsuchiura. We take a before-after approach and estimate linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 \widehat{cartel}\_Tsuchiura$$
$$+ \beta_3 \widehat{cartel}\_Tsuchiura \times policy\_change + \beta controls \qquad (4)$$

on conditional and unconditional auction data. Estimates of (3) and (4) are reported Table 5.

	cartel $\equiv \widetilde{cartel}$		cartel $\equiv \widehat{cartel}$	
	unconditional sample	conditional sample	unconditional sample	conditional sample
cartel	0.012***	-0.000		
	(0.003)	(0.002)		
cartel_Tsuchiura	0.041***	0.026***	0.068***	0.032***
	(0.003)	(0.002)	(0.009)	(0.008)
cartel_Tsuchiura X policy_change	-0.036***	-0.021***	-0.048***	-0.029***
	(0.009)	(0.007)	(0.010)	(0.008)
policy_change	0.023**	-0.008	0.047***	0.009
	(0.010)	(0.007)	(0.014)	(0.011)
N	3705	3459	1748	1660

Table 5: Effect of minimum prices on cartel members and entrants (Tsuchiura).

The findings are consistent with the predictions of our model under collusion. Absent minimum prices, cartel winners obtain contracts at higher prices; the introduction of minimum prices has a disproportionately larger effect on cartel winners than entrant winners.

5.3 Findings for All Cities

To deal with city-level shocks, we extend the analysis to all treatment and control cities listed in Table 2, and aggregate the results. Figure 2 contrasts the before and after distributions of normalized winning bids above .8 for treatment and control cities. It exhibits patterns

consistent with those of Tsuchiura: control cities experience little change; treatment cities experience either little change, or a first order stochastic dominance drop.

For each individual policy group g consisting of one treatment and two control cities we assume that linear models (2) and (3) extend:

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 \log GDP \tag{2g}$$

$$+ \, city\_fe_g + month\_fe_g + year\_fe_g + city\_trends_g$$

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 \widetilde{cartel} + \beta_3 \widetilde{cartel}\_treatment\_city \tag{3g}$$

$$+ \, \beta_4 \widetilde{cartel}\_treatment\_city \times policy\_change + \beta controls + fixed\_effects_g$$

Models (2g) and (3g) are naturally aggregated assuming that the impact of the policy change is the same across cities.[26] For an auction a and a policy group g, we denote, by $a \in g$ the event that auction a is included in the relevant data for policy group g. We define $N_a \equiv \text{card}\{g, \ s.t. \ a \in g\}$ the number of policy group in which auction a is included. For an auction a, averaging over the treatment groups g in which auction a appears yields

$$norm\_winning\_bid_a \sim \beta_0 + \beta_1 policy\_change$$

$$+ \frac{1}{N_a} \sum_{g, \ s.t. \ a \in g} group\_fe_g + city\_fe_g + month\_fe_g + year\_fe_g + city\_trends_g$$

$$\tag{2Agg}$$

$$norm\_winning\_bid_a \sim \beta_0 + \beta_1 policy\_change + \beta_2 \widetilde{cartel} + \beta_3 \widetilde{cartel}\_treatment\_cities$$

$$+ \, \beta_4 \widetilde{cartel}\_treatment\_cities \times policy\_change$$

$$+ \frac{1}{N_a} \sum_{g, \ s.t. \ a \in g} city\_fe_g + month\_fe_g + year\_fe_g + city\_trends_g.$$
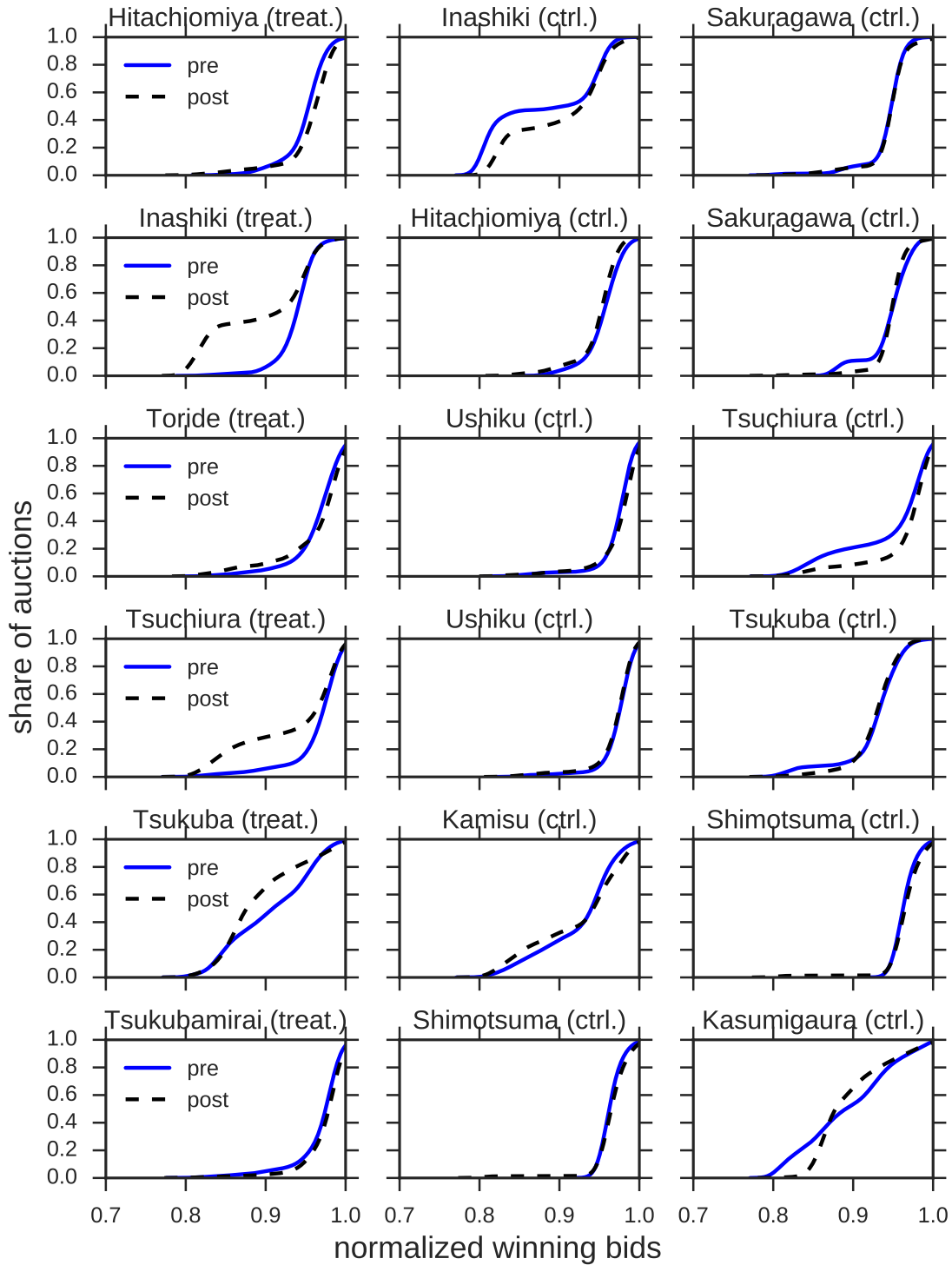
$$\tag{3Agg}$$

Figure 2: Distribution of winning bids, before and after treatment.

norm_winning_bid	unconditional mean effect		conditional mean effect	
policy_change	-0.018	-0.000	-0.025**	-0.009
p-value	0.376	0.938	0.020	0.174
policy_change x cartel		-0.019**		-0.019***
p-value		0.012		0.004
N	8623	8623	7903	7903

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table 6: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

Estimates from (2Agg) and (3Agg), reported in Table 6, corroborate findings from Tsuchiura. The introduction of minimum prices lowers winning above the minimum price, and the effect is disproportionately borne by potential cartel members.

6   Discussion

6.1   Summary

We propose a tractable framework in which to analyze the effect of price constraints on repeated collusion. Our model delivers a simple intuition: price constraints limit the range of continuation equilibrium payoffs, making cartel enforcement and entry deterrence more difficult. The analysis yields a number of transparent empirical predictions that allow us to test whether there is collusion, whether cartel-enforcement constraints are binding, and whether they are affected by minimum prices.

We take those predictions to procurement data from Japan, and confirm that the channel emphasized in this paper is empirically relevant. We find that minimum prices reduce the right tail of winning bids, and that the effect is mainly concentrated among likely car-

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[26]This is true under the H_0 assumption that the policy change has no impact.

tel winners. The implication of these findings is that there is collusion, cartel enforcement constraints are binding, and price constraints can weaken enforcement. This vindicates theoretical mechanisms prominent in the repeated implementation literature (Lee and Sabourian, 2011, Mezzetti and Renou, 2012), and in the relational contracting literature (Bull, 1987, Baker et al., 1994, 2002, Levin, 2003).

Our framework suggests a robust policy implication (Corollary 2). Setting no minimum price is weakly dominated by setting a minimum price at the bottom of the distribution of observed winning bids.

The remainder of this section briefly discusses the robustness of our theoretical and empirical findings. A deeper treatment of these robustness checks is provided in Appendix A and in the Online Appendix.

6.2 Robustness: Theory

Collusion with many firms. The cartel measures we propose, $\widehat{cartel}$ and $\widetilde{cartel}$, classify sizeable proportion of firms as potential cartel members. In Tsuchiura, the quartile of most active firms (which represents 80% of auction $\times$ bidder data) includes 82 firms. This is partly because we want our cartel proxy to be a superset of actual cartel members. In addition, although this is a large number, it is consistent with known cases of collusion among construction firms. In 2008 the United Kingdom's Office of Fair Trading filed a case against 112 firms in the construction sector. At least 80 of these firms have admitted engaging in bid-rigging, and reported the use monetary transfers. Another example is the Dutch construction cartel, which included on the order of 650 firms (Eftychidou and Maiorano, 2015).

It is not implausible that comparable levels of collusion could exist in Japan's construction industry. However, rationalizing collusion within such a large group could be a challenge for the model of Section 2. Pledgeable surplus is bounded, and many bidders must be

compensated for their deviation temptation. We show in the Online Appendix that provided the cartel can control participation in auctions, then, a cartel can continue to collude even as the number of cartel members grows large.

The intuition for this result is the following. When participation is endogenous, the cartel faces two enforcement constraints: (i) bidders participating in an auction must accept to bid according to plan; (ii) bidders instructed not to participate in an auction must comply. While enforcement constraint (i) becomes unsustainable as the number of participants in an auction grow large, enforcement constraint (ii) can be satisfied even as the cartel size grows large.

Indeed, imagine a cartel member that participates in an auction in which she was not supposed to bid. This unauthorized bidder is no different from an entrant. By bidding sufficiently low, other bidders can ensure that the unauthorized bidder makes zero flow profits. This implies that the cartel can control participation very effectively, and therefore keep incentive constraint (i) enforceable. An additional prediction is that introducing minimum prices will make it more difficult to keep cartel members from participating in auctions.

The data is consistent with this theory. In Tsuchiura (where we observe all bidders), participation is limited: the mean and median number of bidders per auction are both between 3 and 4. Table A.2 in Appendix A confirms that introducing a minimum price leads to greater participation by both entrants and cartel members.

Non-performing bidders. The official rationale for introducing minimum prices is that it reduces the incidence of non-performing bidders, i.e. bidders unable to execute the tasks described in the procurement contract. One question is whether explicitly introducing such bidders would change the findings from our analysis.

The answer is no. As we argue in the Online Appendix, non-performing bidders can be modeled within the framework of Section 4: we treat them as short-term entrants whose cost of production is set to 0. As Calveras et al. (2004) suggest, this may be because a

bidder near bankruptcy is protected by limited liability. Proposition 5 and Proposition 6 continue to hold in the presence of such non-performing bidders, since they rely only on the bidder-side of the market.

6.3   Robustness: Empirics

Our model and our interpretation of the data rely on several assumptions which can be motivated from data. We briefly summarize our findings below, and present more detailed results in Appendix A.

Smooth equilibrium adjustments.   Propositions 3 and 6 provide a test of collusion by contrasting the comparative statics of the distribution of winning bids following the introduction of minimum prices, depending on whether we are in a collusive or competitive environment. These comparative statics presume that bidders are in equilibrium given the existing policy, which is necessarily an approximation. Indeed, although communication with city officials suggest that the move to a minimum price format was unexpected, it is still possible that the anticipation of the change may have affected behavior before the change, or that behavior after the change did not immediately move to the equilibrium corresponding to the new policy.

A priori, smooth equilibrium adjustment would bias estimates against our findings. Replicating our analysis excluding auctions occurring in six months period before and after the policy change does not affect our results (see Table A.6).

Separate markets.   Our difference-in-differences analysis presumes that control cities are not affected by the policy change. One potential concern is that some of the cartel members active in a treatment city may also be active in control cities. If that is the case, the introduction of minimum bids in a treatment city may also cause a shift in the distribution of bids in control cities.

This possible effect does not change the interpretation of our findings. Indeed, it should lead to an attenuation bias: part of the treatment effect would be interpreted as a common shock. Furthermore, we argue in Appendix A that the assumption of separate markets is plausible: in Tsuchiura the bulk of active cartel members are geographically much closer to Tsuchiura than its control cities. They should be more or less uniformly distributed if the cities were an integrated market.

Observable participation. Our model assumes that participation is observed at the bidding stage. If participation is unknown at the bidding stage, minimum prices may increase the expected number of competitors and therefore induce firms to place lower bids. However, the assumption that participation is observed can be motivated from data. We test this hypothesis by estimating the effect of entrant participation and cartel participation on realized bids (winning or not). Table A.8 summarizes the results: even controlling for auction size through reserve prices, both entrant and cartel participation have a significant effect on non-winning bids. This suggests that participants must have information about the set of bidders and vindicates our modeling choice.

Are the theory and empirics consistent? In Online Appendix OC, we gauge the potential effect that minimum prices have on bidding behavior in our model by conducting a back-of-the-envelope calibration exercise. We calibrate the model's parameters to match key statistics of bidding data from the city of Tsuchiura.

Our calibration exercise produces three main results. First, the introduction of minimum prices at the levels implemented by Tsuchiura has a negative effect on conditional winning bids (i.e., winning bids above the minimum price), ranging from -28% to -0.03%. Second, the effect of such minimum prices on average winning bids may be negative or positive, ranging from -11% to $+11\%$. Third, consistent with 2, a key factor explaining whether the unconditional treatment effect is negative or positive is the level at which the minimum price

is introduced: average winning bids fall when the minimum price is relatively low, while they increase when the minimum price is high.

Appendix

A   Further Empirical Exploration

A.1   Greater entry, and worse collusion

We are interested in the relative importance of greater entry and worse within-cartel enforcement in explaining the impact of minimum prices. Data from Tsuchiura includes bids from all participants (i.e. includes non-winners) and lets us make progress on these questions. We proceed by assessing the impact of minimum prices on entry, and then, by assessing the impact of minimum prices on winning bids, controlling for entry. Since these are, by force, single city before-after regressions, we first check that before-after regressions yield estimates of the impact of minimum prices that are consistent with estimates obtained from a more reliable difference-in-differences framework.

Policy impact in a single city regression. We perform both OLS and quantile regressions of the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta controls \tag{5}$$

where *controls* (used throughout the analysis) include Japanese $\log GDP$ as well as a time trend and month fixed effects. We report effects for the subsample of auctions such that the normalized winning bid is above .8, as well as the mean effect for the unconditional sample. Table A.1 reports the outcome of regression (5).

While the results are not precisely identical, these magnitudes match those of our difference-in-differences design (Table 4), which gives us some confidence that our controls are sufficient to make a single-city analysis not-implausible.

Entry and participation. We now study the impact of minimum prices on entry and participation by cartel members.

| norm_winning_bid | unconditional sample | sample s.t. $norm\_winning\_bid > .8$ | | | |
	mean effect	mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.016***	-0.026***	-0.061***	-0.015***	-0.006***	-0.005***
	(0.006)	(0.004)	(0.014)	(0.003)	(0.002)	(0.002)
ln_gdp	0.519***	0.226***	0.531***	0.129***	0.092***	0.052**
	(0.079)	(0.059)	(0.196)	(0.050)	(0.031)	(0.022)
year	0.005***	0.005***	0.010***	0.005***	0.003***	0.002***
	(0.001)	(0.001)	(0.003)	(0.001)	(0.000)	(0.000)
N	1748	1660	1660	1660	1660	1660

Table A.1: The effect of minimum prices on winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; regressions include month fixed-effects.

As expected, minimum prices increase both entry and participation. Table A.2 reports the results from OLS estimation of the following linear models:

$$num\_entrants \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta controls \tag{6}$$

$$num\_bidders \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta controls \tag{7}$$

$$\sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants + \beta controls \tag{8}$$

	num_entrants	num_bidders	num_bidders	num_bidders
policy_change	0.243**	0.516***	0.364**	0.410***
	(0.117)	(0.161)	(0.144)	(0.139)
ln_gdp	1.714	2.535	1.462	2.351
	(1.632)	(2.258)	(2.015)	(1.943)
year	-0.024	-0.350***	-0.335***	-0.378***
	(0.022)	(0.030)	(0.027)	(0.026)
num_entrants			0.626***	0.644***
			(0.030)	(0.029)
ln_reserve				0.382***
				(0.033)
N	1748	1748	1748	1748

Table A.2: The effect of minimum prices on entry and participation; regressions include month fixed-effects.

The introduction of minimum prices has a significant effect on both entry and participation by cartel members, adding on average .24 entrants and .52 bidders to auctions. These

numbers are large given that the mean and median number of participants per auction are respectively 3.8 and 3. Note that participation increases even controlling for new entrants, suggesting that participation by cartel members is an endogenous decision. The results are broadly unchanged when controlling for the auction's reserve price. The data suggests that cartel participation itself is affected by minimum prices, which is consistent with the extension of our model discussed in Section 6 and fully exposed in the Online Appendix.

Next, we examine the effect of minimum prices on winning bids controlling for participation, using the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 policy\_change + \beta_2 num\_bidders$$
$$+ \beta_3 num\_bidders\_sqr + \beta controls. \tag{9}$$

whose estimates are reported in Table A.3.

norm_winning_bid	unconditional sample mean effect	sample s.t. $norm\_winning\_bid > .8$				
		mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.010*	-0.023***	-0.039***	-0.016***	-0.008***	-0.006***
	(0.005)	(0.004)	(0.011)	(0.003)	(0.002)	(0.002)
num_bidders	-0.004*	-0.005***	-0.014***	-0.000	-0.002**	-0.001
	(0.002)	(0.002)	(0.005)	(0.001)	(0.001)	(0.001)
num_bidders_sqr	-0.001***	-0.001***	-0.000	-0.001***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
ln_gdp	0.539***	0.250***	0.411***	0.233***	0.115***	0.064***
	(0.074)	(0.054)	(0.153)	(0.044)	(0.031)	(0.023)
year	0.002*	0.002**	0.002	0.002***	0.001**	0.001***
	(0.001)	(0.001)	(0.002)	(0.001)	(0.000)	(0.000)
N	1748	1660	1660	1660	1660	1660

Table A.3: The effect of minimum prices on winning bids, controlling for participation. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; regressions include month fixed-effects.

As Table A.3 shows, the policy change has a negative effect on bids even when controlling for participation. We emphasize that the findings of Table A.3 do not arise naturally from a model of competitive bidding: controlling for the number of bidders, minimum prices should not cause a first-order stochastic dominance drop in the right tail of winning bids under competition (Proposition 5).

A.2 Individual policy group regressions

Aggregate regressions (2Agg) and (3Agg) aggregate results from individual policy group regressions. Tables A.4 and A.5 provide a sense of potential heterogeneity in treatment effects by reporting estimates for (2g) and (3g). With the exception of Tsukubamirai, individual policy group findings are broadly consistent with the aggregate estimates.

We emphasize that setting a threshold of 0.8 is not necessarily appropriate for all treatment cities.[27] In the case of Hitachiomiya, for instance, we find that the policy has a negative effect on the unconditional mean, but no effect on the conditional one. In the case of Tsukuba, we find that the policy has a negative effect on the upper quantiles of the winning bid distribution. This is consistent with Hitachiomiya having set the minimum prices at lower levels than Tsuchiura, and Tsukuba having set minimum prices at higher levels.[28]

A.3 Robustness

Smooth equilibrium transition. A potential concern with the analysis in Section 5 is that it implicitly assumes that firms' bidding behavior prior to the introduction of the minimum price was not affected by expectations of change, and that their behavior after the introduction of minimum prices adjusted immediately to the new environment. We have argued that this should bias results against our findings.

We further address these concerns by running regressions (2Agg) and (3Agg), excluding the data on auctions that were conducted within six months before or after the policy change. Table A.6 reports the results. Findings are unchanged.

Separate markets. We now provide support for the assumption that markets are separate. The argument is geographical and uses the fact that bidder names are publicly available for Tsuchiura. This allows us to geolocate all potential cartel bidders, and compute their (straight line) distance to treatment and control cities. We then compute two measures of proximity indicating that the three markets are not integrated.

The first metric is the proportion of cartel bidders whose closest city is Tsuchiura (treatment) rather than Tsukuba or Ushiku (controls). If the three markets were integrated, given

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[27]The threshold of 0.8 is the mid-point of minimum prices we observe in Tsuchiura. We do not observe minimum prices in other cities.

[28]Table A.9 shows that our results are robust to specifying different thresholds.

norm_winning_bid	unconditional mean effect	sample s.t. $norm\_winning\_bid > .8$				
		mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	-0.008	-0.026***	-0.084***	-0.021***	-0.006*	0.003
Tsuchiura	(0.007)	(0.005)	(0.010)	(0.005)	(0.003)	(0.002)
N	3705	3459				
policy_change	-0.021**	-0.008	-0.004	0.006	0.009**	0.009***
Hitachiomiya	(0.008)	(0.006)	(0.005)	(0.011)	(0.005)	(0.004)
N	2457	2379				
policy_change	-0.040***	-0.032***	-0.112***	-0.023*	0.011*	0.014***
Inashiki	(0.009)	(0.007)	(0.005)	(0.013)	(0.006)	(0.004)
N	1990	1913				
policy_change	-0.029**	-0.021**	-0.026	-0.003	-0.001	-0.002
Toride	(0.012)	(0.008)	(0.022)	(0.007)	(0.004)	(0.003)
N	2348	2272				
policy_change	0.046***	0.014**	0.021*	-0.014**	-0.019***	-0.006
Tsukuba	(0.010)	(0.007)	(0.011)	(0.007)	(0.006)	(0.008)
N	2650	2276				
policy_change	0.001	0.006	0.035***	0.007	0.006	0.004
Tsukubamirai	(0.017)	(0.009)	(0.009)	(0.007)	(0.009)	(0.011)
N	1070	930				

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table A.4: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples and quantile regression estimates for conditional sample; regressions include city fixed-effects, year fixed-effects, month fixed-effects and city specific time-trends.

that the population of Tsuchiura is bracketed by that of the control cities, we should expect roughly 1/3 of cartel bidders to have Tsuchiura as their closest location. Instead the number in our data is 87%.

Our second metric compares the share of bidders within a fixed radius from each city. Given a quantile Q, we compute the Qth quantile radius for Tsuchiura, i.e. the distance d_Q such that a proportion Q of cartel bidders are within distance d_Q of Tsuchiura. We then compute the proportion of cartel bidders within distance d of either control cities. Since the distance between control cities is roughly equal to the distance between Tsuchiura and each control city, if the markets were integrated, we would expect that a proportion Q of cartel bidders would be within distance d_Q of each control city. This is not the case: for $Q = .5$,

norm_winning_bid	unconditional mean effect	sample s.t. *norm_winning_bid* > .8				
		mean effect	$q = .2$	$q = .4$	$q = .6$	$q = .8$
policy_change	0.023**	-0.008	-0.025	-0.013*	-0.000	0.006
Tsuchiura	(0.010)	(0.007)	(0.017)	(0.008)	(0.005)	(0.004)
cartel X policy_change	-0.036***	-0.021***	-0.053***	-0.007	-0.007	-0.004
Tsuchiura	(0.009)	(0.007)	(0.015)	(0.007)	(0.005)	(0.004)
N	3695	3449				
policy_change	-0.016	-0.013*	-0.007	-0.001	0.007	0.012***
Hitachiomiya	(0.010)	(0.008)	(0.007)	(0.013)	(0.006)	(0.004)
cartel X policy_change	-0.008	0.003	-0.001	0.007	0.001	-0.002
Hitachiomiya	(0.007)	(0.005)	(0.005)	(0.009)	(0.004)	(0.003)
N	2457	2379				
policy_change	-0.017*	-0.016**	-0.094***	-0.006	0.012	0.014***
Inashiki	(0.010)	(0.008)	(0.008)	(0.008)	(0.009)	(0.005)
cartel X policy_change	-0.032***	-0.025***	-0.018***	-0.077***	-0.006	0.003
Inashiki	(0.008)	(0.006)	(0.006)	(0.006)	(0.007)	(0.003)
N	1990	1913				
policy_change	-0.040***	-0.028**	-0.049	-0.009	0.001	-0.001
Toride	(0.015)	(0.011)	(0.032)	(0.009)	(0.006)	(0.004)
cartel X policy_change	0.017	0.010	0.022	0.004	-0.003	-0.003
Toride	(0.013)	(0.010)	(0.029)	(0.009)	(0.005)	(0.004)
N	2348	2272				
policy_change	0.088***	0.040***	0.035**	0.014	0.014**	0.032***
Tsukuba	(0.012)	(0.008)	(0.014)	(0.009)	(0.007)	(0.008)
cartel X policy_change	-0.053***	-0.035***	-0.022**	-0.034***	-0.052***	-0.062***
Tsukuba	(0.009)	(0.006)	(0.010)	(0.006)	(0.005)	(0.006)
N	2650	2276				
policy_change	0.010	-0.009	0.005	-0.078***	-0.024	0.013
Tsukubamirai	(0.030)	(0.016)	(0.018)	(0.014)	(0.018)	(0.018)
cartel X policy_change	-0.007	0.023	0.035**	0.085***	0.032*	-0.004
Tsukubamirai	(0.028)	(0.015)	(0.017)	(0.013)	(0.017)	(0.017)
N	1070	930				

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table A.5: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples and quantile regression estimates for conditional sample; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends.

norm_winning_bid	unconditional mean effect		conditional mean effect	
policy_change	-0.029***	-0.009	-0.028***	-0.009
p-value	0.008	0.394	0.002	0.206
policy_change x cartel		-0.020**		-0.020**
p-value		0.046		0.020
N	7899	7899	7278	7278

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table A.6: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids, excluding auctions occurring around the policy change. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

the proportion of cartel bidders within distance d_Q of control cities is exactly equal to 0; for $Q = .75$, it is 13%. This suggests that markets are largely separate.

Controlling for reserve prices. We now show that our empirical results continue to hold if we control for the level of the reserve price. We run the following versions of regressions (2Agg) and (3Agg):

$$\log\_winning\_bid_a \sim \beta_0 + \beta_1 log\_reserve\_price + \beta_2 policy\_change$$
$$+ \frac{1}{N_a} \sum_{g,\ s.t.\ a \in g} group\_fe_g + city\_fe_g + month\_fe_g + year\_fe_g + city\_trends_g$$

$$\log\_winning\_bid_a \sim \beta_0 + \beta_1 log\_reserve\_price + \beta_2 policy\_change + \beta_3 \widetilde{cartel}$$
$$+ \beta_4 \widetilde{cartel\_treatment\_cities} + \beta_5 \widetilde{cartel\_treatment\_cities} \times policy\_change$$
$$+ \frac{1}{N_a} \sum_{g,\ s.t.\ a \in g} city\_fe_g + month\_fe_g + year\_fe_g + city\_trends_g.$$

The results are presented in Table A.7.

log_winning_bid	unconditional mean effect		conditional mean effect	
policy_change	-0.021	0.004	-0.029**	-0.011
p-value	0.454	0.950	0.022	0.162
policy_change x cartel		-0.029***		-0.021***
p-value		0.008		0.004
N	8623	8623	7903	7903

***, ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table A.7: Difference-in-differences analysis of the effect of minimum prices on log winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

Observability of participation. To assess whether the assumption of observable participants is plausible, we compute OLS estimates of linear models

$$norm\_bid \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants$$
$$+ \beta_4 num\_cartel\_participants + \beta controls$$
$$ln\_bid \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants$$
$$+ \beta_4 num\_cartel\_participants + \beta_5 ln\_reserve + \beta controls$$

for all (bidder, auction) pairs using data from Tsuchiura. The results are presented in Table A.8. For concision we do not reports coefficients for control variables (year and log GDP).

	norm_bid	ln_bid
policy_change	-0.025***	-0.024***
	(0.005)	(0.006)
num_entrants	-0.012***	-0.014***
	(0.002)	(0.002)
num_cartel	-0.011***	-0.013***
	(0.001)	(0.001)
ln_reserve		1.008***
		(0.003)
N	6560	6560

Table A.8: Bid (winning or not) as a function of realized participation; clustered by auction id.

The data supports the assumption that participation is observable. Indeed, even conditional on auction size (proxied here by the reserve price), both the realized number of entrants and the realized number of participating cartel members have a significant effect on bids.

Different thresholds for normalized bids. Throughout the paper, we analyzed the effect that the policy change had on the distribution of normalized winning bids truncated at 0.8. Our results are robust to changes in this threshold.

To illustrate this, we estimate equations (2Agg) and (3Agg) using thresholds of 0.78 and 0.82. The results are presented in Table A.9.

norm_winning_bid	$norm\_winning\_bid > 0.78$ mean effect		$norm\_winning\_bid > 0.82$ mean effect	
policy_change	-0.031*	-0.014	-0.011*	0.002
p-value	0.050	0.176	0.084	0.727
cartel_winner x policy_change		-0.018***		-0.016***
p-value		0.004		0.002
N	8085	8085	7725	7725

*** , ** and * respectively denote effects significant at the .01, .05 and .1 level.

Table A.9: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional and conditional samples; regressions include city fixed-effects, year fixed effects month fixed-effects and city specific time-trends. Standard errors are clustered at the (city, year) level and p-values are calculated by wild bootstrap.

B Key Proofs

B.1 Proofs for Section 2

This appendix contains the proofs of Section 2. We start with a few preliminary observations. First, since the game we are studying is a complete information game with perfect monitoring, the set of SPE payoffs is compact (Proposition 2.5.2 in Mailath and Samuelson (2006)). Hence, $\overline{V}_p$ and $\underline{V}_p$ are attained. Fix an SPE σ and a history h_t. Let $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ be the bidding and transfer profile that firms play in this equilibrium after history h_t. Let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and the allocation induced

by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. Let $h_{t+1} = h_t \sqcup (\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$ be the concatenated history composed of h_t followed by $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$, and let $\{V(h_{t+1})\}_{i \in N}$ be the vector of continuation payoffs after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ denote the on-path history that follows h_t when current costs are $\mathbf{c}$. Note that the following inequalities must hold:

(i) for all $i \in \widehat{N}$ such that $c_i \leq \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \delta \underline{V}_p. \tag{10}$$

(ii) for all $i \in \widehat{N}$ such that $c_i > \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{11}$$

(iii) for all $i \in N$,
$$T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{12}$$

The inequality in (10) must hold since a firm with cost below $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by undercutting the winning bid when $\beta^W(\mathbf{c}) > p$, or, by bidding p and choosing $\gamma_i = 1$ when $\beta^W(\mathbf{c}) = p$. Similarly, the inequality in (11) must hold since firms with cost larger than $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by bidding more than $\beta^W(\mathbf{c})$. Finally, the inequality in (12) must hold since otherwise firm i would not be willing to make the required transfer.

Conversely, suppose there exists a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c}))\}_{i \in N}$ that satisfy inequalities (10)-(12) for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $\gamma(\mathbf{c})$ is such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j:x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). Then, $(\beta^W, \mathbf{x}, \mathbf{T})$ can be supported in an SPE as follows. For all $\mathbf{c}$, all firms $i \in \widehat{N}$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$. If no firm deviates at the transfer stage, in the next period firms play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i \in N}$. If firm i deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$; if firm i deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$ (deviations by more than one firm go unpun-

ished). Since (10) holds, under this strategy profile no firm has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (11) holds, no firm with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Upward deviations by a firm i with $c_i < \beta^W(\mathbf{c})$ who wins the auction are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Finally, since (12) holds, all firms have an incentive to make their required transfers.

Proof of Lemma 1. Let σ be an SPE that attains $\overline{V}_p$. Towards a contradiction, suppose there exists an on-path history $h_t = h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ such that $\sum_i V_i(\sigma, h_t) = V(\sigma, h_t) < \overline{V}_p$. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$.

Consider changing the continuation equilibrium at history h_t by an equilibrium that delivers payoff vector $\{V_i\}_{i \in N}$, and changing the transfers after history $h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ as follows. First, for each $i \in N$, let $\hat{T}_i$ be such that $\hat{T}_i + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(\sigma, h_t)$. Note that

$$\sum_i \hat{T}_i = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(\sigma, h_t) - V_i)\} < 0,$$

where we used $\sum_i V_i = \overline{V}_p > \sum_i V_i(\sigma, h_t)$ and $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$. For each $i \in N$, let $\tilde{T}_i = \hat{T}_i + \frac{\epsilon}{n}$, where $\epsilon > 0$ is such that $\sum_i \tilde{T}_i = \sum_i \hat{T}_i + \epsilon = 0$. Replacing transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ and continuation values $V_i(\sigma, h_t)$ by transfers $\tilde{T}_i$ and values V_i relaxes constraints (10)-(12) and increases the total expected discounted surplus that the equilibrium generates. Therefore, if σ attains $\overline{V}_p$, it must be that $V(\sigma, h_t) = \overline{V}_p$ for all on-path histories.

We now prove the second statement in the Lemma. Fix an optimal equilibrium σ, and let $\{V_i\}_{i \in N}$ be the equilibrium payoff vector that this equilibrium delivers, with $\sum_i V_i = \overline{V}_p$. For each $\mathbf{c}$, let $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$ be the bidding profile that firms use in the first period under σ, and let $\mathbf{x}(\mathbf{c})$ denote the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. It follows that

$$\overline{V}_p = \mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - c_i)\right] + \delta\overline{V}_p \iff \overline{V}_p = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - \mathbf{x}(\mathbf{c}))\right].$$

We show that there exists an optimal equilibrium in which firms use bidding profile $(\beta(\cdot), \gamma(\cdot))$ after all on-path histories. For any $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$, let $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ denote the transfer that firm i makes at the end of the first period under equilibrium σ when first period costs, bids and allocation are given by $\mathbf{c}, \mathbf{b}, \gamma$ and $\mathbf{x}$. Let $V_i(h_1(\mathbf{c}))$ denote firm i's continuation payoff under equilibrium σ after first period history $h_1(\mathbf{c}) = (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$. By our arguments above, $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p$ for all $\mathbf{c}$. Since σ is an equilibrium, it must be

that $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$, $\mathbf{x}(\mathbf{c})$, $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ and $V_i(h_1(\mathbf{c}))$ satisfy (10)-(12).

Consider the following strategy profile. Along the equilibrium path, at each period t firms bid according to $(\beta(\cdot), \gamma(\cdot))$. For any $(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$, firm i makes transfer $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ such that $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_1(\mathbf{c}))$. Note that

$$\sum_i \hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(h_1(\mathbf{c})) - V_i)\} = 0,$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p = \sum_i V_i$. If firm i deviates at the bidding stage or transfer stage, then firms revert to an equilibrium that gives firm i a payoff of $\underline{V}_p$. Clearly, this strategy profile delivers total payoff $\overline{V}_p$. Moreover, firms have the same incentives to bid according to (β, γ) and make their required transfers than under the original equilibrium σ. Hence, no firm has an incentive to deviate at any stage and this strategy profile can be supported as an equilibrium. $\blacksquare$

Proof of Lemma 2. Suppose there exists an SPE σ and a history h_t at which firms bid according to a bidding profile (β, γ) that induces winning bid $\beta^W(\mathbf{c})$ and allocation $\mathbf{x}(\mathbf{c})$. Let $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ be firm i's transfers at history h_t when costs are $\mathbf{c}$ and all firms play according to the SPE σ. Let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ be the on-path history that follows h_t when costs are $\mathbf{c}$, and let $V_i(h_{t+1}(\mathbf{c}))$ be firm i's equilibrium payoff at history $h_{t+1}(\mathbf{c})$. Since the equilibrium must satisfy (10)-(12) for all $\mathbf{c}$,

$$\sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\}$$
$$\leq \sum_{i \in N} T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta \sum_{i \in N} (V_i(h_{t+1}(\mathbf{c})) - \underline{V}_p) \leq \delta(\overline{V}_p - n\underline{V}_p),$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_{t+1}(\mathbf{c})) \leq \overline{V}_p$.

Next, consider a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (1) for all $\mathbf{c}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})$ for all $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$). We now construct an SPE that supports $\beta^W(\cdot)$ and $\mathbf{x}(\cdot)$ in the first period. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $i \in N$ and

each $\mathbf{c}$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_p) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) + \epsilon(\mathbf{c}) & \text{if } i \notin \widehat{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Note that, for all $\mathbf{c}$,

$$\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c})$$

$$= -\delta(\overline{V}_p - n\underline{V}_p) + \sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \leq 0,$$

where the inequality follows since β^W and $\mathbf{x}$ satisfy (1). We set $\epsilon(\mathbf{c}) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c}$ all participating firms bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c})/\sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms exchange transfers $T_i(\mathbf{c})$. If no firm deviates at the transfer stage, from $t = 1$ onwards they play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_p$ (if more than one firm deviates, firms punish the lowest indexed firm that deviated). This strategy profile satisfies (10)-(12), and so β^W and $\mathbf{x}$ are implementable. $\blacksquare$

Proof of Proposition 1. By Lemma 1, there exists an optimal equilibrium in which firms use the same bidding profile (β, γ) at every on-path history. For each cost vector $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation induced by this bidding profile under cost vector $\mathbf{c}$.

We first show that $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $\beta^W(\mathbf{c}) \neq b_p^*(\mathbf{c}) > p$. Since $\mathbf{x}^*(\mathbf{c})$ is the efficient allocation, the procurement cost under allocation $\mathbf{x}(\mathbf{c})$ is at least as large as the procurement cost under allocation $\mathbf{x}^*(\mathbf{c})$. Since bidding profile (β, γ) is optimal, it must be that $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Indeed, if $\beta^W(\mathbf{c}) < b_p^*(\mathbf{c})$, then the cartel would strictly prefer to use a bidding profile that allocates the contract efficiently and has winning bid $b_p^*(\mathbf{c})$ under cost vector $\mathbf{c}$ than to use

bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ must satisfy

$$
\begin{aligned}
\delta(\overline{V}_p - n\underline{V}_p) &\geq \sum_{i \in \widehat{N}} \left\{ (1 - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i \right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i \right]^- \right\} \\
&\geq \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i \right]^+ ,
\end{aligned}
$$

which contradicts $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Therefore, $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$.

Next, we show that $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $b_p^*(\mathbf{c}) \leq p$ and $\beta^W(\mathbf{c}) > p$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy

$$
\begin{aligned}
\delta(\overline{V}_p - n\underline{V}_p) &\geq \sum_{i \in \widehat{N}} \left\{ (1 - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i \right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i \right]^- \right\} \\
&\geq \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i \right]^+ ,
\end{aligned}
$$

which contradicts $\beta^W(\mathbf{c}) > p \geq b_p^*(\mathbf{c})$. Therefore, $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Combining this with the arguments above, $\beta^W(\mathbf{c}) = \beta_p^*(\mathbf{c}) = \max\{p, b_p^*(\mathbf{c})\}$.

Finally, we characterize the allocation in an optimal equilibrium. Note first that under an optimal bidding profile the cartel must allocate the procurement contract efficiently whenever $\beta_p^*(\mathbf{c}) > p$. Indeed, by construction, the efficient allocation is sustainable whenever the winning bid is $\beta_p^*(\mathbf{c}) > p$. Therefore, if the allocation was not efficient for some $\mathbf{c}$ with $\beta_p^*(\mathbf{c}) > p$, the cartel could strictly improve its profits by using a bidding profile with winning bid $\beta_p^*(\mathbf{c})$ that allocates the good efficiently.

Consider next a cost vector $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$. In this case, the cartel's bidding profile in an optimal equilibrium induces the most efficient allocation (i.e., the allocation that minimizes expected procurement costs) consistent with (1) when the winning bid is p. ∎

Proof of Corollary 1. Note that, for $\delta = 0$, $b_p^*(\mathbf{c}) = c_{(2)}$ for all $\mathbf{c}$. By Proposition 1, when $\delta = 0$ the winning bid under the best equilibrium for the cartel is equal to $\beta^{\mathsf{comp}}(\mathbf{c}) = \max\{c_{(2)}, p\}$, which is the winning bid under competition. ∎

Fix a minimum price p. For every value $V \geq n\underline{V}_p$ and every $\mathbf{c}$, let

$$b_p(\mathbf{c}; V) \equiv \sup\left\{ b \leq r : \sum_{i \in \widehat{N}}(1 - x_i^*(\mathbf{c}))[b - c_i]^+ \leq \delta(V - n\underline{V}_p) \right\},$$

and let $\beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$. Note that $\beta_p(\mathbf{c}; V)$ would be the winning bid in an optimal equilibrium if the cartel's total surplus was equal to V. Let $\mathbf{x}^p(\mathbf{c}; V)$ be the allocation under an optimal equilibrium when the cartel's total surplus is V. For every $V \geq n\underline{V}_p$, let

$$U_p(V) \equiv \frac{1}{1 - \delta}\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i^p(\mathbf{c}; V)(\beta_p(\mathbf{c}, V) - c_i) \right],$$

be the total surplus generated under a bidding profile that induces winning bid $\beta_p(\mathbf{c}; V)$ and allocation $\mathbf{x}^p(\mathbf{c}; V)$. The winning bid and allocation in an optimal equilibrium are $\beta_p^*(\mathbf{c}) = \beta_p(\mathbf{c}; \overline{V}_p)$ and $\mathbf{x}^p(\mathbf{c}; \overline{V}_p)$, and so $\overline{V}_p = U_p(\overline{V}_p)$. Let

$$\overline{U}_p \equiv \sup\{V \geq n\underline{V}_p : V \leq U_p(V)\}.$$

Lemma B.1. $\overline{V}_p = \overline{U}_p$.

Proof. Since $\overline{V}_p = U_p(\overline{V}_p)$, it follows that $\overline{U}_p \geq \overline{V}_p$. We now show that $\overline{U}_p \leq \overline{V}_p$. Towards a contradiction, suppose that $\overline{U}_p > \overline{V}_p$. Hence, there exists $\tilde{V}$ such that $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$. Let $V = \frac{U_p(\tilde{V})}{n}$, and consider the following strategy profile. For all on-path histories, cartel members use a bidding profile (β, γ) inducing winning bid $\beta_p(\mathbf{c}; \tilde{V})$ and allocation $\mathbf{x}^p(\mathbf{c}; \tilde{V})$. If firm i deviates at the bidding stage, there are no transfers and in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_p$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_p$ to the lowest indexed firm that deviated). If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c})$ given by

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V - \underline{V}_p) + (\rho_i(\beta_p, \gamma, \mathbf{x}^p)(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))(\beta_p(\mathbf{c}; \tilde{V}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta_p(\mathbf{c}; \tilde{V}), \\ -\delta(V - \underline{V}_p) + \epsilon(\mathbf{c}) & \text{otherwise,} \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined.[29] Note that

$$\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = -\delta(U_p(\tilde{V}) - n\underline{V}_p) + \sum_{i \in \widehat{N}}((\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))[\beta_p(\mathbf{c}; \tilde{V}) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta_p(\mathbf{c}; \tilde{V})$ and $x_i^p(\mathbf{c}; \tilde{V})$ are the winning bid and the allocation under an optimal equilibrium when the cartel's total surplus is $\tilde{V} \leq U_p(\tilde{V})$. We set $\epsilon(\mathbf{c}) \geq 0$ such that $\sum_i T_i(\mathbf{c}) = 0$. If firm i deviates at the transfer stage, in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_p$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_p$ to the lowest indexed firm that deviated). Otherwise, in the next period firms continue playing the same strategy as above. This strategy profile generates total surplus $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$ to the cartel. Since firms play symmetric strategies, it gives a payoff $V = \frac{U_p(\tilde{V})}{n}$ to each cartel member. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. This contradicts $U_p(\tilde{V}) > \overline{V}_p$, so it must be that $\overline{U}_p \leq \overline{V}_p$. ∎

Proof of Proposition 2. We first establish part (i). Suppose that $p \leq \underline{c}$ and fix equilibrium payoffs $\{V_i\}_{i \in N}$. Fix $j \in N$ and consider the following strategy profile. At $t = 0$, firms' behavior depends on whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. If $j \in \widehat{N}$, all firms $i \in \widehat{N}$ bid $\min\{c_j, c_{(2)}\}$ (where $c_{(2)}$ is the second lowest procurement cost). Firm $i \in \widehat{N}$ chooses $\gamma_i = 1$ if $c_i = \min_{k \in \widehat{N}} c_k$ and chooses $\gamma_i = 0$ otherwise. Note that this bidding profile constitutes an equilibrium of the stage game. If $j \notin \widehat{N}$, at $t = 0$ participating firms play according to some equilibrium of the stage game. If all firms bid according to this profile, firm j's transfer is $T_j = -\delta V_j$ at the end of the period regardless of whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. The transfer of firm $i \neq j$ is $T_i = \frac{1}{n-1}\delta V_j$ at the end of the period, so $\sum_i T_i = 0$. If no firm deviates at the bidding or transfer stage, at $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}$. If firm i deviates at the bidding stage, there are no transfers and at $t = 1$ firms play the strategy just described with i in place of j. If no firm deviates at the bidding stage and firm i deviates at the transfer stage, at $t = 1$ firms play the strategy just described with i in place of j (if more than one firm deviates at the bidding or transfer stage, from $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}_{i \in N}$). Note that this strategy profile gives player j a payoff of 0. Moreover, no firm has an incentive to deviate at $t = 0$, and so $\underline{V}_p = 0$ for all $p \leq \underline{c}$.

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[29]Recall that $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is the allocation under an optimal equilibrium when continuation payoff is $\tilde{V}$. Therefore, $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is such that $x_i^p(\mathbf{c}; \tilde{V}) = 0$ for all i with $c_i > \beta_p(\mathbf{c}; \tilde{V})$.

Suppose next that $p > \underline{c}$, and note that

$$\underline{V}_p \geq \underline{v}_p \equiv \frac{1}{1-\delta}\mathrm{prob}(i \in \widehat{N})\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i \leq p}(p - c_i)|i \in \widehat{N}\right] > 0,$$

where the first inequality follows since $\underline{v}_p$ is the minmax payoff for a firm in an auction with minimum price p. This establishes part (i).

We now turn to part (ii). Note that $\beta_0^*(\underline{c}) = \inf_{\mathbf{c}}\beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta\overline{V}_0}{n-1} > \underline{c}.$[30] We now show that there exists $\eta > 0$ such that $\overline{V}_p - n\underline{V}_p < \overline{V}_0$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. Fix $\eta > 0$ and $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. For every $V \geq n\underline{V}_p$ and every $\mathbf{c}$, let $\tilde{\beta}_p(\mathbf{c}; V) \equiv \max\{b_0(\mathbf{c}; V), p\}$. Since $\underline{V}_p > 0$ for all $p > \beta_0^*(\underline{c})$, it follows that $b_0(\mathbf{c}; V) \geq b_p(\mathbf{c}; V)$ for all $\mathbf{c}$ and all $V \geq n\underline{V}_p$, and so $\tilde{\beta}_p(\mathbf{c}; V) \geq \beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$ for all $\mathbf{c}$ and all $V \geq n\underline{V}_p$. Define

$$\tilde{U}_p(V) \equiv \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}}x_i^*(\mathbf{c})(\tilde{\beta}_p(\mathbf{c}; V) - c_i)\right],$$

and note that $\tilde{U}_p(V) \geq U_p(V)$ for all $V \geq n\underline{V}_p$. Let $\tilde{V}_p \equiv \sup\{V \geq n\underline{V}_p : \tilde{U}_p(V) \geq V\}$, and note that $\tilde{V}_p \geq \overline{V}_p$. Recall that, for all V, $U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}}x_i^*(\mathbf{c})(b_0(\mathbf{c}; V) - c_i)\right].$[31] Therefore, for all V,

$$\tilde{U}_p(V) - U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[(p - b_0(\mathbf{c}; V))\mathbf{1}_{\{\mathbf{c}:b_0(\mathbf{c};V)<p\}}\right] > 0.$$

Note that for all V and all $\mathbf{c}$, $b_0(\mathbf{c}; V) \geq \underline{c} + \frac{\delta V}{n-1}$. Let $\hat{V} > 0$ be such that $\underline{c} + \frac{\delta\hat{V}}{n-1} = \beta_0^*(\underline{c}) + \eta = \underline{c} + \frac{\delta\overline{V}_0}{n-1} + \eta$; that is, $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_0$. Then, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $V \geq \hat{V}$, $b_0(\mathbf{c}; V) \geq p$ for all $\mathbf{c}$, and so $\tilde{U}_p(V) = U_0(V)$. Since $\hat{V} > \overline{V}_0$ and since $\overline{V}_0 = \sup\{V \geq 0 : U_0(V) \geq V\}$, it follows that $V > U_0(V) = \tilde{U}_p(V)$ for all $V \geq \hat{V}$ and all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, and so $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \tilde{V}_p \geq \overline{V}_p$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$.

Finally, let $\eta > 0$ be such that $\frac{(n-1)\eta}{\delta} = n\underline{v}_{\beta_0^*(\underline{c})} = n\frac{\mathrm{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i \leq \beta_0^*(\underline{c})}(\beta_0^*(\underline{c}) - c_i)|i \in \widehat{N}\right].$[32] Since $\underline{V}_p \geq \underline{v}_p \geq \underline{v}_{\beta_0^*(\underline{c})}$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$,

$$\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_p \Rightarrow \overline{V}_0 > \overline{V}_p - n\underline{V}_p,$$

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[30]Term $\beta_0^*(\mathbf{c})$ attains its lowest value when all cartel members participate in the auction and costs are $\mathbf{c} = (\underline{c})_{i\in N}$ (i.e., all firms have cost $\underline{c}$). For this cost vector, $\beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta\overline{V}_0}{n-1}$.

[31]Indeed, by Proposition 1, $\mathbf{x}^{p=0}(\mathbf{c}; V) = \mathbf{x}^*(\mathbf{c})$ for all V.

[32]Recall that for all p, $\underline{V}_p \geq \underline{v}_p = \frac{\mathrm{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i \leq p}(p - c_i)|i \in \widehat{N}\right]$.

which completes the proof. ∎

B.2 Proofs of Section 3

Proof of Proposition 3. Consider first a collusive environment. By Propositions 1 and 2, there exists $\eta > 0$ such that $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $\mathbf{c}$ such that $\beta_0^*(\mathbf{c}) \geq p$, with strict inequality if $\beta_0^*(\mathbf{c}) < r$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\mathrm{prob}(\beta_p^* \geq q | \beta_p^* \geq p) \leq \mathrm{prob}(\beta_0^* \geq q | \beta_0^* \geq p)$, and the inequality is strict for some $q > p$ whenever $\mathrm{prob}(\beta_0^* < r) > 0$. This proves part (i).

Under competition, for all p and all $q > p$, $\mathrm{prob}(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p) = \mathrm{prob}(c_{(2)} \geq q | c_{(2)} > p) = \mathrm{prob}(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p)$. This proves part (ii). ∎

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