Choice Deferral, Indecisiveness and Preference for Flexibility

Leonardo Pejsachowicz and Severine Toussaert*

November 3, 2016

Indecision is the key to flexibility. Proverb

Abstract

We introduce a model of menu choice in which a person's psychological tastes and revealed preferences may be distinct. The former is modeled by means of a possibly incomplete (but otherwise rational) preference relation $\succcurlyeq$, and the latter by a completion $\succcurlyeq^*$ of that relation. The two relations are connected through an axiom which formalizes the following intuitive rule: "Whenever in doubt about which menu is better, don't commit now; just leave options open." The main result of this paper is that, under the usual assumptions of the menu choice literature, this deferral property alone forces $\succcurlyeq^*$ to exhibit a preference for flexibility. In particular, we find that there is a fundamental tension between non-monotonic preferences, such as preferences for self-control, and tendency to defer choice due to indecisiveness.

KEYWORDS: Incomplete preferences, preference for flexibility, choice deferral.

The primitive of the theory of choice among opportunity sets is a preference relation defined on a collection X of subsets of a given space of alternatives. These subsets, called

---
*Pejsachowicz: Department of Economics, Ecole Polytechnique, Route De Saclay, 91128 Palaiseau, France, Email: leonardo.pejsachowicz@polytechnique.edu. Toussaert: Department of Economics, New York Unviversity, 19 West 4th Street, New York, NY, Email: st1445@nyu.edu.

"menus," are generally interpreted as feasible sets from which an alternative will be selected at some later (unmodeled) stage. With this dynamic interpretation in mind, Kreps [27] introduced in a seminal paper a behavioral property called "preference for flexibility," which is now a fairly common postulate in this literature.[1] According to this postulate, the preference relation of the decision maker ranks any superset of a menu at least as high as that menu. Clearly, this monotonicity property seems particularly meaningful if the decision maker aims to accommodate unforeseen contingencies.[2] However, it is by no means unexceptionable. There are situations in which one may prefer smaller menus to larger ones. Indeed, an agent may well favor commitment if he fears to be tempted by some option as in the model of Gul and Pesendorfer [23]. Alternatively, he might prefer to restrict his choice set if he anticipates regret as in the model of Sarver [40]. In this respect, it seems important to take a closer look at the rationale behind preference for flexibility.

The primary objective of this paper is to show that the inability of an agent to compare two menus at a basic (psychological) level may alone account for his preference for flexibility. The idea of indecisiveness is of course not new in economics; it dates back to Aumann [3], and is captured by dropping the assumption of *completeness* of one's preferences. Moreover, while it is mostly studied in other contexts,[3] indecisiveness is likely to be more pronounced in the context of menu preferences. After all, one of the main justifications for allowing for indecisiveness stems from the complex nature of the objects of choice (such as goods with multiple attributes or lotteries with a large support), which the decision maker may find hard to compare. This source of indecisiveness was originally noted by Aumann (1962) who wrote

---

[1]For instance, Nehring (1999) studies preference for flexibility in a Savagean context. Dekel et al. [?] extend the work of Kreps [27] to a lottery framework. Recently, Ahn and Sarver (2013) combined preference for flexibility ex ante with random choice ex post. This property has also been recently analyzed in a dynamic setting (see, for example, Krishna and Sadowski (2012)).

[2]One common justification for preference for flexibility stems from the idea that the decision maker may feel uncertain about what his future tastes will be, and indeed, the famous representation theorems of Kreps [27] and Dekel et al. [12] give substance to this interpretation.

[3]There is now a sizable literature on the theory of incomplete preferences. The recent contributions can be found on a large spectrum, ranging from the ordinal framework of Peleg [37], Ok [34] or Evren and Ok [18] to the cardinal world of lotteries (Dubra et al. [15] and Baucells and Shapley (2008)) or choice under uncertainty (Bewley (1986), Ok et al. [36], Galaabaatar and Karni (2013), Riella [39]).

that some "... decision problems might be extremely complex, too complex for intuitive "insight", and our individual might prefer to make no decision at all in these problems." In this respect, one would expect the situation to be no easier when the objects of choice are themselves decision problems, as is the case in the menu choice framework. It thus appears that representing the psychological tastes of an agent by a potentially incomplete preference relation $\succcurlyeq$ on X is quite natural in the present setting. By $A \succcurlyeq B$, we understand that there is no doubt in the mind of the agent that the menu A is better than B. Nevertheless, we leave open the possibility that neither $A \succcurlyeq B$ nor $B \succcurlyeq A$ may hold.

Yet, more often than not, we do not observe the inherent tastes of a decision maker; we rather see the choices he makes. Suppose that from the feasible set $\{A, B\}$ of menus, we see him choosing A instead of B. We then say that A is revealed preferred to B by this agent. Assuming that a choice has to be made at all times, this gives rise to a *complete* preference relation, say, $\succcurlyeq^*$, on X. If $A \succcurlyeq^* B$, we understand that the agent declared A better than B through his choice, but we do not know if $A \succcurlyeq B$ actually holds. Perhaps the agent was unable to compare A and B on the basis of his core preferences,[4] and his choice of A over B followed from the recommendation of a second party and/or from the adoption of some ad hoc choice procedure. By contrast, if $A \succcurlyeq B$ actually holds, we surely expect $A \succcurlyeq^* B$ to hold.

The literature on menu preferences usually takes the revealed (complete) preference relation $\succcurlyeq^*$ on X as the primitive of the analysis. By contrast, in this paper, we take two preference relations $\succcurlyeq$ and $\succcurlyeq^*$ on X as the primitives. As discussed above, $\succcurlyeq$ is possibly incomplete and corresponds to the psychological tastes of the agent, while $\succcurlyeq^*$ is a completion of $\succcurlyeq$ and corresponds to the revealed preferences of the agent. Thus, when $\succcurlyeq \, = \, \succcurlyeq^*$, our setting reduces to the standard framework.[5]

---

[4]This might be the case if for instance he cannot foresee how he will feel in the future, when choosing from the selected menu. Alternatively, his tastes might be so genuinely incomplete that he cannot make up his mind between some sets even when they contain a single alternative. The idea that indecisiveness may be understood either as incompleteness in beliefs (here about one's tastes tomorrow) or as incompleteness in tastes was recently formalized by Ok Ortoleva and Riella [36] in the context of decision making under uncertainty.

[5]This way of modeling an individual with two relations, one being a completion of the other, has been

This modeling approach allows us to explore the connections between an agent's indecisiveness and his preference for flexibility. On the one hand, indecisiveness is a psychological phenomenon, which is captured by the potential incompleteness of the core relation $\succsim$. On the other hand, preference for flexibility is a behavioral phenomenon, which is captured by the monotonicity of $\succsim^*$ (which formally means that $A \succsim^* B$ for any two menus A and B with $A \supseteq B$). Our main axiom builds a bridge between those two phenomena and allows us to investigate the implications of indecisiveness (of $\succsim$) for monotonicity (of $\succsim^*$). This axiom formalizes the following intuitive rule: "Whenever in doubt, just leave options open." More formally, this rule says that if A and B are incomparable according to $\succsim$, one would expect that $A \cup B \succsim^* A$ (but we do not know whether $A \succsim^* B$ or $B \succsim^* A$). Intuitively, an indecisive decision maker will often seek to defer choice if, for instance, he expects to be better informed in the future or simply needs additional time for contemplation about a difficult decision. Under such circumstances, choosing not to commit to a given menu can be seen as a cautious attitude. We thus call this property the *Cautious Deferral Axiom*.[6]

Our primary interest is to understand the behavioral consequences of the Cautious Deferral Axiom and its relation to monotonicity. In order to do so, we adopt the standard framework of the theory of menu preferences and impose the usual rationality assumptions of the literature (reviewd in Section 3). Our main result (Section 1.4) highlights an interesting connection between Cautious Deferral and monotonicity. When the agent faces no internal conflict, that is, when $\succsim$ is complete, our rule imposes no restriction on choice behavior. On the other hand, we find that even a minimal amount of indecisiveness has far-reaching consequences: provided that $\succsim$ is not complete, $\succsim^*$ must exhibit preference for flexibility on its entire domain and this, regardless of the extent of the incompleteness of $\succsim$. In this sense, the Cautious Deferral Axiom provides foundations for preference for flexibility

---

an object of recent investigation. For instance, this approach has been adopted by Gilboa et al. (2010), Cerreia-Vioglio (2012), Cerreia-Vioglio et al. (2013), Lehrer and Teper (2013) or Nehring (2009) in a world of uncertainty and by Danan (2003) in a general menu choice setting. (More on this in Section 3.)

[6]Note that this property is specific to the language of menus, which allows the formulation of the notion of "choice deferral" through the agent's preference for flexibility. For this reason, such a connection between a preference relation and its completion has not been explored in decision theory. One exception is Danan (2003), about which more shortly.

and gives credence to the proverb "Indecision is the key to flexibility."

One can also view our main result as an impossibility theorem (Section 1.5). To wit, consider an agent with incomplete tastes who satisfies the basic rationality postulates but may violate set monotonicity because, for instance, he suffers from temptation à la Gul and Pesendorfer (2001). Then our main theorem shows that if this agent also abides by the Cautious Deferral rule, temptation, self-control and flexibility motives all disappear. What we are left with is a standard decision maker who evaluates a set by its maximal elements with respect to the expectation of some utility function. Thus we identify a basic tension in the usual menu choice framework between non-monotonic preferences, indecisiveness and the Cautious Deferral Axiom.

1 The Model

1.1 Preliminaries

We work in the standard framework of the theory of menu preferences as in, say, Dekel et al. (2001) (from now on DLR). In what follows, we let Δ stand for the set of all probability distributions (lotteries) over a finite prize space Z with $|Z| = n$. The generic members of Δ are denoted as $a, b, c, \ldots$ etc. We let X stand for the set of all nonempty closed subsets of Δ. As is standard in this literature, we view X as a metric space relative to the Hausdorff metric. The generic members of X are denoted as $A, B, C, \ldots$ etc., and are referred to as *menus*. These sets are interpreted as opportunity sets from which the decision maker will choose an option at a later (unmodeled) stage.

In what follows, we also consider X as an algebraic entity by imposing on it the mixture operation induced by the Minkowski sum of sets. That is, for any A and B in X and any $0 \leq \lambda \leq 1$, we define

$$\lambda A \oplus (1 - \lambda)B := \{\lambda a + (1 - \lambda)b \mid a \in A \text{ and } b \in B\}$$

which is itself an element of X.

A binary relation $\succcurlyeq$ on X is said to be a *preorder*, or a preference relation, on X if it is reflexive and transitive. The asymmetric part of this relation is denoted by $\succ$, that is, we have $A \succ B$ if and only if $A \succcurlyeq B$ but not $B \succcurlyeq A$. (The symmetric part of $\succcurlyeq$, denoted by $\sim$, is thus $\succcurlyeq \setminus \succ$.) A preorder $\succcurlyeq$ on X is said to be *monotonic* if $A \succcurlyeq B$ holds for every A and B in X with $A \supseteq B$.

We denote the non-comparability part of a preorder $\succcurlyeq$ by $\bowtie$. That is, $\bowtie$ is the binary relation on X such that $A \bowtie B$ if and only if neither $A \succcurlyeq B$ nor $B \succcurlyeq A$. If $\bowtie = \varnothing$, then $\succcurlyeq$ is said to be *complete.* In turn, given a preference relation $\succcurlyeq$ on X, by a *proper completion* of $\succcurlyeq$, we mean a complete preference relation $\succcurlyeq^*$ such that

$$A \succcurlyeq B \text{ implies } A \succcurlyeq^* B \quad \text{and} \quad A \succ B \text{ implies } A \succ^* B$$

We say that an ordered pair $(\succcurlyeq, \succcurlyeq^*)$ is a *preference structure* on X if $\succcurlyeq$ is a preference relation on X and $\succcurlyeq^*$ is a proper completion of $\succcurlyeq$.[7] In this paper, we will consider such structures as the primitives of the environment.[8] Of course, when $\succcurlyeq = \succcurlyeq^*$, the induced preference structure can be identified with a complete preference relation, so the standard framework of menu preferences is a special case of ours.

We interpret a preference structure $(\succcurlyeq, \succcurlyeq^*)$ as follows. The first relation $\succcurlyeq$ represents the decision maker's psychological tastes. Simply put, this core relation captures among which menus the agent is completely decisive. As such, $\succcurlyeq$ is unobservable to the modeler. On the other hand, the second relation $\succcurlyeq^*$ stems from the revealed preferences of the agent through his choice behavior. As we presume that we observe *all* choices of the agent across pairwise problems, this relation is taken as complete. Furthermore, it is consistent with the core relation of the agent, that is, whenever $\succcurlyeq$ ranks menus in a particular manner, $\succcurlyeq^*$ does so in exactly the same way.

---

[7]A weaker version of this assumption without the strict part of the requirement, is present as an axiom in Gilboa et al. (2010) under the name of Consistency.

[8]We note that such preference structures have been previously studied by Danan (2003), a discussion of which is provided in Section 3.

1.2 Basic Axioms

Throughout the paper, we will work with a preference structure $(\succsim, \succsim^*)$ on X which is rational in the standard sense. That is, we will impose the following two axioms on $(\succsim, \succsim^*)$:

Axiom 1 (Independence): For every $A, B, C \in X$ and $0 < \lambda < 1$, we have

$$A \succsim B \quad \text{if and only if} \quad \lambda A \oplus (1 - \lambda)C \succsim \lambda B \oplus (1 - \lambda)C$$

and similarly for $\succsim^*$.

Axiom 2 (Continuity): Both $\succsim$ and $\succsim^*$ are closed in $X \times X$.[9]

Axiom 1 is an adaptation of the standard Independence Axiom to a menu choice setting, which was first introduced by DLR [12] and Gul and Pesendorfer [23] and is now widely used in this literature. The logic for this axiom is the following. First, notice that if an agent satisfies the classical Independence condition, then whenever she prefers A to B, she must also prefer $\lambda A + (1 - \lambda)C$ (i.e. the lottery that gives menu A with probability λ and menu C with probability $(1 - \lambda)$) to $\lambda B + (1 - \lambda)C$. Secondly, if the agent is indifferent as to when uncertainty is resolved, then he should be indifferent between the lottery $\lambda A + (1 - \lambda)C$, which provides randomization before the agent has chosen an item from the menu, and the set mixture $\lambda A \oplus (1 - \lambda)C$, where randomization takes place ex post. Axiom 1 therefore logically follows from the assumptions that the agent satisfies the standard Independence Axiom and is indifferent as to the timing of the resolution of uncertainty.

Axiom 2, also known as Closed-Continuity, is a technical assumption, which is fairly standard in decision theory. For a complete preference relation it is equivalent, under appropriate topological conditions on the choice space, to continuity of the representing functional.[10] It

---

[9]$X \times X$ is here the product metric space with Hausdorff underlying metric on X.

[10]For complete relations, it is also equivalent to openness of the strict part $\succ$. While it is well known since Eilenberg (1947) and Debreu [10] that, for many topological spaces, the condition is sufficient for the existence of a utility representation, it is not always necessary. For example, the expected utility theory for preferences over lotteries relies on a weaker condition, Archimedean Continuity, which only restricts the Bernoulli function to being bounded over the prize space.

is important to highlight that, for an incomplete relation, Axiom 2 also implies that $\bowtie$ will be open. Thus in a certain sense incomparability, when present, will never be too "small".[11] This turns out to be essential for our result, since we extensively use the implied richness of the incomparability relation.[12] We would like to note here that, even though continuity-like assumptions are by nature unfalsifiable, and thus a discussion of their relative value has no empirical relevance, we do in fact feel that requiring openness of $\bowtie$ has descriptive appeal. It is very hard indeed to imagine that a decision maker would declare himself unable to decide between two alternatives a and b and yet be able to compare two very close substitutes of those options. For example, for a worker who is fundamentally undecided between two hardly comparable job offers, say one in the financial industry and one in the public sector, it is very hard to believe that changing minor aspects of the two contracts, such as the level of dental coverage or the number of sick leave days, would lead to a clear preference. In this sense, Axiom 2 captures the idea that incomparability is a "robust" relation.

1.3 Cautious Deferral

We now introduce our main axiom, called the *Cautious Deferral Axiom*, which connects the preference relations $\succsim$ and $\succsim^*$.[13]

Axiom 3 (Cautious Deferral): For every A and B in X,

$$A \bowtie B \quad \text{implies} \quad A \cup B \succsim^* A$$

---

[11]In particular, whenever $A \bowtie B$, there will be two open neighbourhoods $\mathcal{O}_A$ of A and $\mathcal{O}_B$ of B such that $C \bowtie D$ if C is in $\mathcal{O}_A$ and D is in $\mathcal{O}_B$. When the choice space is Euclidean, this implies that the set of options incomparable to an element is either empty or full dimensional.

[12]For an examination of the different implications of closed and open continuity in the context of incomplete relations, see Evren and Ok [18] and Evren [?].

[13]Although very different in their structure and context, several papers present similarities with the approach adopted here by considering a pair of preference relations connected through some behavioral axiom such as our Cautious Deferral rule. For instance in the Anscombe-Aumann framework, Gilboa et al. [22] connect two preference relations, one incomplete and one complete, with an axiom called Caution which captures the agent's behavioral attitude towards uncertainty. That paper, as well as other papers which adopt such an approach, are discussed in Section 3.

Note that one can always view an incomplete preference $\succcurlyeq$ as the intersection of a collection of complete preferences, each of which represents a different criterion of evaluation in the mind of the agent.[14] Thus, the decision maker's inability to compare two menus through $\succcurlyeq$ can be seen as stemming from the conflict between the various considerations that may enter his evaluation of the problem, or equivalently, between his different "selves" where the tastes of each self are represented by a complete preference relation on X. For instance, consider an agent who must decide between two restaurants A and B and evaluates each menu according to how healthy and how appetizing the available options are in each menu. If menu A contains the most healthy option (option a) and menu B contains the most appetizing option (option b), the agent might be unable to decide at which restaurant to dine. This inability to compare two menus A and B is captured in our model by setting $A \bowtie B$.

When intimately torn between two menus A and B, what rule of conduct may the decision maker adopt? If the agent is constrained to choose from the feasible set $\{A, B\}$, our model remains silent on what the final choice will be. All we know is that $A \succcurlyeq^* B$ if this choice is A, and similarly $B \succcurlyeq^* A$ if B is selected. (Recall that $\succcurlyeq^*$ stands for the *revealed* preference of the agent.) As $A \bowtie B$, we also do not have any rationale for why either of these choices might occur. (For instance, the agent may have consulted a second party who suggested $A \succcurlyeq^* B$ or he may have adopted some ad hoc choice procedure.) But now suppose that his choice set is actually $\{A, B, A \cup B\}$. In the context of the example above, our agent may have the additional option of going to a restaurant with a larger menu where the options of both restaurants A and B are available. Alternatively, one might think of $A \cup B$ as waiting until dinner time in order to pick a restaurant rather than making a reservation at either A or B. In either case, it stands to reason that the decision maker would seize this additional opportunity, which would mean $A \cup B \succcurlyeq^* A$ (and by symmetry, $A \cup B \succcurlyeq^* B$), since this leaves all options open at the later stage: by not committing to either menu, deferral is a cautious attitude for the agent. This suggests that perhaps we have $A \cup B \succcurlyeq A$ and therefore, $A \cup B \succcurlyeq^* A$, meaning that the agent intimately prefers the flexibility offered by the wider menu. Or perhaps the agent is still indecisive between A and $A \cup B$, that is,

---

[14]This is a set-theoretic fact; see Section 1.4 of Ok [35] for more details.

$A \bowtie A \cup B$.[15] Even in this case, it seems that committing today to A as opposed to $A \cup B$, that is, $A \succ^* A \cup B$, is unwarranted. For instance, suppose that $\succeq^*$ results from the advice of a second party. Then a natural recommendation of this party would be "if you are not sure what menu to commit to, choose not to commit at all and wait if this is possible". This suggests again $A \cup B \succeq^* A$, which is exactly what the Cautious Deferral Axiom says.

This discussion points to the intuitive nature of the Cautious Deferral Axiom, but it does not say anything about why a conflicted decision maker may find choice deferral desirable. In our view, this may happen for at least two reasons. On the one hand, an indecisive agent might prefer to postpone his choice if he expects his internal conflict to be resolved in the future. For instance, the agent in our example might later find himself in an indulgent mood, thus making restaurant B a clearly more attractive choice than A. In this respect, choice deferral may have an informational value to the decision maker. On the other hand, even when the agent does not expect his indecisiveness to resolve in time, choice deferral might be valued if it helps the decision maker to come to terms with a difficult decision. By leaving options open, the agent allows himself additional time for contemplation about this difficult choice problem.[16]

Finally, we wish to emphasize that the Cautious Deferral Axiom should be interpreted as having a positive, rather than a normative, content. Indeed, introspection and anecdotal evidence seem to provide support for this intuitive rule.[17] Furthermore, several empirical studies

---

[15]To illustrate this possibility, consider an agent who has two "selves". One has self-control preferences à la Gul and Pesendorfer [23] and assesses the value of a menu A as $W_1(A) = \max_{a \in A}[u(a) + v(a)] - \max_{a \in A} v(a)$, while the other evaluates A as a weighted average of the "commitment" and "temptation" utilities, that is, $W_2(A) = \lambda \max_{a \in A} u(a) + (1 - \lambda) \max_{a \in A} v(a)$, where λ is a real number in $(0, 1)$. Now assume that the agent needs to choose between $\{a\}$ and $\{a, b\}$, and that $u(a) > u(b)$ while $v(b) > v(a)$. Then, we have $\{a\} \bowtie \{a, b\}$ because $W_1(\{a\}) > W_1(\{a, b\})$ and $W_2(\{a, b\}) > W_2(\{a\})$.

[16]A dramatic illustration of this case can be found in the novel of William Styron, *Sophie's Choice*. One can think that the choice Sophie faced between saving her son from the gas chamber or saving her daughter might have been anything but a spontaneous decision.

[17]Tversky and Shafir [42] report an anecdote that was shared by Thomas Schelling. The latter had decided to buy an encyclopedia for his children but, to his discontent, he found two in the bookstore. Even though both options seemed satisfactory, the difficulty to make a choice between the two encyclopedias led Schelling to buy neither.

attest to the descriptive appeal of this axiom by documenting a link between choice deferral and the inability to compare alternatives (Tversky and Shafir [42], Dhar [14], Tykocinski and Ruffle [43]; see Section 3 for more details). At the same time, we concede that in some contexts, indecisiveness might instead result in a desire for commitment, for instance if the decision maker suffers from temptation or cognitive load. We return to this issue in Section 1.5 where we discuss the tensions between commitment and flexibility.

1.4 Main Result

The main objective of the present work is to understand the implications of the Cautious Deferral Axiom with respect to the preference for flexibility that may be exhibited by the choice behavior of a decision maker. More precisely, we wish to understand the extent to which the behavioral preference relation $\succcurlyeq^*$ may exhibit a preference for flexibility when the preference structure $(\succcurlyeq, \succcurlyeq^*)$ on X satisfies Axiom 3. It is plain that the answer depends on how incomplete the psychological preference relation $\succcurlyeq$ is. At one extreme of the spectrum is the case where this relation is complete, that is, $\bowtie = \varnothing$. In this case, Axiom 3 becomes a triviality, yielding no clue as to the structure of $\succcurlyeq^*$. At the other extreme is the case where $\succcurlyeq$ is unable to rank any two distinct menus, that is, $\bowtie = \{(A, B) : A \neq B\}$. In that case, it is readily verified that Axiom 3 implies the monotonicity of $\succcurlyeq^*$, that is, $A \succcurlyeq^* B$ for every A and $B \in X$ such that $A \supseteq B$. Therefore, while the Cautious Deferral Axiom enforces some monotonicity on $\succcurlyeq^*$, the scope of this monotonicity, and in particular, whether or not $\succcurlyeq^*$ is monotonic on its entire domain, depends on the degree of indecisiveness exhibited by the core preference relation $\succcurlyeq$.

The main result of this paper shows that, in the context of *rational* preferences structures on X, the restrictions on $\succcurlyeq^*$ imposed by the Cautious Deferral Axiom are actually much stronger than the previous discussion suggests. Indeed, for preference structures $(\succcurlyeq, \succcurlyeq^*)$ that satisfy Axioms 1-2 as well as the Cautious Deferral Axiom, it turns out that the full completeness of $\succcurlyeq$ is the only way in which $\succcurlyeq^*$ may escape from exhibiting a global preference for flexibility. Even when the psychological preference relation $\succcurlyeq$ presents only a minimal amount of incompleteness, that is, whenever $\bowtie \neq \varnothing$, it turns out that $\succcurlyeq^*$ is sure

to be monotonic on its entire domain. Under the usual rationality postulates, the Cautious Deferral Axiom thus implies preference for flexibility so long as the core preferences $\succcurlyeq$ of the individual are incomplete, and this, regardless of the extent of the incompleteness of $\succcurlyeq$.

THEOREM 1: *Let $(\succcurlyeq, \succcurlyeq^*)$ be a preference structure on X which satisfies Axioms 1-3. Then, either $\succcurlyeq$ is complete or $\succcurlyeq^*$ exhibits preference for flexibility.*

On the one hand, this theorem shows that there is a fundamental tension between non-monotonic preferences over menus, incompleteness of one's tastes and the Cautious Deferral Axiom. In particular, an agent with incomplete tastes cannot exert commitment on *any* part of the menu space while at the same time completing his core preferences in concert with the Cautious Deferral Axiom. On the other hand, this theorem appears to provide a psychological foundation for the property of preference for flexibility.

Insofar as one concedes that a rational decision maker may sometimes be indecisive and also that her revealed preferences are consistent with the Cautious Deferral Axiom, then those preferences must exhibit preference for flexibility in all contingencies. In accordance with the opening vignette of the paper, we thus find that indecisiveness (on the part of the psychological preferences $\succcurlyeq$) is the key to preference for flexibility (on the part of the revealed preference $\succcurlyeq^*$) for rational individuals who behave consistently with the Cautious Deferral Axiom.

One natural question at this point is if the core relation $\succcurlyeq$ is also forced to be monotonic (when it is incomplete) under the conditions of Theorem 1. We show next by an example that this need not be the case.

EXAMPLE: Consider the real maps W_1 and W_2 on X defined by

$$W_1(A) := \lambda \max_{a \in A} w(a) - \max_{a \in A} v(a)$$

and

$$W_2(A) := \lambda \max_{a \in A} v(a) - \max_{a \in A} w(a),$$

where w and v are distinct, continuous and affine functions on $\triangle$ and $\lambda > 1$. Then, it can

easily be seen that the relation $\succeq$ defined as

$$A \succeq B \quad \text{if and only if} \quad W_1(A) \geqslant W_1(B) \text{ and } W_2(A) \geqslant W_2(B),$$

is incomplete and satisfies Axioms 1-2. On the other hand, the preference relation $\succeq^*$ on X represented by $W_1 + W_2$ is a completion of $\succeq$ which satisfies the Cautious Deferral Axiom. Therefore, in line with our main theorem, $\succeq^*$ exhibits preference for flexibility but $\succeq$ is not monotone on X.

1.5   Self-Control versus Cautious Deferral

One way of interpreting our main theorem is as a negative result, which shows that non-monotonic preferences must be renounced if one wishes to maintain indecisiveness together with Cautious Deferral. To demonstrate this point, consider the Gul and Pesendorfer [23] model of temptation and self-control. This model takes as a primitive the revealed preference relation $\succeq^*$ on X, and besides the standard rationality assumptions, imposes the Set Betweenness Axiom. This axiom states that $A \succeq^* B$ implies $A \succeq^* A \cup B \succeq^* B$, for any two menus A and B. To understand the first part of the implication, consider an agent for whom $A \succ^* B$ because the best alternative in A is strictly preferred to the best alternative in B. Then, the axiom allows for the agent to strictly prefer committing to A today rather than to $A \cup B$, that is, $A \succ^* A \cup B$, which is rather reasonable if B contains a temptation that the agent wishes to avoid.

Even without appealing to the representation theorem of Gul and Pesendorfer [23], we can show that the Cautious Deferral Axiom goes against the agent's desire for commitment. To this end, suppose that the core preferences of the agent are represented by some incomplete preference relation $\succeq$, and $\succeq^*$ is a completion of $\succeq$. Now take any two menus A and B. If $A \succeq B$, then the motivation behind the Set Betweenness Axiom applies, so we may comfortably posit that $A \succeq A \cup B$. But suppose that $A \bowtie B$ is the case, that is, the agent is unable to decide (today) between A and B on the basis of his core preferences. Then, the Cautious Deferral Axiom tells us that the agent's fear of making a mistake by committing to either A or B would overwhelm any other consideration, prompting him to set $A \cup B \succeq^* A, B$.

At an intuitive level, this behavior seems to make perfect sense: the agent exerts self-control when he can clearly identify the elements of temptation that may be present (i.e. $A \succcurlyeq B$ implies $A \succcurlyeq A \cup B$ and hence $A \succcurlyeq^* A \cup B$), but wishes not to commit when comparing two menus between which he is indecisive (i.e. $A \bowtie B$ implies $A \cup B \succcurlyeq^* A$).

This intuition runs, however, to a severe difficulty, at least in the presence of the standard rationality axioms. For, take any two menus A and B such that $A \succcurlyeq^* B$. By the Set Betweenness Axiom, it must be that $A \succcurlyeq^* A \cup B$. If, however, $\succcurlyeq^*$ is also a completion of $\succcurlyeq$, and both relations satisfy all the standard axioms and the Cautious Deferral Axiom, then we know by Theorem 1 that $\succcurlyeq^*$ must be monotonic on its entire domain, so that $A \cup B \succcurlyeq^* A$. Therefore, $A \succcurlyeq^* B$ implies $A \sim^* A \cup B$, that is, our agent must be a standard decision maker who evaluates a menu by its best elements. Put differently, self-control motives are completely annihilated in the presence of the Cautious Deferral Axiom.

One could reasonably argue that the flip side of Cautious Deferral, which would require that when $A \bowtie B$ then $A \succcurlyeq^* A \cup B$, might be a more appropriate assumption in the context of temptation. Indeed, if the decision maker feels indecisive between two menus because of the respective temptations they contain, then committing to either menu would seem to be a more cautious attitude than leaving options open: by avoiding exposure to one of the temptations, the agent can increase his chances of exercising self-control.[18] We therefore coin this axiom *Cautious Avoidance.* Given the structure of our proof, it can be easily verified that a symmetric result would be obtained if Cautious Deferral were to be replaced by Cautious Avoidance, this time with the conclusion that nonempty incomparability implies $A \succcurlyeq^* B$ whenever $A \subseteq B$, a property we dub *global weak preference for commitment*:[19]

---

[18]To see this in a concrete example, consider an agent who evaluates sets by combining the value of the best normative choice and that of the strongest temptation. Suppose he faces the options: broccoli b, potato chips p and chocolate cake c. Potato chips and chocolate cake are more tempting than broccoli but the chips are more tempting as a salty craving, while the cake is more tempting as a sweet craving. On the other hand, broccoli is healthier than either of the options. If the agent is unable to determine which is stronger between the salty and the sweet craving, it will be the case that $\{b, p\} \bowtie \{b, c\}$. On the other hand, it makes sense that he would always choose either $\{b, p\}$ or $\{b, c\}$ over $\{b, c, p\}$, since the latter set guarantees the worst temptation. Such agent would clearly violate Cautious Deferral (we thank John Stovall for providing this example).

[19]Preference for commitment, as defined by Gul and Pesendorfer [23], requires that there exists at least a

Axiom 4 (Cautious Avoidance): For every A and B in X,

$$A \bowtie B \quad \text{implies} \quad A \succcurlyeq^* A \cup B$$

THEOREM 2: *Let* $(\succcurlyeq, \succcurlyeq^*)$ *be a preference structure on* X *which satisfies Axioms 1,2 and 4. Then, either* $\succcurlyeq$ *is complete or* $\succcurlyeq^*$ *exhibits global weak preference for commitment.*

We can now consider what additional restrictions would appear if Cautious Avoidance were to be combined with Set Betweenness. Take any two menus A and B such that $A \succ^* B$. By the Set Betweenness Axiom, it must be that $A \succcurlyeq^* A \cup B \succcurlyeq^* B$. On the other hand, under our rationality assumptions, Theorem 2 implies that $B \succcurlyeq^* A \cup B$. As a result, $A \succ^* B$ implies that $A \succ^* A \cup B \sim^* B$. In the language of Gul and Pesendorfer (2001), B is therefore an overwhelming temptation, meaning that the agent expects to exert no self-control when facing a temptation in B. Since this can be done for any pair (A, B), the agent faces overwhelming temptation at every set.[20]

2 Discussion

Our framework is parsimonious in that it solely relies on three axioms, two of which are generally considered as the basic assumptions for rational preferences. Yet, we obtain a very strong result regarding the restrictions imposed by these axioms on our preference structure. In this section, we provide some intuition for the proof of our main theorem and discuss the role played by our rationality assumptions. We conclude by discussing the observable implications of our axiomatic framework.

---

pair A, B such that $A \subset B$ and $A \succ B$. Hence the qualifiers: global because in our case it applies to all sets, weak because a strict preference is not required.

[20]It can be verified that the preferences of such a decision maker can be represented by the function $V : X \to \mathbb{R}$ given by $V(A) = -\max_{a \in A} v(a)$, where v is an expected utility function.

2.1   Proof Sketch

The proof relies on the observation that, if the completion $\succeq^*$ establishes that $A \succ^* B$ for some sets A and B such that $A \subset B$, then the Cautious Deferral Axiom must force $A \succ B$. In other words, the underlying psychological relation $\succeq$ must be able to compare every pair (A, B) for which the completion violates monotonicity. The proof builds upon this observation in order to show that the mere existence of a single pair (A, B) at which $\succeq^*$ violates monotonicity is sufficient to guarantee that $\succeq$ is complete.[21]

While we leave the details of the proof to the appendix, we note that the idea is based on an adaptation of the classical constructive proof of the existence of a Von Neumann-Morgenstern utility. Starting with sets A, B and C such that $A \subset B \subset C$ and $A \succ^* C \succ^* B$, one can show that the Cautious Deferral Axiom entails that C must be ranked by $\succeq$ between A and B, and it can be assigned a unique utility value. After this first step, it can be shown that the whole space of lotteries $\triangle$ can be uniquely ranked and assigned a utility value. This step follows from the affinity implied by the Independence Axiom and because Continuity implies that $A \succ^* \alpha C \oplus (1 - \alpha)\triangle \succ^* B$ for large enough α. A third repetition of the argument, this time applied to a generic set D, concludes the proof: $\succeq$ must be able to rank any arbitrary menu (i.e. it must be complete), if $\succeq^*$ is allowed to violate monotonicity even at a single instance.

We wish to emphasize that this simple proof makes almost no use of existing representation theorems for menu preferences, such as the ones established in DLR. All we need are results regarding the existence of continuous and affine representations for $\succeq$ and $\succeq^*$. Beyond these auxiliary results, our argument essentially exploits the topology of the space to show how Continuity and Independence can work together to make the Cautious Deferral Axiom bite under any circumstance.

---

[21]Obviously, thanks to Independence of $\succeq^*$, a violation of monotonicity at a pair (A, B) implies such violations at many other points, a fact we return to at the end of this section and in one of the counterexamples we consider in a supplemental appendix. Also, notice that the above statement, that a violation of monotonicity at a single pair (A, B) ensures completeness of $\succeq$, is the contrapositive of the statement of our theorem.

2.2 Necessity of the Axioms

The Independence Axiom is a key building block of the vast majority of menu preference models. As previously mentioned, this axiom relies on the assumptions that the decision maker a) satisfies the standard Independence Axiom; b) is indifferent as to the timing of the resolution of uncertainty. This last assumption places strong restrictions on the way the decision maker evaluates a given menu. As shown by DLR, any complete preference satisfying Independence and Continuity can be seen "as if" it evaluates each menu in terms of some aggregation of the indirect utilities the set can provide for a certain collection of (Von Neumann-Morgenstern) utilities over lotteries.[22] Now consider an agent who is unable to compare two menus, say, $\{a\}$ and $\{a, b\}$. This must be because of a conflict between his multiple psychological rationales. For example, it might be that one of these, as in Sarver [40], induces him to prefer the singleton $\{a\}$, if he suffers from regret for not picking b from $\{a, b\}$. At the same time, another one of these could suggest picking the larger menu in order to hedge against the possibility of a future change in preferences, as in the original model of Kreps [27]. In any case, we know that because of Independence, the disagreement must boil down to two opposite views on the indirect utility of b: some rationales will value it positively, others will value it negatively.

At this point, one can see how the Cautious Deferral Axiom might be leveraged. For the completion to state that $\{a, b\} \succcurlyeq^* \{a\}$, it must necessarily be giving more weight to those rationales that value b positively than to the rest. If one can then show that this exercise can be repeated for every option, or at least for every relevant indirect utility, we are necessarily led to the result of overall monotonicity. The latter part is not obvious, which is what makes the result non-trivial.[23] This rationalization suggests that if we wish to describe agents for which the cautious behavior suggested by Axiom 3 coexists with some indecisiveness and some preference for commitment, we must consider models in which the value of a menu is

---

[22]In particular, there is a set $\mathcal{U}$ of Bernoulli utilities over prizes and an aggregator V that is affine in $\oplus$ and that assigns to any set A the value $V(\{\max_{a \in A} u \cdot a\}_{u \in \mathcal{U}})$.

[23]There are additional complexities that must be considered. For example one could value positively b when added to $\{a\}$, but not when added to $\{c\}$. This is possible under Independence whenever the collection of indirect utilities is infinite; see Krishna and Chatterjee [?].

not as strictly linked to the indirect utiliy of the available options.

In an online appendix available on the authors' website, we also show that the affinity implied by the Independence Axiom plays an essential role for Theorem 1, because it allows local properties to spread over the entire space. In particular, we consider there the natural relaxation of Independence to Indifference to Randomization, which requires that $A \sim co(A)$ for all $A \in X$. We show that this relaxation, which breaks the linearity behind the Independence Axiom while preserving the convexity, allows the Cautious Deferral Axiom to coexist with the non-monotonicity of $\succsim^*$. As such, our theorem is another illustration of the technical power of the Independence Axiom in a menu choice setting.[24]

From the above discussion, one might be tempted to conclude that the result we obtain is essentially driven by the Independence Axiom. Yet, we show that our theorem is tight and requires the other ingredients of the model. First, we show in the online appendix that closed continuity may not be relaxed or even replaced by the most common alternative for incomplete relations, open continuity. In another counterexample, we show that our requirement that the completion of $\succsim$ be proper is not innocuous, and it cannot be weakened to only requiring that $A \succsim B$ implies $A \succsim^* B$.

2.3 Observability

The testable implications of our model, summarized by the Independence and Cautious Deferral axioms, crucially rely on the observability of the underlying psychological preference $\succsim$. Yet, if an indecisive agent is forced to choose, and if in choosing he always follows a given completion $\succsim^*$, it seems that there is no way in which we might recover $\succsim$ from his choices. We wish to make two remarks about this matter.

First, the fact that the presence of incompleteness poses a challenge for revealed prefer-

---

[24]Other papers discuss the restrictive character of the Independence Axiom in the context of menu choice and propose relaxations. For example, Epstein et al. [17] propose that a decision maker who has a coarse perception of future contingencies will only satisfy a weaker assumption, which is an adaptation of the Uncertainty Aversion Axiom to the menu choice setting. In the temptation literature, Noor and Takeoka [32] and [33] propose models à la Gul and Pesendorfer in which the cost of resisting temptation is either convex or menu dependent, which necessitates a violation of the affinity assumption.

ence data is not a new concern and has been addressed by several papers in the literature. (See Eliaz and Ok [16] and Danan and Ziegelmeyer [11]). Any discussion of incompleteness requires by its very nature to refer to objects other than the final choices made by the decision maker, and as such, we need to extend our notion of observability beyond pure choice data. This caveat, acknowledged also by Gilboa et al. [22] in an earlier version of their paper, is thus not specific to our work. Yet, in concert with Gilboa et al. [22], we believe that this caveat does not diminish the value our contribution, which is to make a conceptual point rather than to obtain a new representation. Our main point is here that *if* one agrees that a decision maker may have preferences that are innately incomplete, then one must be confronted with an unexpected tension between commitment and flexibility. By relying on this *if* exercise, our approach reasonates well with behavioral economic theories, which in general stipulate that a certain behavioral aspect is connected to an unobservable psychological phenomenon and evaluate the consequences of this connection.

Second, although our main axiom cannot be tested directly within our model, we believe that it can be tested, at least indirectly, by extending the dataset, an approach pursued for instance by Danan and Ziegelmeyer [11]. To see how such an indirect test could be performed, consider a student who is asked to make a choice between two universities, say $\{NYU\}$ and $\{Northwestern\}$. If the student does not reveal a strict preference between these menus, then consider offering him the choice between $\{NYU\}$ and $\{Northwestern\} \pm \$10$. If he was originally indecisive, then adding $10 to one of the options should not change his state of mind, while he would now have a clear preference if he was actually indifferent.[25] We contend that a student who still does not reveal a strict preference after the second question would naturally prefer to have more time to think and opt for $\{NYU, Northwestern\}$ for a small fee, which is something we could test.

---

[25]Obviously, if he had originally a clear preference for one option, one would expect his preference to remain unchanged after adding or subtracting $10, by continuity.

3 Related literature

There are a number of recent papers that investigate the completion and/or extension rules for incomplete preference relations in a variety of contexts. In particular, Gilboa et al. [22] and Kopylov [25] investigate such completions in the Anscombe-Aumann framework. The former builds a bridge between the classical models of Bewley [5] and Gilboa and Schmeidler [?] while the latter ties Bewley [5] to a model of ambiguity aversion known as the ϵ-contamination model. Similarly, Cerreia-Vioglio et al. [8] use a completion rule for a preference relation over risky prospects in order to obtain a representation for a particular class of complete risk preferences. Finally, Lehrer and Teper [29] consider a binary relation that is complete on a convex subset of the space of Anscombe-Aumann acts and propose an extension rule that generates Bewley [5] preferences as well as a completion rule that generates the MaxMin preferences. It is worth noting that most of these papers adopt a notion of completion which is weaker than the one we adopt in this paper, for they do not require the strict part of the incomplete relation to be preserved by its completion. This weaker notion of a completion is not suitable to our purposes because we interpret the underlying incomplete preference relation $\succsim$ as representing the basic psychological tastes of the decision maker. In this respect, the revealed preferences of the agent, which represent a "completion" of $\succsim$, must be perfectly in line with his psychological tastes.

More closely related to the present work is that of Danan [10] who also works with a pair of preferences $(\succsim, \succsim^*)$ defined over a generic set of menus (which may or may not consist of lotteries). The interpretation of this structure is the same as ours, that is, $\succsim$ stands for the psychological preferences of the agent while $\succsim^*$ refers to his behavioral (revealed) preferences. The main objective of that paper is however different from ours, namely, finding conditions under which $\succsim$ can be identified through the observation of $\succsim^*$. One of Danan's main identifying conditions, which may appear as being quite similar to our Cautious Deferral axiom, can be expressed as: $A \bowtie B$ if and only if $A \cup B \succ^* A$ and $A \cup B \succ^* B$. This condition is however much stronger than the Cautious Deferral Axiom for the following two reasons. First, preference for flexibility is required to be strict while our axiom also allows for indifference. For instance, consider a decision maker who cannot compare any

two distinct menus (perhaps due to his lack of information and/or interest) and hence is indifferent between any two menus. The preference structure of this agent is $(\succcurlyeq, X \times X)$, where $\bowtie = \{(A, B) : A \neq B\}$; it obviously satisfies the Cautious Deferral Axiom but fails Danan's condition. Secondly, Danan *identifies* incomplete preferences with a strict desire for flexibility by requiring the implication to go in both directions. This identification strategy is used in Danan and Ziegelmeyer [11] in order to test the completeness axiom in a setting involving choices among menus of lotteries. In contrast, we do not impose the "if" part of Danan's axiom which is fairly restrictive. For instance, consider an agent with complete DLR preferences such that $A \cup B \succ^* A$ for at least two sets A and B. While these preferences are standard in the literature on menu preferences, they do not satisfy Danan's axiom. However, when $\succcurlyeq$ is complete, the Cautious Deferral Axiom is trivially satisfied.

In a different setting, Arliegi and Nieto [2] also adopt a connecting condition which presents similarities with the Cautious Deferral Axiom. Their condition, called Restricted Monotonicity, connects an asymmetric binary relation P defined on a finite set X of alternatives to a binary relation $\succcurlyeq$ defined on the set of all menus from X. This condition requires that for any two alternatives x and y, xPy implies $\{x, y\} \sim \{x\}$ and not xPy implies $\{x, y\} \succ \{x\}$. If we identify xPy with $\{x\} \succ \{y\}$, the second part of their axiom can be seen as a stronger version of Axiom 4b when restricted to singleton sets (for, as in Danan [10], preference for flexibility in the Restricted Monotonicity Axiom is required to be strict). Furthermore, we remain silent on what happens when two sets are comparable (the first part of that axiom); in particular, we do not impose that $A \succcurlyeq B$ implies $A \sim^* A \cup B$, the natural analog in our framework.

Finally, we note that several papers in the literature explore the connections between the internal conflict of a person and her tendency to defer choice. On the theoretical side, Gerasimou [21] considers a model in which choice deferral is driven by incomparability. This idea is captured by a choice correspondence which can be empty-valued. Closely related is the work of Buturak and Evren [6] who study the choice deferral phenomenon in a risky setting; they obtain a representation in which the decision-maker defers his choice from a set if and only if the option value of deferring (as an expected utility over subjective states)

is larger than the current utility from the best available alternative in that set.[26]

On the empirical side, Tversky and Shafir [42] show in one study that subjects are more likely to wait when the objects of choice are difficult to compare than when one of the alternatives appears to be clearly superior on all attributes. In a related study, they find that subjects' propensity to wait in order to learn more about the various options available tends to increase if their choice is between two equally attractive options instead of just one option. In another study, Dhar [14] finds that choice deferral tends to increase when the differences in attributes among the available alternatives are small, as this makes alternatives harder to distinguish. Finally, Tykocinski and Ruffle [43] show that subjects may choose to postpone their choice even when they do not expect to receive additional information relevant to their choice, for instance if they need additional time for contemplation. Furthermore, they find a higher propensity to wait among subjects who express low confidence in committing themselves to a specific choice.

Acknowledgments

A   Mathematical Appendix

Proof of Theorem 1. Part 1: Preliminaries Here we collect some results that will be useful for the proof of the main theorem. To begin with, we notice that, as established in DLR, Lemma 1, for any preference $\succcurlyeq$ satisfying Axioms 1 and 2, $A \sim co(A)$ for all $A \in X$, where $co(A)$ is the convex hull of A.[27] Since the space $\hat{X}$ of closed and convex subsets of Δ is a mixture space under $\oplus$, it follows from Herstein and Milnor [24] that $\succcurlyeq^*$ can be represented by a real valued function V that is continuous and affine in $\oplus$, by which we mean

---

[26]While the choice correspondence in Buturak and Evren [6] takes a specific value when deferral is chosen, the standard rationality axioms are only imposed in situations where the decision maker does not defer. In this respect, their approach is equivalent to the empty-set approach adopted by Gerasimou [21].

[27]This property is often referred to as *Indifference to Randomization*, because $co(A)$ can be seen as the set of lotteries that can be obtained from A when we allow the agent to choose using a random strategy.

that:

1) $A \succcurlyeq^* B$ if and only if $V(A) \geq V(B)$.

2) $V(A^n) \to V(A)$ if A^n converges to A in the Hausdorff metric.

3) $V(\alpha A \oplus (1-\alpha)B) = \alpha V(A) + (1-\alpha)V(B)$ for all α in $(0,1)$.

Secondly, Theorem 1 in Galaabaatar [19] ensures that there is an affine multi-utility representation $\mathcal{W}$ for $\succcurlyeq$, where an affine multi-utility representation is a collection $\mathcal{W}$ of continuous and affine (in $\oplus$) real valued functions W on X, such that

$$A \succcurlyeq B \text{ if and only if } W(A) \geq W(B) \text{ for all } W \in \mathcal{W}$$

The following claim is the last preliminary result we need before we can provide a proof of Theorem 1:

Claim A.1 *Let $V : X \to \mathbb{R}$ be affine and continuous, and assume there are sets $A_1, A_2, B_1, B_2 \in X$ such that $A_i \subset B_i$ for $i \in \{1, 2\}$, $V(A_1) > V(B_1)$, and $V(A_2) < V(B_2)$. Then we can find sets $A, B, C \in X$ such that*

$$A \subset B \subset C \text{ and } V(A) > V(C) > V(B)$$

Proof: Let $V(A_1) - V(B_1) = \epsilon$. By affinity of V, we can find an $\alpha \in (0,1)$ such that $V(\alpha B_2 \oplus (1-\alpha)A_2) - V(A_2) < \epsilon$. Let $B_2' = \alpha B_2 \oplus (1-\alpha)A_2$, and notice that $A_2 \subset B_2'$. Now let the three sets A,B,C be

$$A = \frac{1}{2}A_1 \oplus \frac{1}{2}A_2, \ \ B = \frac{1}{2}B_1 \oplus \frac{1}{2}A_2, \ \ \text{and} \ \ C = \frac{1}{2}B_1 \oplus \frac{1}{2}B_2'$$

As required, $A \subset B \subset C$. Moreover $V(C) - V(B) = \frac{1}{2}(V(B_2') - V(A_2)) > 0$ and $V(A) - V(C) = \frac{1}{2}\epsilon - \frac{1}{2}(V(B_2') - V(A_2)) > 0$ so that $V(A) > V(C) > V(B)$. $\blacksquare$

Part 2: Proof of Theorem 1 To prove Theorem 1, it is sufficient to show that given Axioms 1-3, if $\succcurlyeq$ is incomplete then $\succcurlyeq^*$ must be monotone in set inclusion. Let V and $\mathcal{W}$ be the affine representations of $\succcurlyeq^*$ and $\succcurlyeq$ we just introduced, and assume, by contradiction, that $A \succ^* B$ for some $A \subset B$. There are two cases we need to consider:

Case 1: $A \succcurlyeq^* B$ **for all** A, B **such that** $A \subseteq B$. In this case $\succcurlyeq^*$ is always monotonically decreasing w.r.t. set inclusion, thus $V(A) > V(A \cup B)$ for all $A, B \in X$ such that $V(A) > V(B)$. By assumption, there are two incomparable sets A_1, B_1. By Cautious Deferral then it must be that $V(A_1) = V(B_1)$. But since $\succ^*$ is nonempty, we can find A_2, B_2 such that $V(A_2) > V(B_2)$. It then follows, from affinity of V and continuity of $\succcurlyeq$ that we can find $\alpha \in (0,1)$ big enough such that the sets $A_0 = \alpha A_1 \oplus (1-\alpha)A_2$ and $B_0 = \alpha B_1 \oplus (1-\alpha)B_2$ will satisfy $V(A_0) > V(B_0)$ and $A_0 \bowtie B_0$, leading to a contradiction.

23

Case 2: $B \succ^* A$ **for some** $A \subset B$. If there are no $A' \subset B'$ such that $A' \succ^* B'$ we are done. Otherwise, the conditions of Claim A.1 apply, thus we can find sets $A \subset B \subset C$ such that $A \succ^* C \succ^* B$. Since $A \succ^* B$ there are neighborhoods $\mathcal{N}'_A$ of A and $\mathcal{N}'_B$ of B such that $\hat{A} \succ^* \hat{B}$ for each pair $(\hat{A}, \hat{B}) \in \mathcal{N}'_A \times \mathcal{N}'_B$. Since $\cup$ is a continuous binary operation on X, we can find neighborhoods $\mathcal{N}_A \subseteq \mathcal{N}'_A$ of A and $\mathcal{N}_B \subseteq \mathcal{N}'_B$ of B such that $\hat{A} \cup \hat{B} \in \mathcal{N}'_B$ for each pair $(\hat{A}, \hat{B}) \in \mathcal{N}_A \times \mathcal{N}_B$. Notice that for each such pair $\hat{A} \succ^* \hat{A} \cup \hat{B}$. Since $\succcurlyeq^*$ is a proper completion of $\succcurlyeq$, Cautious Deferral implies that $\hat{A} \succ \hat{B}$ for all $(\hat{A}, \hat{B}) \in \mathcal{N}_A \times \mathcal{N}_B$. As a consequence $W(A) > W(B)$ for every non constant W in the affine multi-utility representation of $\succcurlyeq$, which allows us to set $W(A) = 1$ and $W(B) = 0$ for each non constant $W \in \mathcal{W}$.

Since $A \succ^* C \succ^* B$, there is an α' such that $\alpha'A \oplus (1 - \alpha')B \sim^* C$ and $\alpha A \oplus (1 - \alpha)B \succ^* C$ for all $\alpha \in (\alpha', 1)$. Since $\alpha A \oplus (1 - \alpha)B \subset C$ for all $\alpha \in (0, 1)$, Cautious Deferral implies $\alpha A \oplus (1 - \alpha)B \succ C$ for all $\alpha \in (\alpha', 1)$. By continuity $\alpha' A \oplus (1 - \alpha')B \sim C$, which allows us to establish that $W(C) = \alpha'$ for all non constant $W \in \mathcal{W}$.

By continuity of $\succcurlyeq^*$, there is an α' such that $A \succ^* \alpha'C \oplus (1 - \alpha')\triangle \succ^* B$. Once again, we can find an α'' such that $\alpha''A \oplus (1 - \alpha'')B \sim^* \alpha'C \oplus (1 - \alpha')\triangle$ and $\alpha A \oplus (1 - \alpha)B \succ^* \alpha'C \oplus (1 - \alpha')\triangle$ for all $\alpha \in (\alpha'', 1)$. Since $\alpha A \oplus (1 - \alpha)B \subseteq \alpha'C \oplus (1 - \alpha')\triangle$ an argument similar to the previous leads us to conclude that $\alpha''A \oplus (1 - \alpha'')B \sim \alpha'C \oplus (1 - \alpha')\triangle$, so that $W(\triangle) = \frac{\alpha'' - \alpha'W(C)}{1 - \alpha'}$ for all non constant $W \in \mathcal{W}$.

Finally, let D be a generic set in X. We can find an α' such that $A \succ^* \alpha'C \oplus (1 - \alpha')\triangle \succ^* \alpha'B \oplus (1 - \alpha')D$. For some α'' then $\alpha''A \oplus (1 - \alpha'')(\alpha'B \oplus (1 - \alpha')D) \sim^* \alpha'C \oplus (1 - \alpha')\triangle$. Once again we can derive from this that $\alpha''A \oplus (1 - \alpha'')(\alpha'B \oplus (1 - \alpha')D) \sim \alpha'C \oplus (1 - \alpha')\triangle$, which means all the W's assign the same value to D. Since this was done for a generic set D, the implication is that $\succcurlyeq$ is complete, a contradiction to our initial assumption.∎

References

[1] **Ahn, D. S., and T. Sarver** (2013): "Preference for Flexibility and Random Choice," *Econometrica*, 81, 341-361.

[2] **Arlegi R., and J. Nieto** (2001): "Ranking opportunity sets: An approach based on the preference for flexibility," *Social Choice and Welfare*, 18, 23-36.

[3] **Aumann, R. J.** (1962): "Utility Theory Without the Completeness Axiom," *Econometrica*, 30, 445–462.

[4] **Baucells, M., and L. S. Shapley** (2008): "Multiperson Utility," *Games and Economic Behavior*, 62, 329-347.

[5] **Bewley, T**. (1986): "Knightian Uncertainty Theory: Part I," Cowles Foundation Discussion Paper No. 807.

[6] **Buturak G., and O. Evren** (2010): "Rational Choice Deferral," Working Paper.

[7] **Cerreia-Vioglio, S.** (2012): "Objective Rationality and Uncertainty Averse Preferences", Working Paper, Department of Decision Sciences and IGIER, University of Bocconi.

[8] **Cerreia-Vioglio, S., D. Dillenberger, and P. Ortoleva** (2013): "Cautious Expected Utility and the Certainty Effect," Working Paper.

[9] **Chatterjee, K., and R. V. Krishna** (2011): "On Preferences with Infinitely many Subjective States," *Economic Theory*, 46, 85-98.

[10] **Danan, E.** (2003): "A behavioral model of individual welfare," Working Paper, University of Paris 1.

[11] **Danan, E., and A. Ziegelmeyer** (2006): "Are Preferences Complete? An Experimental Measurement of Indecisiveness Under Risk," Working Paper, Max Planck Institute of Economics.

[12] **Dekel, E., B. Lipman, and A. Rustichini** (2001): "Representing Preferences with a Unique Subjective State Space," *Econometrica*, 69, 891-934.

[13] **Dekel, E., B. Lipman, A. Rustichini, and T. Sarver** (2007): "Representing Preferences with a Unique Subjective State Space: A Corrigendum," *Econometrica*, 75, 591-600.

[14] **Dhar, R.** (1997): "Consumer preference for a no-choice option," *Journal of Consumer Research*, 24, 215-231.

[15] **Dubra, J., F. Maccheroni, and E. A. Ok** (2004): "Expected Utility Theory Without the Completeness Axiom," *Journal of Economic Theory*, 115, 118–133.

[16] **Eliaz, K., and E. A. Ok** (2006): "Indifference or indecisiveness? Choice-theoretic foundations of incomplete preferences," *Games and Economic Behavior*, 56, 61-86.

[17] **Epstein, L. G., M. Marinacci and K. Seo** (2007): "Coarse Contingencies and Ambiguity," *Theoretical Economics*, 2, 355-394.

[18] **Evren, O., and E. A. Ok** (2011): "On the Multi-Utility Representation of Preference Relations," *Journal of Mathematical Economics*, 47, 554-563.

[19] **Galaabaatar, T.** (2011) : "Menu Choice Without Completeness," Working Paper.

[20] **Galaabaatar, T., and E. Karni** (2013): "Subjective Expected Utility with Incomplete Preferences," *Econometrica*, 81, 255-284.

[21] **Gerasimou, G.** (2012): "Incomplete Preferences and Rational Choice Avoidance," Working Paper.

[22] **Gilboa, I., F. Maccheroni, M. Marinacci, and D. Schmeidler** (2010): "Objective and Subjective Rationality in a Multi-Prior Model," *Econometrica*, 78, 755-770.

[23] **Gul, F., and W. Pesendorfer** (2001): "Temptation and Self-Control," *Econometrica*, 69, 1403-1435.

[24] **Herstein, I. N., and J. Milnor** (1953): "An Axiomatic Approach to Measurable Utility,"*Econometrica*, 21, 291-297

[25] **Kopylov, I.** (2009a): "Choice Deferral and Ambiguity Aversion," *Theoretical Economics*, 4, 199–225.

[26] **Kopylov, I.** (2009b): "Finite Additive Utility Representations for Preferences over Menus," *Journal of Economic Theory*, 144, 354-374.

[27] **Kreps,D. M.,** (1979) :"A Representation Theorem for "Preference for Flexibility","*Econometrica*, 4, 565-578.

[28] **Krishna, R. V. and P. Sadowski** (2012): "Dynamic Preference for Flexibility," Working Paper, Economic Research Initiatives at Duke.

[29] **Lehrer, E., and R. Teper** (2013): "Extension Rules or What would the Sage do?," Working Paper.

[30] **Nehring, K.** (1999): "Preference for Flexibility in a Savage Framework," *Econometrica*, 67, 101-119.

[31] **Nehring, K.** (2009): "Imprecise probabilistic beliefs as a context for decision-making under ambiguity," *Journal of Economic Theory*, 144, 1054-1091.

[32] **Noor, J. and N. Takeoka** (2010a): "Uphill Self-Control," *Theoretical Economics*, 5, 127-158.

[33] **Noor, J. and N. Takeoka** (2010b): "Menu-Dependent Self-Control," Working Paper.

[34] **Ok, E. A.** (2002): "Utility Representation of an Incomplete Preference Relation," *Journal of Economic Theory*, 104, 429-449.

[35] **Ok, E. A.** (2007):"*Real Analysis with Economic Applications*", Princeton: Princeton University Press.

[36] **Ok, E. A., P. Ortoleva, and G. Riella** (2012): "Incomplete Preferences Under Uncertainty: Indecisiveness in Beliefs versus Tastes," *Econometrica*, 80, 1791-1808.

[37] **Peleg, B.** (1970): "Utility Functions for Partially Ordered Topological Spaces," *Econometrica,* 38, 93-96.

[38] **Riella, G.** (2013): "Preference for Flexibility and Dynamic Consistency," *Journal of Economic Theory*, forthcoming.

[39] **Riella, G.** (2015): "On the Representation of Incomplete Preferences under Uncertainty with Indecisiveness in Tastes and Beliefs," *Economic Theory*, 58(3), 571-600.

[40] **Sarver, T.** (2008): "Anticipating Regret: Why Fewer Options May be Better," *Econometrica*, 76, 263-305.

[41] **Stovall, J. E.** (2010): "Multiple Temptations", *Econometrica,* 78, 349-376.

[42] **Tversky, A., and E. Shafir** (1992): "Choice Under Conflict: The Dynamics of Deferred Decision," *Psychological Science*, 3, 358-361.

[43] **Tykocinski, O. E., and B. J. Ruffle** (2003): "Reasonable reasons for waiting," *Journal of Behavioral Decision Making*, 16, 147-157.