Group-Shift and the Consensus Effect*

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Abstract

Individuals often tend to conform to the choices of others in group decisions, compared to choices made in isolation, giving rise to phenomena such as group polarization and the bandwagon effect. We show that this behavior, which we term the consensus effect, is equivalent to a well-known violation of expected utility, namely strict quasi-convexity of preferences. In contrast to the equilibrium outcome when individuals are expected utility maximizers, quasi-convexity of preferences imply that group decisions may fail to properly aggregate preferences and strictly Pareto-dominated equilibria may arise. Moreover, these problems become more severe as the size of the group grows.

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1 Introduction

Group decision-making is ubiquitous in social, economic, and political life. Empirical evidence suggests that individuals tend to make different choices depending on whether the outcome of interest is a result of their choice alone or also the choice of others in a group. In particular, the existing evidence largely supports the idea that these *choice shifts in groups*, which are prominent in a variety of contexts across fields, are predicted by the preference of the majority of individuals. For example, political scientists often discuss the bandwagon effect, where voters are more likely to vote for candidates who they think will win, i.e., who they believe others will vote for.[1] Another example, from the psychology and sociology literature, is the robust finding that individuals, when voting in a group, will take riskier or safer decisions vis-à-vis those taken by the individuals separately.[2] In the legal realm, jurors and judges tend to be affected by the preferences of other members of the jury or the court.[3] As an influential early article in sociology by Granovetter (1978) summarized it, "collective outcomes can seem paradoxical — that is intuitively inconsistent with the intentions of the individuals who generate them."

Models of group decisions typically analyze either private-value or common-value settings. Because, as will be explained below, with expected utility preferences in a private-value setting we should not observe choice shifts, much of the literature exploring choice shifts has focused on the common-value setting. In this context, group decisions aggregate private information regarding the relative value of possible outcomes.[4] In contrast, in this paper we maintain a private-value setting, but relax the assumption of expected utility.

To see why a violation of expected utility may generate choice shifts in groups, note that an individual choice in a group decision matters only when that individual is pivotal, that is, when his vote actually changes the outcome. However, from an ex-ante perspective, when choosing for which option to vote, an individual does not know whether or not he will be pivotal. Thus, his choice is not a choice between receiving Option 1 or Option 2 for sure, but rather between *lotteries* defined over these two options — where if the individual turned out to be pivotal his selected option will be implemented, and otherwise the probability of each alternative to win depends on the probability that the group chooses it conditional on him not being pivotal. Violations

[1] Goidel and Shields (1994) found that within the United States, independents tend to vote more for a Republican candidate if that candidate is expected to win. Similarly, they found that weak Republican supporters are more likely to vote for a Democrat if that candidate is expected to win. Niemi and Bartels (1984) and Bartels (1988) discuss further evidence for this phenomenon.

[2] Stoner (1961, 1968), Nordhoy (1962) and Pruitt (1971) .

[3] Schkade, Sunstein and Kahneman (2000), Sunstein, Hastie, Payne, Schkade and Viscusi (2002), and Sunstein (2005).

[4] This literature, typified by Feddersen and Pesendorfer (1997), focuses on the ability of group decisions to aggregate private information rather than preferences. In Section 4.2 we contrast our findings with theirs as well as the larger literature on information aggregation in groups.

of the independence axiom of expected utility imply that an individual may prefer Option 1 to Option 2 in isolation, yet prefer the lottery induced in the group context by choosing Option 2 over the one induced by choosing Option 1, thus accounting for the aforementioned choice shift.

In Section 2 we formally link violations of expected utility with the phenomenon of choice shifts in groups. In doing so, we provide a relationship between two types of non-standard behavior, one observed at the individual level and one at the group level. Our first result states that individuals have preferences that are strictly *quasi-convex* in probabilities if and only if they will systematically exhibit a *consensus effect* — an individual who is indifferent between two options when choosing in isolation will actually strictly prefer to vote for the option that is sufficiently likely to be chosen by the group. As discussed, the consensus effect captures the stylized fact that in group contexts individuals want to exhibit preferences that match those of the group as a whole. Quasi-convexity, on the other hand, is a well established preference pattern in decision making under risk, according to which individuals are averse toward randomization between equally good lotteries.[5] Popular models of preferences over lotteries which can exhibit quasi-convexity include rank-dependent utility (Quiggin, 1982, hereafter RDU), quadratic utility (Chew, Epstein, and Segal, 1991), and Kőszegi and Rabin's (2007) model of reference-dependence. Moreover, as observed by Machina (1984), quasi-convexity occurs if, before the lottery is resolved, the individual is allowed to take an action that determines his final utility. As long as the optimal decision is affected by a change in the probabilities, the induced maximum expected utility will be convex in the probabilities, meaning that even if the underlying preferences are expected utility, induced preferences over the 'optimal' lotteries will be quasi-convex.

In a seminal paper discussing choice shifts in groups, Eliaz, Ray, and Razin (2006, hereafter ERR) used the same model of group decision making but focused on group choices between particular pairs of options, safe and risky, where the former is a lottery that gives a certain outcome with probability one. They confined their attention to RDU preferences and established an equivalence between specific types of choice shifts and Allais-type behavior, one of the most documented violation of expected utility at the individual level. Since choice shifts in groups are observed in experiments even when all lotteries involved are non-degenerate, our results suggest that the choice shifts discussed in ERR are actually manifestations of the consensus effect. In Section 2.3 we turn to relating our results to theirs. We extend their results for RDU preferences, but also demonstrate why the relation to Allais paradox is restricted

[5] Our proof shows that having quasi-convex preferences is equivalent to adopting a "threshold" rule towards the level of support that others will exhibit for any given option (i.e., the probability that any given option is chosen when a voter is not pivotal). When the level of support for an option exceeds the threshold, the individual will strictly prefer to choose it in a group situation. These thresholds have similar intuition to the reasons provided for similar consensus effects in other fields; for example, Granovetter (1978) specifically discusses the effect thresholds will have on aggregate versus individual behavior.

to that specific class of preferences. In particular, the consensus effect is in general consistent not only with Allais-type behavior but also with the opposite pattern of choice and, similarly, Allais-type behavior does not rule out the anti-consensus effect.

In Section 3 we analyze what type of observable equilibrium behavior results from quasi-convex preferences in conjunction with strategic considerations. We describe a majority voting game as a collection of individuals, each of whom receiving one vote to cast in favor of option p or option q (no abstentions are allowed). After observing their own preferences (which are drawn i.i.d. from some known distribution), but no other information, individuals vote. Whichever option receives the majority of the votes is implemented.

As previously mentioned, expected utility maximizers do not alter their decisions in the context of group choice. In contrast, the fact that individuals with quasi-convex preferences do not like to randomize implies that voting games take on the properties of *coordination games*. These individuals benefit from coordinating their votes with others because it reduces the amount of "randomness" in the election. They typically face a tradeoff between having the option they prefer selected and reducing the uncertainty regarding the identity of the chosen outcome.

We prove the existence of an equilibrium and describe the main properties of any possible equilibrium. We also examine how the set of equilibria depends on the distribution of types, the voting rule, and the size of the electorate. When individuals exhibit the consensus effect, group decisions may fail to aggregate preferences properly because voters are willing to coordinate on either option, rather than voting for the option they prefer in isolation. Thus, strictly Pareto-dominated equilibria may result. This willingness to coordinate implies that our model features non-uniqueness of equilibrium not due to randomization by indifferent types (as in the expected utility case) but rather because of weak-preference reversals. We further show that some individuals *necessarily* exhibits strict preference reversal when the group becomes large. Therefore — and in contrast to the standard model where a large number of voters tends to ensure that the option favored by the majority when choosing in isolation is also chosen as part of the group — a large number of voters actually impedes preference aggregation in our setting.

In Section 4 we relate our results to commonly discussed phenomena such as group polarization and the bandwagon effect, and provide foundations for the previously discussed empirical findings. For example, our model generates an *endogenous* benefit of conformity (for being in the 'winning side'), which can be behaviorally distinguished from an analogous *exogenous* benefit term that is added to the expected utility of an alternative. We also discuss how our model relates to, and can be distinguished from, alternative models in the larger literature on voting, including both common-value and private-value settings.

2 The Consensus Effect and Quasi-Convex Preferences

2.1 Model

Our aim is to link an individual's private ranking of objects with his ranking of these same objects in a group context. We assume that any individual has preferences over simple lotteries. Formally, let X be the set of outcomes (which is assumed to be a compact metric space) and denote by Δ the set of lotteries with finite support over X. We identify an individual with his complete, transitive, and continuous preference relation $\succcurlyeq$ over Δ, which is represented by some monotone function $V : \Delta \to \mathbb{R}$.[6] Throughout the paper we denote by x, y, z generic elements of X and by p, q, r generic elements of Δ.

In describing group decision problems, we extend the model suggested by ERR (see Section 2.3). There is a group of N individuals. We identify a group decision problem as perceived by any individual i with a quadruple (p, q, α, β), consisting of two lotteries $p, q \in \Delta$ and two scalars $\alpha \in (0, 1)$ and $\beta \in [0, 1]$; α is the probability that individual i's decision is pivotal in choosing between p and q, and β is the probability that the group chooses p conditional on i *not* being pivotal.[7] For now, both α and β are exogenous and fixed; accordingly, we can interpret the choice from any such quadruple as determining an individual's best-response function. In Section 3 they will be derived as part of the equilibrium analysis. Note that the alternatives we consider are lotteries. For example, in a voting context we associate a candidate with a lottery over policies.

If, in the group context, the individual votes for q, the effective lottery he faces is the convex combination of p and q, given by:[8,9]

$$q^* = \alpha q + (1-\alpha)\left(\beta p + (1-\beta) q\right) = \left[\alpha + (1-\alpha)(1-\beta)\right] q + (1-\alpha)\beta p$$

And if the individual votes for p, the effective lottery he faces is:

$$p^* = \alpha p + (1-\alpha)\left(\beta p + (1-\beta) q\right) = (1-\alpha)(1-\beta) q + [\alpha + (1-\alpha)\beta] p$$

A choice shift is thus the joint statement of $p \sim q$ but $q^* \succ p^*$ or $q^* \prec p^*$.[10] An

[6] Monotonicity means that $V(p) \geq V(q)$ whenever p first-order stochastically dominates q; the stochastic dominance order is with respect to the induced relation $\overline{\succcurlyeq}$ on X defined by $x \overline{\succcurlyeq} y \iff \delta_x \succcurlyeq \delta_y$, where for any $z \in X$, δ_z is the Dirac measure at z.

[7] We omit the index i till Section 3, where we explicitly study strategic interactions between members of the group.

[8] For p,$q \in \Delta$ and $\lambda \in (0, 1)$, $\lambda p + (1-\lambda) q$ is the lottery that yields the prize x with probability $\lambda p(x) + (1-\lambda) q(x)$.

[9] We assume the reduction of compound lotteries axiom to only analyze single-stage distributions.

[10] Both p^* and q^* are functions of the group decision-problem, but for simplicity we will suppress the notation depicting this dependence.

individual decision problem can be identified with the case $\alpha \equiv 1$.[11]

Our definition of the *consensus effect* below suggests a specific type of choice shift, whereby an individual tends to draw towards what others would do in the absence of him being pivotal. In particular, it captures the idea that if other members of the group are likely enough to choose p when the individual is not pivotal, then the individual himself will prefer to choose p as well.

Definition 1. *The individual exhibits a consensus effect at (p, q, α, β^*) if $p \sim q$ and $\beta > \beta^*$ (resp. $\beta < \beta^*$) implies that $p^* \succ q^*$ (resp., $p^* \prec q^*$). The individual exhibits the consensus effect if for all p, q, α with $p \sim q$, there exists β^* such that he exhibits the consensus effect in (p, q, α, β^*).*

Anti-consensus effect at (p, q, α, β^*) and general anti-consensus effect are similarly defined.

Observe that if preferences $\succcurlyeq$ satisfies the following *betweenness* property, $p \sim q$ implies $\gamma p + (1-\gamma)q \sim q$, then the individual will never display any choice shift in group. This property is weaker than the standard independence axiom,[12] which suggests that to accommodate such shifts, one needs to go beyond expected utility (or, more generally, beyond the betweenness class of preferences, suggested by Chew, 1983 and Dekel, 1986). To this aim, we consider the following two properties.

Definition 2. *The preference relation $\succcurlyeq$ is strictly quasi-convex if for all $p, q \in \Delta$, with $p \neq q$, and $\lambda \in (0, 1)$,*

$$p \sim q \Rightarrow \lambda p + (1-\lambda) q \prec p$$

and is strictly quasi-concave if

$$p \sim q \Rightarrow \lambda p + (1-\lambda) q \succ p$$

Quasi-convexity implies aversion towards randomization between equally good *lotteries*; whereas quasi-concavity implies affinity to such randomization. (Betweenness preferences satisfy both weak quasi-convexity and weak quasi-concavity.)[13]

[11] The consensus effect is defined where $p \sim q$. By continuity, the choice patterns that we study when the options are indifferent will persist even when one option is strictly preferred to the other.

[12] According to the independence axiom, $p \succcurlyeq q$ if and only if for any lottery r and $\gamma \in [0, 1]$, $\gamma p + (1-\gamma)r \succcurlyeq \gamma q + (1-\gamma)r$.

[13] The experimental evidence on quasi-convexity versus quasi-concavity is mixed. While it is a stylized empirical finding that betweenness is often violated, most of the experimental literature that documents violations of linear indifference curves (e.g., Coombs and Huang, 1976) found deviations in both directions, that is, either preference for or aversion to randomization. Camerer and Ho (1994) find support for a complicated pattern with quasi-convexity over gains and quasi-concavity over losses. A concrete example of the behavioral distinction between quasi-concave and quasi-convex risk preferences is the probabilistic insurance problem of Kahneman and Tversky (1979). They showed that in contrast with experimental evidence, any risk averse expected utility maximizer must prefer probabilistic insurance to regular insurance. Sarver (2014) pointed out that this result readily extends to the case of quasi-concave preferences. In contrast, quasi-convex preferences can accommodate aversion to probabilistic insurance.

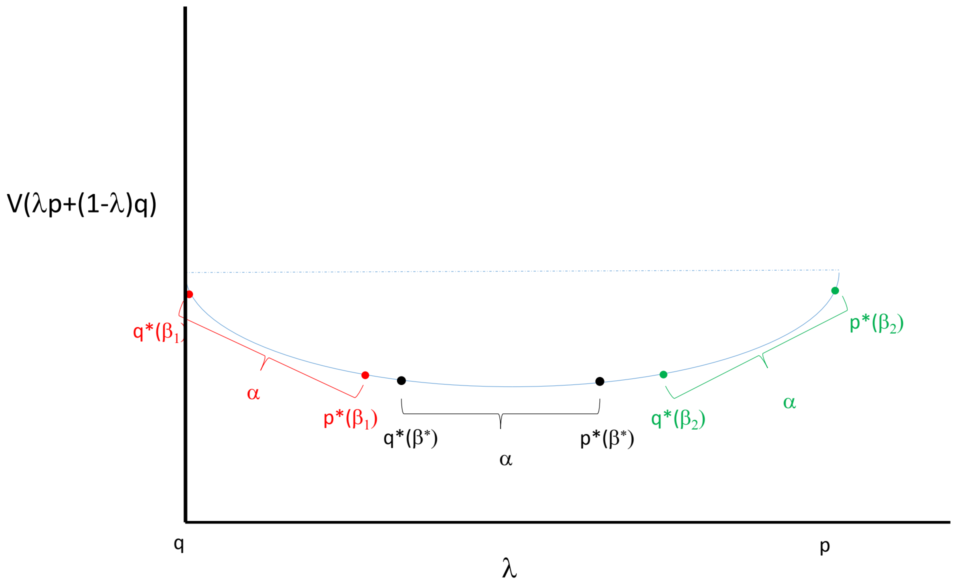

Figure 1: $V(\lambda q + (1-\lambda)p)$ for $\lambda \in [0,1]$.

Our main result links violations of expected utility at the individual level with a specific pattern of choices in group situations.

Proposition 1. *Preferences are strictly quasi-convex (resp., strictly quasi-concave) if and only if they exhibit the consensus (resp., anti-consensus) effect.*

To see the intuition behind Proposition 1, observe first that p^* is always closer to p and q^* is always closer to q, with $p^* - q^* = \alpha(p-q)$. In addition, $\succcurlyeq$ are quasi-convex if and only if they are *single-troughed* between p and q. The proof is established by noting that an increase in β moves both p^* and q^* closer to p. Figure 1 plots the function $V(\lambda q + (1-\lambda)p)$ for $\lambda \in [0,1]$. The lotteries p^* and q^* are depicted for three different values of β ($\beta_1 < \beta^*$, β^*, and $\beta_2 > \beta^*$). Given a fixed α, for $\beta >$(resp., $<$)β^* the two lotteries get closer to p (resp., q), while the distance between them remains intact. More generally, even when the individual is not initially indifferent between p and q, his ultimate choice reflects a tradeoff between his preferred outcome and the desire to avoid randomization across lotteries (i.e. he wants to choose extreme lotteries that are as close to p or q as possible). If the latter effect is strong enough, he may reverse his preferences in a group context.

2.2 Examples

We now discuss the implications of Proposition 1 for some popular non-expected utility models. (We focus throughout the paper on the quasi-convex case, although the results naturally extend, modulo standard reversal, to quasi-concavity.) In all these examples, preferences are defined over monetary lotteries, that is, the underlying set of outcomes is an interval $X \subset \mathbb{R}$.

Rank-Dependent Utility (RDU): Order the prizes $x_1 < x_2 < ... < x_n$. The functional form for RDU is:

$$V_{RDU}(p) = u(x_1) + \textstyle\sum_{i=2}^{n} g\left(\sum_{j\geq i} p(x_j)\right)[u(x_i) - u(x_{i-1})] \quad (1)$$

where the weighting function $g : [0,1] \to [0,1]$ is bijective and strictly increasing. If $g(l) = l$ then RDU reduces to expected utility.

RDU preferences are quasi-convex if and only if the weighting function is convex (see Wakker, 1994). Convexity of the weighting function — which is also a necessary condition for risk-aversion within RDU — is typically interpreted as a type of pessimism: improving the ranking position of an outcome decreases its decision weight. This suggests the following corollary.

Corollary 1. *Suppose preferences are RDU. Then the individual is pessimistic (g is strictly convex) if and only if he exhibits the consensus effect.*

The consensus effect as in Definition 1 is weak, in the sense that it does not determine how likely it has to be that the group chooses p in the absence of the individual being pivotal. However, if we put more structure on preferences we can have stronger results. This motivates introducing the class of quadratic preferences.

Quadratic Utility: A utility functional is quadratic in probabilities if it can be expressed in the form

$$V_{\mathbb{Q}}(p) = \sum_{x}\sum_{y} \phi(x,y)\ p(x)p(y)$$

where $\phi : X \times X \to \mathbb{R}$ is symmetric. The quadratic functional form was introduced in Machina (1982) and further developed in Chew, Epstein, and Segal (1991, 1994).

The following result establishes that in the class of quadratic preferences, the consensus effect becomes a *majority effect*: β^* *always* equals .5, independently of the two options under consideration. So if initially indifferent, the individual simply chooses the option he believes the group is most likely to choose when he is not pivotal.

Proposition 2. *Suppose preferences can be represented by a quadratic functional. Then preferences are strictly quasi-convex if and only if the individual exhibits the consensus effect at* $(p, q, \alpha, .5)$.

The following class of preferences was introduced by Kőszegi and Rabin (2007) and Delquié and Cillo (2006).[14]

Choice Acclimating Personal Equilibrium: The value of a lottery p is

$$V_{\mathbb{CPE}_M}(p) = \underbrace{\sum_x u(x)p(x)}_{\text{consumption utility}} + \underbrace{\sum_x \sum_y \mu(u(x) - u(y))\; p(x)p(y)}_{\text{gain-loss utility}}$$

where u is an increasing utility function over final wealth and

$$\mu(z) = \begin{cases} z & \text{if } z \geq 0 \\ \kappa z & \text{if } z < 0 \end{cases}$$

is a gain-loss function with $0 \leq \kappa \leq 2$ denoting the coefficient of loss aversion. Loss aversion occurs when $\kappa \geq 1$. Masatlioglu and Raymond (2016) show that these preferences are the intersection of RDU and quadratic utility, and that they are quasi-convex if and only if $\kappa \geq 1$.

Corollary 2. *Suppose preferences have a representation $V_{\mathbb{CPE}_M}$. Then individuals are loss averse if and only if they exhibit the consensus effect at $(p, q, \alpha, .5)$.*

Corollary 2 links notions of reference dependence in individual choice with a similar notion (the consensus effect) in group choice. If the group is more likely to choose p than q when an individual is not pivotal, then this likely choice would naturally serve as a reference point when the individual is deciding how to make his own choice (which will only matter in the case where he is pivotal). This almost exactly mirrors the underlying intuition often provided for a preference for conformity — it is a type of external (i.e. based on the actions of others) reference point.

2.3 Risky Shifts, Cautious Shifts, and Allais Paradox

In this section we focus on group choices between particular pairs of options, s(afe) and r(isky), where s is a degenerate lottery, that is, a lottery that yields a certain prize $x \in X$ with probability 1, and r is some nondegenerate lottery. A group decision problem is then (r, s, α, β). In this context, we refer to *risky shift* (resp., *cautious shift*) as the joint statement $r \sim s$ and $r^* \succ s^*$ (resp., $r^* \prec s^*$), where

$$r^* = [\alpha + (1-\alpha)(1-\beta)]\, r + (1-\alpha)\beta s$$

and

$$s^* = (1-\alpha)(1-\beta)r + [\alpha + (1-\alpha)\beta]s$$

[14]These preferences have been widely used in the behavioral literature (Sydnor, 2010; Herweg, Muller and Weinschenk, 2010; Abeler, Falk, Goette and Huffman, 2011; Ericson and Fuster, 2011; Gill and Prowse, 2012; and Barseghyan, Molinari, O'Donoghue, and Teitelbaum, 2013.)

These shifts are clearly a subset of the more general shifts discussed under the consensus effect. For a particular r, s, and α, there exists a β^* where an individual always exhibits a risky shift for $\beta \leq \beta^*$ and a cautious shift for $\beta \geq \beta^*$ if and only if the individual exhibits the consensus effect at (s, r, α, β^*).

ERR used this setting and focused on RDU preferences (see Section 2.2). Below we generalize their contribution within RDU, but also demonstrate that their main message *is not* necessarily valid for other types of non-expected utility preferences. Segal (1987) showed that within RDU, a convex distortion function g in equ.(1) implies (and is implied by) behavior that accommodates a version of Allais paradox — also known as the common consequence effect — which is one of the most prominent evidence against expected utility. Formally, fix any three prizes $x_3 > x_2 > x_1$ and denote by (p_1, p_2, p_3) the lottery that yields the prize x_i with probability p_i. The following definition formalizes this notion of the Allais paradox.[15]

Definition 3. *An individual exhibits the Allais paradox if for every pair of lotteries* $(1-\alpha, \alpha, 0)$ *and* $(1-\beta, 0, \beta)$ *with* $\alpha > \beta$, $(1-\alpha, \alpha, 0) \sim (1-\beta, 0, \beta)$ *implies* $(1-\alpha-\gamma, \alpha+\gamma, 0) \succ (1-\beta-\gamma, \gamma, \beta)$ *for all* $\gamma \in (0, 1-\alpha]$.

Theorem 1 in ERR states that within RDU, an individual exhibits the Allais paradox if and only if for any $r \sim s$ and $\alpha \in (0,1)$ there exists $\beta^* \in (0,1)$ such that he exhibits risky (resp., cautious) shift if $\beta < \beta^*$ (resp., $\beta > \beta^*$). ERR thus suggest an equivalence between a commonly known violation of expected utility and a robust phenomenon in the social psychology of groups when choosing between risky and safe options. Because Allais-type behavior is equivalent to the convexity of the weighting function and therefore to quasi-convexity of preferences, it is also the case that within RDU we have additional equivalences, as the following corollary summarizes.

Corollary 3. *Consider the rank dependent utility model (equ.(1)). The following statements are equivalent:*

1. *An individual exhibits the Allais paradox*

2. *For all* $r \sim s$ *and* α *there exists* β^* *such that the individual exhibits the consensus effect at* (r, s, α, β^*)

3. *An individual's preferences satisfy quasi-convexity*

4. *An individual exhibits the consensus effect*

While these logical equivalences are quite strong (in the sense that they link specific behavior regarding r and s to arbitrary behavior for any p and q) and have

[15] In Allais' original questionnaire, $x_3 = 5M; x_2 = 1M$, and $x_1 = 0$. Subjects choose between $A = (0, 1, 0)$ and $B = (0.1, 0.89, 0.01)$, and also between $C = (0, 0.11, 0.80)$ and $D = (0.1, 0, 0.9)$. The typical pattern of choice is the pair (A, D). Definition 3 is more general than the original paradox proposed by Allais, since it puts behavioral restrictions also when no certain outcome is involved.

an intuitive appeal (in that they link preferences for a risky versus safe option in Allais questionnaire to similar preferences in group choice), they — as well as ERR's original results — are derived in the narrow context of RDU preferences. We will now argue that they are specific to that class and do not hold in general. In other words, empirical evidence that refutes RDU also challenges the aforementioned relationship between Allais-type behavior and consensus effects. The intuition, which we make more concrete in the two examples below and in Figure 2, is the following: Definition 3 puts restrictions on how the slope of indifference curves change as we move *between them* in a specific direction in the probability triangle. Quasi-convexity and quasi-concavity, on the other hand, put restrictions on how the slope of a single indifference curve changes as we slide *along it*. In general, these two restrictions are independent.

To demonstrate this, first observe that the pattern of risky and cautious shifts discussed in ERR is implied by the consensus effect. Thus, in constructing our examples, we show that both quasi-convexity and quasi-concavity are consistent with both Allais-type behavior and with the opposite pattern of individual choice. We further note that any lottery p over fixed three outcomes $l < m < h$ can be represented as a point (p_l, p_h) in a two-dimensional unit simplex, where the probability of l (p_l) is on the x-axis and that of h (p_h) is on the y-axis. Showing that indifference curves become steeper, or fanning out, in the 'north-west' direction is sufficient for Allais-type behavior, while the opposite pattern, fanning in, is sufficient for anti-Allais-type behavior.

Figure 2 shows all possible combinations of attitudes towards randomization and fanning properties of indifference curves. Panel (a) plots indifference curves of individuals who exhibit both the Allais paradox and the consensus effect. Panel (b) demonstrates preferences which exhibit both anti-Allais behavior and the anti-consensus effect (both sets of indifference curves can be generated by the functional form $V_{\mathbb{CPE}_M}$ (Section 2.2), which, as shown in Masatlioglu and Raymond (2016), also admits an RDU representation and thus falls under the results of ERR). In contrast, Panel (c) depicts preferences that exhibit the Allais paradox but the anti-consensus effect, while Panel (d) shows preferences which generate anti-Allais behavior but also the consensus effect.[16]

The following functional forms generate the types of behavior depicted in Panels (c) and (d).

Example 1 (quasi-concavity with Allais-type behavior): Consider the quadratic functional,

$$V(p) = \mathrm{E}\left[v(p)\right] \times \mathrm{E}\left[w(p)\right]$$

which is quasi-concave (since $\log V$ is concave).[17] For three outcomes, $l < m < h$, define v and w as follows: $v(l) = 1, v(m) = 2, v(h) = 4$; $w(l) = 2, w(m) = 3, w(h) = 4$.

[16]Graphically, the indifference curves in Panels (a) and (b) are sections of concentric circles, those in Panel (c) are sections of hyperbolas, and those in Panel (d) are sections of parabolas.

[17]In this example, $\phi(x, y) = \frac{v(x)w(y)+v(y)w(x)}{2}$.

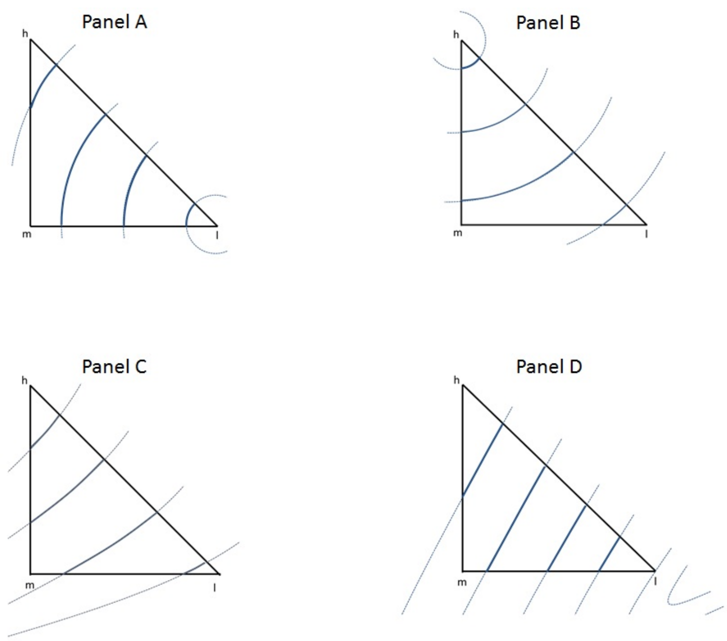

Figure 2: Attitudes towards randomization and fanning properties of indifference curves

We show in the appendix that the indifference curves of this functional fan out.

Example 2 (quasi-convexity with anti-Allais-type behavior): Consider again three fixed outcomes, $l < m < h$, and the utility functional be defined as

$$V(p_l, p_h) = -6p_l + p_l^2 + 7.82p_h - 3.2p_l p_h + 2.56p_h^2$$

We show in the appendix that this functional represents quasi-convex preferences, but its indifference curves fan in.[18]

Examples 1 and 2 show that Allais-type behavior and risky and cautious shifts (and the consensus effect more generally) are not necessarily related outside RDU. In the appendix we provide another example, which demonstrates that even the equivalence between risky and cautious shifts and quasi-convexity (and so the consensus effect) that Corollary 3 describes does not extend. While quasi-convexity is a sufficient condition for ERR's risky/cautious shifts, it is not necessary.

[18] V is a quadratic functional, with $\phi(l,l) = -5$, $\phi(m,l) = -3$, $\phi(h,l) = 2.51$, $\phi(m,m) = 0$, $\phi(h,m) = 3.91$, and $\phi(h,h) = 10.38$.

3 The Consensus Effect in Equilibrium

Our analysis so far has been restricted to understanding the behavior of an individual who is facing a fixed, exogenous decision process. While, similar to ERR, our interpretation of the environment is of a group decision problem, the exact same analysis would apply also if the environment reflects a situation where the individual gets to choose with some probability, and with the remaining probability a computer chooses for him. To explicitly captures the strategic interaction, in this section we extend our analysis to a full equilibrium setting, and in doing so refer to individuals as voters.

We will show that, in contrast to settings where voters are expected utility maximizers, quasi-convex preferences can lead to phenomena such as group polarization, the bandwagon effects, preference reversals, and multiple equilibria. This is driven by the fact that quasi-convex preferences give the voting game properties of a *coordination game*.

We describe a majority voting game as a collection of N individuals (where N is odd), each of whom receives one vote to cast in favor of either p or q (no abstentions are allowed).[19,20] Whichever option receives the majority of the votes is implemented (in Proposition 8 we relax the assumption of a simple majority rule). Throughout this section when we refer to equilibria we mean "voting equilibria", that is, Nash equilibria where voters do not use weakly dominated strategies.[21]

Voters come in three *types*: Those that prefer p to q (Type P), those that prefer q to p (Type Q), and those that are indifferent (Type I). Each individual is drawn at random from each of the three types with probabilities f_P, f_Q, and f_I, respectively, where $f_P + f_Q + f_I = 1$. We denote the vector of probabilities by F. Each individual observes his own type and votes for either option p or option q.

As a benchmark, we first review the set of equilibria that emerge if all voters have expected utility preferences.

Proposition 3. *An equilibrium always exists. Moreover, a set of strategies is an equilibrium if and only if*

1. *Type Ps vote for p*
2. *Type Qs vote for q*
3. *Any given i of Type I votes for p with probability $r_i \in [0, 1]$*

[19] An alternative assumption, often taken in the voting literature, is that the number of voters is a random variable that has a Poisson distribution. Such an assumption will not change our results.

[20] Identical results will be obtained if voting is assumed instead to be voluntary but costless. Costless/required voting is a reasonable assumption in many settings, such as committee votes, where members are required to be present regardless of whether they choose to vote. We discuss costly voting in Section 4.

[21] While there are equilibria that involve coordination motives even when individuals are expected utility maximizers, they rely on voters using weakly dominated strategies and are ruled out by this assumption.

Observe that in this equilibrium people vote for the option they favor in individual choice, or arbitrarily randomize between outcomes they are indifferent between.

We now turn to voters with quasi-convex preferences. Types P and Q can now come in two different sub-types. We call them $P1$, $P2$ (and $Q1$, $Q2$). Types $P1$ and $Q1$ have monotone preferences between q and p. For example, $P1$ (resp., $Q1$) strictly prefers $\lambda p + (1-\lambda)q$ to $\delta p + (1-\delta)q$ if and only if $\lambda > \delta$ (resp., $\lambda < \delta$).[22]

In contrast, $P2$'s preferences are non-monotonic between q and p. By strict quasi-convexity, $P2$'s preferences are single-troughed between p and q and there exists a unique λ^* such that $\lambda^* p + (1-\lambda^*)q \sim q$. Thus, for all $\lambda < \lambda^*$ we have $\lambda p + (1-\lambda)q \prec q$, which means that even though a $P2$ type prefers p to q, so long as p^* and q^* are both close enough to q he will prefer q^* to p^*. Similarly, for $Q2$, there exists a λ^* such that $\lambda^* p + (1-\lambda^*)q \sim p$, and $\lambda p + (1-\lambda)q \prec p$ for all $\lambda > \lambda^*$. We will refer to types $P1$ and $Q1$ as monotone types and to the others as non-monotone types. Figure 3 illustrates the utility of each type over all convex combinations of q ($\lambda = 0$) and p ($\lambda = 1$).[23] We assume that the distribution F has a full support, that is, it places strictly positive probabilities on all possible types.

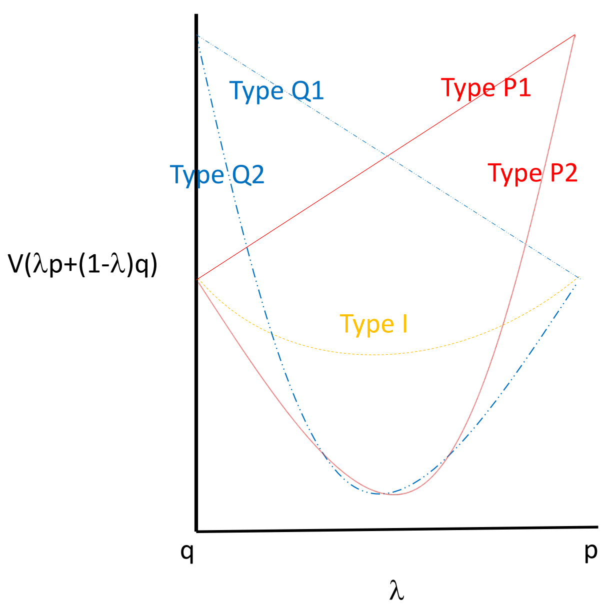

Figure 3: Five types of preferences

We assume that individuals within each type have the same preferences, so that given a group problem, λ_i^* is the same for all i of type $P2$ (similarly for $Q2$ and I)

[22] Expected utility preferences are always monotonic between q and p.

[23] The normalization of the utility levels from p and from q in Figure 3 and having the same convex combination give the minimum utility for both $P2$ and $Q2$ are unnecessary.

and hence β_i^* is the same as well.[24]

A key property, which will be the main driving force behind many of the formal results below, is that with quasi-convex preferences, the majority voting game takes on aspects of a *coordination game* — non-monotone types experience benefits from coordinating their votes with others because it reduces the amount of "randomness" in the election, in the sense that it pushes p^* (resp., q^*) towards p (respectively, q).[25]

We turn now to studying some of the properties of the Nash equilibria of the voting game. First, we demonstrate that an equilibrium always exists. In particular, we prove the existence of an "anonymous Nash equilibrium" that is, a Nash equilibrium in which each individual's strategy depends only on his preferences (i.e. his type) and not on his identity. Although the exact set of equilibria will depend on the distribution F, we will highlight some of the salient features that differ from the expected utility case.

Proposition 4. *An anonymous Nash equilibrium always exists. Moreover, in any equilibrium (not necessarily anonymous)*

1. *Generically,*[26] *all individuals strictly prefer to vote for one option or the other. Moreover, no individuals randomize*
2. *Type* $P1$*s vote for* p
3. *Type* $Q1$*s vote for* q

In contrast to Proposition 3, here no individual randomizes and, in fact, strictly dislikes randomizing. Thus, we will expect to observe choice shifts in the group — individuals who are indifferent between p and q in individual situations strictly prefer one or the other in a group setting.[27] Proposition 4, however, does not specify whether the shift would be towards q or towards p.

In order to provide intuition for the actual pattern of voting that can be observed in equilibrium, we will analyze the best response function of a voter. We index the number of possible voting combinations by m. Consider voting pattern $\mathbb{V}^m$ and suppose individual i is a member of type Γ. Given this, observe that F and $\mathbb{V}^m$

[24] We focus on the situation where all individuals in each type have the same preferences for analytic convenience, although the results naturally extend to situations where they do not.

[25] A key technical aside; as Crawford (1990) points out, games in which individuals have quasi-convex preferences may oftentimes admit no Nash equilibrium. He suggest a new notion "equilibrium in beliefs" which coincides with standard Nash equilibrium under expected utility, but also exists when players have quasi-convex preferences. We simply focus on Nash equilibrium, which, as we show in Proposition 4, always exists because of the benefits of coordination.

[26] Generically here, as well as in Propositions 6 and 7, is in the set of distributions, using the standard weak* topology.

[27] However, it is not necessarily the case that if we sum up the total number of choices for p less the total number of choices for q in the individual choice problem, and compare it to the vote totals in the majority voting game, that the latter is farther from 0.5 than the former (see Section 4).

generate a probability $\alpha(\mathbb{V}^m, F)$ of an individual being pivotal, and so a threshold probability $\beta^*(\mathbb{V}^m, \Gamma, F)$. Denote the set of types that vote for p (resp., q) given $\mathbb{V}^m$ as $\mathbb{P}(\mathbb{V}^m)$ (resp., $\mathbb{Q}(\mathbb{V}^m)$).[28]

The probability that p is chosen when i is not pivotal is:

$$\beta_{i,\mathbb{V}^m,F} = \frac{\sum_{k=\lceil \frac{N}{2}+2 \rceil}^{N} \binom{N}{k} (\sum_{\tau \in \mathbb{P}(\mathbb{V}^m)} f_\tau)^k (\sum_{\tau \in \mathbb{Q}(\mathbb{V}^m)} f_\tau)^{N-k}}{1 - \sum_{k=\lceil \frac{N}{2}-1 \rceil}^{\lfloor \frac{N}{2}+1 \rfloor} \binom{n}{k} (\sum_{\tau \in \mathbb{P}(\mathbb{V}^m)} f_\tau)^k (\sum_{\tau \in \mathbb{Q}(\mathbb{V}^m)} f_\tau)^{N-k}}$$

Individual i's best response is to choose p if $\beta_{i,\mathbb{V}^m,F} > \beta^*(\mathbb{V}^m, \Gamma, F)$ and q if the inequality is reversed. Thus, a voting pattern is an equilibrium if it is the case that $\mathbb{V}^m$ generates $\beta_{i,\mathbb{V}^m,F}$ that are consistent with it.[29]

The question of whether there is a unique equilibrium depends on the exact preferences and parameters of the problem. Because of the coordination nature of the majority voting game, there will often be multiplicity. However, the next proposition provides a sufficient condition for a unique pattern of voting. It states that whenever there are enough voters that strongly favor one of the options (i.e. in a monotone fashion), it is the case that all non-monotone types vote for that option as well.

Proposition 5. *There exists an N^*, such that for all $N > N^*$ there is a f_{P1} (resp., f_{Q1}) sufficiently close to 1 for which the unique equilibrium is for all non-monotone types to choose p (resp., q).*

Proposition 5 thus predicts group polarization to such an extent that it actually causes *preference reversals* — individuals who in an individual problem would choose q over p will now actually choose p in the group problem (e.g., type $Q2$). The result generates an intuitive type of preference reversal — individuals coordinate on voting for an outcome strongly favored by many others.

However, individuals can also coordinate on equilibria that are not necessarily strongly favored, as shown by the next proposition. This proposition highlights how benefits from coordination generate multiple equilibria.

Proposition 6. *For any given N, if there exists a small enough proportion of non-monotone types, then generically there is a unique equilibrium.[30] In contrast, for large enough N, if the proportion of non-monotone types is sufficiently close to 1, then there are always at least two equilibria.*

[28] In defining $\beta_{i,\mathbb{V}^m,F}$, we assume that all individuals of the same type behave the same; a similar construction — albeit more complicated — can be performed without assuming anonymity.

[29] Quasi-convexity of preferences alone provides no restrictions on the ordering of the thresholds $\beta^*(\mathbb{V}^m, \Gamma, F)$ across different non-monotone types. Additional restrictions, such as that all preferences are in the quadratic class, do ensure that the thresholds are ordered in the "intuitive" fashion.

[30] Moreover, if preferences are quadratic and if f_{P1} and f_{Q1} are sufficiently close to one another, then $P2$ (resp., $Q2$) types all vote for p (resp., q).

That is, if non-monotone types form a large enough proportion of the population, they can all vote the same to ensure an outcome gets elected with very high probability. Voting against the group leads to additional uncertainty, which reduces ex-ante utility. In other words, when there is a sufficient number of any non-monotone type, the benefits of coordination become so large that multiple equilibria must exist. This can have counter-intuitive effects on voting outcomes. For example, imagine that all individuals are of type $P2$ and hence, when choosing individually, will choose p. However, when choosing as a group they could not only coordinate on an equilibrium where everyone votes for p but also on one where everyone votes for q. The latter is clearly Pareto sub-optimal, but exists because of the benefits of coordination. Thus, we can observe preference reversal not just because an individual knows many other voters have "extreme" preferences, but also because an individual knows that many other voters have preferences where they would like to coordinate.

Proposition 6 also demonstrates that when there are enough monotone types, uniqueness of an equilibrium is guaranteed. Importantly, the proposition does not state that in this equilibrium non-monotone types will coordinate on their actions, but only that it will be unique. The intuition is that with very few non-monotone types, although individuals may not know with certainty which option will be chosen, they know with near certainty, regardless of the voting behavior of non-monotone types, what the probability that p is chosen, that is, they know β with near certainty, which implies uniqueness.

Because voters care about what happens when they are not pivotal, and the uncertainty about what will happen hinges on the number of voters, the size of the group has important implications on behavior. If there are too few voters, then any given individual can have a large impact on the election.In contrast, as N grows large, the probability of being pivotal goes to 0, but also the chosen outcome when a voter is not pivotal becomes known with (almost) certainty. Intuitively, as N grows large, both the proportions of each type of voters and (since voters generically do not randomize) the proportions of votes for each option are known with almost certainty. This implies that p^* and q^* are arbitrarily close to either p or q, and so all individuals will prefer to vote for either one or the other. The following proposition formalizes these assertions.

Proposition 7. *For sufficiently small N, types $P2$ and $Q2$ always vote for p and q respectively. For a sufficiently large N, generically in all equilibria, all non-monotone types take the same action.*

Proposition 7 says that in large elections we should always expect to see preference reversals, meaning that large elections will almost surely fail to aggregate preferences.

The proposition, however, says nothing about the structure of the set of equilibria. With expected utility preferences, and no indifferent types, there is always a unique equilibrium regardless of N. The only change in outcomes is that as N increases in size, the type that F places more weight on is more likely to have its choice implemented.

In contrast, suppose individuals have quasi-convex preferences and that the distribution F puts positive weight only on types $P2$ and $Q2$. In this case, with a sufficiently large N, there are two possible equilibria, each of which features all individuals taking the same action. Moreover, and unlike in the case of expected utility, the model features "jumps" in the probability of which option is chosen. Below some threshold (in terms of the number of voters N) there exists an equilibrium where the probability of the ex-ante favored option (i.e. the option favored by the majority of voters when choosing in isolation) being chosen increases with N but is bounded away from 1. At the threshold, the probability of one of the options being chosen (in at least one equilibrium) jumps to certainty, and the option may not be the one favored ex-ante. This also suggests that the probability of the initially favored option to be chosen need not increase with N.

We can also consider what happens as the voting rules shift. Denote one option, without loss of generality p, as the status quo, and assume the threshold T needed to replace p with q increases from fifty percent in favor of q.[31] Intuitively, as the threshold increases, the probability of q being chosen falls, and so non-monotone types become less likely to vote for it. Eventually, the unique equilibrium is for non-monotone types to votes for p. (Of course a similar result holds if q is made the default option.)

Proposition 8. *There exists an N^*, such that for all $N > N^*$ there is a T sufficiently close to 1 for which the unique equilibrium is for all non-monotone types to choose p.*

With expected utility preferences, and again with no indifferent types, an increase in the threshold T will not change strategies, but will decrease the probability that q is actually chosen (as it decreases the change that there are enough Q types to overcome the threshold requirement). Moreover, this change in probability will be a continuous function of T. This is distinct from the prediction with quasi-convex preferences, where — similarly to what happens when N increases — there exists a threshold value below which the probability of choosing p (in at least one equilibria) is increasing with T, and above which the probability of choosing p jumps to 1 discontinuously.

4 Discussion

Our discussion of quasi-convex preferences has focused on preferences that are explicitly non-expected utility. However, as we mentioned in the introduction and was initially pointed out by Machina (1984), if an expected utility maximizer is allowed to take a payoff-relevant action before the lottery he faces is resolved, then his induced preferences over the ex-ante 'optimal' lotteries will be quasi-convex. To see this, suppose there are two individuals, facing two lotteries, p and q, between which they are

[31] ERR consider an extreme form of voting rule, where a unanimous agreement must be made to shift away from the status quo.

both indifferent. There are three outcomes, and p is a binary lottery over the best and middle outcomes while q is a binary lottery over the best and worst outcomes. Both individuals are indifferent between p and q. The individuals vote as in our voting game. After voting, but before the chosen alternative is revealed, each individual can take one (and only one) of two 'insurance' action; a_1 or a_2. Action a_1 fully insures against the realization of the middle outcome, but not the low outcome, while a_2 insures against the realization of the low outcome, but not the middle outcome. Thus, even if the two individuals have expected utility preferences over lotteries, they have a strict incentive to coordinate their votes, because they would like to know which insurance action to take.

Because many applications focus on groups choosing between two options, we have also restricted our analysis to binary choices. Our results, however, are readily extended. For example, individuals will still exhibit a consensus effect. Imagine that the group must choose over Ω possible lotteries, denoted $p_1, ...p_\Omega$, and that an individual is indifferent between all of them. Then, so long as β_j is sufficiently large, the individual will vote for option j.

One way of interpreting our results is in line with notions of reference dependence. Kőszegi and Rabin (2007) discuss how an individual may prefer p to q if expecting p, and q to p if expecting q. In line with this, our non-monotone individuals choose p if they think it is sufficiently likely that they will receive p regardless of their choice, and similarly for q. One way of interpreting this behavior is that individuals are disappointed when they receive outcomes in q and were expecting better outcomes in p, and vice versa. This complements the intuition given in Section 2.

The link between quasi-convex preferences and the consensus effect has implications beyond standard voting situations. For example, suppose two candidates are running against one another. The election just happened and Candidate A was declared the winner by a small margin. However, there is the possibility of a recount. Our results imply that individuals with quasi-convex preferences who voted for Candidate B may actually prefer, after learning that it is likely A will be elected, to avoid a recount (which add additional randomness to the outcome).

4.1 Relation to Stylized Facts

The bandwagon effect, as described in the introduction, is discussed in Simon (1954), Fleitas (1971), Zech (1975), and Gartner (1976), among others. It captures the idea that if individuals believe others will vote for a certain option, they themselves are more likely to vote for that option as well. Thus, it reflects the change in the individual's behavior as a function of the group composition. Abusing notation slightly, we will denote the fraction of individuals voting for p given voting pattern V^m as $\mathbb{P}(\mathbb{V}^m)$ and for q as $\mathbb{Q}(\mathbb{V}^m)$. Let $Z = \mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)$, and observe that $Z \in (-1+f_{P1}, 1-f_{Q1})$. The bandwagon effect describes the fact that if Z is close enough to 1 (resp.,-1), then any non-monotone type individual will strictly prefer to vote for p (resp., q). For example, the results of Proposition 5 demonstrate the bandwagon effect: Z approaches 1 (resp., -1) as f_{P1} (resp., f_{Q1}) approaches 1. In Proposition 6 , as $f_{P1} + f_{P2}$ goes to 0, Z can take on any number in $(-1, 1)$, thus the bandwagon effect guarantees two equilibria. The second part of Proposition 7 can be interpreted in a similar manner — for a large enough N, Z is always sufficiently close to -1 or 1.

Much of the discussion regarding group shifts focuses on group polarization, where the group ends up having a more extreme decision than the aggregate of individuals' decisions in isolation. This has been documented in a variety of settings — for example, Isenberg, (1986), Myers and Lamm (1976), McGarty et al. (1992), Van Swol (2009), and Moscovici and Zavalloni (1969) — and has been of particular interest to researchers examining the effects of decisions by juries (such as Main and Walker, 1973, Bray and Noble, 1978, and Sunstein, 2002). This phenomenon is distinct from a related phenomenon of individual-belief polarization, according to which individuals' beliefs become more extreme.

In our simple stylized setting, we can analyze when, and why, group decisions may be more polarized than individual decisions. In some settings, where groups have access to a range of ordered alternatives, polarization can take the form of the group choosing a more extreme alternative than what the average individual choice would be. In our context, where there are only two alternatives to choose between, polarization is a measure of how extreme the choice proportions are. For simplicity, suppose there are no I-types. We can then measure the degree of polarization by the difference between the proportion of people who choose p relative to those who choose q. Thus, in individual choice settings $|(f_{P1} + f_{P2}) - (f_{Q1} + f_{Q2})|$ is the relative strength of the support for p over q, while the corresponding measure of polarization in the group setting is $|\mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)|$. Group polarization occurs if $|f_{P1} + f_{P2} - (f_{Q1} + f_{Q2})| < |\mathbb{P}(\mathbb{V}^m) - \mathbb{Q}(\mathbb{V}^m)|$. Although it seems intuitive that the consensus effect generates group polarization, this is not necessarily true. While Proposition 5 guarantees that with large enough N the equilibrium exhibits group polarization, for relatively small groups we can observe the opposite phenomenon. In particular, for a small enough N, fixing the distribution of types, we can always find preferences for $P2$ and $Q2$ types so that we get group *de-polarization*, that is, $|f_{P1}+f_{P2}-(f_{Q1}+f_{Q2})| > |\mathbb{P}(\mathbb{V}^m)-\mathbb{Q}(\mathbb{V}^m)|$. In other words, our model has the feature that polarization will be more robust phenomenon as the group size becomes larger.

4.2 Alternative Approaches to Group Choice

Our results are related to the large literature on voting and the aggregation of preferences or information in elections.[32] The literature has made different assumptions

[32] An important distinction is that while we assume that alternatives in the voting game are lotteries, most papers suppose they are final outcomes. Of course, this complicates thinking about our results in relation to the pre-existing literature; for example, in a common-values setting, private signals would then need to be about a particular outcome in the support of p or q. We nevertheless

regarding how individuals value outcomes and about the cost of voting. We have focused on the situation where voters have private values and have either compulsory or costless voting. As mentioned, with expected utility preferences and either compulsory or costless voting, all voters vote sincerely (i.e., individuals vote as part of the group in the same way they would choose in isolation) and all individuals vote, meaning that preferences are aggregated. By contrast, with quasi-convex preferences we find Pareto-dominated equilibria where preferences are not aggregated properly.

However, there are alternative models of group decisions, some of which can generate behavior that is consistent with our model. We now summarize these alternative specifications and compare their predictions to ours.

Costly Voting: Many papers in the literature on voting with private values assume that voting is costly, meaning that, as Ledyard (1981, 1984) and Palfrey and Rosenthal (1983, 1985) point out, individuals need to compare the cost of voting to the benefit of voting, namely the chance of being pivotal. This implies that the proportion of votes cast for each side will now depend not only on the fraction of supporters for each option, but also on the cost and benefit distributions of both types of supporters (Taylor and Yildrim, 2010). However, as in models with compulsory voting and in contrast to the preference reversals we predict, conditional on voting, voters will still truthfully reveal their preferences over the options.

To build intuition, consider the case of an expected utility maximizer with utility function V_{EU}, who favors option p. In a private values setting, the only two factors affecting the decision of whether or not to vote is the utility gap between p and q and the probability of being pivotal, α; the benefit of voting is then $\alpha(V_{EU}(p)-V_{EU}(q)) = V_{EU}(p^*)-V_{EU}(q^*)$. This implies the following two properties: (i) $V_{EU}(p^*)-V_{EU}(q^*) > 0$ if and only if $V_{EU}(p) - V_{EU}(q) > 0$, so that *if* individuals vote, they vote for the candidate they would favor in isolation; and (ii) $V_{EU}(p^*) - V_{EU}(q^*)$ is increasing in α, so that as the individual's probability of being pivotal increases, he requires a lower cost of voting to actually go to the poll. Note that the decision of whether to cast any vote at all (and whom to vote for) is independent of β.

For individuals with quasi-convex preferences (who favor p^*), the benefit of going to the poll is again $V_{EU}(p^*)-V_{EU}(q^*)$, which now depends not only on p, q, and α, but also on β. Individuals will compare this benefit to the cost of voting, and vote only when benefit exceeds the cost. The dependency on β can generate some interesting dynamics in voting. Fix a cost of voting $c > 0$. If the utility gap between p and q is large enough and α is small enough, then there will be intermediate values of β for which the individual will abstain, but if β is more extreme (closer to either 1 or 0, but not necessarily both), the individual will vote. Since the effects of α and β can be manipulated independently (via changes in F or N), we *may* observe individuals who

believe our assumption is natural in many instances; for example, if voters value candidates by what policies they will implement and there is a degree of uncertainty about what campaign promises candidates will actually follow through with.

would not vote even if they knew that they are very likely to be pivotal ($\alpha \cong 1$), or if they believe the election is likely to be close ($\beta \cong 0.5$); but will vote when they are less likely to be pivotal, but think the election will be a blow-out (β is close enough to either 0 or 1).

Common Values: The other major assumption in the literature is that outcomes have a common-value component and voters receive signals about it.[33] With compulsory voting, as Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) first noted, sincere voting is in fact not an equilibrium. Surprisingly, despite this, information is still aggregated in large elections, other than in knife-edge situations. The prediction is slightly different with voluntary, instead of compulsory, participation. Krishna and Morgan (2012) demonstrate that if participation is voluntary (either free or costly) then although some individuals may not vote, individuals who do vote will do so sincerely, and information is aggregated in large elections.

In a related environment, with common values but private signals, Sobel (2014) shows that without restricting the informational environment, any action is rationalizable. Roux and Sobel (2015) impose additional restrictions on the environment to identify when group decisions are more variable than individual decisions; a notion of group polarization.

A key difference between the predictions of our model and the common-value literature is that in our model individuals may vote insincerely to avoid randomness, whereas with a common-value component, individuals vote insincerely to help ensure the selected option is optimal given the (unknown) state. These two motivations can imply very different behaviors in some circumstances. For example, adding partisan individuals who will always vote for p will push uninformed quasi-convex voters who want to match the state towards choosing p. However, as the example below illustrates, in a common-value setting individuals will instead want to more often vote against p; as Feddersen and Pesendorfer (1996) note "[uninformed independent agents] vote to compensate for the partisans". More generally, in a model with both features (quasi-convexity preferences and a common-value component) individuals may vote insincerely not only for strategic reasons but also for reasons related to their desire to reduce the randomness of the election. These mixed motivations for insincere voting will impede information aggregation.

A simple example can highlight these issues. Suppose there are five voters in a majority rule election. Voters 1 and 2 are partisans and will always vote for option p (as specified below). Voters 3, 4, and 5 care about both what state will be realized and what alternative was chosen; in particular, they want to match the chosen alternative to the state. There are two equally likely states, s_p and s_q. Suppose there are three final outcomes $\bar{x} > x > \underline{x}$. The alternatives are two lotteries p and q. p (resp., q) gives

[33] Although we consider the two assumptions about values separately, Ghosal and Lockwood (2009), Feddersen and Pesendorfer (1999), Feddersen and Pesendorfer (1997) and Krishna and Morgan (2011) consider elections in the presence of both common-value and private-value components.

x with probability ρ regardless of the state, $\bar{x}$ with probability $1-\rho$ if the state is s_p (resp., s_q), and $\underline{x}$ with probability $1-\rho$ if the state is s_q (resp., s_p). Finally, voters 3 and 4 receive perfectly revealing private signals about the state prior to voting, while voter 5 receives no signal at all.

If all voters have expected utility preferences, then consider a situation where voters 3 and 4 always vote in accordance with their perfectly revealing signals. Voter 5 now wants to condition his vote on being pivotal. Voter 5 knows that the only time he is pivotal is when the state is s_q (otherwise all other four voters are voting for p). Thus, he should always cast her vote for q. It is easy to show that such behavior on the parts of voters 3, 4, and 5 constitute an equilibrium which aggregates information.

Now, to make the minimal deviation from the standard model, suppose only voter 5 has quasi-convex preferences (everyone else still has expected utility preferences), that are non-monotone between p and q. Since states are equally likely, $p^* = p$ and $q^* = \frac{1}{2}p + \frac{1}{2}q$.[34] One can easily construct preferences such that p^* is preferred to q^*. In this case there will be no equilibrium that aggregates information.

This intuition readily extends even when the number of voters becomes large (as in the results in the literature on information aggregation) if any given voter has an equal chance of having preferences like that of voter 1, 2, 3, 4 and 5 in our simple example. In this case, information aggregation will still fail, even with many voters.

Exogenous Conformity Costs: One explanation for group shifts is an explicit benefit of conformity or for being on the winning side (for example, Callander, 2007, Hung and Plott, 2001, Goeree and Yariv, 2007, and Moreno and Ramos-Rosa, 2015). Our model generates an endogenous benefit of conformity; individuals are willing to vote against what they would choose in isolation in order to reduce the uncertainty of the outcome, or in other words to conform to what they expect to happen. Their interest in doing so is not explicit, but rather depends on the distribution of types and expected number of voters.

One way of understanding the distinct predictions of our model relative to both the standard model and to models that incorporate exogenous benefits of conformity is to look at the appropriate utility differences between options.

The simplest model of *exogenous* conformity benefits maintains the assumption of expected utility, but allows individuals to directly care about the choices of others. We can capture exogenous benefit of conformity in two steps. First, we suppose that the utility gap between choosing p and q in isolation is, as before, simply $V(p)-V(q)$. However, when choosing as part of a group, an individual's payoff for voting for an option is directly affected by the probability that option is chosen when the individual is not pivotal (i.e. β or $1-\beta$), as well as by his probability of being pivotal, α.

We can represent this distortion using a function $\varsigma_i(\beta, \alpha)$ for option $i \in \{p^*, q^*\}$ that is *added* to $V_{EU}(i^*)$, the value of an option to a standard expected utility maximizer, that is, $\widetilde{V}(i^*) = V_{EU}(i^*) + \varsigma_i(\beta, \alpha)$.Suppose $\varsigma_{p^*}(\beta, \alpha)$ is increasing in β and

[34]Note that in this example $\alpha = \frac{1}{2}$ and $\beta = 1$.

$\varsigma_{q^*}(1-\beta,\alpha)$ is increasing in $1-\beta$. Moreover, we suppose that $\varsigma_{p^*}(\beta,\alpha)-\varsigma_{q^*}(1-\beta,\alpha)$ is decreasing in α, and $\varsigma_{p^*}(\beta,1)-\varsigma_{q^*}(1-\beta,1)=0$. This captures the intuition that as a voter becomes more likely to be pivotal (and in the limit becomes the only relevant voter) then others' behavior do not influence his relative ranking of options. Given this distortion, the utility difference between casting a vote for either option becomes $\widetilde{V}(p^*)-\widetilde{V}(q^*)=\alpha(V_{EU}(p)-V_{EU}(q))+(\varsigma_{p^*}(\beta,\alpha)-\varsigma_{q^*}(1-\beta,\alpha))$. Since the utility gap now depends also on the size of the conformity distortion, we can capture the size and direction of conformity benefits by the term $\widetilde{V}(p^*)-\widetilde{V}(q^*)-\alpha(\widetilde{V}(p)-\widetilde{V}(q))$.[35] Unlike under expected utility, this gap need not be zero (and will be either positive or negative depending on β). Moreover, our assumptions imply that this difference is increasing in β and falling in α.

The utility gap in our model of quasi-convex preferences, which captures a type of *endogenous* conformity benefits, is given by $V(p^*)-V(q^*)-\alpha(V(p)-V(q))$. Similar to the exogenous case above, the gap need not be zero and will be rising in β. However, and unlike the exogenous model of conformity, $V(p^*)-V(q^*)-\alpha(V(p)-V(q))$ may be non-monotone (and so non-decreasing) in α.

More generally, the motivation for conformity in our model, i.e., reducing randomness, is distinct from potential other motivations, which often rely on a desire to feel socially integrated and so may depend on factors such as the observability of one's vote, or the extent to which the choice is being made by other voters (versus an objective randomization device).

Voting with Subjective Uncertainty: Ellis (2016) also relaxes the assumption of expected utility in a voting setting. He considers a common-value voting game with subjective uncertainty, where voters have max-min utility as in Gilboa and Schmeidler (1989). Because in a subjective environment max-min utility implies a preference for hedging, he shows that voters have a desire to randomize; i.e. they exhibit an anti-consensus effect.

[35] Note that $\widetilde{V}(p)-\widetilde{V}(q)=V_{EU}(p)-V_{EU}(q)$.

5 Appendix: Proofs

Before we discuss the proofs, whenever we consider two arbitrary options p and q, we adopt the following normalization: Recall that for all values of $\alpha, \beta \in [0,1]$, q^* and p^* are on the line segment connecting q and p in some multidimensional simplex. In order to simplify notation, we will rotate the probability simplex so that for any given p and q under consideration, this line segment runs from the origin through $e_1 = (1, 0, 0, 0...)$ and associate q with the origin. Moreover, we can now focus on the 1 dimensional case, and think of the line segment connecting 0 and 1 where we associate q with 0 and p with 1. We will thus associate a lottery $zp + (1-z)q$ for $z \in [0,1]$ with the point z. Note that since $p^* - q^* = \alpha(p-q) = \alpha$, we have that $p^* \geq q^*$ given our normalization.

Moreover, we fix representation of the preference relation $\succsim$ for each given type V_Γ, which can depend on the type Γ (we will frequently omit the dependence on Γ to simplify notation). For $z', z'' \in [0,1]$, let $\gamma(z', z'') = V(z') - V(z'')$ measure the utility gap between z' and z''. Observe that γ depends on the exact representation V. However, we will be concerned with ordinal rather than cardinal properties of γ and V.

Lemma 1 $\succsim$ satisfies strict quasi-convexity if and only if for all p and q such that $p \sim q$ there exists a $z^* \in (0,1)$ such that V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$.

Proof of Lemma 1: First we show the *if* part. Observe that the assumption implies that $V(z) < V(p) = V(q)$ for all $z \in (0,1)$. This implies quasi-convexity since it holds for arbitrary p and q such that $p \sim q$.

We now show the *only if* part. Suppose not. Then for some pair p and q such that $p \sim q$ there is no z^* with the properties as in the premise. This implies that there exists at least one interior local *maximum*, denoted $Z \in (0,1)$. Then, by continuity, there exists a neighborhood $[\underline{z}, \bar{z}] \ni Z$ such that $V(\underline{z}) = V(\bar{z}) \leq V(Z)$, violating strict quasiconvexity. $\square$

Lemma 2 For all p and q such that $p \sim q$ there exists a $z^* \in (0,1)$ such that V is strictly decreasing on $[0, z^*]$ and strictly increasing on $[z^*, 1]$, if and only if for all p and q such that $p \sim q$ and $\alpha \in (0,1)$, there exists a pair $z', z'' \in [0,1]$ with the following three properties:

1. $z' - z'' = \alpha$ and $\gamma(z', z'') = 0$.
2. For all $\widetilde{z}' > z'$, $\widetilde{z}'' > z''$, and $\widetilde{z}' > \widetilde{z}''$, $\gamma(\widetilde{z}', \widetilde{z}'') > 0$.
3. For all $\widetilde{z}' < z'$, $\widetilde{z}'' < z''$, and $\widetilde{z}'' < \widetilde{z}'$, $\gamma(\widetilde{z}', \widetilde{z}'') < 0$.

Proof of Lemma 2: We prove the only if part first. To see that 1 is implied, first consider all pairs z', z'' such that $z' - z'' = \alpha$. Observe that both $\gamma(1, 1-\alpha) > 0$ and

$\gamma(\alpha, 0) < 0$ hold by definition. By continuity there must be a point $z \in [\alpha, 1]$ such that $\gamma(z, z-\alpha) = 0$.

To see that 2 is implied, observe that since $\gamma(z', z'') = 0$, $z^* \in [z', z'']$ (if not, then the line $[0, 1]$ would have at least two local minima, a contradiction). There are two cases. If $\widetilde{z}'' > z^*$, then by Lemma 1 we have $\gamma(\widetilde{z}', \widetilde{z}'') > 0$. In contrast, if $\widetilde{z}'' < z^*$ then $V(\widetilde{z}'') < V(z'')$, and since $V(\widetilde{z}') > V(z')$, we have $V(\widetilde{z}') > V(\widetilde{z}'')$, or $\gamma(\widetilde{z}', \widetilde{z}'') > 0$. The proof that 3 is implied is exactly analogous.

To prove the if part, suppose it is not the case so that there is an interior local maxima in the interval, denoted $Z \in (0, 1)$. Then, by continuity, there exists a neighborhood $[\underline{z}, \bar{z}] \ni Z$ such that $V(\underline{z}) = V(\bar{z})$. Thus there exists an α' such that $\underline{z} - \bar{z} = \alpha'$. Observe that the pair $\underline{z}, \bar{z}$ satisfies condition 1, but not conditions 2 or 3. $\square$

Proof of Proposition 1: By construction $p^* - q^* = \alpha$. Given that, Condition 1 implies that at β^* we have $p^* = z'$ and $q^* = z''$. By Conditions 2 and 3 of Lemma 2, $\beta > \beta^*$ (resp., $\beta < \beta^*$) implies that $\gamma(p^*, q^*) > 0$ (resp.,< 0). Conversely, the pair p^*, q^* at β^* satisfies the properties of $z', z'' \in [0, 1]$ in Lemma 2. $\square$

Proof of Corollary 1: Wakker (1994) shows that convexity of g is equivalent to quasi-convexity of preferences. The result follows from Proposition 1. $\square$

Proof of Proposition 2: Recall that quadratic preferences imply mixture symmetry (Chew, Epstein, and Segal, 1991). The preference relation $\succeq$ satisfies mixture symmetry if for all $p, q \in \Delta$ and $\lambda \in [0, 1]$,

$$p \sim q \Rightarrow \lambda p + (1-\lambda)\, q \sim \lambda q + (1-\lambda)\, p$$

Suppose $q \sim p$. By mixture symmetry, we have

$$q^* = [\alpha + (1-\alpha)(1-\beta)]\, q + (1-\alpha) bp \sim (1-\alpha) bq + [\alpha + (1-\alpha)(1-\beta)]\, p \equiv \widehat{q}$$

If $\beta < 0.5$, $k = \frac{(1-\alpha)(1-2\beta)}{\alpha+(1-\alpha)(1-2\beta)} \in (0, 1)$ and we have $p^* = kq^* + (1-k)\widehat{q}$. By strict quasi-convexity $q^* \succ p^*$.

Moreover, by mixture symmetry we have

$$p^* = (1-\alpha)(1-\beta) q + [\alpha + (1-\alpha)\beta] p \sim [\alpha + (1-\alpha)\beta] q + (1-\alpha)(1-\beta) p \equiv \widehat{p}$$

If $\beta > 0.5$, $l = \frac{(1-\alpha)(2\beta-1)}{\alpha+(1-\alpha)(2\beta-1)} \in (0, 1)$ and we have $q^* = lp^* + (1-l)\widehat{p}$. By strict quasi-convexity $p^* \succ q^*$.

And if $\beta = 0.5$ and $q \sim p$ then, by mixture symmetry,

$$q^* \sim \widehat{q} = (1-\alpha)\beta q + [\alpha + (1-\alpha)(1-\beta)]\, p = (1-\alpha)(1-\beta) q + [\alpha + (1-\alpha)\beta]\, p = p^*$$

and hence $q \sim p \Rightarrow q^* \sim p^*$

To show the other direction, suppose preferences do not satisfy strict quasi-convexity everywhere. If preferences satisfy betweenness someplace, then in that region the decision-maker is indifferent to convexification. If preferences satisfy strict quasi-concavity somewhere, then we observe an anti-consensus effect in that region. □

Proof of Corollary 2: Masatlioglu and Raymond (2016) show that under $\mathbb{CPE}_M$, individuals are loss averse if and only preferences are strictly quasi-convex. Moreover, they show that if preferences can be represented with $V_{\mathbb{CPE}_M}$ then they also have a quadratic representation. The result follows. □

Proof of Corollary 3: The equivalence of 1, 2, and 3 is shown by ERR. The equivalence of 3 and 4 is Proposition 1. □

Proof of Example 1: This utility functional does not exhibit Allais-type behavior. To see this, denote the probability of h by p_h and the probability of l by p_l. The utility of a lottery $(h, p_h; m, 1 - p_l - p_h, l, p_l)$ is then

$$\begin{aligned}
&p_l^2[\phi(m,m) - 2\phi(m,l) + \phi(l,l)] \\
&+ p_l p_h[-2\phi(h,m) + 2\phi(h,l) + 2\phi(m,m) - 2\phi(m,l)] \\
&+ p_h^2[\phi(h,h) - 2\phi(h,m) + \phi(m,m)] \\
&+ p_l[-2\phi(m,m) + 2\phi(m,l)] \\
&+ p_h[2\phi(h,m) - 2\phi(m,m)] \\
&+ \phi(m,m)
\end{aligned}$$

First, we will normalize the utility values. Chew, Epstein, and Segal (1991) show that ϕ is unique up to affine transformation. So we will set $\phi(m,m) = 0$ and $\phi(m,l) = \phi(l,m) = -1$ (recall that $\phi(m,m) \geq \phi(l,m)$ by monotonicity). The other relevant values of ϕ will be stated below.

Second, recall that Allais-type behavior is equivalent to indifferent curves fanning out in the probability simplex, where the value of p_l is on the horizontal axis and that of q_h on the vertical axis. Fanning out is equivalent to the slopes of the indifference curves becoming less steep moving horizontally in the simplex. The slope of the indifference curves is equal to

$$\mu(p_l, p_h) = -\frac{2p_l[2 + \phi(l,l)] + p_h[-2\phi(h,m) + 2\phi(h,l) + 2] - 2}{p_l[-2\phi(h,m) + 2\phi(h,l) + 0 + 2] + 2p_h[\phi(h,h) - 2\phi(h,m)] + [2\phi(h,m)]}$$

Taking the derivative $\frac{\partial \mu(p_l, p_h)}{\partial p_l}$ and observing that its denominator is always positive, we know that to determine its sign (which tells us whether we get fanning out or fanning in) we only need to consider its numerator.

First, we focus on fanning out along the $p_l - axis$, and so will set $p_h = 0$ after calculating $\frac{\partial \mu(p_l, p_h)}{\partial p_l}$. Note that the derivative of the numerator of $\mu(p_l, p_h)$ with respect to p_l is $2[2 + \phi(l,l)]$, while the derivative of the denominator of $\mu(p_l, p_h)$ with respect to p_l is $[-2\phi(h,m) + 2\phi(h,l) + 2]$. We also have that at $p_h = 0$, the numerator of $\mu(p_l, p_h)$ equals $2p_l[2+\phi(l,l)]-2$ and the denominator of $\mu(p_l, p_h)$ equals $p_l[-2\phi(h,m)+2\phi(h,l)+2]+[2\phi(h,m)]$. Therefore, the numerator of $\frac{\partial\mu(p_l,p_h)}{\partial p_l}$ equals $-4\phi(h,m)-4\phi(l,l)\phi(h,m)-4\phi(h,l)-4$, meaning that we get fanning out horizontally along $q=0$ if and only if

$$-\phi(h,m)-\phi(h,l)-1-\phi(l,l)\phi(h,m)<0$$

Given our specified v and w functions, we can represent ϕ using a matrix

$$\begin{pmatrix} \phi(l,l) & \phi(l,m) & \phi(l,h) \\ \phi(l,m) & \phi(m,m) & \phi(m,h) \\ \phi(l,h) & \phi(m,h) & \phi(h,h) \end{pmatrix}$$

Substituting in our actual values (only for the lower triangle, because of the symmetry of ϕ) gives

$$\begin{pmatrix} 2 & \phi(l,m) & \phi(l,h) \\ 3.5 & 6 & \phi(m,h) \\ 6 & 10 & 16 \end{pmatrix}$$

To normalize $\phi(m,m)=0$ and $\phi(m,l)=-1$, we subtract 6 from all payoffs and then divide by 2.5. This yields the ϕ matrix

$$\begin{pmatrix} -8/5 & \phi(l,m) & \phi(l,h) \\ -1 & 0 & \phi(m,h) \\ 0 & 8/5 & 4 \end{pmatrix}$$

We then have $-\phi(h,m)-\phi(h,l)-1-\phi(l,l)\phi(h,m)=-1/25<0$, so indifference curves are fanning out. This proves fanning out along the line $p_h=0$.

In order to extend fanning out throughout the unit simplex, we use the notion of expansion paths, defined by Chew, Epstein, and Segal (1991). We will use their definition, tailored to our example, which is as follows.

Given three outcomes $l<m<h$, consider the probability simplex (i.e. triangle) over those three outcomes, as described in the text (where p_h denotes the probability of h and p_l the probability of l). Suppose that indifference curves in this space are always differentiable inside the simplex, where, as above, $\mu(p_l, p_h)$ denotes the slope of the indifference curve passing through any given point (p_l, p_h). An expansion path collects the set of all points, the indifference curve through which have the same slope (that is, (p_l, p_h) and (p_l', p_h') are on the same expansion path if $\mu(p_l, p_h)=\mu(p_l', p_h')$.)

Chew, Epstein, and Segal (1991) show that for quadratic preferences which are not expected utility, expansion paths are linear (in the case of expected utility all points in the simplex are in the same expansion path). Moreover, they show that either[36]

[36] See Lemmas A2.2-5 in their paper.

- no two expansion paths intersect (in other words expansion paths are parallel); or
- all expansion paths intersect at a single point (i.e., if two expansion paths intersect at (p_l', p_h') then all expansion paths must intersect there), which may or may not be inside the unit simplex (i.e., it is possible that the point where they intersect has p_l and p_h values greater than 1 or less than 0)

We now turn to applying expansion paths to our example. In Example 1, the "reduced form" utility function over lotteries defined over the three outcomes (taking into account our normalized values) is:

$$V(p_l, p_h) = -2p_l + \frac{2p_l^2}{5} + \frac{16p_h}{5} - \frac{6p_l p_h}{5} + \frac{4p_h^2}{5}$$

Observe that $\left(\frac{-6}{5}\right)^2 - 4 \times \frac{2}{5} \times \frac{4}{5} = \frac{36}{25} - \frac{32}{25} = \frac{4}{5} > 0$, and so we know the indifference curves take the shape of *hyperbolas*, and thus all expansion paths intersect at a single point.[37] To find this point of intersection, we simply need to find the critical point of the utility function.[38] The first order conditions demonstrate that this is at $p_l = 4, p_h = 1$. Thus, all expansion paths must intersect there, which in turns implies that, within the unit simplex, all expansion paths are positively sloped (and do not intersect within the simplex).

Consider moving from some point (p_l, p_h) to (p_l', p_h) in the probability simplex, with $p_l < p_l'$. Denote the expansion path (p_l, p_h) is on as E_1 and the expansion path (p_l', p_h) is on as E_2. Then we can find points $(\hat{p}_l, 0)$ and $(\hat{p}_l', 0)$ such that the former is on expansion path E_1 and the latter is on expansion path E_2. Since the expansion paths cannot cross anywhere other than $(4, 1)$, $\hat{p}_l < \hat{p}_l'$. But we know from our previous reasoning that, regardless of the initial value of p_l, when increasing p_l and moving along the line $p_h = 0$, the slopes of the indifference curves decrease. So the slope of the indifference curve is lower at $(\hat{p}_l', 0)$ than $(\hat{p}_l, 0)$, meaning that the slope of the indifference curve must be lower at (p_l', p_h) than (p_l, p_h). Therefore, we get fanning out as p_l increases, regardless of p_h, so long as we are inside the probability simplex. $\square$

Proof of Example 2: We consider the functional over (p_l, p_h) given by

$$V = -6p_l + p_l^2 + 7.82p_h - 3.2p_l p_h + 2.56p_h^2$$

Since $3.2^2 - 4 \times 2.56 = 10.24 - 10.24 = 0$, the indifference curves of V take the shape of *parabolas*, which have the same axis of symmetry. Thus all indifference curves either have lower contour sets that are (strictly) convex or upper contour sets that are (strictly) convex. In our case, because the axis of symmetry has a positive

[37]For details, see Chew, Epstein, and Segal (1991). Intuitively, the expansion paths all must intersect at center of the hyperbolas, or, in other words, at the point of intersection of the asymptotes.

[38]This follows from the fact that the asymptotes of the hyperbola must be on the same level set.

slope and lies below the unit simplex, preferences have convex lower contour sets and hence satisfy quasi-convexity.

Moreover, $\frac{\partial V}{\partial p_l} = -6 + 2p_l - 3.2p_h$ and $\frac{\partial V}{\partial p_h} = 7.82 - 3.2p_l + 5.12p_h$. Thus, the slope of the indifference curves is $\mu(p_l, p_h) = -\frac{-6+2p_l-3.2p_h}{7.82-3.2p_l+5.12p_h}$.

Along the set of lotteries where $p_h = 0$, $\mu(p_l, p_h)$ reduces to $-\frac{-6.+2p_l}{7.82-3.2p_l}$. Taking the derivative of this with respect to p_l gives $\frac{0.347656}{(2.44375-p_l)^2} > 0$, so indifference curves are fanning in. This proves fanning in along the line $p_h = 0$.

In order to extend fanning in throughout the probability simplex, we use expansion paths in a similar way to Example 1. Since the indifference curves are parabolas, it is the case that the expansion paths are parallel.[39] Moreover, because the axis of symmetry of the indifference curves is an expansion path, the expansion paths have positive slopes.

Consider moving from some point (p_l, p_h) to (p_l', p_h), where $p_l < p_l'$. Denote the expansion path (p_l, p_h) is on as E_1 and the expansion path (p_l', p_h) is on as E_2. Then we can find points $(\hat{p}_l, 0)$ and $(\hat{p}_l', 0)$ such that the former is on expansion path E_1 and the latter is on expansion path E_2. Since the expansion paths cannot cross $\hat{p}_l < \hat{p}_l'$. But we know from our previous reasoning that, regardless of the starting value of p_l, when increasing p_l and moving along the line $p_h = 0$ the slope of the indifference curves increase. So the slope of the indifference curves is higher at $(\hat{p}_l', 0)$ than at $(\hat{p}_l, 0)$, which, in turns, implies that the slope of the indifference curves must be higher at (p_l', p_h) than (p_l, p_h). So we get fanning in as p_l increases, regardless of p_h. □

Proof of Proposition 3: For any distribution F over types, consider the strategies as specified in the Proposition. Type I voters are indifferent between all possible outcomes and hence will be indifferent between any randomization over p and q. Because we focus on the equilibria where no individuals play a weakly dominated strategy, it must be the case that types P vote for p and Q for q in equilibria.□

Before proceeding to the rest of the proofs, we denote the induced lotteries faced by individual i of type Γ given voting pattern m and distribution F by $p_i^*((\mathbb{V}^m), \Gamma, F)$ and $q_i^*((\mathbb{V}^m), \Gamma, F)$. We sometime refer to non-monotone types, that is, types $P2$, $Q2$, or I, by NM.

Proof of Proposition 4: First, by same arguments as in the proof of Proposition 3, it is clear that in any equilibrium, $P1$ and $Q1$ types will behave like expected utility maximizers, which implies points 2 and 3.

To show the existence of an anonymous equilibrium, notice that actions can't depend on an individual's identity, just their type. Thus $\alpha_i(\mathbb{V}^m, F) = \alpha(\mathbb{V}^m, F)$ and so $\beta_i^*(\mathbb{V}^m, \Gamma, F) = \beta^*(\mathbb{V}^m, \Gamma, F)$ for all i. We prove existence by contradiction, that is we will suppose no such equilibrium exists and show a contradiction occurs. We do this in several steps.

[39] Again, see Chew, Epstein, and Segal (1991).

- Initially we suppose all NM types vote for p. Call this voting pattern $(1:1)$.[40] We will order the three NM types by increasing order of the threshold required to vote for q (given this voting pattern): $\mathbb{I}$, $\mathbb{II}$ and $\mathbb{III}$. Thus, if type $\mathbb{III}$ wants to switch their vote to q then all other NM types would as well. Since, by assumption, we are supposing this is not an equilibrium, then at least one of the three NM types wants to deviate to voting for q. Clearly individuals of type $\mathbb{I}$ must want to switch (because of our ordering assumption).

  We now order all possible individuals $1, 2, ..N$. We will consider each individual's strategy, conditional on him being of type $\mathbb{I}$ and induct on the order of the individuals. Begin with individual 1. By construction, in the proposed voting pattern, $\beta^*(\mathbb{V}^{(1:1)}, \mathbb{I}, F) > \beta(\mathbb{V}^{(1:1)}, \mathbb{I}, F)$ or, equivalently, $q^* \succ p^*$. So individual 1 in type $\mathbb{I}$ would prefer to switch to voting for q. Denote this voting pattern $(1:2)$.

  Observe that under voting pattern $(1:2)$, we have that for all other individuals both $p^*(\mathbb{V}^{(1:2)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(1:2)}, \Gamma, F)$ are closer to q than $p^*(\mathbb{V}^{(1:1)}, \Gamma, F)$ and $p^*(\mathbb{V}^{(1:1)}, \Gamma, F)$, respectively. Therefore, because all individuals in type $\mathbb{I}$ preferred to deviate from voting for p to voting for q under voting pattern $(1:1)$, it is now the case that $q^*(\mathbb{V}^{(1:2)}, \mathbb{I}, F)$ is strictly preferred to $p^*(\mathbb{V}^{(1:2)}, \mathbb{I}, F)$. Thus individual 2, if realized as type $\mathbb{I}$, will also have a strict incentive to switch his vote from p to q.

  We continue by simply inducting on the number of individuals. After all individuals with index smaller than k have switched, we have voting pattern $(1:k)$. It is clear using the reasoning described above that all individuals in type $\mathbb{I}$ with index greater than k strictly prefer $q^*(\mathbb{V}^{(1:k)}, \mathbb{I}, F)$ to $p^*(\mathbb{V}^{(1:k)}, \mathbb{I}, F)$ and the same for those with index less than k, which guarantees that they will not switch back to vote for p. Thus, we conclude this step by having a potential anonymous equilibrium where of the NM types, types $\mathbb{I}$ vote for q and the other NM types vote for p.

- Suppose again, continuing our contradiction, that this voting pattern (where of the NM types, types $\mathbb{I}$ vote for q and the other NM types vote for p) isn't an equilibrium. Denote this voting pattern by $(2:1)$. Now, we re-order the two remaining NM types that are voting for p under voting pattern $(2:1)$, calling them types $\mathbb{II}$ and $\mathbb{III}$.[41] Under our assumption that voting pattern $(2:1)$ is not an equilibrium, it must be the case that $\mathbb{II}$ types want to switch from voting for p to q.

[40] In the proof we induct on the number of types (the number on the left), and within each type, on the number of individuals within it (the number on the right).

[41] Types $\mathbb{II}$ and $\mathbb{III}$ need not correspond to the same groups as under voting pattern $(1:1)$; the ranking of the threshold to switch from p to q may be lower in one group under $(1:1)$ but higher under $(2,1)$.

We now repeat the inductive process from the previous step but for individuals in type $\mathbb{III}$; order all individuals, and conditional on them drawing that type, switching them one by one from voting for p to voting for q. Observe that after individual k in type $\mathbb{III}$ switches from voting for p to q, that for all other individuals both $p^*(\mathbb{V}^{(2:k+1)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(2:k+1)}, \Gamma, F)$ are both closer to q than $p^*(\mathbb{V}^{(2:k)}, \Gamma, F)$ and $q^*(\mathbb{V}^{(2:k)}, \Gamma, F)$ respectively. This means that (i) conditional on drawing type $\mathbb{III}$ no individual has an incentive to switch their votes, and (ii) conditional on drawing type $\mathbb{I}$ no individual would want to switch their vote back to p after any step in the inductive process. We conclude this step by having a potential equilibrium where of the NM types, types $\mathbb{I}$ and $\mathbb{III}$ vote for q and the type $\mathbb{IIII}$ vote for p.

- Lastly, we repeat the same exercise above, applying to type $\mathbb{IIII}$ voters. We will then conclude that we have an equilibrium in which all NM types vote for q, and have a strict preference to do so. This equilibrium is obviously anonymous, contradicting the assumption that no such equilibrium exists.

We now turn to proving the properties of the equilibrium. We have already proved parts 2 and 3. Suppose that an equilibrium exists with voting pattern $\mathbb{V}^m$ which induces, for each individual i, a pivot probability $\alpha(\mathbb{V}^m, F)$ and a threshold $\beta^*(\mathbb{V}^m, \Gamma, F)$. To see that 1 is true, observe that in the space of distributions F (using the weak* topology), generically $\beta^*(\mathbb{V}^m, \Gamma, F) \neq \beta_{i,\mathbb{V}^m,F}$. If in fact $\beta_i^*(\mathbb{V}^m, \Gamma, F) = \beta_{i,\mathbb{V}^m,F}$ then because of quasi-convexity the decision-maker still prefers not to randomize between the two. □

Before proceeding, we prove another useful Lemma.

Lemma 3 For all $\epsilon > 0$ there exists an N^*, such that $N \geq N^*$ implies $0 < \alpha = p^* - q^* \leq \epsilon$.

Proof of Lemma 3: Because F has full support and the fact that types $P1$ and $Q1$ vote for p and q respectively, it is always the case that any individual has a non-zero chance of being pivotal. However, as N goes to infinity the probability of being pivotal goes to 0, and thus α approaches to 0. □

Proof of Proposition 5: Because F has full support and the fact that types $P1$ and $Q1$ vote for p and q respectively, it is always the case that any individual has a non-zero chance of being pivotal. Let $\bar{z}^*$ indicate the highest value of z_Γ^* across all NM types, which means $\bar{z}^* \in (0, 1)$.[42] For a large enough f_{P1} and large enough N, in any voting pattern it is very likely, for each individual i, that p is chosen whenever i is not pivotal. Thus, for all individuals $p^*(\mathbb{V}^m, \Gamma, F)$ and $q^*(\mathbb{V}^m, \Gamma, F)$ are both in $(\bar{z}^*, 1)$, meaning that all NM individuals will choose p^*. We can conduct a similar exercise for f_{Q1}. □

[42] Recall that for any type Γ, z_Γ^* is such that V_Γ is strictly decreasing on $[0, z_\Gamma^*]$ and strictly increasing on $[z_\Gamma^*, 1]$.

Proof of Proposition 6: Proof of the first part: Each voting pattern generates a $\beta_{i,\mathbb{V}^m,F}$. Observe that since types $P1$ and $Q1$ always vote for p and q, respectively, as the proportion of NM types goes to 0 it is the case that $\beta_{i,\mathbb{V}^m,F}$ approaches some $\hat{\beta}$ regardless of the voting pattern of the NM types. Similarly, $\beta^*(\mathbb{V}^m,\Gamma,F)$ approaches $\beta^*(\Gamma)$. But since generically it is not the case that $\beta^*(\Gamma)=\hat{\beta}$ (for example, any small change in the distribution F which shifts small weight from $P1$ types to $Q1$ types will alter it), each NM type will have a unique best response regardless of the strategy of any other NM type.

Proof of the second part: Observe that if $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to 1 then all individuals will vote p. Similarly if it is arbitrarily close to 0, all individuals will vote q. If the proportion of NM types goes to 1 and all NM types vote for p, then $\beta_{i,\mathbb{V}^m,F}$ goes to 1 and so we have an equilibrium. Similar logic applies if all NM types vote for q. $\square$

Proof of Proposition 7: For the first part: Clearly when N is small enough, conditional on being realized as an actual voter, an individual puts arbitrarily high probability on being the only person, and so their vote is pivotal. Thus $P2$ and $Q2$ will almost surely determine the outcome and so always vote for p and q respectively.

We prove the second part of in two steps. First, we show that it holds for all anonymous equilibria. Recall that in all anonymous equilibria, all individuals of the same type take the same action. For large enough N, the proportion of each type in the total number of voters is known with near certainty (equals f_Γ). Moreover, fixing an equilibrium it is known exactly what action each type takes. This means that with near certainty we know what proportion of the total number of voters choose p and what proportion choose q, and hence the outcome of the voting game is known with near certainty; in other words, for all individuals $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to either 1 or to 0. Without loss of generality suppose $\beta_{i,\mathbb{V}^m,F}$ is arbitrarily close to 1. Then $p^*(\mathbb{V}^m,\Gamma,F)$ is arbitrarily close to p and since N is large, Lemma 3 implies that $q^*(\mathbb{V}^m,\Gamma,F)$ is also arbitrarily close to $p^*(\mathbb{V}^m,\Gamma,F)$ (and so to p). This immediately implies that for any Γ, $q^*(\mathbb{V}^m,\Gamma,F)$ and $p^*(\mathbb{V}^m,\Gamma,F)$ are both greater than z^*_Γ, and so all NM types prefer to choose p. This proves the second part in the case of anonymous equilibria.

The next step is to prove that with large N, generically all equilibria are anonymous. Consider two different individuals, i and j, who are considering their strategies, conditional on drawing the same type Γ. For large enough N, even if they choose different strategies, $\alpha_i(\mathbb{V}^m,F)$ is arbitrarily close to $\alpha_j(\mathbb{V}^m,F)$ (and both are arbitrarily close to 0). Moreover, $\beta^*_i(\mathbb{V}^m,\Gamma,F)$ is arbitrarily close to $\beta^*_j(\mathbb{V}^m,\Gamma,F)$. Thus, for large enough N if $p^*_i(\mathbb{V}^m,\Gamma,F)\succ q^*_i(\mathbb{V}^m,\Gamma,F)$ then $p^*_j(\mathbb{V}^m,\Gamma,F)\succ q^*_j(\mathbb{V}^m,\Gamma,F)$. We can iterate this argument over all individuals of a given type, and we obtain an anonymous equilibrium.

Thus, the only situation where we may have non-anonymous equilibria is where we have an (infinite) sequence of N along which $p^*_i(\mathbb{V}^m,\Gamma,F)\sim q^*_i(\mathbb{V}^m,\Gamma,F)$ holds. But using similar arguments as given above, it can be shown that this generically will not happen. □

Proof of Proposition 8: Suppose $f_{P1} > \epsilon > 0$. Recall we need for a proportion of at least T people to vote for q in order for it to be chosen. But even if all NM types vote for q, as T goes to 1 the probability that the proportion of votes for q is greater than T goes to 0. Thus $p^*(\mathbb{V}^m, \Gamma, F)$ and $q^*(\mathbb{V}^m, \Gamma, F)$ both go to p, so p^* is preferred over q^* by all NM types. Thus in equilibrium all NM must vote for p. □

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