Leisure-Enhancing Technological Change*

Łukasz Rachel†

November 19, 2020

click here for the latest version

JOB MARKET PAPER

Abstract

Modern economies are awash with leisure-enhancing technologies: products supplied in exchange for time and attention, rather than money. This paper studies how such technologies interact with the broader macroeconomy. The theory provides a technology-based account for the decades-long downward trend in hours worked and lackluster productivity growth observed across developed economies. In particular, since leisure technologies crowd out 'traditional' innovation, the theory sheds new light on the modern manifestation of the Solow Paradox. I show that the adverse productivity effect dominates the utility gain from the free products, leaving societies persistently worse-off. The market equilibrium is inefficient: the ad-based business model of leisure innovators means that the wrong price values leisure technologies in equilibrium; moreover, the adverse impact of leisure-enhancing innovations on future productivity is external to individual choices.

*I am deeply indebted to my advisors, Ricardo Reis, Ben Moll and Francesco Caselli for their continuous guidance and support. I am very grateful to Daron Acemoglu, Philippe Aghion, Ufuk Akcigit, Robert Barro, Charlie Bean, Olivier Blanchard, Erik Brynjolfsson, Tiago Cavalcanti, Svetlana Chekmasova, Maarten De Ridder, Nicola Fontana, Emmanuel Farhi, Matthew Gentzkow, Austan Goolsbee, Shane Greenstein, Gene Grossman, Wouter den Haan, Elhanan Helpman, Chad Jones, Per Krusell, Robin Lee, Josh Lerner, Rachel Ngai, Thomas Sampson, Larry Summers and John Van Reenen and for helpful conversations and comments. I would like to thank participants at seminars and conferences including NBER Summer Institute, LSE, Harvard, VMACS Junior Macro Conference and the RES Annual Meeting for the many comments that significantly improved the paper. I am grateful for the hospitality of the Department of Economics at Harvard University where I worked on this project as a Fulbright Visiting Fellow. Financial support of US-UK Fulbright Commission and the Bank of England are gratefully acknowledged. The views in this paper are solely mine and do not reflect the views of the Bank of England or its policy committees.

†LSE and the Bank of England. *Email:* l.p.rachel@lse.ac.uk. *Web:* https://sites.google.com/site/lukaszrachel/

1 Introduction

In models of economic growth, technological change is a catch-all generalization of a large and diverse set of innovations undertaken in the real world. In this paper I distinguish between 'traditional' product- or process-innovations and inventions that are *leisure-enhancing*.

The defining difference between the traditional and leisure-enhancing technologies is the way they are monetized. Improving a production process or introducing a new product tends to raise the profits of the innovator directly. Instead, leisure-enhancing products are often available for free, and are instead monetized indirectly through harnessing consumers' time and attention. Because of this, leisure-enhancing innovations are designed to capture consumers' time: they are *time-biased*.[1] The main insight of this paper is that the traditional and the time-biased technologies interact in ways that shed new light on important macroeconomic phenomena, such as dynamics of hours worked and a modern incarnation of the Solow (1987) Paradox,[2] with associated implications for welfare and efficiency.

Examples of leisure-enhancing technologies range from free newspapers given out on the metro and TV channels to smartphone apps.[3] These ever-more-present products have changed the nature of leisure dramatically. Social media is a telling example. With nearly 4 billion users worldwide – including 70-80% of the industrialized world's population – and average daily use in excess of two hours as of 2020, social media are ubiquitous.[4] In factor markets social media platforms attract top talent and have little trouble sourcing financing, with market valuations putting some of the firms among the world's most valued businesses. Social media platforms are also innovation hubs (Figure 1). The 'like' button, the scroll-down newsfeed, various photo filters and the like have kept populations across the globe engaged for trillions of hours over the last decade. Consumers can tap into those services largely without reaching for their wallets: it is their time, attention and data that buys them access.[5] Industry estimates suggest that over

[1] Traditional innovations do not exhibit such systematic bias. It is true that consumption of many goods and services takes time; but while some traditional innovations are increasingly time intensive (e.g. better movies), many are specifically designed to save time (e.g. a robot hoover, a high-speed train or a tax return software).

[2] In 1987 Bob Solow famously quipped that "computer age is visible everywhere except for the productivity statistics". Computer age eventually made an appearance in the mid-90s, driving much of the pick-up in growth in capital intensity and total factor productivity in the United States (see Jorgenson (2005) for a summary). However, this revival was ultimately short-lived, and TFP growth since the early 2000s has again been puzzlingly sluggish. The perception that we live in an era of rapid technological change appears to be, once again, at odds with the official statistics. The theory of leisure technologies can help explain the puzzle as it predicts that lower (traditional and well-measured) productivity growth is accompanied by potentially rapid leisure-enhancing technical change.

[3] Following an average annual growth of 160% over the past decade, the number of apps available to be downloaded from the Google Play Store has reached almost 3 million in June 2020, with a vast majority – around 96% – available free of charge (see Appendix A).

[4] Source: Globalwebindex, a consultancy which runs a large scale (550,000 participants) survey of online behaviors.

[5] In the model I focus on the role of consumers' time spent on the technologies, which I view as correlated with attention. Arguably time spent on using technologies is a pre-requisite to gathering data, so in that sense I also capture the data gathering motive. However, data gathering has some distinct features, and is more relevant for

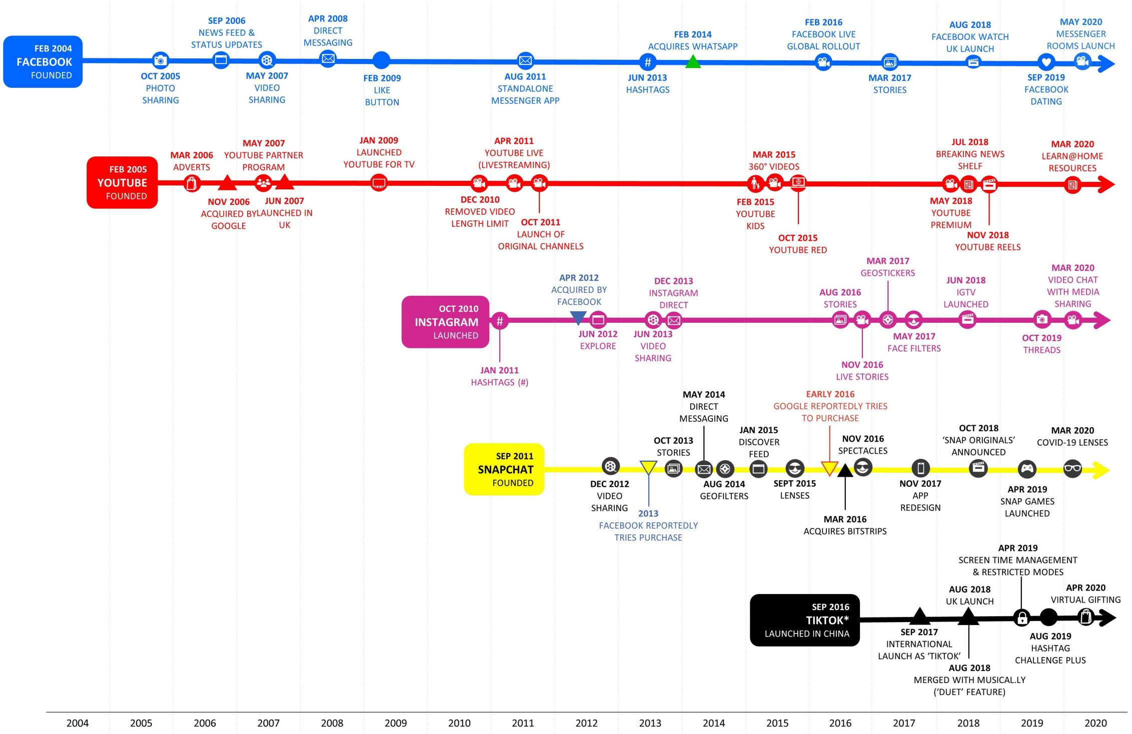

Figure 1

Timeline of Selected Innovations in the Social Media Sector

Source: Ofcom.

90% of social media firms' revenues comes from advertising (OfCOM, 2019).

These characteristics carry beyond the handful of social media platforms operating in recent years. The 'leisure sector' as a whole is an important cluster of innovation and discovery, and has become more so over the recent past. For example, a proxy for its share in overall R&D spending across the industrialized world has more than doubled between 2005 and 2014, according to the data produced by the OECD.[6] Furthermore, monetizing time and attention is hardly a new phenomenon. Using data for the United States, the first panel of Figure 2 shows that free ad-financed products have been around since at least 1950s. The share of advertising revenues in GDP follows a similar pattern. And historically leisure technologies have been instrumental in shifting time allocation patterns: for example, Aguiar and Hurst (2007a) and Gentzkow (2006) find evidence that the introduction of the television in the 1950s and 1960s had a large impact on time allocation patterns in the United States, and Falck *et al.* (2014) document the significant impact on leisure time of the roll-out of the internet in Germany in the 2000s. Both episodes constituted an expansion of free-of-charge, ad-financed services available to consumers.

the digital technologies. I discuss these issues further in the concluding section.

[6]This refers to the share in total private sector R&D of the sectors that can be loosely classified as leisure sectors. See Figure A.2 in Appendix A.

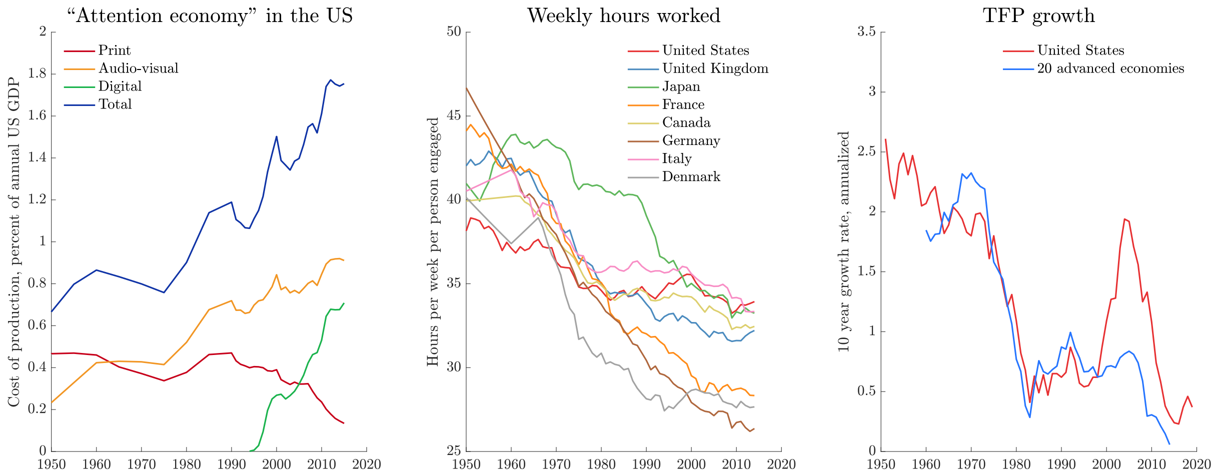

Figure 2

Motivating Trends: Free Products in the United States, and Cross-Country Trends in Hours Worked and Total Factor Productivity

Notes: Estimates of the cost of production of free consumer services are from the Bureau of Economic Analysis (Nakamura *et al.* (2017)). The figure shows the ratio of free consumer content, measured by the costs of production, to GDP. Thus, for example, it does not attempt to capture utility benefit of Facebook, but only the cost of providing it. Hours worked are from Penn World Tables 9.0 (Feenstra *et al.* (2015)). The US TFP growth rate is the utilization-adjusted series following Basu *et al.* (2006). The TFP growth rate for advanced economies is constructed by the IMF and is PPP-weighted (Adler *et al.* (2017)). Both series show 10-year growth rates.

The technological developments in leisure have occurred against the backdrop of a trend decline in hours worked (Figure 2, middle panel) and slowing growth of labor- and total factor productivity (the right panel). How, if at all, are these trends linked?

To begin thinking about this question, I use an illustrative setup with exogenous growth in leisure technology, modeled as a trend in the weight on leisure in the utility function. While this exercise puts aside the crucial question of what might bring about such a trend, it allows for a preview of the interactions between the leisure-enhancing and traditional technologies. I show that with an exogenously increasing weight on leisure utility, hours worked decline at a constant rate and yet this decline is consistent with balanced growth and thus with the Kaldor Facts. More importantly, if the development of traditional technology is endogenous and relies on human input, the decline in hours worked has a negative effect on productivity growth in the 'traditional' sector. These insights suggest that, to the extent that leisure technologies can be thought of as shifting the relative weight on the utility of leisure relative to consumption, they provide a candidate explanation for the joint dynamics of hours and productivity elsewhere in the economy. But why is it sensible to think of leisure technologies in this way? Where do these technologies come from? And are there any other ways through which they interact with the macroeconomy?

To study these issues more closely I develop a tractable general equilibrium theory of an *attention economy* – the economic ecosystem that supports the existence of leisure-enhancing innovations. The essence of an attention economy is that brand equity – a form of intangible capital acquired by firms through advertising – requires consumers' time and attention.

On the consumer side, the model builds on Becker (1965), with leisure utility generated from combining users' time with market goods and services. The novel aspect is that I focus on services that are *free* (available at zero prices) and *strongly non-rival* (the marginal cost of supplying an extra user is zero) – such as TV channels, web content or social media. This focus is justified given the proliferation of such services; it also plugs the gap in the existing literature, which has focused on the role of durable goods (such as TV sets, computers, smartphones) and fixed-cost expenditure (e.g. broadband subscription) in household production of leisure. I show that within such a framework the index of leisure technology naturally shows up as a time-varying shifter in the household utility function.

On the firm side, I derive a tractable extension to the canonical monopolistic competition setup in which firms demand brand equity in equilibrium.

Between the consumers seeking free entertainment and firms demanding brand equity are the platforms. On one side they innovate on leisure technologies in order to capture 'eyeballs', on the other they supply businesses with ads. I derive a closed-form expression for the equilibrium supply of leisure technologies to the market, hence endogenizing leisure-enhancing technological change.

Embedding these features in a setting with endogenous 'traditional' innovation brings out the following insights.

Leisure-enhancing technologies emerge endogenously on the growth path, once the economy is sufficiently developed. This is driven by the interaction between a feature of household preferences (leisure technologies must be sufficiently developed for households to use them) and a market size effect (the economy must be sufficiently large to support platforms' business model). The steady-state equilibrium thus takes a form of a *segmented balanced growth path* (sBGP). The remaining results of the paper concern the changing nature of economic growth between the two segments of the sBGP, elucidating this new kind of structural change.

One feature of the equilibrium is that hours worked decline in the presence of leisure-enhancing innovations. Ever-improving leisure options tilt the balance towards more free leisure and less work and traditional consumption. This prediction matches the trend in time use observed across countries over long periods (Aguiar and Hurst, 2007b) and provides a new way to interpret the recent dramatic shifts in time allocation (Appendix A presents more evidence on these shifts).

The growth rate of productivity in the 'traditional' sectors of the economy declines following the entry of the platforms. There are three channels through which this effect operates. The first channel underlines the heightened competition for time and attention that is characteristic of the attention economy: better leisure leaves less time for productive activities.[7] Second, the new sector competes with the traditional R&D sector in factor markets (e.g. for talent). Third, brand equity competition results in profit shifting, away from competing firms and towards the platform sector. To delineate these effects I derive analytical expressions for the steady state shares of labor employed in the two (traditional- and leisure-) R&D sectors. I find that the emergence of the attention economy accounts for between a third and a half of the slowdown in TFP growth observed in the data.[8]

The theory also generates novel insights with regards to the measurement of the attention economy. Two questions arise in the context of leisure-enhancing technological change: first, is GDP mismeasured? And second, does GDP do a good job of capturing changes in welfare? I answer the first question in the negative: the components that are missing from GDP are too small to make a difference. The answer to the second question is a qualified 'yes': to the extent that increases in usage go hand-in-hand with increases in utility,[9] GDP does miss a potentially sizeable welfare effect of leisure technologies. Leisure-enhancing technologies introduce a systematically *growing* wedge between GDP and welfare.[10] All in all, they are associated with declining GDP growth, but this decline might be less of a concern since GDP does not reflect the positive welfare effects.

An obvious next question is what happens to welfare on net: does the leisure economy make us better off? Do the leisure technologies make up for the loss in traditional technologies? I show analytically that in the period immediately after leisure technologies emerge, the welfare

[7]The framework for analysis of leisure technologies in this paper builds on the semi-endogenous growth paradigm (Jones, 1995), in which the long-run growth rate of total factor productivity is tied to the growth rate of the labor input used to generate ideas. The decline in hours worked translates into slower growth of the pool of resources that are devoted to generating new ideas and knowledge. But as I explain below, the insights carry over into any model in which innovation and adoption of ideas require human cognition.

[8]The quantitative exercise holds structural parameters of the model fixed. But it is likely that these parameters might have changed over time: for example, digital technologies of the 2010s are in some 'structural' respects quite different from audio-visual technologies of the 1970s. In Appendix E I consider the response of the economy to shifts in the preference and technology parameters that could be thought of as capturing some of the features of the digital revolution. I show that these shifts may have further contributed to slowdown in traditional productivity more recently.

[9]This qualification is an important one. For example, there is growing evidence of an association between greater social media use and higher depressive and anxiety scores, poor sleep, low self-esteem and body image concerns (Kelly *et al.* (2018); Royal Society for Public Health (2017)). There is evidence that at least for some users social media is addictive (see e.g. https://www.addictioncenter.com/drugs/social-media-addiction/ who explain that "social media platforms produce the same neural circuitry that is caused by gambling and recreational drugs. Studies have shown that the constant stream of retweets, likes, and shares from these sites have affected the brain's reward area to trigger the same kind of chemical reaction as other drugs, such as cocaine."). Nonetheless, at a broader level, measures of leisure and leisure time are correlated with higher life satisfaction and well-being (OECD (2009)). So without detracting from the potential importance of these concerns, the present paper abstracts from the direct negative effects of the leisure technologies on wellbeing and focuses on the case where revealed preference argument holds. Note that even with this assumption it finds negative welfare effects. Introducing habit formation and addiction into the analysis would only strengthen these results.

[10]These findings suggest that leisure time enhanced with technology should be an important component in the measures of economic wellbeing, in the spirit of Nordhaus and Tobin (1972) and Stiglitz *et al.* (2009).

response is determined by the response of consumption utility. I also demonstrate in simulations that the net-negative welfare effect is persistent. Two complementary intuitions are available for this result. First, when the *level* of consumption utility is greater than the *level* of utility derived from leisure technologies, a negative effect on the former is likely to dominate any positive effect of the latter. The second intuition is that when consumers allocate their time they take wages as given; yet in an economy with endogenous growth more leisure today drives down future growth in wages – a dynamic externality.

The final set of results goes deeper into the efficiency properties of the decentralized equilibrium, studying the dynamic externality described above and the static inefficiency, which arises because of the zero-price nature of the leisure services. Intuitively, the equilibrium supply of leisure technologies is guided by the wrong price (the price of brand equity and not the marginal rate of substitution between consumption and leisure).

Related literature. In proposing a directed-technology explanation for the trend in hours worked, this paper brings together the literatures on endogenous innovation[11] with that on the long-run shifts in time allocation.[12] Since the seminal paper of King *et al.* (1988) which derive the 'balanced growth' preference class, most growth models have featured constant hours worked along the balanced growth path. This prediction is rejected by the historical data which show a steady long-run decline of around -0.4% per annum across a wide range of countries (Jones, 2015).[13] In contributions closely related to this paper, Ngai and Pissarides (2008) and Boppart and Krusell (2020) provide two alternative accounts for this trend: the former paper highlights the role of differential sectoral growth rates and non-separability of preferences while the latter characterizes the preference class that delivers an income effect larger than the substitution effect along the BGP. Both of these papers and other related contributions assume growth is exogenous. Instead, this paper assumes separable balanced growth preferences and instead focuses on the profit-driven equilibrium rise of the attention economy as an explanation for the shifts in time use.

The present paper extends the line of research that starts from the work of Becker (1965), recently summarized in Aguiar and Hurst (2016), which develops a unified theory of consumption and time allocation. The contribution is to develop a tractable model for analysis of zero price

[11]It is impossible to cite all, or even most, of the contributions in this vein. Some of the prominent examples include Romer (1990), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Segerstrom (1998), Acemoglu (2002) Acemoglu and Guerrieri (2008), and Aghion *et al.* (2014).

[12]Prominent contributions include Aguiar and Hurst (2007a), Ramey and Francis (2009), Aguiar *et al.* (2017), Vandenbroucke (2009), Aguiar *et al.* (2012) and Scanlon (2018).

[13]The aggregate trend masks the underlying heterogeneity, not least by income and education groups. Leisure inequality has increased as the less well-off, less educated households increased their leisure time by more than the rich (Aguiar and Hurst (2008), Boppart and Ngai (2017a)). The free leisure technologies could be considered an important component driving this trend. Analysis of this hypothesis is beyond the scope of this paper and is left for future work.

services and study the implications. The focus on leisure technologies brings the paper close to Aguiar *et al.* (2017) who investigate how video games have altered the labor supply of young men in the United States. Relative to that paper I cast the net more broadly: intertemporally, cross-sectionally, and in terms of the scope of analysis.[14]

The paper also contributes to the literature on the productivity slowdown and the mismeasurement hypothesis.[15] Within that literature the structural model developed here can serve as a useful organizing framework for the analysis of a narrower issue of 'free economy'. The paper shows that while mismeasurement of GDP (a production-based metric) is second order, a growing disconnect between GDP and measures of economic wellbeing is likely. The decline in productivity growth is thus less concerning than it otherwise would be. Nonetheless, the persistent negative welfare impact demonstrates that the attention economy poses challenges that go far beyond mismeasurement alone.

Finally, this paper builds on the literature on two-sided markets, intangible capital and advertising in industrial organization and in macroeconomics.[16] Relative to this literature my contribution is to point out a crucial feature of advertising, namely that its production requires consumers' time and attention as inputs. Viewed from this perspective, competition through ads underlies the very existence of the attention economy. The paper embeds this relationship in a general equilibrium setting and shows that it opens up new ways of interpreting salient trends observed in the data.

Roadmap. Section 2 sets the scene by illustrating the growth effects of exogenous leisure technologies. Section 3 outlines the model of the attention economy and defines the equilibrium. The main results characterizing the balanced growth equilibrium are presented in Section 4.

[14]My theory speaks to historical events such as the roll-out of the TV in the 1950s as well as the more recent digital trends. Also, I consider the whole swathe of free technologies which are used by a vast majority of the population, whereas Aguiar *et al.* (2017) focus on computer games which are used primarily by young men. Finally, Aguiar *et al.* (2017) focus on the labor supply aspect, whereas I venture beyond that, also exploring the implications for total factor productivity, measurement and welfare.

[15]Useful references include Brynjolfsson and Oh (2012), Byrne *et al.* (2016a), Bean (2016), Bridgman (2018), Syverson (2017), Coyle (2017), Aghion *et al.* (2017), Nakamura *et al.* (2017), Hulten and Nakamura (2017), Brynjolfsson *et al.* (2018) and Jorgenson (2018).

[16]Classic references on the economics of platforms are Rochet and Tirole (2003) and Anderson and Renault (2006) who study the equilibrium pricing decisions in two-sided markets. Relative to that literature I explore the implications of the two-sided market structure in a macro setting, drawing on the lessons that this literature has offered on optimal pricing. Specifically I assume that platforms do not charge for the leisure technologies. In Appendix D I show how the insights from this literature help rationalize this assumption. There is an extensive literature on the economics of advertising, going back to Marshall (1890) and Chamberlin (1933), and summarized in the IO Handbook Chapter by Bagwell (2007). Several papers analyzed theoretically the way in which ads enter the consumer problem, and what the positive and normative implications are (Dorfman and Steiner (1954), Dixit and Norman (1978), Becker and Murphy (1993), Benhabib and Bisin (2002)) as well as the businesses decisions to invest in and accumulate intangible capital (Hall (2008), Corrado and Hulten (2010), Corrado *et al.* (2012), Gourio and Rudanko (2014)). The present paper assumes a neutral formulation in terms of direct utility impact of advertising, and focuses instead on the indirect impact of the advertising production process. On the firm side it proposes a simple and tractable way to incorporate advertising in a model of monopolistic competition.

Section 5 illustrates the magnitudes. Section 6 discusses the measurement challenges. Section 7 studies the efficiency properties of the market equilibrium. Section 8 concludes with a historical narrative through the lens of the model and a discussion of areas for future work.

2 Growth effects of exogenous leisure-enhancing technological change

To illustrate the long-run growth effects of leisure technologies and to set the stage for the analysis that follows I begin with a simple setup with *exogenous* leisure-enhancing technologies (denoted M) and *endogenous* growth of 'traditional' total factor productivity (denoted A). The economy is populated by $N = N_0 e^{nt}$ households with balanced growth preferences (King *et al.*, 1988) who maximize their lifetime utility subject to a usual flow budget constraint:[17]

$$\max_{c,h} \int_0^\infty e^{-\rho t} \left(\log c + M^\psi (1-h)\right) dt \quad \text{s.t. } \dot{a} = ra + wh - c. \tag{1}$$

where c is consumption and $1-h$ is leisure time. The only difference to the standard setup is that instead of being a *parameter*, M is a *variable*.[18] One of the contributions of this paper is a theory of M, which elucidates why M enters the utility function in this way, and when, why and how it changes over time. I develop this theory in the following section. For now, I simply assume that M grows at an exogenous rate γ_M satisfying $n > \psi\gamma_M$ (throughout the paper notation γ denotes net growth rates).

The supply side of the economy is standard, with a constant-returns final good production function and profit-driven innovation, as in Romer (1990) and Jones (1995). On the balanced growth path wages increase with A, which expands as a result of R&D activity. New ideas are developed by researchers, whose success rate depends on the stock of knowledge:

$$\underbrace{\dot{A}}_{\text{new ideas}} = \underbrace{L^A}_{\text{researcher-hours}} \cdot \underbrace{A^\phi}_{\text{success rate}} \tag{2}$$

where $L^A := N \cdot h \cdot s_A$ and s_A is the share of labor input in R&D. I assume that $\phi < 1$.[19]

This coarse description of the economy omits many relevant details but is sufficient to gain insights into the interactions between leisure- and traditional- technologies.

[17] In the budget constraint, w is the wage rate and a denotes the level of assets. I normalize the time endowment to 1.

[18] Parameter ψ controls the elasticity of utility to leisure technologies. In the full model below ψ will be derived as a combination of the underlying structural preference parameters.

[19] This places my benchmark framework within the semi-endogenous class of growth models (Jones, 1995). The evidence does indeed suggest that ideas "are getting harder to find", supporting the assumption of $\phi < 1$ (Bloom *et al.*, 2020). But the lessons here are more general and extend beyond this particular underlying growth paradigm. I discuss this issue further below.

Solving problem (1) we obtain that households choice of hours satisfies

$$h = \min\left\{1, \frac{\Phi}{M^{\psi}}\right\}, \tag{3}$$

where $\Phi := \frac{1-\alpha}{1-s_A}\frac{Y}{C}$ is a variable that is constant on the balanced growth path (BGP) – an equilibrium where all variables grow at constant rates. Note that the choice of hours is independent of wage w: with balanced growth preferences, income and substitution effects of rising wages cancel out. Balanced growth preferences thus conveniently isolate the effect of leisure technology alone. Indeed, for M sufficiently large, (3) implies

$$\gamma_h = -\psi\gamma_M, \tag{4}$$

that is, hours worked decline at a rate proportional to the growth rate of M.

Differentiating (2) with respect to time yields the expression for the growth rate of A on the BGP:

$$\gamma_A = \frac{n + \gamma_h}{1-\phi}. \tag{5}$$

Combining (4) and (5) gives the following result:

Proposition 1. ***Growth effects.*** *Suppose M_0 is large and $n > \psi\gamma_M$. Then growth is balanced, with hours declining at a constant rate given by (4) and A increasing at a constant rate given by*

$$\gamma_A = \frac{n - \psi\gamma_M}{1-\phi}.$$

The growth rate of A is decreasing in γ_M.

The result is simple yet striking: leisure-enhancing technology weighs down on the growth rate of the 'traditional' economy not just directly through the labor input, but also indirectly through TFP growth. The mechanism is straightforward: the long-run growth rate of A is pinned down by the growth rate of the pool of resources devoted to generating ideas. Leisure technologies effectively reduce the growth of that pool.

Beyond the specifics of the ideas production function in (2), a broader interpretation of this result is that productivity-enhancing improvements and discoveries rely on human input, making time and attention important determinants of long-run growth. Note also that the adverse effect on productivity is external to the individual choices: consumers choose h taking wages (and so the level of A) as given.

Balanced growth preferences with growing M. The setup above assumed that the utility function is linear in leisure (defined as $l := M^{\psi}(1-h)$), which is a particular case of separable “balanced growth preferences”:

$$\log c + \frac{l^{1-\eta}}{1-\eta}, \quad 0 \leq \eta < 1. \tag{6}$$

The restriction $\eta = 0$ imposed above turns out to be important for generating balanced growth with growing M. To see why, consider the intratemporal optimality condition implied by (6):

$$\frac{w}{c} = M^{(1-\eta)\psi} (1-h)^{-\eta}. \tag{7}$$

Suppose there exists a balanced growth path with variables increasing at constant rates. Condition (7) implies that the following must hold: $\gamma_w - \gamma_c = (1-\eta)\psi\gamma_M - \eta\gamma_{1-h}$, where again γ_x denotes the net growth rate of x. Furthermore, the budget constraint implies that on the balanced growth path consumption and labor income must grow at the same rate: $\gamma_c = \gamma_w + \gamma_h$. Together these imply:

$$-\gamma_h = (1-\eta)\psi\gamma_M - \eta\gamma_{1-h}. \tag{8}$$

Since clearly it is impossible for h and $1-h$ to simultaneously grow at constant non-zero rates, there are only two scenarios under which (8) holds: it must be that either $\gamma_M = \gamma_h = \gamma_{1-h} = 0$ (the standard case without leisure technologies) or that $\eta = 0$. This proves that for the case with $\gamma_M > 0$, $\eta = 0$ is the necessary restriction for the model to be consistent with exact balanced growth.[20]

Note however that a closely related function which takes disutility of work as an argument does not suffer from the same issue. Letting ω denote the disutility of labor, we have that

$$\log c - \frac{\omega^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} \quad \omega := M^{\psi}h, \ \theta > 0, \tag{9}$$

which gives the equivalent to equation (8):

$$-\gamma_h = \left(1 + \frac{1}{\theta}\right)\psi\gamma_M - \frac{1}{\theta}\gamma_h,$$

which reduces to (4) for *any* value of the Frisch elasticity θ. Thus in most applications formulation (9) can be used without imposing any parametric restrictions on the Frisch elasticity and still being consistent with balanced growth. The two formulations – the one with linear utility of leisure as in (1) and the one with convex disutility of work in (9) – yield identical conclusions in terms of the elasticity of hours worked to leisure technology (in either case this elasticity is equal

[20]In the case with $\gamma_M, \eta > 0$ growth can still be balanced asymptotically, since in the long-run $1-h$ converges to a constant ($= 1$) and so γ_{1-h} converges to zero. However, hours worked approach zero at that point limiting applicability in practice.

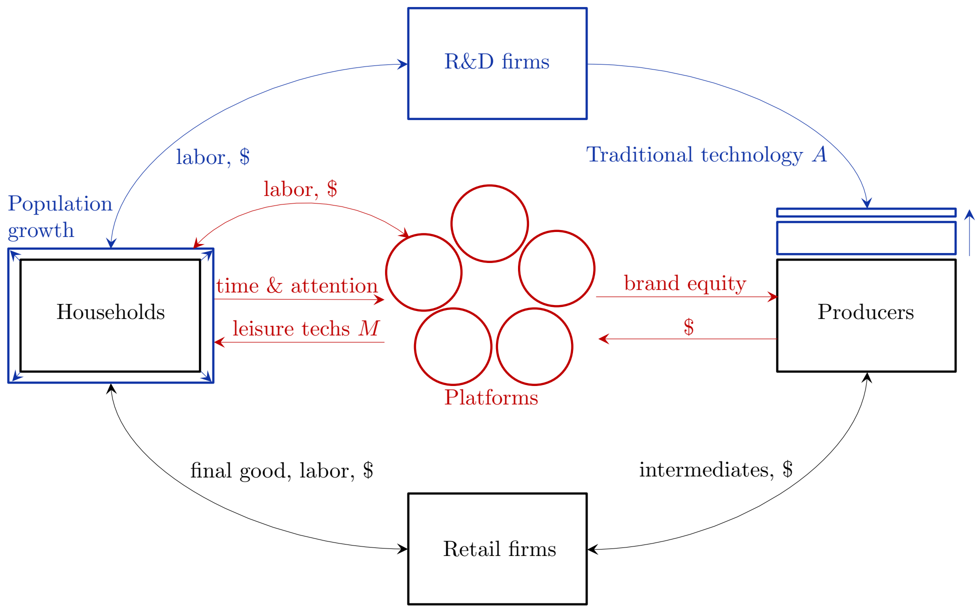

Figure 3
The Model Structure

to $-\psi$). I work with the more flexible (9) throughout, except for the welfare analysis in Sections 6 and 7 where (6) is more appropriate.

3 Endogenous M: the attention economy

The previous Section provided a preview of the interactions between leisure- and traditional technologies but it assumed that the leisure technologies are exogenous. I now turn to the all-important question of what M is and how it is determined in equilibrium.

The framework builds on the classic monopolistic competition setting (Dixit and Stiglitz, 1977) with endogenous horizontal innovation as in Romer (1990) and Jones (1995). Figure 3 illustrates the structure. Two ingredients are necessary to capture the main mechanisms: consumers ought to engage with the available leisure technologies, and this engagement ought to make the production of brand equity possible. These are represented by the arrows to the left and to the right of *platforms* in the middle of Figure 3, respectively. The platforms are the two-sided businesses that are at the centre of the attention economy. I now lay out the assumptions on the behavior of different agents in this economy, starting with households, then describing firms in the business sector, before finally turning to the platforms.

3.1 Households

3.1.1 Activity-based framework

If l denotes leisure and ℓ denotes time spent on leisure, then in a standard macro model $l = \ell$ by the equivalent definitions of the two variables.

Analysis of leisure technologies requires a more careful treatment of leisure. To address this, I develop a tractable formulation in which households derive utility from a range of *leisure activities*:

$$l = \left(\int_0^M \underbrace{[\min\{\ell_\iota, m_\iota\}]}_{\text{activity } \iota}^{\frac{\nu-1}{\nu}} d\iota \right)^{\frac{\nu}{\nu-1}}. \tag{10}$$

where ℓ_ι is the time spent on activity ι and m_ι denotes the leisure services required for that activity. There is a continuum of M activities so that total leisure time is $\ell := 1 - h = \int_0^M \ell_\iota d\iota$. Parameter $\nu > 1$ is the elasticity of substitution across activities.[21]

Note the assumptions embodied in this formulation: first, $\nu > 1$ implies that different activities are substitutes. As long as ν is finite, there is *love of variety* in leisure options. Second, within each activity, there is no substitutability between time and free services: enjoying a TV show or browsing the web requires both, in fixed proportions. This is a natural assumption given zero prices: a positive elasticity would lead to a complete substitution towards free services.[22]

To see how this formulation affects household's dynamic optimization problem, it is useful at this stage to consider optimal behavior given (10). Households choose how much time and leisure services to devote to each individual activity. Clearly, the Leontief structure implies that $m_\iota = \ell_\iota \forall \iota$[23] and, given the symmetry of the problem, it is easy to see that the optimal choice is to spread leisure time evenly across the activities, which implies the following Lemma:

Lemma 1. ***Leisure and leisure time.*** *Optimal allocation of time across activities implies:*

$$l = \ell M^{\frac{1}{\nu-1}}. \tag{11}$$

Proof. Appendix B. $\square$

Relative to the standard formulation where $l = \ell$, the framework highlights the importance

[21] Activities that do not involve free leisure technologies, such as walking in a park or hiking, are outside of the benchmark model for simplicity, but they are straightforward to incorporate. Appendix H presents the extension along these lines.

[22] In practice, besides time and leisure services, paid-for consumption goods – broadband charges, TV sets, phones or computers, for example – are inputs in leisure production. Appendix C proposes a more general leisure production function in which there are complementarities between leisure and consumption goods, and shows that the insights continue to hold in that more general formulation.

[23] This is the case even if leisure services are supplied at positive prices. Positive prices would only alter the budget constraint of the household and not the form of leisure utility.

of technology for generating leisure utility. We can define the disutility from labor ω in an analogous fashion, by multiplying hours worked by the same factor:

Definition 1. Instantaneous disutility from labour is $\omega := hM^{\frac{1}{\nu-1}}$.

We're now in the position to spell out the dynamic optimization problem of a representative consumer.

3.1.2 Representative household's problem

The representative household chooses the path of consumption and hours worked to maximize discounted lifetime utility:

$$\begin{aligned} &\max_{c,h} \int_0^\infty e^{-\rho t} \log c - \frac{\omega^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} dt \quad \text{subject to} \\ &\dot{K} = whN + \text{asset income} - cN, \\ &\omega = hM^{\frac{1}{\nu-1}} \end{aligned} \tag{12}$$

where c is consumption, K is the aggregate capital stock, w is the hourly wage rate.[24] Note that, since $m(\iota)$ are available at zero prices, they do not show up in the household budget constraint.

3.2 Traditional production and brand equity competition

3.2.1 Final good

Competitive final good producers combine labor with differentiated intermediate goods $x_i,\ i \in [0, A]$. The sole departure from the benchmark expanding variety framework is that the desirability of product i is determined by the brand equity capital of its producer:[25]

$$Y = \int_0^A \left(\left(\frac{b_i}{\bar{b}} \right)^{\chi \cdot \Omega} x_i \right)^{\alpha} L_Y^{1-\alpha} di, \tag{13}$$

where $b_i \geq 0$ is the brand equity associated with product i and $\bar{b}$ is the average brand equity across all firms: $\bar{b} := \frac{1}{A} \int_0^A b_i di$. Fraction $\frac{b_i}{\bar{b}}$ measures the relative advantage of firm i due to

[24] Asset income is equal to $rK + A\Pi + J\Pi_B - V\dot{A}$, where V, Π and Π_B are value of the blueprint, profits of a firm in the intermediate sector and profits of a platform, respectively, and $\dot{A}$ is the flow of patents purchased by the household each period.

[25] In this simple setting each producer operates a single production line and sells only one product. With multiple production lines, large firms may find advertising more profitable than smaller firms since advertising one product has spillover effects to other products under the same brand (a phenomenon known as *umbrella advertising*). See Cavenaile and Roldan-Blanco (2020) who study this aspect of brand equity competition using a variant of Akcigit and Kerr (2018) model with advertising.

its holdings of brand equity, as compared to its competitors.[26] Parameter $\chi \geq 0$ measures the perceived effectiveness of ads. Indicator variable Ω equals to 1 when $\bar{b} > 0$ and 0 otherwise, making (13) well-defined when no firm invests in brand equity.

The implication of (13) is that only by investing in brand equity by more than its competitors can a firm boost demand for its product: brand equity is all about *relative* advantage. This assumption is supported by empirical evidence on advertising, starting from the early studies such as Borden (1942) and Lambin (1976), and through more recent work summarized in Bagwell (2007). This literature suggests that marketing may have some positive short-lived impact on individual firm's sales, but that the effect tends to disappear once the unit of observation is expanded to a broader sector (or to the macroeconomy).

Beyond its simplicity and empirical relevance, an advantage of this formulation is its neutrality: in a symmetric equilibrium, brand equity investments have no *direct* impact on aggregate productivity or consumer welfare (since in such equilibrium $b_i = \bar{b}\, \forall i$ and the $\frac{b_i}{\bar{b}}$ term vanishes). This is a neutral stance since there are many possible channels outside of the model but analyzed in the literature, both positive and negative, through which brand equity might affect aggregate output and welfare.[27] To give just a few examples: on the positive side, brand equity investments can provide consumers with useful information about available products, which might lead to fiercer competition, lowering the distortion that arises from monopoly power (Nelson, 1974; Butters, 1977; Grossman and Shapiro, 1984; Milgrom and Roberts, 1986; Stahl, 1989; Rauch, 2013); they can complement consumption goods (Becker and Murphy, 1993); or, when interpreted as accumulation of information and data, they can help firms better target consumer needs (Jones and Tonetti, 2019; Farboodi and Veldkamp, 2019). Examples of negative effects include the possibilities that the brand equity competition may lead to greater product differentiation, raising markups and exacerbating the monopoly distortion (Molinari and Turino, 2009); that aggressive advertising might become a nuisance to consumers (Johnson, 2013); that collection of mass datasets might raise privacy concerns (Tucker, 2012); and that advertising leads to envy and supports 'conspicuous consumption', ultimately diminishing consumers' utility (Veblen, 1899; Benhabib and Bisin, 2002; Michel *et al.*, 2019). Incorporating some of these channels into the theory would necessarily be somewhat ad-hoc and would detract from the focus of the paper. Consequently, the formulation in (13) puts these considerations aside and allows the paper to focus on teasing out the *indirect* macro effects of brand equity competition

[26] The final-good firms anticipate any shifts in relative demand due to firms' intangible capital investments and demands more of the varieties with higher brand equity. This setup is isomorphic to the model where consumers were choosing the products directly and their relative taste for specific varieties was driven by brand equity.

[27] The literature has distinguished three broad views of advertising: the *persuasive view* which sees advertising as primarily shifting demand curves outwards or lowering the elasticities of substitution across goods; the *informative view* according to which ads help consumers make better choices; and the *complement view* which sees ads as complements to the advertised consumption goods. The formulation in this paper is most closely aligned with the persuasive view; but the relative competition assumption implies that direct negative effects associated with ads in the persuasive view wash out in equilibrium.

and the attention economy.[28]

3.2.2 Intermediate goods

The differentiated goods are produced by a continuum of monopolistically competitive firms. Every firm has to invest in a blueprint as the prerequisite of production. The owner of a blueprint is the only producer of the respective good. Technology is such that each unit of capital, which can be rented at net rate r and depreciates at rate δ, can be used to produce a unit of the intermediate good. Furthermore, each producer can invest in intangible capital in the form of brand equity, which can be purchased at price p_B. For simplicity, I assume that brand equity depreciates fully after use, so that producer i's problem remains static. The profit maximization problem of producer i is:

$$\max_{x_i,p_i,k_i,b_i} p_i x_i - (r+\delta)k_i - p_B b_i \tag{14}$$

subject to the linear technology $x_i = k_i$, the demand curve for its product and taking the average intangible investment of its competitors $\bar{b}$ as given.

3.2.3 Traditional R&D sector

New designs of differentiated goods are invented by the R&D sector employing researchers (equation (2)). The value of a blueprint at time t is:

$$V(t) = \int_t^\infty \Pi(\tau) e^{-\int_t^\tau r(u)du} d\tau. \tag{15}$$

There is free entry to the R&D sector so that

$$V \cdot A^\phi = w. \tag{16}$$

3.3 Platforms

3.3.1 Market structure

Platforms are the centerpiece of the attention economy. I assume that there are J of them, with J exogenous and constant. Without loss of generality, I assume that platforms engage in Cournot competition in the brand equity market, implying that J determines the degree of competition and mark-ups in that market. I also make the following assumption:

Assumption 1: Platforms do not charge for leisure services: their price is zero.

[28] Appendix J considers two non-neutral ways of modeling brand equity competition.

Assumption 1 underlies the focus of this study on zero price services. The introduction and Appendix A present empirical basis for this assumption. From a theoretical standpoint it can be motivated in several ways. Zero prices can be a result of optimal pricing behavior in two-sided markets characterized by asymmetric externalities and differing elasticities of demand (and likely some transaction costs which prevent prices for going negative). To explore this possibility in more detail, Appendix D derives the optimal pricing strategy of a monopoly platform in a two-sided market and shows that the optimal price charged on the consumer side might be zero or negative when consumer demand is highly elastic and when the interaction externalities are asymmetric. These are exactly the conditions that are likely to be satisfied in the context of the attention economy: consumers exert positive externalities on the advertisers and on each other (ad watching and network effects, respectively), while advertisers do the opposite (if ads are a nuisance to consumers, and in the likely scenario when congestion limits their effectiveness). There are other possible microfoundations too.[29] Incorporating these features into the model is not the focus of this article, which instead studies the consequences of this business model on the macroeconomy.

3.3.2 Technologies

Platforms are endowed with two technologies. To produce brand equity, they must capture consumers' time:

$$B_j = \ell_j \qquad \text{where} \qquad \ell_j = \ell \cdot \frac{M_j}{M}. \tag{17}$$

The amount of brand equity produced is linear in consumers' time captured by platform j,[30] and j's share of consumers' time is determined by the share of leisure technologies that platform j supplies. Second, platforms operate the technology for generating M: *the leisure ideas production function*. I consider two formulations:

$$\text{Dynamic: } \dot{M}_j = L_j^M \cdot A^\phi \tag{18}$$
$$\text{Static: } M_j = L_j^M \cdot A^\phi. \tag{19}$$

[29] A complementary explanation relies on competition and strong non-rivalry. Since the marginal cost of providing an extra user with a leisure technology that already exists is zero, a high degree of competition between platforms could depress prices towards the marginal cost and possibly beyond (again, transaction costs might account for exactly zero prices in equilibrium). Another explanation could be that, in a model with firm life-cycle, entry and exit, firms may find it optimal to charge zero prices during a certain period to build customer base.

[30] The particular form of (17) is chosen for parsimony. The production function of brand equity could also include other inputs, such as labor or capital, without altering the conclusions of the analysis. Clearly, the important point is that consumers' leisure time is an input in production of brand equity.

where $-1 \leq \phi < 1$ and A is the stock of existing knowledge in the economy.[31] The dynamic formulation follows the tradition in growth theory literature and assumes that new leisure technologies are added to the existing stock, mirroring the ideas production function used in the traditional R&D sector and hence putting leisure technologies on an equal modeling footing with the traditional technologies. The static alternative assumes that leisure technologies depreciate every period. In this case M can be interpreted as a measure of content, such as TV shows or news websites. The main long-run results of the paper hold for either of these two formulations (see Proposition 4). The static formulation makes it possible to derive closed-form solution for equilibrium M which is useful to gain the intuition for how leisure economy operates. For that reason I use the static formulation (19) in the main text, and I delegate the analysis of the dynamic formulation to Appendix G.[32]

3.3.3 Aggregation

Brand equity output is homogenous across platforms, so that the aggregate supply is simply $B = \sum_J B_j$. Similarly, we have: $M = \sum_J M_j = L^M A^\phi$.

3.4 Equilibrium definition

Definition 2. The almost-perfect foresight equilibrium is a set of paths of aggregate quantities $\{Y, C, K, h, \ell, s_A, s_M, L_Y, A, M, B\}_{t=0}^{\infty}$; micro-level quantities $\{x_i, b_i, m_\iota, h_\iota, B_j\}_{t=0}^{\infty} \forall i, \iota, j$; prices $\{p_i, p^B, w, r\}_{t=0}^{\infty} \forall i$ and platform activity indicator $\{\Omega\}_{t=0}^{\infty}$ such that: households choose consumption and hours to maximize utility taking all aggregate variables as given; final-output producers choose $\{x_i\}$ and L_Y to maximize profits taking all aggregate variables, $\{b_i\}\forall i$ and $\bar{b}$ as given; intermediate producers choose p_i and b_i to maximize profits, taking $\bar{b}$ and other aggregate variables as given; platform j chooses B_j to maximize profits taking actions of all other platforms $B_k \forall k \neq j$, the average level of ads $\bar{b}$, the households' leisure policy function and all aggregate variables as given; there is free entry to the traditional R&D sector; wages are equal across sectors; labour, goods and brand equity markets clear so that $L_Y = (1 - s_A - s_M) Nh$, $Y = \dot{K} + C + \delta K$, $A\bar{b} = B$; if $B_j(t) = 0$ then $\Omega(t) = 0$ and $= 1$ otherwise; if $B_j(s) = 0$

[31] For the sake of transparency I assume that parameter ϕ which governs the magnitude of increasing returns to R&D is the same in the traditional- and the leisure-enhancing sector.

[32] The long-run growth results are also robust to alternative formulations of the leisure production function; for example, M could be produced using final output. Note that (18) and (19) imply that there is a knowledge spillover from the traditional sector towards the leisure sector, reflecting the fact that the production of leisure technologies draws on all the existing technologies. The long-run results of the paper are unchanged also if the spillover comes from both the traditional innovations and the leisure technologies. In this case we would have $M_j = L_j^M \cdot (A+M)^\phi$ and perhaps $\dot{A} = L^A \cdot (A+M)^\phi$ if the leisure technologies can affect innovation in the traditional sector. It is straightforward to show that A and M grow at the same rate in equilibrium: if $X := A+M$ then $\frac{\dot{A}}{A} = L^A \cdot X^\phi / A$, and so $0 = n + \gamma_h + \phi\gamma_X - \gamma_A$. We also have $\gamma_M = n + \gamma_h + \phi\gamma_X$. These two equations imply that $\gamma_M = \gamma_A = \gamma_X$ and $\gamma_A = \frac{n+\gamma_h}{1-\phi}$. But while the long-run results are unchanged, the formulation in (19) is slightly more convenient as the equilibrium supply of leisure technologies can be expressed in closed form.

$\forall j$ and $\forall s \leq t$, and all firms and households expect $B_j(s') = 0$ for all $j, s' > t$. Otherwise agents have perfect foresight.

The equilibrium definition follows naturally from the economic environment; the only non-standard feature is that agents do not anticipate leisure technologies if no leisure technologies had ever existed. This arguably makes the concept of equilibrium more realistic: it is unlikely that firms and consumers at the start of the 20th century had anticipated the invention and the rise of television or that they had acted upon these expectations. It is also convenient since it allows for a tractable analysis of endogenous entry of the platforms along the growth path.

This completes the description of the environment. I now solve the model and characterize the equilibrium.

4 The segmented balanced-growth path

Goal and strategy. In most models of economic growth the balanced growth path can be characterized by computing the constant growth rates of model variables. The balanced growth path in this paper instead consists of *two segments* along which growth is balanced, with a transition in between. When the economy is smaller than a certain threshold, platforms are inactive and there is no leisure-enhancing technological change (segment 1); as the economy grows, at some point leisure innovations appear, the economy adjusts, and asymptotically growth is again balanced (segment 2). The goal of this Section is to prove that the growth path indeed takes this segmented form, and to characterize segments 1 and 2 analytically (the following Section then quantifies the effects described here and numerically computes the transition path).

The strategy for characterizing the equilibrium is as follows. I first guess that some platforms are active. Under this guess I compute the equilibrium as an intersection of (1) the household optimal choice of hours for a given level of leisure technologies, with (2) the platforms' optimal supply of leisure technologies for a given level of hours. This approach lends itself to a graphical analysis which gives the intuition on the equilibrium dynamics. I then find the conditions under which it is indeed optimal for the platforms to operate.

4.1 Equilibrium time allocation and the leisure technologies

Appendix B contains the solution to the representative household's problem (12); the main result is summarized in the following lemma:

Lemma 2. ***Hours worked and leisure technology.*** *Optimal hours worked satisfy:*

$$h = 1 - \ell = \min\{1, \Phi M^{\frac{1}{1-\nu}}\} \tag{20}$$

where $\Phi := \left(\frac{Y}{C}\frac{1-\alpha}{1-s_A-s_M}\right)^{\frac{\theta}{1+\theta}}$ *is a variable that is constant when growth is balanced.*

Proof. Appendix B. □

When leisure technologies are not well developed, households optimally choose the corner solution $h = 1$, with no time spent on marketable leisure. For M sufficiently large, hours worked vary inversely with the measure of available leisure options (recall that $\nu > 1$).[33]

I now turn to the supply side of the attention economy to pin down the equilibrium level of M.

4.2 Equilibrium supply of leisure technologies

4.2.1 Demand for brand equity

Equilibrium M is ultimately determined by the equilibrium supply of brand equity B: for the platforms, leisure technologies are strictly a means to an end. This and the next subsection compute equilibrium B.

Starting on the demand side, solving (14) gives the following results:

Lemma 3. ***Demand curve and intermediate profits.*** *Firm i's (inverse) demand for brand equity b_i satisfies:*

$$p_B = \alpha^2 \chi \frac{Y}{A}\frac{1}{\bar{b}}\left(\frac{b_i}{\bar{b}}\right)^{\frac{1}{\varepsilon}} \tag{21}$$

where $\varepsilon = -\frac{1}{1-\frac{\alpha}{1-\alpha}\chi}$.

In a symmetric equilibrium all firms choose identical brand equity investments: $b_i = \bar{b}$ and thus the equilibrium price satisfies

$$p_B = \alpha^2 \chi \frac{Y}{B}. \tag{22}$$

Equilibrium prices, quantities and revenues in the intermediate sector are "as if" there was no brand equity competition. Equilibrium profits of an intermediate firm are

$$\Pi = \alpha \frac{Y}{A}\left(1 - \alpha - \alpha\chi\right), \tag{23}$$

which is lower than $\alpha\frac{Y}{A}(1-\alpha)$, the value of profits with no brand equity competition.

[33] The implication of the theory that there is a causal link between leisure technology and total leisure time receives strong empirical support. For example, Gentzkow and Shapiro (2008) and Aguiar and Hurst (2007b) show how the introduction of television substantially raised leisure consumption in the United States. Falck *et al.* (2014) identify exogenous geographical variation in the speed of the roll-out of broadband internet in Germany and document the significant boost to leisure consumption as high-speed internet became available. Using proprietary data on television and internet subscriptions, Reis (2015) documents that television shows and internet content are imperfect substitutes, supporting the prediction that more plentiful leisure varieties increase overall leisure time.

Proof. Appendix B. □

The hoped-for revenue-boosting effects of brand equity investments wash-out in equilibrium. Consequently, from an individual firm's perspective brand equity competition lowers profits. This will have important implications for the innovation incentives, the issue I return to below.[34]

4.2.2 Platforms' cost structure

Equations (17) and (19) imply:

$$B_j = \frac{\ell}{M} L_j^M A^\phi \tag{24}$$

Using this together with equation (20), platform $j's$ cost function is:

$$\mathbb{C}(B_j) = B_j \cdot w \cdot \frac{M}{\ell A^\phi}. \tag{25}$$

That is, at any t platform j faces a constant marginal cost $\mathcal{M}^B = w\frac{M}{\ell A^\phi}$. Note that this cost will be changing *over time*, but it is independent of the quantity produced at any instance. This feature makes the platform problem extremely tractable.

4.2.3 Platform's problem

Platform j solves

$$\max_{B_j \geq 0} p_B\left(B_j + \sum_{k \neq j} B_k\right) \cdot B_j - B_j \cdot \mathcal{M}_B \tag{26}$$

where B_k is the output level of platform k, $k \neq j$. This is a textbook Cournot competition problem: each platform acts as a monopolist facing the demand curve $p_B\left(B_j + \sum_{k\neq j} B_k\right)$, taking the actions of its competitors as given. Since in equilibrium $b_i = \frac{\sum B_j}{A}$ by the symmetry of the choices of the intermediate firms, equation (21) implies that the demand curve can be written as follows:[35]

$$p_B\left(B_j + \sum_{k\neq j} B_k\right) = \alpha^2 \chi \frac{Y}{B_j + \sum_{k\neq j} B_k}\left(\frac{B_j + \sum_{k\neq j} B_k}{A\bar{b}}\right)^{\frac{\alpha}{1-\alpha}\chi}. \tag{27}$$

Solving the Cournot game in (26) given (27) yields the next lemma:

[34] Lemma 3 shows that competition through brand equity can be easily incorporated in the monopolistically competitive setting with tractable closed-form results such as the constant elasticity demand function in (21). Since the monopolistically competitive setting is present in a vast number of application in economics, it would be straightforward to consider brand equity competition in those models as well, demonstrating a potentially wider applicability of the formulations developed here.

[35] Note that, in line with Definition 2, each platform takes the average brand equity investments in the economy $\bar{b}$ as given. This setting applies more directly to the case where there are several platforms and J is not too low. It is however without loss of generality, in that had platforms internalized their impact on $\bar{b}$, the mark-up would be $(1 - 1/J)^{-1}$ and all the results would continue to hold.

Lemma 4. ***Supply of brand equity.*** *Each platform faces a constant (independent of quantity) marginal cost of production equal to*

$$\mathcal{M}_B = w\frac{M}{\ell A^{\phi}}. \tag{28}$$

The price of brand equity in the Nash equilibrium of the Cournot competition game is equal to markup over the marginal cost, where the markup is:

$$\Psi := \frac{p_B}{\mathcal{M}_B} = \frac{1}{1 - \left(1 - \frac{\alpha}{1-\alpha}\chi\right)\frac{1}{J}}. \tag{29}$$

The markup depends on the degree of competition (the number of firms J) active in the market. As J gets large, the markup converges to zero.

Lemma 4 shows that the seemingly complicated problem of platform optimization is in fact straightforward to characterize and yields intuitive outcomes. The framework can accommodate varying degree of competition in the platform market, from high levels of concentration and high markups for low J, to perfect competition as J becomes large.

4.2.4 Equilibrium supply of leisure technologies

Combining equations (17), (22), (28) and (29) yields the following lemma:

Lemma 5. ***Equilibrium supply of leisure technologies.*** *When $h = 1$, platforms are inactive: $B_j = M_j = 0 \forall j$ and $\Omega = 0$.*

Whenever $h < 1$ platform j's profits are non-negative:

$$\Pi_j^B = B_j \mathcal{M}_B \left(\Psi - 1\right) \geq 0,$$

and the equilibrium supply of leisure technologies M satisfies:

$$M = \Upsilon A^{\phi} h N. \tag{30}$$

where $\Upsilon := \frac{\alpha^2}{1-\alpha}(1 - s_A - s_M)\chi$ is a variable that is constant when growth is balanced.

Lemma 5 states that whenever households choose to spend positive amount of time on leisure, platforms can make positive profit. In that case the equilibrium supply of leisure technologies depends positively on the size of the economy (hours worked, population and technical advancement), because a larger economy generates more demand for brand equity and because it makes the leisure technologies cheaper to produce. If households spend no time on leisure, platforms have no way of making a positive return and they remain inactive.

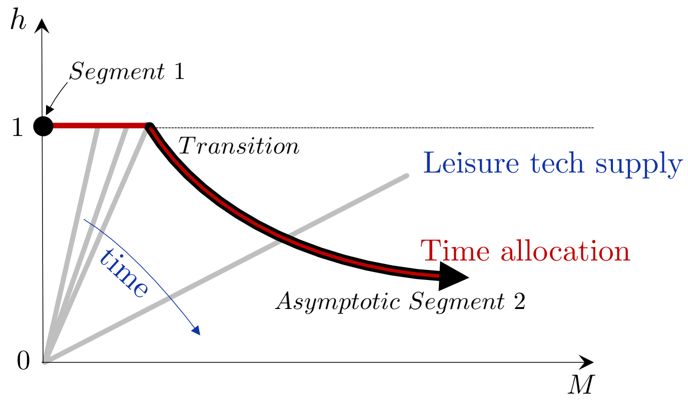

Figure 4
Equilibrium Over Time

4.2.5 Existence and uniqueness

Equations (20) and (30) readily give the following result:

Proposition 2. ***Existence and uniqueness.*** *The equilibrium exists and is unique.*

4.3 Graphical representation

Figure 4 illustrates the equilibrium graphically. Households' choice of leisure hours (equation (20)) is a downward sloping curve with a flat section for low values of M. This "Time allocation" curve is stable through time if growth is balanced (since then Φ is a constant). Equation (30) is a ray from the origin for $h \in [0, 1)$; for $h = 1$, it is the point on the y-axis since platforms are inactive in that case. Since its slope depends on the levels of N and A which are growing variables, this ray continuously rotates clockwise over time.[36]

As long as the economy is small and the two curves cross on the flat section of the "Time allocation" curve, we have $M = 0$ in equilibrium. Once N and A get sufficiently large and the "Leisure tech supply" is sufficiently flat, the two lines cross at $h < 1$, and the equilibrium coincides with the crossing point of the two curves. The point labelled 'Segment 1' and the thick arrow labelled 'Transition' and 'Asymptotic Segment 2' trace out the dynamics of the equilibrium over time.[37]

[36]The intuition for why this curve is upward sloping is simply that higher level of hours worked translates into higher output and thus to greater demand for brand equity, thus supporting a higher supply of leisure technologies in equilibrium. The curve rotates for similar reasons.

[37]This representation is illustrative only, since it ignores the shift in the "Time allocation" curve during transition as a result of changes in Φ. I compute the full transition path numerically below.

4.4 Origins of the attention economy

Proposition 3. ***The condition for leisure-enhancing technological change.*** *Platforms are active and there is leisure-enhancing technological change if*

$$N(t) \geq \Gamma, \tag{31}$$

where Γ is a variable that is constant.

Proof. See Appendix B. □

Proposition 3 describes a watershed moment for an economy, which occurs when the "Leisure tech supply" curve first crosses the "Time allocation" curve at the interior value of h (i.e. $h < 1$). Since N grows exponentially and Γ is constant, the proposition shows that it is only a matter of time when the leisure technologies emerge in equilibrium, confirming the graphical intuition above. In that sense leisure-enhancing technologies are an integral part of the growth process. The emergence of the attention economy brings about a new kind of structural change: consumers are steadily moving away from material consumption and towards 'consumption' of technologically-enhanced leisure time.

Given this result, the balanced growth equilibrium can be formally defined as follows:

Definition 3. A *segmented balanced growth path* (sBGP) is an equilibrium trajectory along which: (i) when (31) is not satisfied, per capita consumption, output and the measure of varieties A all grow at a constant rate; (ii) as $t \to \infty$, per capita consumption, output, A and M grow at possibly distinct but constant rates, and h decreases at a constant rate.

To facilitate a tractable characterization of the sBGP I make the following assumption about the initial levels of the state variables in this economy:

Assumption 3: Initial levels of capital and technology K_0 and A_0 are such that growth is balanced for all $t < \hat{t}$.[38]

Assumption 3 helps with the exposition and is without loss of generality – an alternative and weaker condition that yields identical results would be to require the initial values of the states to be such that the economy is (approximately) on the balanced growth path by $\hat{t}$. One could of course also analyze the evolution of the economy starting outside of the steady state.

4.5 Long-run growth effects of leisure technologies

How does the nature of growth change as a result of leisure-enhancing technological change? The following proposition shows what happens to the growth rates in segments 1 and 2 of the sBGP.

[38] In the formulation with a dynamic leisure ideas production function in (18), I further assume that M_0 is zero.

Proposition 4. ***Growth along the sBGP.*** *Suppose Assumption 3 holds. There exists $\hat{t}$ such that (31) holds $\forall t \geq \hat{t}$: $N(\hat{t}) = \Gamma$.*

For $t \leq \hat{t}$, platforms are inactive, there is no leisure enhancing technological change, hours worked are constant and equal to 1, and per capita consumption, per capita output, wages and TFP all grow at the same constant rate given by:

$$g := \frac{n}{1-\phi}. \tag{32}$$

For $t \geq \hat{t}$, platforms are active and the economy transitions to segment 2 of the sBGP. Asymptotically, hours worked decline at a constant rate

$$\gamma_h = -\frac{n}{(\nu-1)(1-\phi)+1} \tag{33}$$

and the growth rates of traditional- and leisure technologies are equal and given by:

$$\gamma_A = \gamma_M = \frac{n}{1-\phi+\frac{1}{\nu-1}} < g. \tag{34}$$

Per-capita output and consumption grow at:

$$\gamma_y = \gamma_A \left(\frac{\nu-2}{\nu-1}\right) < g \tag{35}$$

which is positive if $\nu > 2$.

These long-run results hold irrespectively of whether the leisure ideas production function assumes a dynamic (18) or a static (19) formulation.

Proof. See Appendix B. □

The first part of Proposition 4 derives the growth rate of the economy on segment 1 of the sBGP. The expression in (32) is familiar from the canonical semi-endogenous growth model first formulated by Jones (1995).[39] Along segment 1 platforms are inactive, M is zero, labor supply is constant, and firms and households do not anticipate the future entry of platforms (in line with the equilibrium definition above). Segment 1 thus serves as a convenient benchmark against which to compare the economy once the leisure sector emerges.

The second part of the Proposition mirrors the results in Proposition 1. In segment 2 hours worked are no longer constant but are instead falling at a constant rate. The speed of the decline is governed by the elasticity of substitution across leisure varieties ν. The effect vanishes in the limit as $\nu \to \infty$ and leisure varieties become perfect substitutes.

The next result is that along the sBGP leisure technologies grow at the same rate as 'traditional' technologies. This implication is a straightforward corollary of the fact that the leisure

[39]The only difference is that I have implicitly assumed no R&D duplication externalities.

ideas production function (equations (18) or (19)) takes the same form as the ideas production function in the traditional sector (equation (2)).

The emergence of leisure-enhancing technologies is associated with a decline in the long-run growth rate of traditional technology. The mechanics of this effect are the same as those that underlie Proposition 1, and the economic intuition is that the heightened competition for time and attention that results from leisure technologies leaves less resources available for productive activities.

How plausible is this mechanism? Recent research has highlighted the importance of the resources devoted to innovation. For example, Bloom *et al.* (2020) show that research productivity (defined as $\frac{\dot{A}/A}{L_A}$) is declining, both at the aggregate and at the industry- and firm-level. This suggests that the growth of inputs into research is the crucial determinant of the long-run growth rate of innovation. A related idea is that technologies and leisure content available today may act to divert peoples' time and attention away from creative thinking. Some suggest that more distracted minds may lower workers' productivity (Klein (2016), Nixon (2017)). Consistent with that, the nascent experimental evidence shows that leisure technologies occupy our limited cognitive resources, significantly decreasing cognitive performance (Ward *et al.*, 2017). The growth channel highlighted in Proposition 4 can be thought of as broadly capturing some of these ideas.

It is important to note that the result that leisure technologies lead to a decline in productivity growth is arguably more general and goes beyond the particular growth paradigm considered here. In any model in which human labor is important for generating frontier ideas or facilitating the process of technology diffusion, the heightened competition for peoples' time and attention is likely to have adverse effects on the growth of ideas. For example, in endogenous growth models with scale effects (such as the expanding variety models in the tradition of Romer (1990), or the Schumpeterian economies in the spirit of Aghion and Howitt (1992)) an increasing share of time and attention devoted to leisure translates into a decrease in the supply of labor engaged in research as well as diminished market size, as households consume relatively fewer 'traditional' goods and services. In either framework these effects will tend to weigh on the growth rate of the traditional economy.[40]

The last result in Proposition 4 shows that the declining hours worked and slower growth in hourly productivity combine to deliver a severe slowdown in growth of per-capita output and consumption. Indeed, for low values of ν the effect can be so powerful as to reduce the growth rate to zero or below. This is the first sign of the externalities present in the model which may lead to excessive leisure and growth of consumption and output that are too low from a welfare

[40] In fact in models with scale effects the economy's growth rate is also impacted through another channel, namely the decreased equilibrium profitability of the intermediate sector. Appendix F sketches out and solves a Schumpeterian model with leisure technologies and illustrates this effect explicitly. In the semi-endogenous benchmark model employed in this paper this effect only affects the growth rates temporarily, and has a long-run effect in levels. I elaborate on these effects in the following section.

perspective – the subject to which I return in Section 7.

4.6 Allocative effects of leisure technologies

The advent of the attention economy changes how labor is allocated across sectors. Recall that workers can be employed in the production sector of the economy, as well as in the two research sectors: the traditional R&D (the A) sector and the platform R&D (the M) sector. Denote the shares of labor employed in each of these three sectors with $1-s_A-s_M$, s_A and s_M respectively. The following proposition pins down the values of these shares in steady state.

Proposition 5. ***Allocation of labor on the sBGP.*** *For $t < \hat{t}$ (in segment 1 of the sBGP) the share of labor employed in the platform sector s_M is zero. The share of labor in the A sector is:*

$$s_A = \frac{1}{1+\frac{1-\alpha}{\Delta_1}} \quad \textit{where} \quad \Delta_1 = \alpha(1-\alpha)\frac{g}{\rho+g} \tag{36}$$

This share is increasing in Δ_1 and thus increasing in g.

In segment 2 of the sBGP the share of labor employed in the R&D sector converges to

$$s_A = \frac{1}{1+\frac{1-\alpha}{\Delta_2}} \quad \textit{where} \quad \Delta_2 = \alpha(1-\alpha-\alpha\chi)\frac{\gamma_A}{\rho+\gamma_A}\left(1+\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}. \tag{37}$$

Since $\chi > 0$ and $\gamma_A < g$, the share of labor in traditional R&D is lower once the attention economy emerges. The share of labor employed by platforms in leisure-enhancing research is:

$$s_M = \frac{1-s_A}{1+\left(\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}}. \tag{38}$$

Proof. Appendix B. □

The leisure-driven structural change shifts the allocation of labor away from traditional R&D and towards leisure-enhancing R&D via three channels, which can be seen directly in the closed-form expression for Δ_2 in Proposition 5:

$$\Delta_2 = \alpha\left(1-\alpha-\underbrace{\alpha\chi}_{\text{lower profits}}\right)\cdot\underbrace{\frac{\gamma_A}{\rho+\gamma_A}}_{\text{fewer inventions}}\cdot\underbrace{\left(1+\frac{\alpha^2}{\Psi(1-\alpha)}\chi\right)^{-1}}_{\text{competition for researchers}}.$$

The first channel reflects the hit to intermediate producers' profitability. A share of firm revenues is shifted in equilibrium to the platform sector. Each newly invented blueprint – whose value is a discounted sum of future profits – is worth less as a consequence, lowering the marginal revenue product of traditional research in equilibrium. The 'fewer inventions' channel operates via lowering the pace at which new ideas are being invented and hence diminishing the productivity of researchers.[41] Finally, the 'competition for researchers' channel denotes the effect that traditional R&D firms must now compete for workers with platforms. A share of workers who in the absence of leisure technologies would work in the traditional R&D sector find employment in the leisure sector instead.

Each of these three channels lowers the long-run value of s_A. Within the semi-endogenous framework this has no impact on the long-run growth rate of technology – the causation runs from γ_A to s_A and not vice-versa, and the effect on growth is temporary, generating persistent *level* effects.[42] In models with scale effects in which the level of profitability or the size of the pool of potential researchers has an impact on growth rates, these effects would affect the long-run growth rate.

5 Quantification

The analysis so far concerned the initial and the asymptotic segments of the sBGP. The analytical results allowed for a sharp characterization of the key variables along the growth path. This Section quantifies the long-run effects and solves for full transitional dynamics between the two segments. It also contrasts the model predictions with data.

5.1 sBGP as a dynamic system

If an economy admits a balanced growth path its equilibrium can be written as a system of differential-algebraic equations in normalized variables that are constant on the BGP. The challenge in the context of the present model is that the balanced growth path is segmented. The following proposition presents the model in its stationary form:[43]

Proposition 6. ***Equilibrium as a dynamic system.*** *Let* $\gamma_A := \frac{n}{1-\phi+\Omega\frac{1}{\nu-1}}$, $\gamma_Y := n + \left(\frac{\nu-2}{\nu-1}\right)^{\Omega}\gamma_A$, $\beta_A := \gamma_A/n$ *and* $\beta_Y := \gamma_Y/n$ *where* $\Omega = 0$ *if* $t < \hat{t}$ *and* $\Omega = 1$ *otherwise. Let the lower case letters denote the variables constant along the sBGP:* $a := \frac{A}{N^{\beta_A}}$, $k := \frac{K}{N^{\beta_Y}}$, $c := \frac{C}{N^{\beta_Y}}$, $v := \frac{V}{N^{\beta_Y-\beta_A}}$, $\pi := \frac{\Pi}{N^{\beta_Y-\beta_A}}$, $y := \frac{Y}{N^{\beta_Y}}$, $\tilde{h} := \frac{h}{N^{\frac{1}{1-\nu}\beta_A}}$. *The dynamic equilibrium is the solution to the following system:*

[41] This is only partly offset by a less crowded market that results from the slowdown in growth. The first of these two effects shows up in the nominator and the second in the denominator of $\frac{\gamma_A}{\rho+\gamma_A}$.

[42] Although since the transition takes a long time, the effects on the growth rates can persists for some time.

[43] In Appendix G I derive the stationary form under the assumption that the leisure ideas production is given by equation (18).

$$\dot{k} = y - c - \delta k - \gamma_Y k \tag{39}$$
$$\dot{a} = a^{\phi} s_A \tilde{h} - \gamma_A a \tag{40}$$
$$\dot{c} = c\left(r - \rho - \gamma_Y\right) \tag{41}$$
$$\dot{v} = v\left(r - (\gamma_Y - \gamma_A)\right) - \pi \tag{42}$$
$$(1-\alpha)\frac{y}{1 - s_A - s_M} = v a^{\phi} \tilde{h} \tag{43}$$
$$y = k^{\alpha}\left((1 - s_A - s_M)\tilde{h} a\right)^{1-\alpha} \tag{44}$$
$$\tilde{h} = \left(h^{\hat{t}}\right)^{\frac{\Omega-1}{\nu}}\left(\Phi^{1-\nu}\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1 - s_A - s_M)a^{\phi}\right)^{-\frac{\Omega}{\nu}} \tag{45}$$
$$r = \alpha^2\frac{y}{k} - \delta \tag{46}$$
$$\pi = \alpha\frac{y}{a}\left(1 - \alpha - \alpha\chi\Omega\right) \tag{47}$$
$$s_M = \Omega\frac{1 - s_A}{1 + \left(\frac{\alpha^2\chi}{\Psi(1-\alpha)}\right)^{-1}} \tag{48}$$

where $h^{\hat{t}} := \Phi(\hat{t})^{1-\nu}\frac{\alpha^2}{\Psi(1-\alpha)}\chi(1 - s_A(\hat{t}) - s_M(\hat{t}))a(\hat{t})^{\phi}$ *follows from the fact that hours worked do not jump at* $\hat{t}$.[44]

Proof. Appendix B. □

The intuition for why at $\hat{t}$ hours worked do not jump is that, first, hours worked are already at their feasible upper bound prior to $\hat{t}$ so they could potentially drop but cannot increase. But if there was a drop in hours at $\hat{t}$ then any platform could make additional positive profit by entering the market earlier. Thus platform entry must occur at $\hat{t}$ such that hours do not jump.

The system can be used to compute the transitional dynamics following $\hat{t}$. Once the model is parametrized, the transition path can be computed as a response of the system to a change in Ω from 0 to 1 and in the pair ν, χ from $[+\infty, 0]$ to the calibrated parameter values.[45] This gives the values of the normalized variables over the transition. The final step is to convert the normalized variables back into original units. For this we need to compute $N(\hat{t})$, the population size at which leisure enhancing technological change first emerges. At $\hat{t}$ the optimal "shadow"

[44] Equation (39) describes the evolution of the capital stock in the economy; (40) is a stationary ideas production function; (41) is the Euler equation; (42) is the Bellman equation for the value of the blueprint; (43) denotes equality of wages across sectors; (44) is the production function; (45) is an equation that pins down equilibrium hours worked; (46) is the capital demand equation; (47) are the profits of intermediate firms; and (48) is the share of labor in the platform sector. Note that when $\Omega = 0$, the model collapses to a stationary representation of the Jones (1995) economy. Note also that the equation for normalized hours worked follows from the fact that there can be no jump in hours worked at $\hat{t}$: if there was a jump, each platform could increase its profits by entering at $t < \hat{t}$.

[45] I compute the transition of the model using the relaxation algorithm developed by Trimborn *et al.* (2008).

choice of hours worked crosses unity from above – in other words, there is no jump in h at $\hat{t}$. Therefore, by the definition of h, $N(\hat{t})$ solves

$$N(\hat{t}) = \left(h^{\hat{t}}\right)^{(1-\nu)(1-\phi)-1}. \tag{49}$$

5.2 Calibration

Parametrization corresponds to annual frequency, with the discount rate of 1% and population growth of 1% per annum. Several parameters are calibrated to standard values: the capital share α equals 0.35, the depreciation rate δ is 5% per year and the Frisch elasticity of labor supply θ is 1.

Parameter ϕ guides the degree of increasing returns to innovation and determines the steady state growth rate of the economy. Recent work by Bloom *et al.* (2017) has found that the ϕ parameter varies widely across sectors in the US economy, but is likely to be well below 1. I set it to 0.5, targeting the growth rate of the economy in segment 1 of $g = 2\%$.

The elasticity across leisure varieties ν pins down the strength of the link between leisure technologies and time allocation choices (a higher ν makes this link weaker). To get a sense of the plausible magnitudes it is useful to consider the estimates of other elasticities from the existing literature. For example, Goolsbee and Klenow (2006) estimate the elasticity between internet vs. everything else of about 1.5, highlighting that such elasticies can be quite low when the categories are broadly defined. The calibrated value of ν needs to be significantly higher than this. Elsewhere, Broda and Weinstein (2006) study the welfare gains from increased variety as a result of the rising trade penetration in the US economy. In the process, the authors estimate thousands of elasticities of substitution between similar products imported from different countries. For example, they establish that the elasticity of substitution across cars (apparel and textiles) imported from different countries is around 3 (6). Within products classified as differentiated, the median elasticity is around 2 and the mean is about 5. Given these estimates I set $\nu = 4$. This is a cautious calibration, in part on account of the fact that leisure technologies substitute for non-marketable leisure activities and not solely for work hours (see Appendix H). I explore the robustness of the numerical results to different values of ν in Appendix K.

Parameter χ corresponds to the perceived effectiveness of brand equity. I set this parameter to 0.08, which means that for each producer a unilateral doubling of brand equity is expected to increase quantity sold by 5%.[46] This is motivated by the consensus in the empirical literature that estimates the effectiveness of advertising (Bagwell (2007), DellaVigna and Gentzkow (2010), Lewis and Reiley (2014), Lewis and Rao (2015)). Finally, I set J equal to 5, to capture the high

[46] To see this, note that equilibrium quantity sold is $x(i) = [\alpha^2 \left(\frac{b(i)}{b}\right)^{\alpha\chi} L_Y^{1-\alpha}]^{\frac{1}{1-\alpha}}$, thus the elasticity to intangible capital is $\frac{\alpha\chi}{1-\alpha}$, which is roughly 0.05 for $\alpha = 0.35$ and $\chi = 0.08$.

Parameter	Description	Value	Target / source
ρ	Household discount rate	0.01	$r \approx 4\%$
n	Population growth	0.01	AEs data
α	Capital share	0.35	standard calibration
δ	Capital depreciation	0.05	standard calibration
θ	Frisch elasticity	1	standard calibration
ϕ	Returns to ideas in R&D	0.5	Bloom *et al.* (2020)
J	Number of platforms	5	high degree of concentration
χ	Perceived effectiveness of brand equity	0.08	empirical elasticities
ν	Elasticity of substitution between leisure activities	4	see text

Table 1
Model calibration

degree of concentration in the market. The results are insensitive to this choice.

5.3 The magnitude of long-run growth effects

Plugging in the parameter values into the formulas in Propositions 4 and 5 revels that leisure technologies can have substantial macroeconomic effects. The model predicts hours worked declining by around -0.4% per annum. The growth of traditional technology falls from $g = 2\%$ to $\gamma_A = 1.2\%$ along the sBGP. The share of workers in traditional R&D sector is 4% initially and falls to 2.5%, with the platforms employing 1.5% of the workforce. I now turn to how these magnitudes compare to the trends observed in the data.

5.4 Confronting the model with the data

In order to compare the simulated transition path with the observed trends we must first decide on the counterpart to $\hat{t}$ in the data. A plausible candidate is 1950, the year when a mass roll-out of television has started in the United States (Figure 5). Television is widely recognized to have revolutionized the world of mass-available leisure. Adoption along the extensive margin was rapid, and the time-use data show that television had dramatically altered the way people spend their time, with average daily watching time north of 1.5 hours by mid-1960s.[47]

Taking 1950 as a benchmark for $\hat{t}$, Figure 6 plots the model's growth path against the trends observed in the data. Consider first the black lines with empty circles, which show the transition path of the economy without any further structural shifts. This path is thus conditional on the

[47]The mass adoption of the radio that had begun in the mid 1920s would be another candidate. Adoption of the radio along the extensive margin was slower, however, and there is no time-use data that would allow for a systematic analysis of its effects on time allocation. And while the adoption of radio receivers occurred before World War 2, the top right panel in Figure 6 below shows that the explosion in the number of radio stations occurred after the war, again supporting the choice of 1950 as the point of departure. Furthermore, listening to the radio is more amenable to multitasking, so it likely has less of an impact on the allocation of attention than television does. Consistent with this, Vandenbroucke (2009) considers the period 1900-1950 and finds that only about 7% of the shift in time allocation over that period was due to leisure technology.

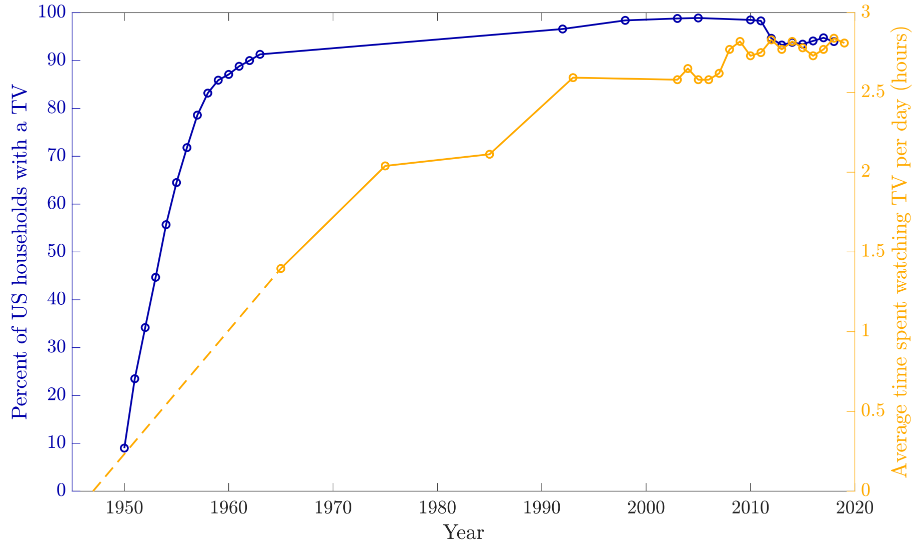

Figure 5

Adoption of TV in the United States

Sources: American Time Use Survey, Aguiar and Hurst (2007b) and Comin and Hobijn (2009). Notes: the dashed line joins the first point available in the data on time use (1965) together with 1947, when fewer than 0.5% of households had a TV set installed at home – a proportion clearly too limited to show up in average time use across the population (source: Televisor Monthly, 1948, accessed via http://earlytelevision.org/us_tv_sets.html).

view that the newer leisure technologies are simply improved versions of the existing ones, and the structure of the attention economy remains fixed. Panel A shows the size of the attention economy measured at cost. The model matches the small size of the sector as a share of the aggregate economy. But clearly, the model does not replicate the increase in the share. Panel B shows the growth rate of leisure technologies. It is not obvious what the correct empirical counterpart to M is: as usual, it is difficult to measure the level of innovation directly. For illustrative purposes the Figure plots the growth rates of the number of TV and radio stations in the United States. The model can replicate the broad shape of these curves. But this is hardly a success, since in reality there has been an explosion of available leisure varieties related to the digital revolution which are not captured in this Figure. Panel C plots the average hours worked across several advanced economies and the (appropriately rescaled) path along the transition in the model. The common downward trend across these countries is similar to the path implied by the model. Finally, panel D shows the model's trajectory for the growth rate of A against measured TFP growth, for the United States and for the aggregate of 20 advanced economies. The emergence of leisure-enhancing technologies accounts for around half – or 0.8 percentage points – of the slowdown in the growth rate of traditional technology.

The model is stylized and these quantitative predictions must be viewed accordingly. Nonetheless, Figure 6 contains two broad lessons that are likely to be robust to the inherent modeling- and data uncertainty.

First, the effects of the mechanisms captured by the theory are quantitatively large. The

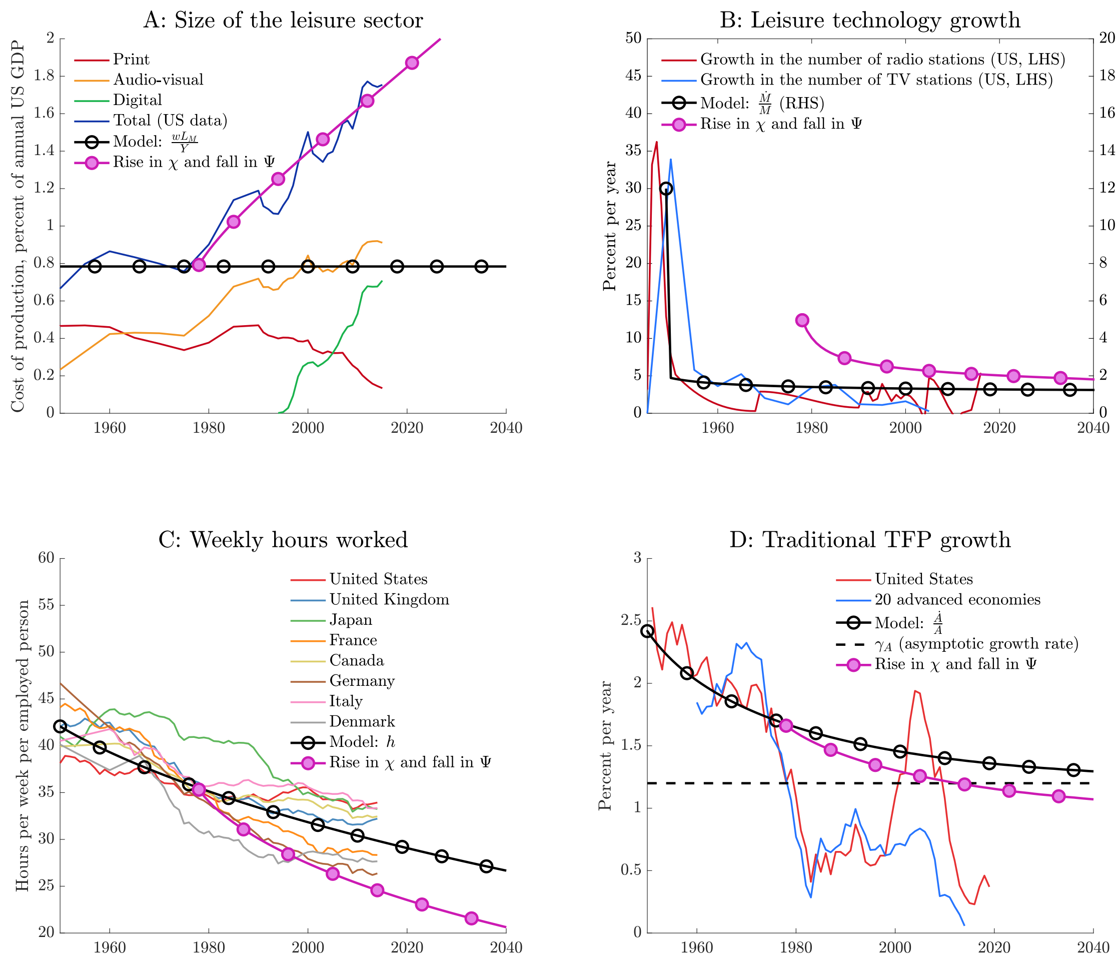

Figure 6

The Model's Growth Path versus the Trends Observed in the Data

Data sources as in Figure 2, except for the top-right panel in which the data are from the Federal Communications Commission and Statista. These data are interpolated over the missing values. The black lines with empty circles show transition following the entry of platforms in 1950. The pink line with filled circles show the transition with additional shocks to parameters χ (increase) and Ψ (decrease) calibrated in such a way that the size of the leisure sector matches the data in the top-left panel.

simple model, which features only two additional parameters (ν and χ) relative to a standard benchmark, can potentially account for a significant proportion of the decline in traditional sector total factor productivity and for most of the cross-country decline in hours worked. All this despite the attention economy being minuscule in the aggregate. The lesson here is not to discount the attention economy as important for macro merely because it is small: the macroeconomic effects come about not through its size, but indirectly through its influence on the time allocation decisions of the consumers.

Second, one needs to go beyond the endogenous entry of the leisure sector that I studied so far to better capture the developments observed in the data. This is not at all surprising: clearly, the attention economy itself has undergone dramatic technological shifts over the decades, some of which go well beyond the steady pace of progress in M captured in the baseline simulation considered so far. For example, panel A of Figure 6 shows that the balanced growth path with constant preference- and technological parameters is a good description of the first few decades following the adoption of television, but struggles to account for what happened in the digital sphere over the past thirty years or so. Put differently, in some ways the leisure technologies we see today represent a natural progression from those that we saw in the 1950s, but in others there are structural differences that, in the context of the model, would show up as changes in model's parameters.

To provide an illustration of such shifts, consider the pink lines with filled circles in Figure 6 which show the transitional dynamics if, in addition to the entry of the platforms in 1950, the economy experiences a shift in parameters that is sufficient to match the rise in the relative size of the attention economy since late 1970s shown in panel A of the Figure. Specifically, I consider a steady increase in the perceived effectiveness of brand equity χ and a rise in the degree of competition among the platforms (a fall in the mark-up Ψ),[48] starting in the late 1970s. Panel B shows that the expanded production of brand equity can in equilibrium support faster growth of leisure technologies, which leads to a sharper decline in hours worked in the market, shown in panel C. This more rapid decline in hours leads to an additional downward drag on traditional productivity (panel D). The model's prediction is thus that the rapid leisure-enhancing technological change co-exists (and causally leads to) low traditional productivity growth.

Future work could usefully explore ways of capturing some of the structural differences between the leisure technologies of today and those available in the past. These differences include data gathering, user-generated content, or the portability of devices through which leisure technologies can be accessed.[49] Once these shifts are mapped to model parameters, the framework can be used to study the potential effects of these changes for the phenomena observed in the data. Appendix E provides some further analysis along these lines.

5.4.1 Cross-country evidence on the market-size effect

In light of the theory, the equilibrium level of leisure technologies depends on market size. Figure 7 provides a simple test of this prediction, by regressing the number of TV channels across countries on GDP per capita, population and the level of aggregate GDP separately in the three consecutive panels. The number of TV channels is a useful metric of M in the context of cross-country analysis, because of the language- and culture- barriers tend to limit the market to national borders.[50] The results suggest that both the level of development and the size of the population matter in determining how many options individuals have when they switch on their

[48]Which is isomorphic to an increase in platforms' productivity in turning consumers' time into brand equity.

[49]This portability enables leisure consumption during all hours of the day, including at work – see Figure A.5 in Appendix A.

[50]For some other leisure technologies such as mobile phone apps the market is global and cross-country exercise may be less useful. This concern could also apply to the English-speaking countries in the case of TV though.

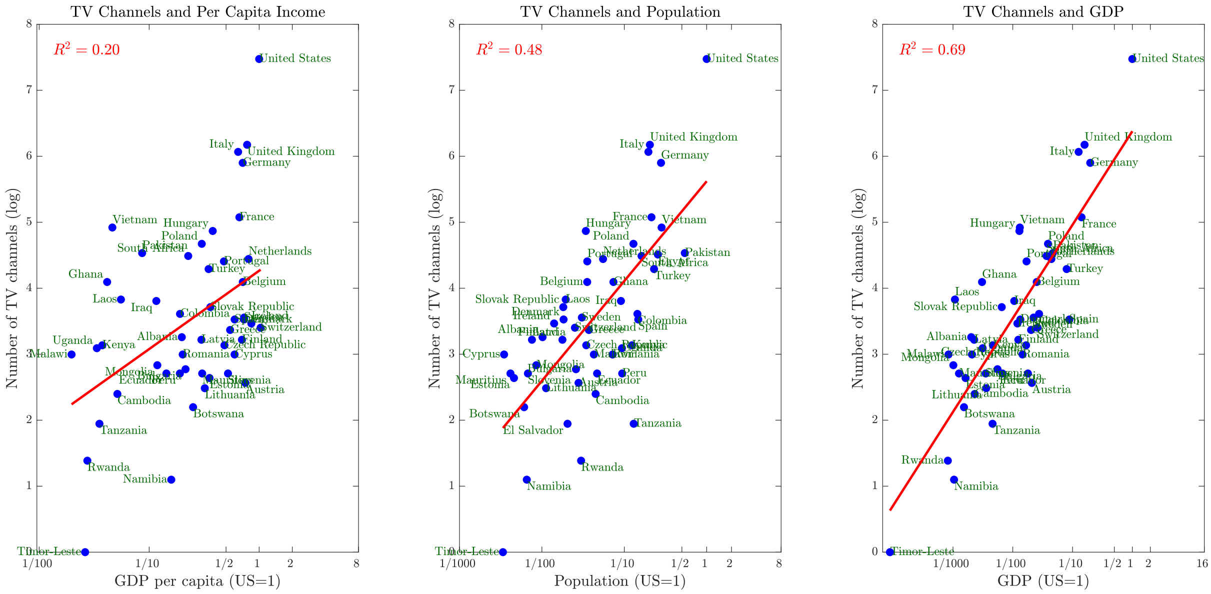

Figure 7

The Number of TV Channels and Market Size

Sources: Data on GDP per capita are from Penn World Tables 9.0 (Feenstra *et al.* (2015)). Data on population are from the World Bank. Data on the number of TV channels in each country has been hand collected from online sources. As such these data are subject to some measurement error. TV channels include state-run channels.

TV. In effect, the number of TV channels is best predicted by aggregate GDP (the final panel), with R^2 of nearly 70%.

Having discussed the impact of leisure technologies on the attention economy, I now proceed to the analysis of the implications for measurement and welfare.

6 Measuring the leisure economy

Leisure technologies and the services that they provide are not captured in headline GDP statistics. The 2008 UN System of National Accounts views platforms as advertising agencies: their output is ads, which serve as intermediate inputs of the ad-buyers (Byrne *et al.* (2016b), (Bean, 2016)).[51] Two questions arise in this context. First, does this mean that GDP is mismeasured? And second, does the attention economy make GDP a less reliable guide to welfare? In this

[51] To illustrate this, it is useful to demonstrate measuring GDP using the output-, expenditure- and income approaches in the model economy. The output measure is the sum of value added in the final, intermediate and platform sectors:

$$GDP(O) = (Y - Ax) + (Ax - B) + B = Y.$$

This equation shows explicitly that brand equity output of the platforms is netted out as an intermediate in the production of differentiated goods. The expenditure measure is the sum of consumption, investment and capital consumption:

$$GDP(E) = C + \dot{K} + \delta K = Y$$

Section I explain why the answers to these questions are 'no' and 'yes', respectively.[52]

6.1 Production cost-based value of leisure technologies

To answer the first question it is useful to be precise about what GDP is designed to capture. There is an ongoing debate about this issue: while GDP has been designed as a measure of (market) production, in practice it includes elements that are outside of the production boundary, such as home production of goods, or services from owner-occupied housing, and, for the lack of an agreed more comprehensive measure of economic wellbeing, it is often used as measure of welfare (see Jorgenson (2018) for an overview of the history and the vast literature on these issues, and Coyle (2017) for an extensive discussion of the production boundary in the context of digital goods). Sticking to the use its originators, including Simon Kuznets, intended for GDP, does the attention economy as described in this paper lead to mismeasurement? Since leisure technologies are used for consumption by households, they should be included: the view of the platforms as solely ad-agencies misses out on their role as leisure-innovators. One striking example of this is that if a consumer switches from a paid-for to a zero price product, GDP declines. To deal with this issue, Nakamura *et al.* (2017) propose valuing the production of these services at cost, consistent with the usual treatment in the National Accounts. In the context of the present model this cost is:

$$V_1 := w \cdot L_M = \frac{\alpha^2 \chi}{\Psi(1-\alpha)} Y \tag{50}$$

where the second equality follows from substituting the equilibrium value of wages. An immediate implication of (50) is that V_1 grows at the same rate as output.

6.2 Consumption-based measures of value

The production-based measure discussed above does not attempt to capture the consumer surplus from free technologies. For that, one must turn to measures on the consumption side. The

where the final equality follows from the resource constraint. Finally, the income measure is the sum of wage payments, profits, rent payments and households' outlays on patents:

$$\begin{aligned} GDP(I) &= \left((1-s_A-s_M)w_Y + s_A w_A + s_M w_M\right) hN + J\Pi_B + A\Pi + (r+\delta)K - V\dot{A} = \\ &= (1-\alpha)Y + V\cdot A^{\phi}L_A + \frac{\alpha^2\chi}{\Psi(1-\alpha)}Y + \left(\alpha^2\chi Y - \frac{\alpha^2\chi}{\Psi(1-\alpha)}Y\right) + \alpha Y\left(1-\alpha-\alpha\chi\right) + \alpha^2 Y - V\cdot A^{\phi}L_A = \\ &= Y. \end{aligned}$$

GDP in this economy is simply equal to final output Y. Clearly, GDP does not include leisure technologies.

[52] To meaningfully talk about welfare effects of leisure technologies, in the remainder of the paper I assume that households' utility follows (6): that is, more leisure technologies raise utility. Specifically, I use the formulation of instantaneous utility that is consistent with balanced growth: $u = \log c + M^{\frac{1}{\nu-1}}(1-h)$.

literature has considered valuing leisure time at an ongoing wage (Goolsbee and Klenow (2006), Brynjolfsson and Oh (2012)) to capture the surplus. In addition to this measure, the model can be deployed to directly compute the compensating variation measure which answers the following question: how much would aggregate consumption had to increase to compensate consumers for a lack of access to leisure technologies? The two measures are given by:

$$V_{2a} := N \cdot w \cdot \int_0^M \ell(\iota) d\iota = \frac{1-\alpha}{1-s_A-s_M}\frac{1-h}{h}Y \tag{51}$$

$$V_{2b} := N(\bar{c}-c) = \left(\exp\left((1-h)M^{\frac{1}{\nu-1}}\right)-1\right)C. \tag{52}$$

where again the second set of equalities follows from substituting in the equilibrium values.

6.3 Analysis

Proposition 7. ***Measurement of the growth rates.*** *In equilibrium, the production cost-based measure V_1 is proportional to output. The value of leisure technologies derived from the consumption side V_{2a} and V_{2b} grow faster than output even in the long-run:*

$$\gamma_{V_1} = \gamma_Y < \gamma_{V_2}$$

Proof. Appendix B. □

The Proposition says that production-cost adjustments will leave growth of the enhanced measure of activity unchanged, relative to measured GDP. Conversely, the consumption based-adjustments will alter the growth rate. The underlying reason for why the production- and consumption-based measures give a different read is strong non-rivalry: in the attention economy, consumption is detached from production.

To illustrate these results quantitatively, Figure 8 plots the level and the growth rate of GDP per capita following the current measurement methodology, together with the 'enhanced' GDP metrics that include the values computed above. The Figure corresponds to the simulation discussed in Section 5.

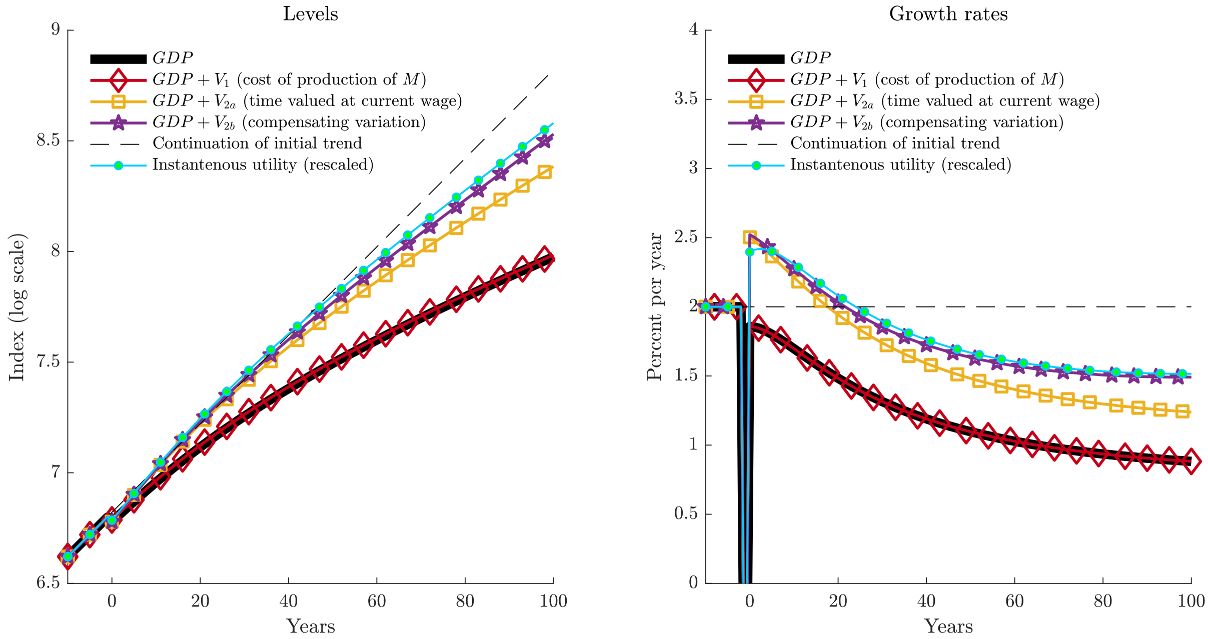

Figure 8

The Level and the Growth Rate of GDP and the Enhanced Measures of Activity

Because the leisure sector is small in the aggregate, the production cost-based measures do not make much difference to the level of activity. Including consumption-side measures on the other hand makes a significant difference not only to the level but also to the growth rate, in line with Proposition 7. Quantitatively, the consumption-based measures boost the growth rate by 0.4% per annum, which is roughly the negative of the growth rate of the hours worked. This magnitude of the quantitative effects is similar to some of the estimates reported in the earlier work.[53]

The time-cost approach tracks the path of instantaneous utility of the representative agent closely, suggesting that it can be a useful measure in practice. Quantitatively, the welfare gains from free technologies can make-up around half of the slowdown in activity.

What with the other half? Despite accounting for the utility benefits of free leisure, the growth rate of utility declines and stays depressed for many decades.[54] This may appear surprising at first: the time-cost based measure effectively 'relabels' leisure time as productive time, it might thus appear that once it is added to GDP, the resulting measure of economic activity would not deviate from the continuation of its initial trend. The reason why this logic is incorrect is that it ignores the general equilibrium feedback from hours worked and the growth rate of TFP. Even as one values hours spent on leisure at the current marginal product of labor, this does not recover the lost ground in activity because the marginal product of labor declines relative

[53]E.g. Brynjolfsson and Oh (2012) find the effect of around 0.3 percentage points per annum.

[54]The effect ultimately turns around, since the consumption utility increases linearly and leisure utility increases exponentially. But the negative effect dominates for more than 300 years in the simulations.

to the no-leisure-economy counterfactual. Furthermore, the negative productivity effect always dominates the positive leisure utility effect for some time after $\hat{t}$, since the level of consumption utility is much higher than the level of leisure utility. To see this, note that in equilibrium, for $t \geq \hat{t}$, utility is:[55]

$$u = \log(c) + \Phi\left(\frac{1}{h} - 1\right). \tag{53}$$

Clearly, the second term is small when h is close to 1. And the corollary of Proposition 3 is that consumption is relatively large when the leisure sector emerges. Together, these imply that the first term on the right hand side of (53) is much larger than the second. So while the consumers are willingly substituting away from consumption and towards leisure *on the margin*, the adverse productivity effect lowers the growth rate of consumption, hurting consumers a lot since consumption is a much larger component of utility *in levels*.

The productivity effect is clearly external to the individual choices. In the next Section I discuss the efficiency aspects of the attention economy.

7 Efficiency

Leisure-enhancing technological change introduces two additional inefficiencies to the market equilibrium.[56] The *static inefficiency* arises because platforms only benefit from the provision of M indirectly: the supply of leisure technologies is in equilibrium driven by the price of brand equity rather than the marginal rate of substitution between leisure technologies and consumption. The *dynamic inefficiency* results from the adverse effects of leisure today on productivity growth in the future. Before discussing those inefficiencies in some detail, I briefly remark on the optimal level of advertising.

7.1 Brand equity in socially-optimal allocation

Given the combative nature of brand equity competition in the benchmark model, the planner is indifferent as to the level of brand equity as long as its provision requires no extra resources:

Lemma 6. ***Optimal brand equity.*** *For a given (optimal) choice of leisure hours ℓ^*, the planner is indifferent between producing any amount of brand equity between 0 and $B(\ell^*)$. If there was an infinitesimal real resource cost to production of brand equity then the planner would choose to produce none.*

Proof. Follows immediately from the fact that for a given level of ℓ^* the choice of B^* in $[0, B(\ell^*)]$ has no impact on the resource constraint or utility. $\square$

[55] To see this, use equation (20) in $u = \log c + M^{\frac{1}{\nu-1}}(1-h)$.

[56] In addition to the inefficiencies familiar from the literature on optimal growth: the presence of monopolistic competition, externalities to R&D and possibly duplication externalities in research. For the analysis of these externalities see Jones (2015).

In what follows I assume that $B^* = 0$, but this is clearly without the loss of generality.

7.2 Static inefficiency

Consider a static setup in which an economy is endowed with exogenously given levels of capital stock K and knowledge A.[57] The supply side is identical to before, except there is no traditional R&D sector (since A is fixed). In particular, intermediate firms advertise and platforms supply brand equity and leisure technologies. Households maximize a separable utility function:

$$U = u(C) + v(1-h, M)$$

Since there is no saving, market clearing requires $C = Y$. Equilibrium conditions naturally mirror those in the dynamic model and I compare them to the social optimality conditions momentarily.

I study the following unconstrained problem of the social planner:

$$\max_{h, s_M} u(C) + v(1-h, M) \text{ s.t. } Y = F_Y(K, A, h, s_M),\ \ M = F_M(A, h, s_M) \text{ and } Y = C. \tag{54}$$

The planner maximizes social welfare subject to technology and resource constraints. (54) is an unconstrained problem in the sense that the planner has control over time allocation of the representative household: h is one of the two choice variables. Another is the share of labor employed in the M sector. These two controls determine traditional output and consumption, as well as the level of leisure technologies.

Socially optimal h^* satisfies the necessary condition

$$\frac{\partial u(C)}{\partial C}\frac{\partial Y}{\partial h} + \frac{\partial v(1-h, M)}{\partial M}\frac{\partial M}{\partial h} \geq \frac{\partial v(1-h, M)}{\partial h} \tag{55}$$

which holds with equality if h^* is interior. This condition compares the utility benefit of additional time spent working – higher consumption and more leisure technologies – with the cost: the loss of leisure utility as a result of working more. The equivalent condition in the decentralized equilibrium is:[58]

$$\frac{\partial u(C)}{\partial C}\frac{\partial Y}{\partial h} + \frac{\partial u(C)}{\partial C}\frac{p_B}{\Psi}\frac{1-h}{M}\frac{\partial M}{\partial h} \geq \frac{\partial v(1-h, M)}{\partial h}. \tag{56}$$

The difference between the two conditions lies in the way M is valued. In condition (55) this

[57] I assume that the level of capital is such that the capital-to-output ratio at full employment is constant and equal to 3 – that is, I consider pairs of A and K such that capital varies linearly with A: $K = 3^{\frac{1}{1-\alpha}}A$.

[58] To see this, note that households' labor income is $(w_Y(1-s_M) + w_M s_M)\, h$. Wages in the two sectors are $w_Y = \frac{\partial Y}{\partial L_Y} = \frac{\partial Y}{\partial h}\frac{1}{1-s_M}$ and $w_M = p_B \cdot \frac{1}{\Psi} \cdot \frac{1-h}{M} \cdot \frac{\partial M}{\partial L_M} = p_B \cdot \frac{1}{\Psi} \cdot \frac{1-h}{M} \cdot \frac{\partial M}{\partial h}\frac{1}{s_M}$. Combining these expressions with the optimality condition $\frac{\partial u}{\partial C}\,(w_Y(1-s_M) + w_M s_M) \geq \frac{\partial v}{\partial h}$ yields (56).

value is $\frac{\partial v(1-h,M)}{\partial M}$: the utility that this technology brings to the consumer on the margin. But in condition (56) it is the marginal revenue product of a platform, and that revenue comes from the sales of brand equity, not leisure technology. Put differently, the wage in the leisure sector does not reflect the marginal revenue product of the free technologies – instead, it reflects the marginal revenue product of brand equity. Since workers choose their labor supply based on the ongoing wage, the distortion in the level of wages translates into the distortion in hours worked.

The same distortion also affects the allocation of labor across sectors. The optimal share of labor employed in the platform sector satisfies

$$-\frac{\partial Y}{\partial s_M} \geq \frac{\partial M}{\partial s_M} \underbrace{\frac{\partial v/\partial M}{\partial u/\partial C}}_{MRS_{M,C}}. \tag{57}$$

Condition (57) compares the marginal cost and the marginal benefit of shifting labor away from production of the consumption good and towards the platform sector. The corresponding condition in equilibrium is:[59]

$$-\frac{\partial Y}{\partial s_M} \geq \frac{\partial M}{\partial s_M} \frac{1-h}{M} \left(\underbrace{p_B}_{\text{wrong price}} + \underbrace{\frac{\partial p_B}{\partial B_j}}_{\text{market power}} \right). \tag{58}$$

Comparing conditions (57) and (58) we see that the exact same distortion drives misallocation. In addition, the allocation of labor across industries is distorted due to the market power of platforms: since the term $\frac{\partial p_B}{\partial B_j}$ is non-positive, market power has the usual effect of making the sector too small relative to the optimum. But the market power distortion is insignificant relative to the distortion that stems from the platforms' business model.

To illustrate what this inefficiency means for the calibrated model, Figure 9 compares the optimal and equilibrium allocations for different levels of A (and implicitly K). Clearly, in a static economy calibrated with parameter values in Table 1 the optimal allocation features a much higher share of labor devoted to leisure innovation, compared to the equilibrium. This distortion affects both the point at which leisure technologies become available (the planner chooses positive M at comparatively much lower levels of A) and the extent, or intensive margin, of the provision of leisure technologies.

[59] This follows from differentiating $\frac{\partial}{\partial s_M}(p_B \cdot B_j)$.

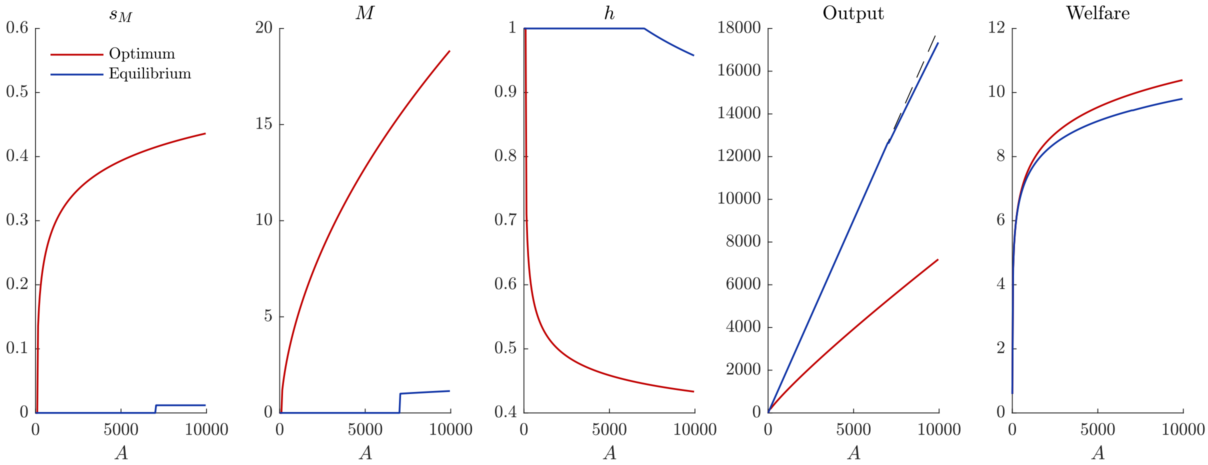

Figure 9
Socially optimal allocation and equilibrium in the static model

Note: The Figure plots the socially optimal allocation and equilibrium allocation for different values of A and implicitly K, holding the capital-to-output ratio constant at 3.

7.3 Dynamic inefficiency

The dynamic inefficiency arises because leisure technologies translate into lower future productivity – an effect that is external to the individual choices. Consider the planner's constrained dynamic optimization problem. When maximizing social welfare, the planner must respect households' time allocation decisions

$$h = \min\{1, \Phi M^{\frac{1}{1-\nu}}\} \quad \text{where} \quad \Phi = \left(\frac{1-\alpha}{1-s-s_M}\frac{Y}{C}\right)^{\frac{\theta}{\theta+1}}.$$

This implies that h is a function of s_M (since h depends on M). The Hamiltonian of this problem is:

$$\mathcal{H}(K, A, C, s, s_M; \mu_K, \mu_A) = u(c) + v(1-h, M) + \mu_K(Y - C - \delta K) + \mu_A \dot{A},$$

where μ_K and μ_A are the costate variables associated with capital and the level of traditional technology. The relevant necessary condition is:

$$\underbrace{\frac{\partial v}{\partial h}\frac{\partial h}{\partial s_M} + \frac{\partial v}{\partial M}\left(\frac{\partial M}{\partial s_M} + \frac{\partial M}{\partial h}\frac{\partial h}{\partial s_M}\right)}_{\text{utility benefit}} + \underbrace{\mu_K\left(\frac{\partial Y}{\partial s_M} + \frac{\partial Y}{\partial h}\frac{\partial h}{\partial s_M}\right)}_{\text{static output cost}} + \underbrace{\mu_A\left(\frac{\partial \dot{A}}{\partial h}\frac{\partial h}{\partial s_M}\right)}_{\text{dynamic TFP cost}} = 0.$$

The final term reflects the dynamic cost that is absent from the decentralized problem.

I collect the efficiency results in the following proposition.[60]

[60]The dynamic externality highlighted here is a separate issue to the externality that has been at the center of the literature on endogenous growth: the positive externalities to R&D. Specifically, the dynamic inefficiency would still be present with an R&D subsidy that internalizes the learning externalities from product innovation. The two

Proposition 8. ***Efficiency of market equilibrium.*** *Platform's ad-based business model results in a static inefficiency since the wrong price values leisure technologies in equilibrium. This distortion suggests that there is insufficient supply of leisure technologies in equilibrium. There is also a dynamic inefficiency as more time spent on leisure lowers future productivity. This inefficiency means the supply of leisure technologies in equilibrium is too high.*

8 Conclusion

In this paper I formalized the idea of leisure-enhancing technologies: products that are available for free and are thus specifically designed to capture our time and attention. Using the theory I studied how these technologies shape the growth patterns and what the welfare consequences are. The main takeaway is that these endogenously time-biased technologies can simultaneously explain shifts in hours worked and account for the low growth observed in the data. The effect on GDP growth reflects both the measurement difficulties and the 'real' slowdown, since leisure technologies fail to fully compensate for the crowding out of traditional productivity.

The analysis can be extended in many interesting directions. To organize thoughts, this concluding Section presents an account of how one might interpret developments in leisure economy over the past century through the lens of the model. Such historical narrative is necessarily more speculative than the analysis thus far, but it is interesting, and it helps spot gaps and spur ideas.

The theory delivered the conclusion that the size of the economy determines whether leisure technologies are viable or not. But other factors play a role too.

One such factor is the share of time that households can feasibly allocate to marketable leisure. The first half of the 20th century saw a substantial increase in this share. Two historical events were key: the introduction of the two-day weekend in the 1930s[61] and the gradual adoption of household appliances – the washing machine, the flush toilet, the vacuum and others – from 1920s through 1940s and beyond. Both of these freed up time for other activities.[62] It is plausible that these developments have acted to direct resources towards inventions and activities that complement leisure and leisure time, in the spirit of the directed technical change literature (Acemoglu, 2002). Incorporating these mechanisms in the model and assessing their empirical validity is an interesting avenue for future work.

Second, the audio-visual entertainment revolution that started in the 1920s and rapidly accelerated in the 1950s has been propelled by the introduction of general purpose technologies

externalities may interact in interesting ways, and the optimal R&D subsidy policy in the context of both is an interesting area for future work.

[61] In the US, Henry Ford made Saturday and Sunday days off for his staff as early as 1926, and the US as a whole adopted the five-day system in 1932 (in part to counter the unemployment caused by the Great Depression).

[62] This is especially interesting since the model presented here abstracted from home production – a margin that is clearly important in practice.

that allowed for the signal to be transmitted to households: the radio receiver and the television set. More recently, invention of PCs, smartphones and tablets made it possible for the free leisure technologies to spread far and wide (Figure 10).

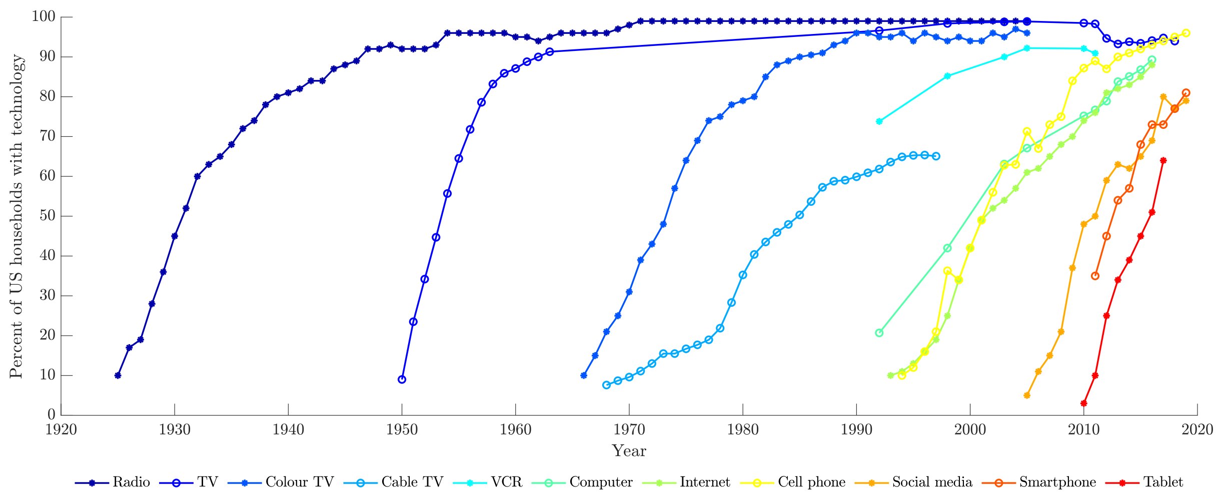

Figure 10

Adoption of leisure-linked general purpose technologies

Sources: Comin and Hobijn (2009).

To analyze these one could bring in the insights from Fernald and Jones (2014) on modeling general purpose technologies into the framework developed here. An alternative that stays closer to the current model is to view the arrival of these technologies as exogenous shifts in the productivity parameters of the leisure technology production functions: each of these GPTs made the delivery of free leisure services easier and cheaper and boosted the ability of platforms to turn consumers' time and attention into brand equity.[63] Appendix E presents analysis along those lines. It shows that such changes effectively turbo-charge the development of leisure technologies, exacerbating the effects discussed in the main body of the paper and potentially explaining the rise of large and highly profitable platforms.

Relatedly, the arrival of the internet, social media and smartphones has arguably been associated with a culture shift towards online activities among consumers. One particularly intriguing feature is the ubiquity of user-generated content: for example, social media are largely popu-

[63] Another effect of the GPTs such as computers and smartphones is that they blur the boundary between leisure and work, enabling leisure consumption at the workplace and making it easier for workers to engage in productive activities at home. Eldridge and Pabilonia (2010) study how 'bringing work home' affects the level and the growth rate of hours worked. They identify 0.7-0.9% downward bias in the level of hours worked due to prevalence of unpaid work at home in the period 2003-08, much of which has been enabled by the technologies mentioned above. However, they find no evidence that the growth rate over the period 2003-08 is affected, and consequently conclude that unpaid work from home over that period did not alter the observed trend in productivity growth. In a recent contribution, Restrepo and Zeballos (2020) study workers who work from home regularly (one day a week for example), and find that working from home cuts the time spent on grooming and commuting and leaves more time for food preparation and for leisure.

lated by content created by consumers, whereby a platform is just that: a platform that connects people and facilitates content generation.

An open question is how the rise of the leisure sector interacts with heterogeneity, both at the household and at the firm level. On the household side, it is interesting to study how time allocation decisions interact with income and wealth inequality. For example, disaggregated evidence on time allocation across the income distribution shows that poor individuals increased their leisure more than the rich (Boppart and Ngai, 2017b). Allowing for household and income heterogeneity in the presence of leisure-enhancing technologies could bring out new insights and aid the debate on leisure-inequality and the welfare implications. Considering firm heterogeneity may be important, too: the current setting is well suited to analyze equilibrium outcomes when heterogenous firms compete not only in prices but also in intangible assets. More productive firms may devote more resources to brand building, cementing their market share, with interesting implications for market power (De Loecker and Eeckhout, 2017). To tackle these issues, the framework developed here could be usefully incorporated into a model of firm dynamics and growth in the tradition of Klette and Kortum (2004).

As leisure economy becomes ever more important going forward, the framework built here can be used as a base for explorations of some of the pressing policy questions, such as optimal taxation of platforms or competition- and anti-trust policy in presence of zero-price services. These ideas formulate an exciting research agenda for economics in general, and macroeconomics specifically, in the years to come.

References

Acemoglu, D. (2002). Directed Technical Change. *Review of Economic Studies*, **69** (4), 781–809.

— and Guerrieri, V. (2008). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy*, **116** (3), 467–498.

Adler, G., Duval, R., Furceri, D., Kiliç Çelik, S., Koloskova, K. and Poplawski-Ribeiro, M. (2017). Gone with the Headwinds: Global Productivity. *IMF Staff Discussion Notes*, **17** (04), 1.

Aghion, P., Akcigit, U. and Howitt, P. (2014). What Do We Learn From Schumpeterian Growth Theory? *Handbook of Economic Growth*, **2**, 515–563.

— and Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, **60** (2), 323–351.

—, Jones, B. and Jones, C. (2017). Artificial Intelligence and Economic Growth.

Aguiar, M., Bils, M., Charles, K. K. and Hurst, E. (2017). *Leisure Luxuries and the Labor Supply of Young Men*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and Hurst, E. (2007a). Measuring Trends in Leisure: The Allocation of Time Over Five Decades. *The Quarterly Journal of Economics*, **122** (3), 969–1006.

— and — (2007b). Measuring trends in leisure: the allocation of time over five decades. *The Quarterly Journal of Economics*, **122** (3), 969–1006.

— and — (2008). *The Increase in Leisure Inequality*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and — (2016). The Macroeconomics of Time Allocation. In *Handbook of Macroeconomics*, vol. 2, pp. 1–44.

—, — and Karabarbounis, L. (2012). Recent developments in the economics of time use. *Annual Review of Economics*, **4**, 373–397.

Akcigit, U. and Kerr, W. R. (2018). Growth through heterogeneous innovations. *Journal of Political Economy*, **126** (4).

Anderson, S. P. and Renault, R. (2006). Advertising Content. *American Economic Review*, **96** (1), 93–113.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, **00** (3), 668–691.

Bagwell, K. (2007). Chapter 28 The Economic Analysis of Advertising. In *Handbook of Industrial Organisation*, pp. 1701–1844.

Basu, S., Fernald, J. G. and Kimball, M. S. (2006). Are technology improvements contractionary? *American Economic Review*, **96** (5), 1418–1448.

Bean, C. (2016). *Independent Review of UK Economic Statistics*. Tech. rep.

Becker, G. S. (1965). A Theory of the Allocation of Time. *The Economic Journal*, **75** (299), 493–517.

— and Murphy, K. M. (1993). A Simple Theory of Advertising as a Good or Bad. *The Quarterly Journal of Economics*, **108** (4), 941–964.

Benhabib, J. and Bisin, A. (2002). Advertising, Mass Consumption and Capitalism. *Working Paper*, pp. 1–38.

Bloom, N., Jones, C., Van Reenen, J. and Webb, M. (2017). *Are Ideas Getting Harder to Find?* Tech. rep., National Bureau of Economic Research, Cambridge, MA.

—, Jones, C. I., van Reenen, J. and Webb, M. (2020). Are ideas getting harder to find? *American Economic Review*, **110** (4), 1104–1144.

Boppart, T. and Krusell, P. (2020). Labor Supply in the Past, Present, and Future: A Balanced-Growth Perspective. *Journal of Political Economy*, **128** (1), 118–157.

— and Ngai, L. (2017a). Rising inequality and trends in leisure. (DP12325).

— and Ngai, L. R. (2017b). Rising inequality and trends in leisure.

Borden, N. H. (1942). Findings of the Harvard Study on the Economic Effects of Advertising. *Journal of Marketing*, **6** (4), 89–99.

Bridgman, B. (2018). Is Productivity on Vacation? The Impact of the Digital Economy on the Value of Leisure.

Broda, C. and Weinstein, D. E. (2006). Globalization and the Gains From Variety. *The Quarterly Journal of Economics*, **121** (2), 541–585.

Brynjolfsson, E., Eggers, F. and Gannamaneni, A. (2018). *Using Massive Online Choice Experiments to Measure Changes in Well-being*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

— and Oh, J. H. (2012). The attention economy: Measuring the value of free digital services on the internet. *International Conference on Information Systems, ICIS 2012*, **4**, 3243–3261.

Butters, G. R. (1977). Equilibrium Distributions of Sales and Advertising Prices. *The Review of Economic Studies*, **44** (3), 465.

Byrne, D. M., Fernald, J. G. and Reinsdorf, M. B. (2016a). Does the United States have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*.

—, — and — (2016b). Does the united states have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*, **2016** (SPRING), 109–182.

Cavenaile, L. and Roldan-Blanco, P. (2020). Advertising, Innovation and Economic Growth.

Chamberlin, E. (1933). *The theory of monopolistic competition*. Harvard University Press.

Christensen, M. A., Bettencourt, L., Kaye, L., Moturu, S. T., Nguyen, K. T., Olgin, J. E., Pletcher, M. J. and Marcus, G. M. (2016). Direct Measurements of Smartphone Screen-Time: Relationships with Demographics and Sleep. *PLOS ONE*, **11** (11).

Comin, D. A. and Hobijn, B. (2009). *The CHAT Dataset*. Tech. rep.

Corrado, C., Haskel, J., Jona-Lasinio, C. and Iommi, M. (2012). Intangible Capital and Growth in Advanced Economies: Measurement Methods and Comparative Results.

Corrado, C. A. and Hulten, C. R. (2010). How Do You Measure a Technological Revolution? *American Economic Review*, **100** (2), 99–104.

Coyle, D. (2017). Do-it-yourself digital: the production boundary, the productivity puzzle and economic welfare Diane Coyle Professor of Economics, University of Manchester, and ONS Fellow 1. pp. 1–38.

De Loecker, J. and Eeckhout, J. (2017). *The Rise of Market Power and the Macroeconomic Implications*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

DellaVigna, S. and Gentzkow, M. (2010). Persuasion: Empirical Evidence. *Annual Review of Economics*, **2** (1), 643–669.

Dixit, A. and Norman, V. (1978). Advertising and Welfare. *The Bell Journal of Economics*, **9** (1), 1.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, **67** (3), 297–308.

Dorfman, R. and Steiner, P. O. (1954). Optimal Advertising and Optimal Quality. *The American Economic Review*, **44** (5), 826–836.

Eldridge, L. P. and Pabilonia, S. W. (2010). Bringing work home: Implications for BLS productivity measures. *Monthly Labor Review*, **133** (12), 18–35.

Falck, O., Gold, R. and Heblich, S. (2014). E-lections: Voting behavior and the internet. *American Economic Review*, **104** (7), 2238–2265.

Farboodi, M. and Veldkamp, L. (2019). A Growth Model of the Data Economy.

Feenstra, R. C., Inklaar, R., Timmer, M. P., Aten, B., Atkeson, A., Basu, S., Deaton, A., Diewert, E., Heston, A., Jones, C., Rao, P., Rodríguez-Clare, A. and Végh, C. (2015). The Next Generation of the Penn World Table. *American Economic Review*, **105** (10), 3150–3182.

Fernald, J. G. and Jones, C. I. (2014). The future of US economic growth. *American Economic Review*, **104** (5), 44–49.

Gentzkow, M. (2006). Television and Voter Turnout*. *Quarterly Journal of Economics*, **121** (3), 931–972.

— and Shapiro, J. M. (2008). Preschool Television Viewing and Adolescent Test Scores: Historical Evidence from the Coleman Study. *Quarterly Journal of Economics*, **123** (1), 279–323.

Goolsbee, A. and Klenow, P. J. (2006). Valuing consumer products by the time spent using them: An application to the Internet. In *American Economic Review*, vol. 96, pp. 108–113.

Gourio, F. and Rudanko, L. (2014). Customer capital. *Review of Economic Studies*, **81** (3), 1102–1136.

Grossman, G. M. and Shapiro, C. (1984). Informative Advertising with Differentiated Products. *The Review of Economic Studies*, **51** (1), 63.

Hall, R. E. (2008). General equilibrium with customer relationships: a dynamic analysis of rent-seeking. *Working Paper*, pp. 1–35.

Hulten, C. and Nakamura, L. (2017). Accounting for Growth in the Age of the Internet: The Importance of Output-Saving Technical Change. *Working Paper*, pp. 1–36.

Johnson, J. P. (2013). Targeted advertising and advertising avoidance. *RAND Journal of Economics*, **44** (1), 128–144.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, **103** (4), 759–784.

— (2015). The Facts of Economic Growth. *The Handbook of Macroeconomics Volume 2*, pp. 0–81.

— and Tonetti, C. (2019). Nonrivalry and the economics of data.

Jorgenson, D. W. (2005). Chapter 10 Accounting for Growth in the Information Age. *Handbook of Economic Growth*, **1** (SUPPL. PART A), 743–815.

— (2018). Production and welfare: Progress in economic measurement. *Journal of Economic Literature*, **56** (3), 867–919.

Kelly, Y., Zilanawala, A., Booker, C. and Sacker, A. (2018). Social Media Use and Adolescent Mental Health: Findings From the UK Millennium Cohort Study. *EClinicalMedicine*, **6**, 59–68.

King, R. G., Plosser, C. I. and Rebelo, S. T. (1988). Production, growth and business cycles. *Journal of Monetary Economics*, **21** (2-3), 195–232.

Klein, E. (2016). Technology is changing how we live, but it needs to change how we work.

Klette, T. J. and Kortum, S. (2004). Innovating Firms and Aggregate Innovation. *Journal of Political Economy*, **112** (5).

Kortum, S. (1997). Research, Patenting, and Technological Change. *Econometrica*, **65** (6), 1389–1419.

Lambin, J. J. (1976). *Advertising, Competition and Market Conduct in Oligopoly Over Time*. Amsterdam: North-Holland.

Lewis, R. A. and Rao, J. M. (2015). The unfavorable economics of measuring the returns to advertising. *Quarterly Journal of Economics*, **130** (4), 1941–1973.

— and Reiley, D. H. (2014). Online ads and offline sales: Measuring the effect of retail advertising via a controlled experiment on Yahoo! *Quantitative Marketing and Economics*, **12** (3), 235–266.

Marshall, A. (1890). *Principles of Economics*. London: MacMillan and Co. Ltd, 8th edn.

Michel, C., Sovinsky, M., Proto, E. and Oswald, A. J. (2019). Advertising as a major source of human dissatisfaction: Cross-national evidence on one million europeans. *The Economics of Happiness: How the Easterlin Paradox Transformed Our Understanding of Well-Being and Progress*, **i** (November), 217–239.

Milgrom, P. and Roberts, J. (1986). Price and Advertising Signals of Product Quality. *Journal of Political Economy*, **94** (4), 796–821.

Molinari, B. and Turino, F. (2009). Advertising and Business Cycle Fluctuations.

Nakamura, L., Samuels, J. and Soloveichik, R. (2017). Measuring the "Free" Digital Economy within the GDP and Productivity Accounts.

Nelson, P. (1974). Advertising as Information. *Journal of Political Economy*, **82** (4), 729–754.

Ngai, L. R. and Pissarides, C. A. (2008). Trends in hours and economic growth. *Review of Economic Dynamics*, **11** (2), 239–256.

Nixon, D. (2017). Is the economy suffering from the crisis of attention?

Nordhaus, W. D. and Tobin, J. (1972). *Is Growth Obsolete?*, vol. 5.

OECD (2009). *Special Focus: Measuring Leisure in OECD Countries*. Tech. rep.

OfCOM (2019). *Online Nation 2019 Report*. Tech. Rep. May.

Rachel, L. (2020). Leisure-enhancing technologies and structural change.

Ramey, V. A. and Francis, N. (2009). A century of work and leisure. *American Economic Journal: Macroeconomics*, **1** (2), 189–224.

Rauch, F. (2013). Advertising expenditure and consumer prices. *International Journal of Industrial Organization*, **31** (4), 331–341.

Reis, F. (2015). Patterns of Substitution between Internet and Television in the Era of Media Streaming-Evidence from a Randomized Experiment.

Restrepo, B. J. and Zeballos, E. (2020). The effect of working from home on major time allocations with a focus on food-related activities. *Review of Economics of the Household*.

Rochet, J.-C. and Tirole, J. (2003). Platform Competition in two-sided markets. *Journal of the european economic association 1 (4), 990-1029*.

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy*, **98** (5), S71–S102.

Royal Society for Public Health (2017). Social media and young people' s mental health and wellbeing. *Royal Society for Public Health,*, (May), 32.

Scanlon, P. (2018). *Why Do People Work So Hard?* 2018 Meeting Papers 1206, Society for Economic Dynamics.

Segerstrom, P. S. (1998). Endogenous Growth without Scale Effects. *American Economic Review*, **88** (5), 1290–1310.

Solow, R. M. (1987). We'd Better Watch out...

Stahl, D. O. (1989). Oligopolistic Pricing with Sequential Consumer Search. *The American Economic Review*, **79** (4), 700–712.

Stiglitz, J. E., Sen, A. and Fitoussi, J.-P. (2009). *Report by the Commission on the Measurement of Economic Performance and Social Progress*. Tech. rep.

Syverson, C. (2017). Challenges to Mismeasurement Explanations for the US Productivity Slowdown. *Journal of Economic Perspectives*, **31** (2), 165–186.

Trimborn, T., Koch, K. J. and Steger, T. M. (2008). Multidimensional transitional dynamics: A simple numerical procedure. *Macroeconomic Dynamics*, **12** (3), 301–319.

Tucker, C. E. (2012). The economics of advertising and privacy. *International Journal of Industrial Organization*, **30** (3), 326–329.

Vandenbroucke, G. (2009). Trends in hours: The U.S. from 1900 to 1950. *Journal of Economic Dynamics and Control*, **33** (1), 237–249.

Veblen, T. (1899). *The Theory of the Leisure Class*. MacMillan and Co. Ltd.

Wallsten, S. (2013). *What Are We Not Doing When We're Online*. Tech. rep., National Bureau of Economic Research, Cambridge, MA.

Ward, A. F., Duke, K., Gneezy, A. and Bos, M. W. (2017). Brain Drain: The Mere Presence of One's Own Smartphone Reduces Available Cognitive Capacity. *Journal of the Association for Consumer Research*, **2** (2), 140–154.

Appendices

A Illustrative evidence

This Appendix further motivates the focus of this paper and forms a background to the analysis.

Evidence on leisure-enhancing innovations

Figure 2 in the main text illustrated the increased importance of the digital sub-sector of the attention economy since the mid-1990s. The available industry statistics reinforce this message. For example, Figure A.1 shows the dramatic rise in the number of smartphone apps, with the majority available free of charge to the consumer. The fact that millions of apps have been created over the past decade is a testament to the innovative efforts of firms in the attention economy.[64]

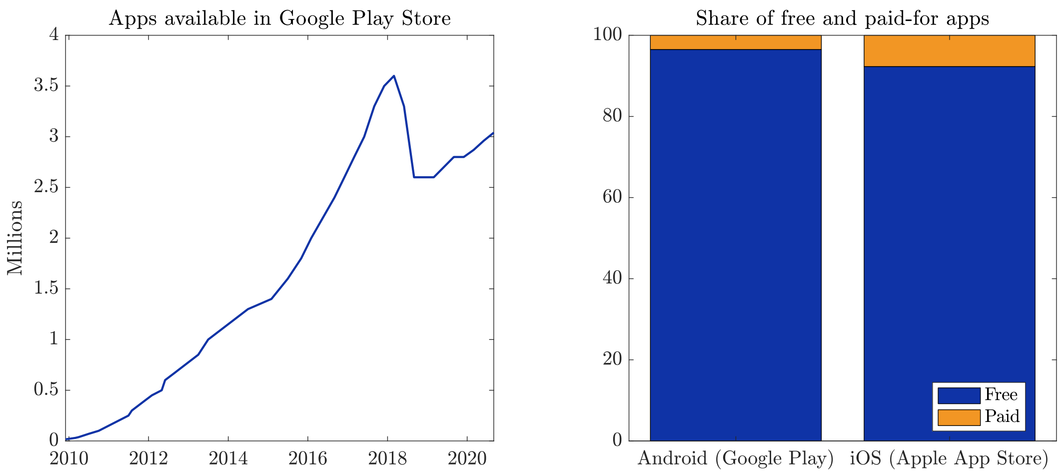

Figure A.1
Smartphone Apps

Source: The number of apps in Google Play Store is from Google, App Annie and AppBrain. The paid vs free apps breakdown is from 42matters, an app analytics company.

Consistent with the rapid technological progress within it, the leisure sector appears to be an increasingly important driver of the overall R&D spending. No exact measure for the share of attention economy in overall R&D spending is available; but it is possible to construct rough proxies by considering a subset of industries which are most likely engaged in leisure-enhancing

[64] The market structure in the app market is more complex than in the model presented in the main text. Apps are available on platforms such as Google Play Store or Apple App Store, but are produced by many firms, not just Google or Apple. This additional layer of intermediation does not change the economics of the paper though: the incentives to capture the time and attention of the end-user remains unchanged. Future work could usefully explore the competition, business stealing and firm dynamics aspects of the app producers or other firms within the broadly defined leisure sector.

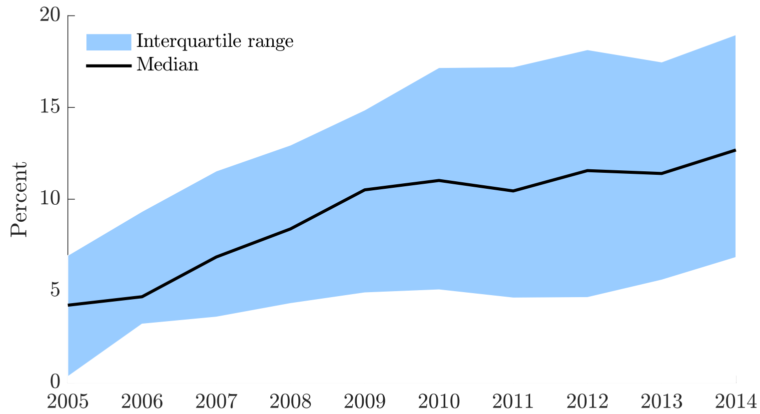

Figure A.2

R&D Expenditure Share of the (Proxy for the) Leisure Economy

Source: OECD. Includes data for an unbalanced panel of 39 countries. The figure shows the median and the interquartile range of the country-level share of R&D spending in the following sectors: publishing; motion picture, video and television program production; sound recording; programming and broadcasting activities; telecommunications services; computer programming, consultancy and related activities; information service activities; data processing, hosting and related activities; and web portals.

innovations. Figure A.2 shows that the share of R&D spending accounted for by the sectors such as video and TV program production, sound recording, broadcasting and web portals has been rising over time.

Recent changes in time allocation patterns

While hours worked in the United States have fallen by less than in other countries (recall the middle panel of Figure 2), the trend in leisure time has been clearly upwards. Data from the annual American Time Use Survey, available from 2003, show that the largest increase in any category has been recorded in the "relaxing and leisure" category. Significantly, the breakdown of the increase reveals that this rise is more-than-accounted-for by changes in the categories most directly related to leisure technologies, such as watching TV or using a computer (Figure A.3).

Nonetheless, there are reasons why the time use survey data may underestimate the time that actually spent on modern leisure technologies, and perhaps overestimate the time spent working (or at least working attentively). First, the survey aims to uncover a person's main activity at any given point in time during a day, and so if some of the leisure technologies are used during other activities (for example during work hours), their use will not be recorded. This is important since the evidence (which I discuss below) suggests that smartphones in particular are being used with a constant frequency throughout the day, including during work hours, and some of that use is likely to be related to leisure. For the same reason the BLS acknowledges that ATUS is not a good source of information on time spent online and/or using a computer or a smartphone: the survey is designed in such a way that time is split across the many traditional categories

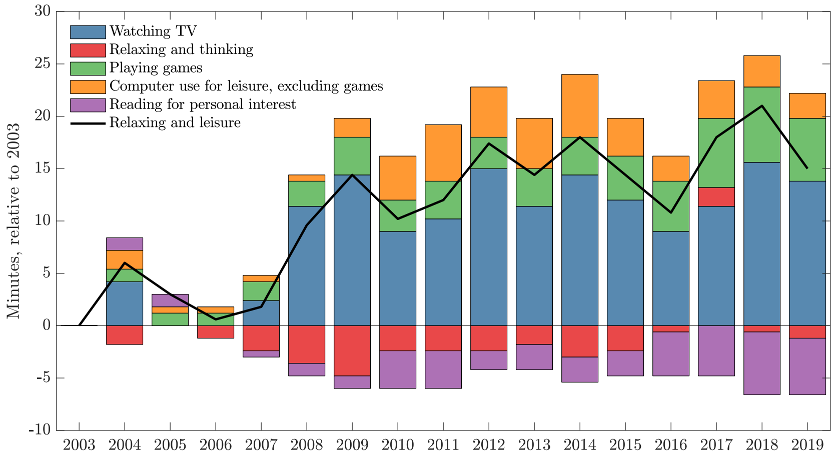

Figure A.3

Decomposition of the Increase in Relaxing and Leisure – the Category in the American Time Use Survey that has Experienced the Largest Increase Since 2003

Source: American Time Use Survey.

such as working, socializing, etc.[65] This could give a misleading steer on the use of the leisure technologies if, for example, socializing today is different to socializing in the past (in particular if socializing today involved the use of leisure technologies). A related point is that, since people tend to check their phone very frequently (numerous estimates available online suggest that we pick-up our phones between 50 and 100 times a day), it is likely that the responders under-report usage when they fill in the survey. Consistent with that, some anecdotal evidence and the popularity screen-time-tracking software suggests that users may find it hard to control the frequency of use and overall amount of time they spend on their devices. That could suggest possible underreporting in the surveys.

Given these possible shortcomings of the time-use survey data, the device-tracking data from Nielsen offers useful cross-check (even as it is not without drawbacks). The data paints a picture of a much more dramatic changes in time-use linked to technology (Figure A.4). For example, the data suggest that the amount of time spent on a smartphone *more than quadrupled* over the last 7 years, reaching over 3 hours daily. One of the limitations of these data is that they are not additive: a person can engage in multiple activities at once (e.g. watching TV and engaging on social media on the smartphone). Another is that the time spent on the devices could be productive time. Nonetheless, these data are a useful complement to the traditional time use surveys which naturally struggle to capture the short-but-frequent spells of usage.

The evidence on how people spend time at work (and indeed how much work is being carried out at home) is imperfect. For that reason it is useful to consider experimental tracking data on the frequency of use throughout the day. In one particularly prescient study, Christensen *et al.* (2016) measured smartphone screen time over the course of an average day among

[65] See https://www.bls.gov/tus/atusfaqs.htm#24 for the discussion of this point.

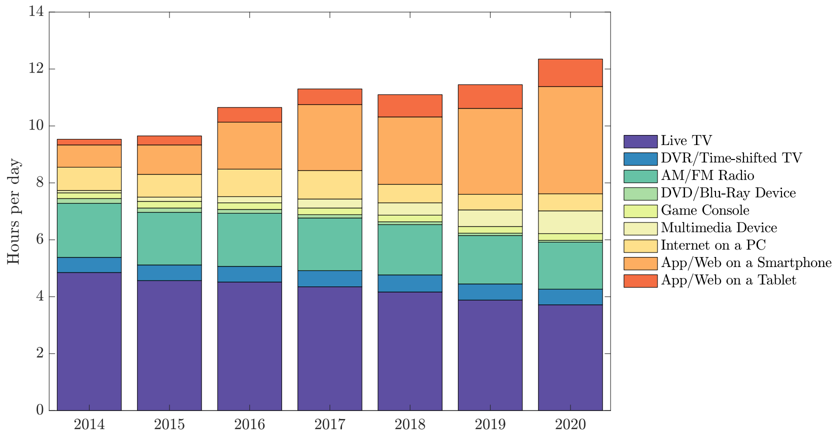

Figure A.4
Average Time Spent on Media Consumption per Adult in the US

Source: Nielsen. Note: Figures for representative samples of total US population (whether or not have the technology). More than one technology may be used at any given time, thus the total is indicative only. Data on TV and internet usage, and the usage of TV-connected devices are based on 248,095 individuals in 2016 and similar sample sizes in other years. Data on radio are based on a sample of around 400,000 individuals. There are approximately 9,000 smartphone and 1,300 tablet panelists in the U.S. across both iOS and Android smartphone devices.

a sample of 653 people in 2014 (Figure A.5). Time spent on the phone averaged 1 hour and 29 minutes per day, a little higher than what the Nielsen data suggests (which makes sense since the study included only users, while Nielsen aim to weight their results to capture non-adopters). Most strikingly, the mobile phone usage appears to be uniformly distributed throughout the day, suggesting that leisure time is, in part, substituting for time spent working. In a different study, Wallsten (2013) uses time use surveys to estimate that each minute spent on the internet is associated with loss of work-time of about 20 seconds.

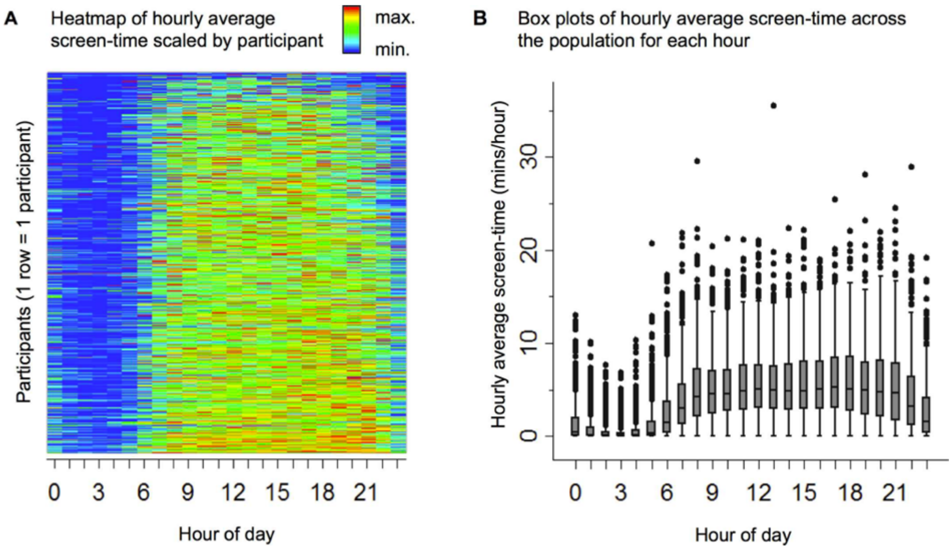

Figure A.5
Mobile Phone Use Over the Course of an Average Day

Source: Christensen *et al.* (2016).

Indeed, one feature of the latest technology is that it allowed leisure to "compete" with work much more directly than has been the case in the past. While it may not have been possible to watch TV at work, online entertainment is available during the work hours. This is, at least in part, balanced by the possibility of accessing work emails at home (see Footnote 63). Future research should consider ways of measuring in more detail how people spend time at work and how much work is done at home.

B Proofs

Proof of Lemma 1

Proof. The total amount of marketable leisure time is defined as $\ell := \int_0^M \ell_\iota d\iota$. By the symmetry of household's problem, it is immediate that the optimal choice is to spread free time evenly across the M available leisure options and therefore $\ell = M\ell_\iota$ which implies $\ell_\iota = \frac{\ell}{M}$. Plugging this back into (10) yields the result. $\square$

Proof of Lemma 2

Proof. The Hamiltonian associated with household problem (12) is:

$$\mathcal{H}(K, C, h; \mu_K) = \log(C/N) - \frac{\left(hM^{\frac{1}{\nu-1}}\right)^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}} + \mu_K\left(whN + rK - C - V\dot{A} + A\Pi + J\Pi_B\right).$$

The necessary conditions for an interior optimum are:

$$C^{-1} = \mu_K \tag{59}$$

$$\left(hM^{\frac{1}{\nu-1}}\right)^{\frac{1}{\theta}} M^{\frac{1}{\nu-1}} = \mu_K wN \tag{60}$$

$$\rho\mu_K - \dot{\mu}_K = \mu_K r. \tag{61}$$

together with the transversality and the no-Ponzi game conditions:

$$\lim_{t\to\infty} e^{-\rho t}\mu_K K = 0.$$

Combining these conditions with equation (65) derived below yields:

$$h^{\frac{\theta+1}{\theta}} M^{\frac{1}{\nu-1}\frac{\theta+1}{\theta}} = \frac{(1-\alpha)Y}{(1-s_A-s_M)C}$$

where $s_A := \frac{L_A}{L_Y+L_A+L_M}$ and $s_M := \frac{L_M}{L_Y+L_A+L_M}$ are the shares of labor employed in the two R&D sectors. Letting $\Phi := \left(\frac{1-\alpha}{1-s_A-s_M}\frac{Y}{C}\right)^{\frac{\theta}{\theta+1}}$, we obtain the interior solution:

$$h = \Phi M^{\frac{1}{1-\nu}}.$$

Since hours worked are bounded from above by 1, the solution is:

$$h = \min\{1, \Phi M^{\frac{1}{1-\nu}}\} \tag{62}$$

and since $\ell = 1 - h$:

$$\ell = \max\{0, 1 - \Phi M^{\frac{1}{1-\nu}}\} \tag{63}$$

□

Proof of Lemma 3

Proof. Suppose that the platform sector is active: $\Omega = 1$. The maximization problem of the final good producer is:

$$\max_{x_i, L_Y} \int_0^A \left(\left(\frac{b_i}{\bar{b}}\right)^{\chi} x_i\right)^{\alpha} L_Y^{1-\alpha} di - \int_0^A x_i q_i di - wL_Y \tag{64}$$

The first order conditions are:

$$(1-\alpha)\frac{Y}{L_Y} = w \tag{65}$$

$$\alpha \left(\frac{b_i}{\bar{b}}\right)^{\alpha\chi} x_i^{\alpha-1} L_Y^{1-\alpha} = p_i \tag{66}$$

where p_i is the price of variety i of the intermediate good. In a symmetric equilibrium $x_i = x\forall i$, $k_i = k\forall i$ and $p_i = p\forall i$. Furthermore, since the linear production technology implies that each intermediate's capital is equal to its output, we can define the aggregate capital stock as $K := Ax$. The final output can be written as

$$Y = \left(\frac{b}{\bar{b}}\right)^{\alpha\chi} K^{\alpha} \left((1 - s_A - s_M)hNA\right)^{1-\alpha}. \tag{67}$$

Condition (66) can then be re-written as:

$$\alpha\frac{Y}{K} = p. \tag{68}$$

The problem of each intermediate producer is (dropping the i subscript):

$$\max_{x,b} px - (r+\delta)x - p_B b \tag{69}$$

subject to (66) and technology $x = k$. The first order conditions are:

$$\alpha p = r + \delta \tag{70}$$

$$p_B = \alpha^2 \chi \frac{b^{\alpha\chi-1}}{\bar{b}^{\alpha\chi}} x^{\alpha} L_Y^{1-\alpha} \tag{71}$$

Together with equation (68), the first of these conditions gives the familiar capital demand condition:

$$\alpha^2 \frac{Y}{K} = r + \delta. \tag{72}$$

The optimal output of each producer can be derived from plugging the first order condition into the demand curve:

$$x = \left(\frac{\alpha^2}{r+\delta} \left(\frac{b}{\bar{\bar{b}}} \right)^{\alpha\chi} \right)^{\frac{1}{1-\alpha}} L_Y \tag{73}$$

which gives the following expression for capital stock:

$$K = Ax = \left(\frac{\alpha^2}{r+\delta} \right)^{\frac{1}{1-\alpha}} AL_Y. \tag{74}$$

Equations (73), (71) and (74) yield:

$$p_B = \alpha^2 \chi \frac{Y}{A} \frac{1}{\bar{\bar{b}}} \left(\frac{b}{\bar{\bar{b}}} \right)^{\frac{\alpha}{1-\alpha}\chi - 1}. \tag{75}$$

In equilibrium $b_i = \bar{b}$ and the results in the Lemma follow immediately from (68), (73) and (75). Equation (75) also implies

$$p_B b = \alpha^2 \chi \frac{Y}{A}. \tag{76}$$

Equilibrium profits of intermediate firms are thus:

$$\Pi = \alpha \frac{Y}{A} (1 - \alpha - \alpha\chi). \tag{77}$$

□

Proof of Lemma 4

Proof. The optimality condition to problem (26) is:

$$p_B - \left(1 - \frac{\alpha}{1-\alpha}\chi\right) p_B \frac{A\bar{b}}{B_j + \sum_{k \neq j} B_k} \frac{B_j}{A\bar{\bar{b}}} = \mathcal{M}^B.$$

In a symmetric equilibrium $B_j = B_k \forall j, k$ hence:

$$p_B \left(1 - \left(1 - \frac{\alpha}{1-\alpha}\chi\right) \frac{1}{J}\right) = \mathcal{M}^B.$$

□

Proof of Lemma 5

Proof. In equilibrium price equals markup over marginal cost:

$$p_B = \Psi w \frac{M}{\ell A^{\phi}}. \tag{78}$$

Using (22) we get:

$$\alpha^2 \chi \frac{Y}{B} = \Psi w \frac{M}{\ell A^{\phi}} \tag{79}$$

Since $B = \ell$ by the platforms' technology and $w = (1-\alpha)\frac{Y}{(1-s_A-s_M)Nh}$ we have:

$$M = \underbrace{\frac{\alpha^2 \chi}{(1-\alpha)\Psi}(1-s_A-s_M)}_{\Upsilon :=} hNA^{\phi}. \tag{80}$$

□

Proof of Proposition 3

Proof. Suppose that $\ell > 0$. Combining (62) and (80) and solving for M:

$$M = \left(\Upsilon \Phi N A^{\phi}\right)^{\frac{\nu-1}{\nu}}. \tag{81}$$

Note that under Assumption 3, $A = A_0 e^{\frac{n}{1-\phi}t} = A_0 \left(\frac{N}{N_0}\right)^{\frac{1}{1-\phi}}$. Equation (63) implies that $\ell > 0$ if and only if $M > \Phi^{\nu-1}$or equivalently that

$$\left(\Upsilon \Phi N^{\frac{2-\phi}{1-\phi}} \left(\frac{A_0}{N_0^{\frac{1}{1-\phi}}}\right)^{\phi}\right)^{\frac{1}{\nu}} > \Phi.$$

So there is leisure-enhancing technical change if

$$N(t) > \left(\frac{\Phi(\hat{t})^{\nu-1}}{\Upsilon(\hat{t})}\left(\frac{N_0^{\frac{1}{1-\phi}}}{A_0}\right)^{\phi}\right)^{\frac{1-\phi}{2-\phi}}$$

where Φ and Υ are evaluated at $\hat{t}$ once platforms have entered. Letting $\Gamma := \left(\frac{\Phi(\hat{t})^{\nu-1}}{\Upsilon(\hat{t})}\left(\frac{N_0^{\frac{1}{1-\phi}}}{A_0}\right)^{\phi}\right)^{\frac{1-\phi}{2-\phi}}$ completes the proof. □

Proof of Proposition 4

Proof. The first part of the Proposition follows directly from Proposition 3.

In segment 1, platforms are inactive: $\Omega = 0$, $s_M = 0$ and $h = 1$. Differentiating equation (2) with respect to time gives the formula for g. Equation (67) implies that output per capita is given by:

$$\bar{y} = \bar{k}^{\alpha}((1 - s_A)A)^{1-\alpha}$$

where $\bar{k} := \frac{K}{N}$ and $\bar{y} := \frac{Y}{N}$. $\bar{k}$ and $\bar{y}$ both grow at equal rate $\gamma_{\bar{y}}$ on the BGP, and thus $\gamma_{\bar{y}} = \alpha\gamma_{\bar{y}} + (1 - \alpha)g$ which implies $\gamma_{\bar{y}} = g$.

Turning to segment 2, equation (62) implies that asymptotically:

$$\gamma_h = \frac{1}{1 - \nu}\gamma_M \tag{82}$$

Differentiating the ideas production functions (2) and (19) with respect to time and assuming balanced growth gives the following two equations

$$0 = (\phi - 1)\gamma_A + n + \gamma_h$$
$$0 = \phi\gamma_A - \gamma_M + n + \gamma_h$$

which imply $\gamma_A = \gamma_M$ as well as the formulas in the proposition. □

Proof of Proposition 5

Proof. The share of workers in the R&D sector is pinned down by the expected zero-profit condition $wL_A = V\dot{A}$ where V is given by (15). Differentiating equation (15) with respect to time yields a standard Bellman equation:

$$\dot{V} = Vr - \Pi. \tag{83}$$

Thus $r = \frac{\Pi}{V} + \frac{\dot{V}}{V}$ and $V = \left(r - \frac{\dot{V}}{V}\right)^{-1}\Pi$. On the balanced growth path, r and $\frac{\dot{V}}{V}$ are constant and so V and Π must grow at the same rate. Equation (23) implies that the growth rate of Π and V is equal to $\gamma_Y - \gamma_A$. Plugging this into the zero profit condition above we get:

$$(1 - \alpha)\frac{s_A}{1 - s_A - s_M} = \frac{\alpha\,(1 - \alpha - \alpha\chi)}{r - (\gamma_Y - \gamma_A)}\gamma_A \tag{84}$$

From $M = A^{\phi}L_M$ and equation (80) we get

$$L_M = \frac{\chi\alpha^2}{\Psi(1 - \alpha)}L_Y$$

so that

$$s_M = \frac{\alpha^2}{\Psi(1-\alpha)\alpha_F}\chi\left(1 - s_A - s_M\right).$$

Solving for s_M:

$$s_M = \frac{\frac{\alpha^2}{\Psi(1-\alpha)\alpha_F}\chi\left(1-s\right)}{1+\frac{\alpha^2}{\Psi(1-\alpha)\alpha_F}\chi} = \frac{1-s_A}{1+\frac{\Psi(1-\alpha)}{\alpha^2\chi}} \tag{85}$$

Plugging this into (84) and solving for s_A yields the result. □

Proof of Proposition 6

The resource constraint of the economy is $\dot{K} = Y - C - \delta K$. On the BGP, capital and output grow at the same rate so that the capital to output ratio and the interest rate are constant. Taking logs and differentiating the expression for k with respect to time gives $\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \gamma_Y$, so that $\dot{K} = \frac{\dot{k}}{k}K + \gamma_Y K = \dot{k}N^{\beta_Y} + \gamma_Y K$ and

$$\dot{k}N^{\beta_Y} + \gamma_Y K = Y - C - \delta K.$$

Dividing through by N^{β_Y} and rearranging yields equation (39).

To obtain equation (40), differentiate the definition of a with respect to time to obtain $\dot{A} = \dot{a}N^{\beta_A} + \gamma_A A$. Solving for $\dot{a}$ gives:

$$\dot{a} = \frac{A^{\phi}L_A}{N^{\beta_A}} - \gamma_A a = a^{\phi}N^{\beta_A\phi}s_A\tilde{h}N^{\frac{1}{1-\nu}\beta_A}N^{1-\beta_A} - \gamma_A a.$$

Noting that $\beta_A\phi + \frac{1}{1-\nu}\beta_A + 1 - \beta_A = 0$ we obtain equation (40).

Differentiating c with respect to time we get so that $\frac{\dot{C}}{C} = \frac{\dot{c}}{c} + \beta_Y n$. Optimality conditions (59) and (61) give $\frac{\dot{C}}{C} = r - \rho$. Together these yield (41).

Taking logs and differentiating the expression for v gives $\frac{\dot{v}}{v} = \frac{\dot{V}}{V} - (\gamma_Y - \gamma_A)$. Thus$\dot{V} = \dot{v}(N^{\beta_Y-\beta_A}) + (\gamma_Y - \gamma_A)V$. Plugging this into equation (83) yields $\dot{v}N^{\beta_Y-\beta_A} + (\gamma_Y - \gamma_A)V = Vr - \Pi$. Dividing by $N^{\beta_Y-\beta_A}$ yields the result.

Wages in the final goods sector and in the R&D sector are equal in equilibrium: $(1-\alpha)\frac{Y}{L_Y} = \frac{V\dot{A}}{L_A}$. By definition of the stationary variables, this equation can be written as:

$$(1-\alpha)\frac{yN^{\beta_Y}}{1-s_A} = vN^{\beta_Y-\beta_A}\frac{a^{\phi}\left(s\tilde{h}\right)^{\lambda}N^{\beta_A}}{s_A}$$

which simplifies to equation (43).

Equilibrium output is $Y = K^{\alpha}((1-s_A-s_M)hNA)^{1-\alpha}$. Dividing through by N^{β_Y} and noting that $\alpha\beta_Y + (1-\alpha)(\frac{1}{1-\nu}\beta_A + 1 + \beta_A) = \beta_Y$ we obtain the expression for normalized output.

To get the expression for normalized hours, consider first the equilibrium conditions for $t > \hat{t}$. Equation (80) implies

$$M = \left(\frac{\alpha^2 \chi}{(1-\alpha)\Psi} (1 - s_A - s_M) \Phi N A^{\phi} \right)^{\frac{\nu-1}{\nu}}$$

Thus

$$h = \Phi M^{\frac{1}{1-\nu}} = \Phi \left(\frac{\alpha^2}{\Psi(1-\alpha)} \chi (1 - s_A - s_M) \Phi N A^{\phi} \right)^{-\frac{1}{\nu}}$$

and, using the definitions of stationary variables,

$$\tilde{h} = \left(\Phi^{1-\nu} \frac{\alpha^2}{\Psi(1-\alpha)} \chi (1 - s_A - s_M) a^{\phi} \right)^{-\frac{1}{\nu}}.$$

Since in equilibrium there can be no jump in hours worked, we need $\lim_{t \to \hat{t}^-} \tilde{h} = \lim_{t \to \hat{t}^+} \tilde{h}$, from which equation (45) follows.

Equations (46) and (47) follow immediately from equations (72) and (23), respectively. Equation (48) follows from equation (85).

Proof of Proposition 7

Proof. V_{2a} grows faster than output because $\frac{1-h}{h}$ grows over time. The growth rate of $V_{2b} = (\exp(M^{\frac{1}{\nu-1}} - \Phi) - 1)C$ is

$$\frac{\dot{V}_{2b}}{V_{2b}} = \gamma_Y + \frac{\exp(M^{\frac{1}{\nu-1}} - \Phi) \frac{1}{\nu-1} M^{\frac{2-\nu}{\nu-1}} \dot{M}}{\exp(M^{\frac{1}{\nu-1}} - \Phi) - 1}$$

The result follows because the final term on the right-hand side is positive. □

C Leisure-consumption complementarities

Consider a more general model where each leisure activity requires leisure time $\ell(\iota)$, free leisure services $m(\iota)$ and leisure consumption goods $c(j)$. For simplicity, assume that elasticity of substitution between time or leisure services and leisure consumption within activity is equal to one, so that:

$$l := \left(\int_0^M \left[\left((\min\{\ell(\iota), m(\iota)\})^{\varphi} (c(\iota))^{1-\varphi} \right) \right]^{\frac{\nu-1}{\nu}} d\iota \right)^{\frac{\nu}{\nu-1}}$$

where $\varphi \in (0,1]$. We recover the formulation in the main text by setting $\varphi = 1$. A symmetric allocation of time and consumption across activities implies that

$$l = \left(M \left(\left(\frac{\ell}{M} \right)^{\varphi} \left(\frac{C_L}{M} \right)^{1-\varphi} \right)^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}} = M^{\frac{1}{\nu-1}} \ell^{\varphi} C_L^{1-\varphi}$$

To see the consequences of this formulation for the leisure supply of the household, consider the simple static time allocation problem:

$$\max_{C_L,h} \log(wh - p_L C_L) + M^{\frac{1}{\nu-1}} (1-h)^{\varphi} (C_L)^{1-\varphi}$$

The first order conditions are:

$$\frac{1}{C} p_L = (1-\varphi) M^{\frac{1}{\nu-1}} (1-h)^{\varphi} (C_L)^{-\varphi} \tag{86}$$

$$\frac{1}{C} w = \varphi M^{\frac{1}{\nu-1}} (1-h)^{\varphi-1} (C_L)^{1-\varphi} \tag{87}$$

Thus the expenditure shares are constant and:

$$C_L = \frac{1-\varphi}{\varphi} \frac{w}{p_L} (1-h).$$

Plugging this into (87):

$$C = \frac{w}{\varphi M^{\frac{1}{\nu-1}} \left(\frac{1-\varphi}{\varphi} \frac{w}{p_L} \right)^{1-\varphi}}. \tag{88}$$

Combining 88 with the budget constraint we obtain the solution:

$$h = \min\left\{1, 1-\varphi + M^{\frac{1}{1-\nu}}\left(\frac{\varphi}{1-\varphi}\frac{p_L}{w}\right)^{1-\varphi}\right\}$$
$$C = \varphi^{\varphi}(1-\varphi)^{\varphi-1}M^{\frac{1}{1-\nu}}w^{\varphi}p_L^{1-\varphi}$$
$$C_L = \frac{1-\varphi}{\varphi}\frac{w}{p_L}(1-h)$$

The first equation shows that the time that households allocate to leisure continues to depend positively on leisure technologies M, so that the main mechanisms and hence the implications of the paper go through with that more general formulation. Note that taking the limit as φ goes to 1 we obtain the result in the main text (equation (20)). It also shows that in presence of leisure consumption goods, hours worked do not converge to zero but instead to a lower bound of $1-\varphi$. This is intuitive: in the limit, households must afford to buy leisure consumption goods therefore they work more than in the baseline model. Moreover, the final two equations show that an expansion in leisure technologies acts as a relative demand shifter, boosting demand for consumption goods that are complimentary with leisure and reducing demand for traditional consumption. A more in-depth analysis of these shifts is beyond the scope of this article; I pursue this avenue in a forthcoming paper (Rachel (2020)).

D The platform pricing decision

The model developed in this Appendix builds on Rochet and Tirole (2003) and Armstrong (2006). The environment is simpler than the problem considered in the main text, but it serves to highlight the important issues when it comes to the optimal pricing strategy of a monopoly platform operating in two-sided leisure markets. In particular, it shows what kind of considerations may be important in driving low or zero prices of leisure services supplied to the consumers by such platforms. In short, high elasticities of consumer demand and substantial benefits to the other side of the market (advertisers) can lead to the optimal pricing strategy that features zero-price leisure services in equilibrium. These basic insights extend beyond the simple monopoly structure to models of platform competition.

Suppose there are two groups: a unit measure of consumers (group 1) and measure-A of firms / advertisers (group 2), interested in interacting with each other through a monopoly platform. In particular, suppose that the platform provides consumers with leisure technologies of value M and charges them price p_1 for accessing the service. Furthermore, consumers may care about how many firms advertise on the platform (with ambiguous sign). The platform charges firms price p_2 for accessing the platform. Since firms use the platform to build brand equity capital, their benefit from using the platform depends on the total time that consumers spend on the platform. Consistent with this description, suppose that the utilities of the two groups are linear:

$$u_1 = \alpha_1 A - p_1 + M + \epsilon \qquad u_2 = \alpha_2 \ell - p_2$$

where $\alpha_2 > 0$, ϵ mean-zero random component, and ℓ is the number / share of consumers that end up using the service. I assume that all agents for whom utility is non-negative participate.

The sign of α_1 is ambiguous as consumers could derive benefits from greater visibility of brands and extra information about their products, but could also find advertising tiresome. To maintain a neutral stance and to make the assumption consistent with the rest of the text, suppose that $\alpha_1 = 0$.

The share of consumers using the platform is a non-decreasing function of utility:

$$\ell = f(u_1) = \phi(\underset{-}{p_1}, \underset{+}{M}).$$

Suppose it costs the platform $\mathbb{C}(M)$ to produce leisure services and brand equity. The platform then chooses prices and quantity M to maximize profits:

$$\max_{p_1, p_2, M} \Pi_B = p_1 \ell + p_2 A - \mathbb{C}(M).$$

Given no random component in the utility of the firms, the platform extracts all surplus from the firm side by charging:

$$p_2 = \alpha_2 \ell.$$

Substituting in from the expressions above yields:

$$\Pi_B = p_1\phi(p_1, M) + A\alpha_2\phi(p_1, M) - \mathbb{C}(M).$$

Profit maximization also implies the following optimality conditions:

$$\phi + p_1\phi_{p_1} + A\alpha_2\phi_{p_1} = 0$$

$$p_1\phi_M + A\alpha_2\phi_M - \mathbb{C}'(M) = 0.$$

Together these imply:

$$p_1 = \frac{\phi + \mathbb{C}'(M)}{\phi_{p_2} - \phi_{p_1}} - A\alpha_2 \tag{89}$$

Equation (89) pins down the optimal price that the platform charges the consumers. Derivatives ϕ_{p_1} and ϕ_{p_2} are negative and positive respectively, so the first term on the right hand side is positive. The optimal price can be zero or negative if the term $A\alpha_2$ is larger than $\frac{\phi+\mathbb{C}'(M)}{\phi_{p_2}-\phi_{p_1}}$. This is more likely when: (i) demand for platform services is low (low ϕ); (ii) it is cheap to produce leisure services (low $\mathbb{C}'(M)$); (iii) consumer demand is highly elastic to prices and leisure technologies (high $\phi_{p_2} - \phi_{p_1}$); and (iv) when there are many advertisers whose utility is highly sensitive to the number of consumers using the service (high A and α_2, respectively). Many of these conditions are likely to be satisfied in the context of leisure platforms, hence the proliferation of zero-price services that we observe in the real world. This analysis underlies the logic of focusing on free leisure services in the rest of the paper. See Appendix C for how to incorporate paid-for leisure consumption goods into the model.

E Leisure technology- and preference shocks

The first exercise in this Appendix is motivated by the finding that the simulation in the main text – which focuses on the endogenous entry of the platforms – struggles to account for the increase in the aggregate cost of supplying the free products (recall the first panel in Figure 6). The increase in this cost in the context of the model could be brought about by changes in two structural parameters of the attention economy: χ and J (via Ψ). Parameter χ encodes the perceived effectiveness of investing in brand equity: an increase in χ raises the demand for brand equity and hence the size of the platform sector. Parameter J denotes the degree of competition in the market for brand equity. Changes to this parameter have a similar effect to changes in the productivity of platforms in turning consumers' time and attention into brand equity.[66]

Figure A.6 shows the response of the economy to permanent changes in these two parameters (a rise in χ and a fall in Ψ, both of 15%), starting from the steady state of segment 2. The solid lines plot the transitional dynamics of the variables while the dotted lines plot the long-run effect. Both changes have a similar impact on the economy: they lower profits in the traditional economy, depressing traditional innovation and growth, with capital, traditional TFP, consumption and output all falling significantly relative to the pre-shock trends. At the same time these changes act as a boost to the attention economy, propping up the share of labor employed in leisure R&D, raising the level of leisure technologies permanently, and through that further depressing hours worked.

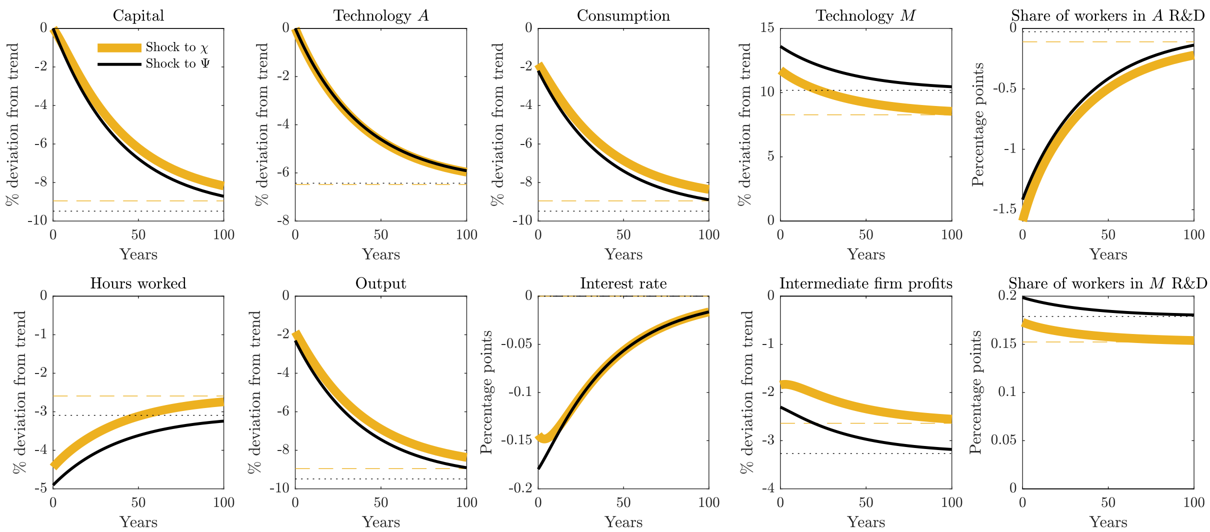

Figure A.6

15% Increase in Businesses' Perceived Effectiveness of Advertising χ and a 15% Increase in Platform Productivity (or a Fall in Markup Ψ).

[66]To see this note that if the platform brand equity technology is $B_j = \eta_B \ell$ with η_B being a technology parameter, then its marginal cost is $\mathcal{M}_B = \frac{w}{\eta_B}\frac{M}{\ell A^{\phi}}$. The equilibrium condition becomes $p_B = \Psi \frac{w}{\eta_B}\frac{M}{\ell A^{\phi}}$. Thus a fall in the markup has the same effect on the supply of leisure technologies as the increase in the technology parameter η_B.

This exercise shows that, on top of the endogenous structural change that was the focus of the main text, shocks to leisure technologies and firm perceptions can have a material impact on the economy. In particular, the model suggests that the shifts that we have seen over the past couple of decades – such as brand equity becoming more important in businesses' strategy – likely contributed to the decline in hours worked, TFP, capital, output, consumption and the interest rate, and contributed to the rapid pace of leisure-enhancing technological change.

The second exercise is motivated by the discussion in the concluding section of the paper, which speculates that the arrival of digital media and the smartphone may have made consumers more dependent on leisure technologies. One way to capture this in the model is to consider a change in the structural elasticity ν. A lower elasticity means that, all else equal, a given change in leisure technologies results in a larger shifts in how time is allocated between market hours and leisure. Figure A.7 illustrates the effect of such change. Hours worked drop on impact, as the stock of currently available leisure technologies becomes more attractive to consumers. The rate of decline of hours worked increases (i.e. hours decline at a faster rate). Both of these changes drag down on traditional productivity growth. The model provides a lens through which one can analyze the structural shifts in the leisure sector and shows that the patterns observed in the data most recently can potentially be driven by these shifts in the attention economy.

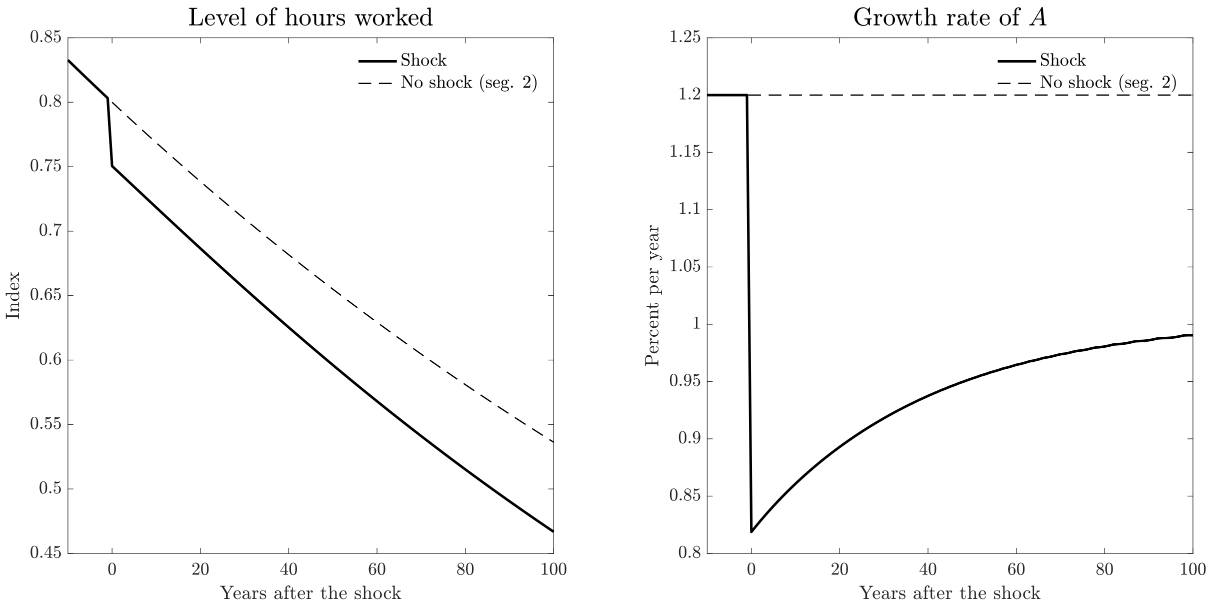

Figure A.7
Effects of a decline in ν from 4 to 3.

F Schumpeterian economy with leisure-enhancing technological change

Consider the basic Schumpeterian growth model with constant population of size N. Each household works h hours, with $h = \min\{1, \Phi M^{\frac{1}{1-\nu}}\}$, as in the model in the main text. Final output is given by:

$$Y = \int_0^1 A_i^{1-\alpha} \left(\left(\frac{b_i}{\bar{b}} \right)^{\chi} x_i \right)^{\alpha} di \cdot L_Y^{1-\alpha}$$

where x_i are the intermediate inputs and A_i is the input-specific productivity. Intermediate product demand is:

$$p_i = \alpha (A_i L_Y)^{1-\alpha} \left(\frac{b(i)}{\bar{b}} \right)^{\alpha\chi} x_i^{\alpha-1}.$$

Thus intermediate producer's problem is to

$$\max_{x_i, b_i} \ \alpha (A_i L_Y)^{1-\alpha} \left(\frac{b_i}{\bar{b}} \right)^{\alpha\chi} x_i^{\alpha} - (r+\delta) x_i - p_B b_i$$

which implies equilibrium quantity:

$$x_i = \left(\frac{\alpha^2}{r+\delta} \right)^{\frac{1}{1-\alpha}} A_i L_Y$$

We can thus write the final output as:

$$Y = \left(\frac{\alpha^2}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}} \left(\int_0^1 A_i di \right) L_Y$$

Equilibrium spend on ads is the same as in the main text. Equilibrium profits are therefore:

$$\Pi_i = x_i (p - (r+\delta) - \chi(r+\delta)) = \left(\frac{1}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} A_i L_Y (1 - \alpha - \alpha\chi) = \pi A_i L_Y.$$

Research

Assume that research costs R_i of final output every period. Research is risky. Denote by μ the probability that research succeeds, and by $A^* := \gamma A$ the target productivity level of the successful innovation. Finally, define $n := R/A^*$ as the productivity adjusted expenditure. Then assume that the success function follows:

$$\mu_i = \lambda n_i^{\sigma} = \lambda \left(\frac{R_i}{A_i^*} \right)^{\sigma}$$

Note that $\mu_i' = \lambda\sigma n_i^{\sigma-1}$. Assume for simplicity that a successful innovator operates the technology for one period, and is subsequently removed either by another innovator or, if no innovator succeeds, by a randomly chosen individual. Thus the reward from pursuing research is $\mu_i \Pi_i$ and the entrepreneur maximizes

$$\max_{R_i} \lambda \left(\frac{R_i}{A_i^*}\right)^\sigma \Pi_i - R_i$$

The optimality condition yields:

$$\lambda\sigma n_i^{\sigma-1} \frac{\Pi_i}{A_i^*} = \lambda\sigma n_i^{\sigma-1} \pi L_Y = 1$$

Solving for n_i gives

$$n_i = (\lambda\sigma\pi L_Y)^{\frac{1}{1-\sigma}}$$

and the optimal frequency of success is $\mu_i = \lambda^{\frac{1}{1-\sigma}} (\sigma\pi L_Y)^{\frac{\sigma}{1-\sigma}}$.

Growth

Growth rate of A is computed as follows:

$$A_{t+1} = \mu A_{t,success} + (1-\mu) A_{t,failure} = \mu\gamma A_t + (1-\mu) A_t$$

Thus

$$\gamma_A = \mu(\gamma - 1) = \lambda^{\frac{1}{1-\sigma}} (\sigma\pi L_Y)^{\frac{\sigma}{1-\sigma}} (\gamma - 1).$$

Clearly, there are two channels through which leisure-enhancing technologies affect γ_A. First, since $L_Y = hN$ by labor market clearing, declining hours worked lead to a declining growth rate of traditional TFP. Second, since π is diminished by $\alpha^2\chi$ per unit sold, this also lowers the incentives to R&D and thus lowers economic growth. This latter effect is analogous to the level effect working through the lower share of R&D workers in the baseline model.

G Dynamic ideas production function

Suppose

$$\dot{M}_j = L_M^j \cdot A^\phi$$

where A^ϕ is taken as given by the platforms. Aggregate new leisure technologies are

$$\dot{M} = \sum \dot{M}_j = \sum \left(L_M^j \cdot A^\phi \right) = A^\phi \sum L_M^j = A^\phi L_M$$

Each platform solves a dynamic optimal control problem:

$$\max_{L_M^j} \int_0^\infty e^{-rt} \left(p_B \cdot M_j \frac{\ell}{M} - w L_M^j \right) dt \text{ subject to}$$

$$\dot{M}_j = L_j^M \cdot A^\phi$$

and

$$p_B \left(B_j + \sum_{k \neq j} B_k \right) = \alpha^2 \chi \frac{Y}{B_j + \sum_{k \neq j} B_k} \left(\frac{B_j + \sum_{k \neq j} B_k}{A\bar{b}} \right)^{\frac{\alpha}{1-\alpha}\chi}$$

The Hamiltonian is:

$$\mathcal{H} = p_B \cdot M_j \frac{\ell}{M} - w L_j^M + \mu \left[L_M^j \cdot A^\phi \right]$$

Optimality conditions are:

$$\mathcal{H}_{L_j^M} = -w + \mu A^\phi = 0 \tag{90}$$

$$\mathcal{H}_{M_j} = p_B \frac{\ell}{M} + M_j \frac{\ell}{M} \left(\frac{\alpha}{1-\alpha}\chi - 1 \right) p_B \frac{1}{\sum M_{j'} \frac{\ell}{M}} \frac{\ell}{M} = r\mu - \dot{\mu} \tag{91}$$

Equation (91) yields:

$$p_B \frac{\ell}{M} \left(1 + \left(\frac{\alpha}{1-\alpha}\chi - 1 \right) \frac{M_j}{M} \right) = r\mu - \dot{\mu}$$

In a symmetric equilibrium $\frac{M_j}{M} = \frac{1}{J}$ so that

$$p_B \frac{\ell}{M} \left(1 + \left(\frac{\alpha}{1-\alpha}\chi - 1 \right) \frac{1}{J} \right) = r\mu - \dot{\mu}$$

We know that $p_B = \alpha^2 \chi \frac{Y}{B}$ and $\ell = B$ so:

$$\alpha^2 \chi \frac{Y}{M} \left(1 + \left(\frac{\alpha}{1-\alpha}\chi - 1 \right) \frac{1}{J} \right) = r\mu - \dot{\mu}$$

Equilibrium requires wages are equalized across sectors thus

$$(1-\alpha)\frac{Y}{L_Y} = V\frac{\dot{A}}{L_A} = \mu\frac{\dot{M}}{L_M}$$

To sum up, relative to the case with the static formulation considered in the main text, there are three equations that are different:

$$\dot{M} = M^{\phi} L_M \tag{92}$$

$$\alpha^2\chi\frac{Y}{M}\left(1+\left(\frac{\alpha}{1-\alpha}\chi-1\right)\frac{1}{J}\right) = r\mu - \dot{\mu} \tag{93}$$

$$\mu\frac{\dot{M}}{s_M} = V\frac{\dot{A}}{s_A} \tag{94}$$

where the final equation replaces (48).

Clearly $\gamma_M = \gamma_A$ so that $m := \frac{M}{N^{\beta_A}}$ is stationary on the balanced growth path. Thus $\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \beta_A n$. Therefore $\dot{M} = \frac{\dot{m}}{m}M - \beta_A nM = \dot{m}N^{\beta_A} - \gamma_A mN^{\beta_A}$. Using these results in equation (92) yields

$$\dot{m}N^{\beta_A} - \gamma_A mN^{\beta_A} = a^{\phi}N^{\phi\beta_A}s_M hN = a^{\phi}N^{\phi\beta_A}s_M\tilde{h}N^{\frac{1}{1-\nu}\beta_A}N,$$

which simplifies to

$$\dot{m} = a^{\phi}s_M\tilde{h} - \gamma_A m.$$

Define $\tilde{\mu} := \frac{\mu}{N^{\beta_\mu}}$ to be the normalized level of the costate. We have $\frac{\dot{\tilde{\mu}}}{\tilde{\mu}} = \frac{\dot{\mu}}{\mu} - \beta_\mu n$ and so $\dot{\mu} = \frac{\dot{\tilde{\mu}}}{\tilde{\mu}}\mu + \beta_\mu n\mu = \dot{\tilde{\mu}}N^{\beta_\mu} + \beta_\mu n\tilde{\mu}N^{\beta_\mu}$. Therefore:

$$\alpha^2\chi\frac{yN^{\beta_Y}}{mN^{\beta_A}}\left(1+\left(\frac{\alpha}{1-\alpha}\chi-1\right)\frac{1}{J}\right) = r\tilde{\mu}N^{\beta_\mu} - \dot{\tilde{\mu}}N^{\beta_\mu} - \beta_\mu n\tilde{\mu}N^{\beta_\mu}$$

Thus

$$\beta_Y - \beta_A = \beta_\mu$$

and therefore the middle equation in stationary form is:

$$\dot{\tilde{\mu}} = \frac{y}{m}\Psi + (r - (\beta_Y - \beta_A)n)\tilde{\mu}$$

Equation (94) yields

$$\tilde{\mu}m^{\phi} = Va^{\phi}.$$

To summarize, the system of equations that pins down the equilibrium with a dynamic leisure production function in (18) is:

$$
\begin{aligned}
\dot{k} &= y - c - \delta k - \gamma_Y k && (95)\\
\dot{a} &= a^{\phi} s_A \tilde{h} - \gamma_A a && (96)\\
\dot{c} &= c\,(r - \rho - \gamma_Y) && (97)\\
\dot{v} &= v\,(r - (\gamma_Y - \gamma_A)) - \pi && (98)\\
\dot{\tilde{\mu}} &= \frac{y}{m}\Psi + (r - (\beta_Y - \beta_A)n)\tilde{\mu} && (99)\\
(1-\alpha)\frac{y}{1 - s_A - s_M} &= v a^{\phi} \tilde{h} && (100)\\
y &= k^{\alpha}\left((1 - s_A - s_M)\tilde{h} a\right)^{1-\alpha} && (101)\\
\tilde{h} &= \left(h^{\hat{t}}\right)^{\frac{\Omega-1}{1-\nu}} (\Phi m)^{\frac{\Omega}{1-\nu}} && (102)\\
r &= \alpha^2 \frac{y}{k} - \delta && (103)\\
\pi &= \alpha \frac{y}{a}\,(1 - \alpha - \alpha\chi\Omega) && (104)\\
\tilde{\mu} m^{\phi} &= v a^{\phi} && (105)
\end{aligned}
$$

where $h^{\hat{t}} := \Phi(\hat{t}) m(\hat{t})$.

H Non-marketable leisure

Suppose leisure output is a combination of marketable and non-marketable leisure, such as hiking or walking in the park. For simplicity, assume that the elasticity of substitution between marketable and non-marketable leisure is one so that:

$$l = l_M^{\eta} \ell_N^{1-\eta}$$

where ℓ_N is time spent hiking, $l_M := \left(\int_0^M \underbrace{[\min\{\ell(\iota), m(\iota)\}]}_{\text{activity}(\iota)}^{\frac{\nu-1}{\nu}} d\iota \right)^{\frac{\nu}{\nu-1}}$ and ℓ is total marketable leisure time as before. Since $\frac{\ell_N}{\ell} = \frac{1-\eta}{\eta}$ and $l_M = \ell M^{\frac{1}{\nu-1}}$ we get

$$l = \ell M^{\frac{\eta}{\nu-1}} \left(\frac{1-\eta}{\eta} \right)^{1-\eta}$$

which is similar to before ($l = \ell M^{\frac{1}{\nu-1}}$). Labor supply is in this case

$$h = M^{\frac{\eta}{1-\nu}} \left(\frac{\eta}{1-\eta} \right)^{1-\eta}.$$

Thus all the results of the benchmark framework go through after parameter ν is recalibrated to reflect the fact that leisure technologies crowd out not just time at work but also time spent on non-marketable leisure.

I Leisure technologies as substitutes for consumption goods

Some leisure technologies may substitute for consumption goods. For example, playing freely available games online might substitute for games previously paid for. This Appendix provides a simple illustration of this hypothesis. Consider the static problem, with the utility function:

$$u = \left(\psi c^{\frac{\sigma-1}{\sigma}} + (1-\psi)\, l^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where assume now that $l := M(1-h)$ is leisure utility. Abstracting from unearned income, the static budget constraint is $wh = c$. The maximization problem is

$$\max_{h} \left(\psi (wh)^{\frac{\sigma-1}{\sigma}} + (1-\psi)\left(M(1-h)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}.$$

Some algebra yields the solution:

$$h = \frac{1}{1 + \left(\frac{1-\psi}{\psi}\right)^{\sigma} \left(\frac{M}{w}\right)^{\sigma-1}}.$$

That is, hours worked are declining with M if $\sigma > 1$, that is, when consumption and leisure are substitutes.

J Alternative ways of modeling advertising

This Appendix sketches out two alternative ways to incorporate brand equity competition into the monopolistic competition framework. Note first that the final good production function (imposing symmetry in advertising) can be written as

$$Y = \left(\left(\int_0^A x_i^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \right)^{\alpha} L_Y^{1-\alpha} di$$

where $\epsilon := \frac{1}{1-\alpha}$ is the elasticity of substitution across the intermediate goods. Here I consider two alternatives to the combative advertising assumption outlined in the main text: that advertising shifts the intensity of tastes towards consumption goods (equivalently raises total factor productivity in the final sector); and that advertising makes products more differentiated.

Non-combative advertising

Consider first a formulation where advertising is not combative, but instead shifts the intensity of preferences as follows:

$$Y = \left(\left(\int_0^A (b_i^{\chi} x_i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \right)^{\alpha} L_Y^{1-\alpha} di = \int_0^A (b_i^{\chi} x_i)^{\alpha} \, di L_Y^{1-\alpha}$$

In a symmetric equilibrium with $K = Ax$ and $B = Ab$ we have:

$$Y = (B^{\chi} K)^{\alpha} A^{1-\alpha-\alpha\chi} {L_Y}^{1-\alpha}.$$

This equation shows that this alternative formulation will have important implications for growth rate of output. Recall that by equation (17) $B = \ell$, so that output growth will be fastest at low levels of B, when ad spending and leisure hours are growing the fastest. Over time ads cease to be a source of growth; instead, the formulation suggests that output will be growing more slowly (the exponent on A is $1 - \alpha - \alpha\chi$ instead of $1 - \alpha$). Demand for good i is

$$\alpha b_i^{\alpha\chi} x_i^{\alpha-1} L_Y^{1-\alpha} = p_i. \tag{106}$$

It is straightforward to show that the demand for brand equity is:

$$p_B = \alpha^2 \chi \frac{Y}{A} b^{\frac{\alpha}{1-\alpha}\chi - 1}. \tag{107}$$

Equation (79) becomes

$$\alpha^2\chi\frac{Y}{A}\frac{A}{B}\left(\frac{B}{A}\right)^{\frac{\alpha}{1-\alpha}\chi}=\Psi w\frac{M}{\ell A^{\phi}},$$

which yieldsthe supply of leisure technologies:

$$M=\frac{\alpha^2\chi}{(1-\alpha)\Psi}(1-s_A-s_M)h(1-h)^{\frac{\alpha}{1-\alpha}\chi}NA^{\phi-\frac{\alpha}{1-\alpha}\chi}$$

Assuming that $\phi-\frac{\alpha}{1-\alpha}\chi>0$, it is clear that the structure of equilibrium is similar to the model in the main text.

Advertising that alters elasticity of substitution across goods

Consider now the following formulation:

$$Y=\int_0^A(x_i-b_i)^{\alpha}\,diL_Y^{1-\alpha}$$

Demand is:

$$x_i=\left(\frac{\alpha}{p_i}\right)^{\frac{1}{1-\alpha}}L_Y+b_i$$

Clearly, advertising shifts demand. But now it also makes demand more inelastic. The elasticity of demand is

$$\left|\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}\right|=\frac{1}{1-\alpha}\left(1-\frac{b_i}{x_i}\right)$$

In this model, brand equity competition exacerbates the monopoly power of firms, raising prices and lowering output, moving the economy further away from the competitive benchmark.

K Alternative calibration of elasticity ν

To illustrate the robustness of the main findings to alternative values of elasticity across leisure varieties ν, Figure A.8 shows the paths for hours worked and traditional TFP growth for the calibration in the main text, as well as a lower and a higher value of ν. While the qualitative conclusions are unchanged, the different calibrations do matter for the quantitative implications. The parameter enters non-linearly, so that a lower value changes the results considerably more.

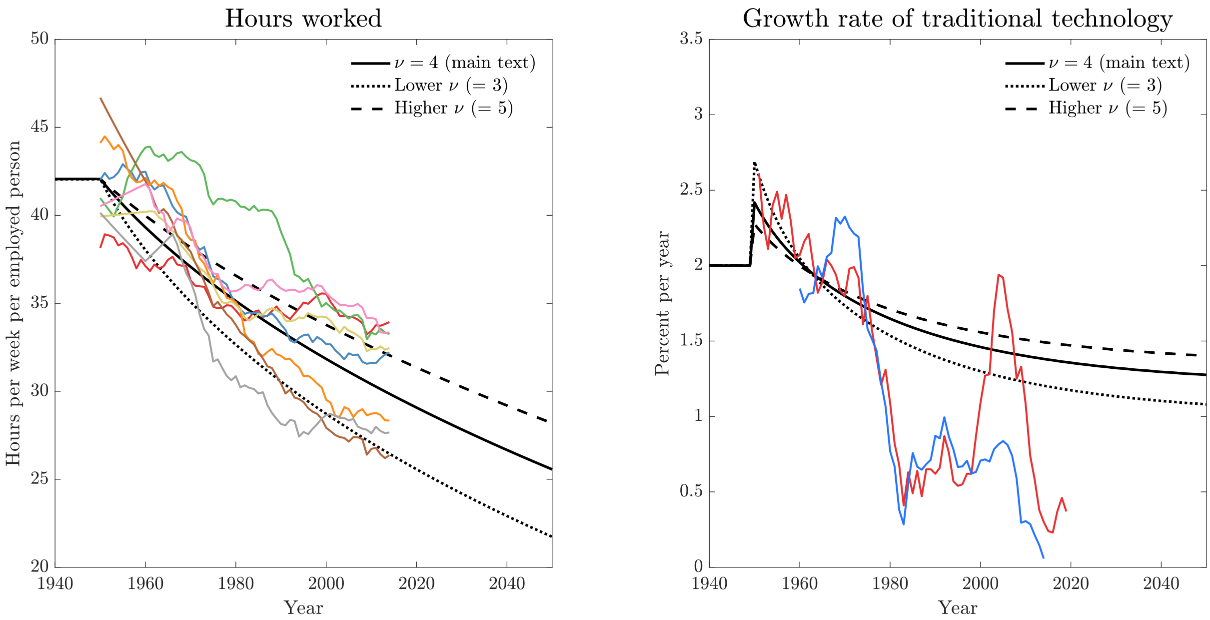

Figure A.8
Robustness to Higher and Lower Values of Elasticity ν.

L Anticipated entry of the leisure platforms

The equilibrium concept in Definition 2 incorporates the assumption that the entry to leisure platforms is unanticipated, and therefore acts as a shock when it happens at $\hat{t}$. Figure A.9 presents the transition path of the economy if the entry is instead anticipated for 50 years prior. Naturally, segment 1 no longer features exact balanced growth: for example, the share of labor in traditional R&D is not constant. However, the broad dynamics of the economy are little changed relative to the model in the main text, suggesting all the results in the paper are robust to this change.

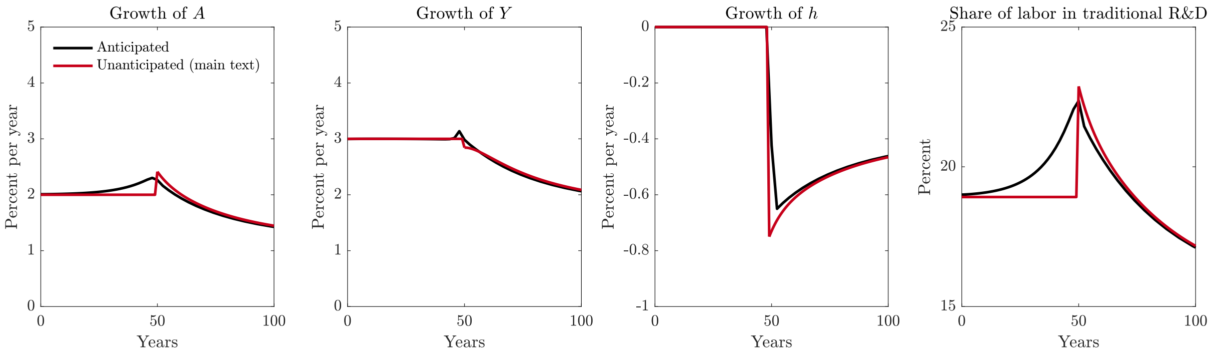

Figure A.9

Transition Dynamics When the Entry of the Platforms is Anticipated