Screening for breakthroughs*

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19 November 2020
[link to latest version]

Abstract

An agent privately observes a technological breakthrough that expands utility possibilities, and must be incentivised to disclose it. The principal controls the agent's utility over time. Optimal mechanisms keep the agent only just willing to disclose promptly. In an important case, a *deadline mechanism* is optimal: absent disclosure, the agent enjoys an efficient utility before a deadline, and an inefficiently low utility afterwards. In general, optimal mechanisms feature a (possibly gradual) transition from the former to the latter. Even if monetary transfers are permitted, they may not be used. We apply our results to the design of unemployment insurance schemes.

1 Introduction

Society advances by finding better ways of doing things. When such a technological breakthrough occurs, it frequently becomes known only to certain individuals with particular expertise. Only by incentivising such individuals to share their knowledge can the promise of progress be unlocked.

For example, information about a firm's production process is often concentrated in the hands of a small number of its employees. When a lower-cost production method becomes available, only these specialist workers learn about it. If the new method also lowers marginal cost, then a conflict of interest arises: the firm wishes to increase output as soon as the new method

*We are grateful to Eddie Dekel, Alessandro Pavan and Bruno Strulovici for guidance and comments, and to Miguel Ballester, Iván Canay, Théo Durandard, Piotr Dworczak, Jeff Ely, George Georgiadis, Yingni Guo, Ian Jewitt, Matt Notowidigdo, Wojciech Olszewski, Harry Pei, Dan Quigley, Marciano Siniscalchi and Asher Wolinsky and audiences at Northwestern and Oxford for helpful comments and suggestions.

becomes available, requiring its employees to work harder or to put in more hours. Were the firm to pursue this naïve course of action, its specialists would prefer not to disclose when the new method becomes available. To incentivise disclosure, the firm must increase the attractiveness of production with the new method relative to the status quo.

The same incentive problem appears in the political arena. Governments often contemplate bold new policies, such as restructuring the civil service to improve its efficiency or enacting economic reforms meant to increase growth. Whether or not these policies will work is difficult to determine in advance.[1] Bureaucrats with policy expertise can help, but may prefer the status quo. If a government wishes to initiate reform only when it appears promising to experts, then it must adjust its policy-making to encourage prompt disclosure.

Both of these examples are instances of a general incentive problem in which an agent privately observes the arrival of a new productive technology. This breakthrough expands utility possibilities for the agent and principal, but generates a conflict of interest between them. The agent decides whether and when to disclose the breakthrough.

In this paper, we study this incentive problem, and apply our findings to the design of unemployment insurance schemes. In our model, a *breakthrough* occurs at a random time, making available a new technology that expands utility possibilities. There is a conflict of interest: were the principal to operate the old and new technologies in her own interest, the agent would be better off under the old one. The agent privately observes when the breakthrough occurs, and (verifiably) discloses it at a time of her choosing. The principal controls a physical allocation that determines the agent's utility over time. (The description of a physical allocation may include a specification of monetary payments to the agent.)

Our results constitute a sharp, distribution-free description of the qualitative features of optimal mechanisms. In particular, we characterise *undominated* mechanisms, meaning those such that no alternative mechanism is weakly better for the principal under every arrival distribution of the breakthrough and strictly better for some distribution. We further describe the quantitative details of the principal's optimal choice among undominated mechanisms for any given breakthrough distribution.

Our first result (Theorem 1) fully determines how incentives are provided in undominated mechanisms: the agent must be kept indifferent at all times

[1] For example, the 'Universal Credit' reform of the British welfare system continues to face technical problems, whereas the earlier the German 'Hartz IV' reform did not.

between prompt and delayed disclosure. This is surprising. It would be obvious if the principal could extract payments from the agent, since she could then improve upon a mechanism in which the agent strictly prefers prompt to delayed disclosure by charging the agent when she discloses. But in our model, the principal can lower the agent's post-disclosure utility only by adjusting the physical allocation, which may lower her own payoff, too.

We next consider an important special case of our model in which the pre-breakthrough technology has an affine shape. (The new technology remains arbitrary.) In this case, we show that only *deadline mechanisms* are undominated (Theorem 2). Absent disclosure, these mechanisms give the agent an efficient utility u^0 before a deadline, and an inefficiently low utility $u^\star$ afterwards. These two utility levels are simple functions of the technologies, leaving the deadline as the only free parameter. The proof of Theorem 2 argues (loosely) that any mechanism may be improved by *front-loading* the agent's pre-disclosure utility, making it higher early and lower late while preserving its total discounted value. We further characterise the principal's optimal choice of deadline as a function of the breakthrough distribution (Proposition 2).

Outside of the affine case, optimal mechanisms continue to enjoy all of the qualitative features of deadline mechanisms, except for their abruptness (Theorem 3): the utility of an agent who has not yet disclosed declines over time, starting at the same high efficient level u^0 and converging to the same inefficiently low level $u^\star$. For a given breakthrough distribution, we describe the optimal transition path (Proposition 3).

What role do monetary transfers play, if permitted? We show that undominated mechanisms never pay the agent before disclosure, and may not use transfers at all (Proposition 4). This finding contrasts with most of the mechanism design literature, where optimal mechanisms typically rely heavily on monetary transfers when these are available.

We conclude by applying our results to the optimal design of unemployment insurance schemes. An unemployed worker (agent) privately receives a job offer at a random time, and chooses whether and when to accept. Offers eventually expire, and the worker may subsequently receive further offers.[2] The state controls the worker's consumption and labour supply through taxes and benefits,[3] but does not observe whether the worker has a job offer.

[2]This fits into our general model, which allows for the new technology eventually to become unavailable if unused, perhaps re-appearing subsequently.

[3]The assumption that the state controls labour supply is standard in the literature on unemployment insurance. The idea is that the state can set work requirements, and enforce these by threatening non-compliant workers with sanctions (such as fines).

Society cares both about the worker's welfare and about net tax revenue.

Undominated unemployment insurance schemes gradually reduce unemployment benefits from a high efficient level toward subsistence. If the worker's utility of consumption has moderate curvature, then a *deadline scheme* is approximately optimal: generous unemployment benefits before a deadline, and mere subsistence benefits for those remaining unemployed beyond the deadline. This result rationalises the German 'Hartz IV' system introduced in 2005, which has exactly this form. Furthermore, it is optimal to set later deadlines for workers with worse job-finding prospects, justifying Hartz IV's use of more lenient deadlines for older workers.

1.1 Related literature

We contribute to the literature on dynamic mechanism design.[4] Our model differs from most in the literature in that the agent cannot secretly benefit from the breakthrough: if she delays disclosure, then the old technology will be used.[5] This feature does appear in work on revenue management, where customers arrive unobservably over time.[6] Another difference is that our agent's disclosures are verifiable, whereas most of this literature studies cheap-talk communication. For example, B. Green and Taylor (2016) and Madsen (2020) study how a principal may elicit truthful non-verifiable progress reports using monetary transfers and (random) termination.[7]

Unlike most of this literature, we do not insist that the principal can make unrestricted use of monetary transfers, placing our paper in the *delegation* literature initiated by Holmström (1977, 1984). Most of the early work on

[4]Seminal papers on this topic include Roberts (1983), Baron and Besanko (1984), Besanko (1985), Courty and Li (2000), Battaglini (2005), Eső and Szentes (2007a, 2007b), Board (2007) and Pavan, Segal and Toikka (2014). A closely related literature studies optimal dynamic taxation and insurance: for example, Townsend (1982), E. J. Green (1987), Thomas and Worrall (1990), Atkeson and Lucas (1992), Fernandes and Phelan (2000), Kocherlakota (2005), Williams (2011), Farhi and Werning (2013) and Golosov, Troshkin and Tsyvinski (2016).

[5]This contrasts with e.g. dynamic Mirrleesian taxation models, where the agent can enjoy an unreported increase in her productivity by working fewer hours while earning the (observable) income expected of a less productive type.

[6]See Pai and Vohra (2013), Board and Skrzypacz (2016), Mierendorff (2016), Garrett (2016, 2017), Gershkov, Moldovanu and Strack (2018) and Dilmé and Li (2019).

[7]In the former paper, the agent privately observes a signal indicating that a payoff-relevant breakthrough is (likely to be) imminent. The breakthrough causes no conflict of interest in our sense; the difficulty is instead that the agent must be incentivised to exert unobservable effort to hasten the breakthrough's arrival. In the pure moral-hazard benchmark (no signal), a deadline-type mechanism is optimal. Our result on the optimality of deadline mechanisms is quite different, as our model is one of pure adverse selection.

this topic focussed on static settings,[8] but a rapidly-growing strand studies dynamic environments.[9] In nearly all of this literature, the agent is privately informed about the relative payoffs of the available actions. We assume instead that the agent is privately informed about *which* actions are available. This approach was pioneered by Armstrong and Vickers (2010) in a static setting. They showed in an example that monetary transfers may play no role in optimal mechanisms even if permitted, a result which we obtain in a general dynamic setting.

Bird and Frug (2019) study a dynamic variant of the Armstrong–Vickers model in which productive opportunities arrive over time (with varying associated payoffs for the agent and principal), but vanish instantly if not exploited. Thus their agent cannot delay disclosure, which substantially slackens the incentive constraints and leads to very different optimal dynamics, as we detail in supplemental appendix M.

Our agent discloses verifiably at a time of her choosing, and our principal can commit to a mechanism. Models of verifiable disclosure were first studied by Grossman and Hart (1980), Milgrom (1981) and Grossman (1981). A strand of the subsequent literature examines the role of commitment in static models,[10] while another studies the timing of disclosure absent commitment (Acharya, DeMarzo & Kremer, 2011; Guttman, Kremer & Skrzypacz, 2014; Campbell, Ederer & Spinnewijn, 2014).[11] Poggi and Sinander (in progress) study a model in which an agent privately learns over time about the viability of a project, and the principal incentivises participation and prompt disclosure using monetary transfers and the threat of termination.

[8]For example, Melumad and Shibano (1991), Dessein (2002), Martimort and Semenov (2006), Alonso and Matouschek (2008), Amador and Bagwell (2013), Frankel (2014, 2016a) and Ambrus and Egorov (2017).

[9]See Jackson and Sonnenschein (2007), Frankel (2016b), Li, Matouschek and Powell (2017), Lipnowski and Ramos (2020), Guo and Hörner (2020) and de Clippel, Eliaz, Fershtman and Rozen (forthcoming). A related literature studies favour exchange without commitment (Möbius, 2001; Hauser & Hopenhayn, 2010; Olszewski & Safronov, 2018a, 2018b).

[10]See, for example, J. R. Green and Laffont (1986), Glazer and Rubinstein (2004, 2006), Bull and Watson (2004, 2007), Deneckere and Severinov (2008), Sher (2011), Kartik and Tercieux (2012), Ben-Porath and Lipman (2012), Hart, Kremer and Perry (2017) and Ben-Porath, Dekel and Lipman (2019).

[11]The last paper also features 'breakthroughs', but these engender no conflict of interest in our sense; the incentive problem is instead that of deterring unobservable shirking. The authors characterise the optimal choice of deadline, but do not show that deadline-type schemes are optimal among all mechanisms.

1.2 Roadmap

We introduce and discuss the model in the next section, then formulate the principal's problem in §3. In §4, we show that undominated mechanisms incentivise the agent by keeping her always indifferent between prompt and delayed disclosure. Next, in §5, we show that only deadline mechanisms are undominated in an important special case that nests our application to unemployment insurance, and further describe the optimal choice of deadline. We then (§6) characterise optimal mechanisms in general. Finally, in §7, we show that transfers play at most a limited role if permitted. We derive the implications of our results for unemployment insurance schemes in §8.

2 Model

There is an agent and a principal, whose utilities are denoted by $u \in [0, \infty)$ and $v \in [-\infty, \infty)$, respectively. A frontier $F^0 : [0, \infty) \to [-\infty, \infty)$ describes utility possibilities: $F^0(u)$ is the highest utility that the principal can attain subject to giving the agent utility u.[12] We assume that F^0 is concave and upper semi-continuous, and that it has a unique peak u^0 (namely, $F^0(u^0) > F^0(u)$ for every $u \neq u^0$).

Time $t \in \mathbf{R}_+$ is continuous. At a random time τ, a *breakthrough* occurs: a new technology becomes available which expands the utility possibility frontier to $F^1 \geq F^0$.[13] The new frontier is likewise concave and upper semi-continuous, with a unique peak denoted by u^1. The breakthrough causes a conflict of interest: the new frontier peaks at a strictly lower agent utility ($u^1 < u^0$), so that the breakthrough would hurt the agent were the principal to operate both technologies in her own interest. We further assume that the gap $F^1 - F^0$ admits a strict local maximum on $[0, u^0]$. Two such frontiers are depicted in Figure 1.

The agent and principal discount their flow payoffs by $r > 0$ and have expected-utility preferences, so that their respective payoffs from random

[12] In the background, there is a compact set $\mathcal{A}^0$ of physical allocations over which the agent and principal have upper semi-continuous utility functions $\alpha, \pi : \mathcal{A}^0 \to \mathbf{R}$. (Physical allocations may include a specification of monetary payments.) The frontier is defined by $F^0(u) := \max_{a \in \mathcal{A}^0} \{\pi(a) : \alpha(a) = u\}$, with $F^0(u) := -\infty$ whenever there is no allocation $a \in \mathcal{A}^0$ such that $\alpha(a) = u$.

[13] The new technology necessarily increases the frontier pointwise because the old technology may still be used when superior. In the language of footnote 12, the new technology enlarges the set of available allocations to some $\mathcal{A}^1 \supset \mathcal{A}^0$.

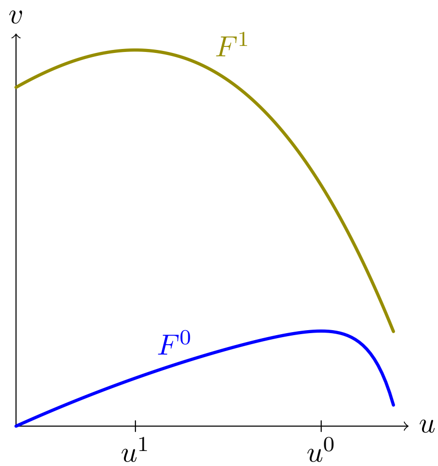


Figure 1: Utility possibility frontiers. The new technology expands utility possibilities ($F^1 \geq F^0$), but causes a conflict of interest ($u^1 < u^0$).

flow utilities $t \mapsto x_t$ and $t \mapsto y_t$ are

$$\mathbf{E}\left(r \int_0^\infty e^{-rt} x_t \mathrm{d}t \right) \quad \text{and} \quad \mathbf{E}\left(r \int_0^\infty e^{-rt} y_t \mathrm{d}t \right).$$

The random time τ at which the breakthrough occurs is distributed according to a CDF G. Our only assumption on G is that the breakthrough has not already occurred at the outset ($G(0) = 0$).

The breakthrough is observed only by the agent. At any time $t \geq \tau$ after the breakthrough, she can verifiably disclose to the principal that it has occurred. (That is, she can *prove* that the new technology is available.)

The principal controls the agent's flow utility over time. This need not be interpreted literally: in most applications, payoffs derive at least in part from observable actions taken by the agent, but the principal can enforce her action recommendations by committing to give the agent zero utility forever if she deviates. The principal is able to commit.

2.1 Discussion of assumptions

Two of our assumptions are economically substantive. First, the agent privately observes a technological breakthrough, but cannot utilise the new technology without disclosing it. Many economic environments have this feature: in unemployment insurance, for example, the worker's employment status is observable. Secondly, there is a conflict of interest, captured by $u^1 < u^0$. Such conflicts arise naturally in applications, such as production (next section) and unemployment insurance (§8).[14] The remaining model assumptions are innocuous: each can be relaxed with little or no effect, as we next relate.

Expiring offers. We assumed that the new technology arrives once and remains available forever. As we explain in Remark 1 in the next section, our analysis applies unchanged if the new technology is available only for some (proper) interval of time, perhaps even disappearing and re-appearing repeatedly. This is natural in the application to unemployment insurance, where job offers expire and new offers arrive subsequently.

Breakthrough distribution. The distribution G of the breakthrough time is nearly unrestricted: it can have atoms, for example, and need not have full support. Our (sole) assumption that $G(0) = 0$ plays a purely expository role, and we relax it in supplemental appendix I.1.

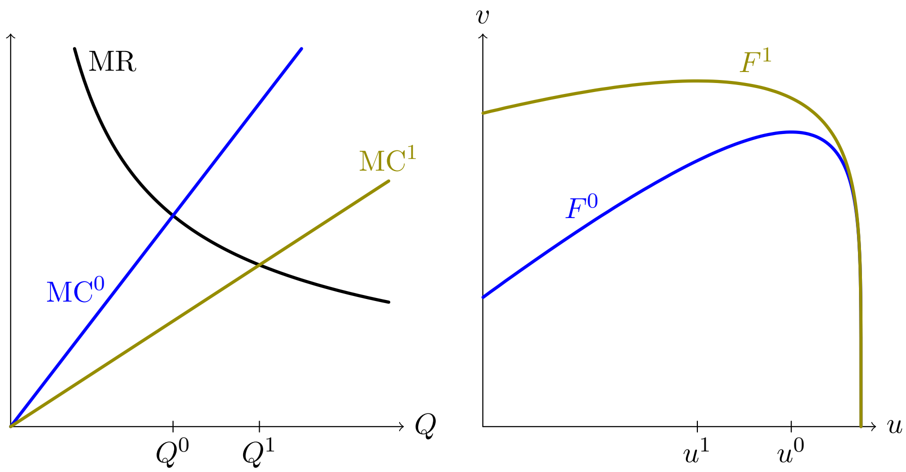
Uncertain technology. Our analysis applies unchanged if the new technology F^1 is random, provided the agent does not have private information about its realisation. We demonstrate this in supplemental appendix I.2.

Verifiability. The verifiability of the agent's disclosures entails no loss of generality. For if communication were cheap talk, then the principal could ask the agent to report the breakthrough, whereupon she receives utility u^1 for a short time. The principal then learns whether the agent lied: her flow payoff is $F^1(u^1)$ if the breakthrough really did occur, and $F^0(u^1) < F^1(u^1)$ if not. The cost of this verification scheme can be made arbitrarily small (for both agent and principal) by choosing the 'short time' appropriately.

Utility domains. Our formalism allows for technological limitations on the agent's utility: if technology $j \in \{0, 1\}$ cannot provide the agent with some utility $u \in [0, \infty)$, then we let $F^j(u) := -\infty$, ensuring that u is never chosen by the principal. We give further details in supplemental appendix I.3.

We have supposed that the agent's flow utility u must be non-negative, meaning that there is a bound (normalised to zero) on how much misery the principal can inflict on the agent. This assumption may be replaced with a participation constraint without affecting our results, as we relate in supplemental appendix I.4.

[14] Absent a conflict of interest, the principal can attain first-best (see Remark 2 below).



(a) Marginal cost and revenue.

(b) Utility possibilities.

Figure 2: Production example.

Frontier shapes. The assumption that the frontiers are concave is without loss of generality: if one of them were not, then the principal could get arbitrarily close to any point on its *concave upper envelope* by rapidly switching back and forth between agent utility levels.[15] Upper semi-continuity is similarly innocuous.

The supposition that $F^1 - F^0$ admits a strict local maximum on $[0, u^0]$ is a weak genericity assumption. Dropping it merely complicates the statement of Theorem 2, as we explain in supplemental appendix I.5.

2.2 Example: production

To illustrate the model, return to the production example from the introduction. A firm (principal) uses labour L to produce output $Q = Y(L)$, with marginal revenue MR and marginal cost MC^0 (depicted in Figure 2a). Its worker (agent) values leisure, so prefers to work fewer hours.

Utility possibilities are described by the frontier F^0 in Figure 2b. As output $Q = Y(L)$ is increased from zero, profit increases and the worker's utility declines, tracing out the downward-sloping portion of the frontier F^0. At the profit-maximising output level Q^0, the worker earns utility u^0. If

[15] The *concave upper envelope* of a function is the smallest concave function pointwise above it.

output is increased further, then both profit and the worker's utility decline, tracing out the upward-sloping part of the frontier.

The worker privately observes when a new production method becomes available, which lowers marginal cost to MC^1. This increases profit at every output level, expanding utility possibilities. It also raises the firm's profit-maximising output from Q^0 to Q^1, causing a conflict of interest: the worker's utility u^1 at the new profit-maximising output level is lower than at the old.

The worker chooses whether and when to disclose the availability of the new method. The firm controls hours worked (and thus utility) by e.g. setting shift lengths and committing to dismiss the worker if she fails to complete a shift. For simplicity, the worker's salary is fixed by (say) a union contract or regulation. We may alternatively assume that only the baseline salary is fixed, so that the firm can pay the worker more if desired.

2.3 Mechanisms and incentive-compatibility

A utility flow $t \mapsto x_t$ specifies the utility x_t enjoyed by the agent in each period t. Given a (Lebesgue-measurable) flow x, we write X_t for the time-t present value of the remainder of the flow:

$$X_t \coloneqq r \int_t^\infty e^{-r(s-t)} x_s \mathrm{d}s.$$

A *mechanism* specifies, for each period t, the flow x_t^0 that the agent enjoys at t if she has not yet disclosed, as well as the continuation utility X_t^1 that she earns by disclosing at t. Formally, a mechanism is a pair (x^0, X^1), where $x^0 : \mathbf{R}_+ \to \mathbf{R}_+$ is Lebesgue-measurable and X^1 is a function $\mathbf{R}_+ \to [0, \infty]$. We call x^0 the *pre-disclosure flow*, and X^1 the *disclosure reward*.

Note that the description of a mechanism does not specify what utility flow $s \mapsto x_s^{1,t}$ the agent enjoys after disclosing at t, only its present value

$$X_t^1 = r \int_t^\infty e^{-r(s-t)} x_s^{1,t} \mathrm{d}s.$$

The definition also does not specify which technology is to be used when both are available. These omissions do not matter for the agent's incentives, so we shall address them when we formulate principal's problem in the next section.

A mechanism is *incentive-compatible ('IC')* iff the agent prefers disclosing promptly to (a) disclosing with a delay or (b) never disclosing. To state this formally, observe that the agent's period-t continuation utility if she decides to *never* disclose is simply the present value

$$X_t^0 \coloneqq r \int_t^\infty e^{-r(s-t)} x_s^0 \mathrm{d}s$$

of the remainder of the pre-disclosure flow x^0.

Definition 1. A mechanism $\left(x^0, X^1\right)$ is incentive-compatible ('IC') iff for every period $t \in \mathbf{R}_+$,

(a) $X_t^1 \geq X_t^0 + e^{-rd}\left(X_{t+d}^1 - X_{t+d}^0\right)$ for every $d > 0$, and

(b) $X_t^1 \geq X_t^0$.

By a revelation principle, we may restrict attention to incentive-compatible mechanisms. (See supplemental appendix L for details.)

Remark 1. Suppose that the new technology comes and goes: it becomes available at τ, but disappears if not disclosed before $\tau + \varepsilon$ (where $\varepsilon > 0$), perhaps re-appearing later. This does not slacken IC: the agent still has to be deterred from delaying by any $d \in (0, \varepsilon)$, and this suffices to ensure that she does not wish to delay by $d \geq \varepsilon$, either.[16] Our analysis therefore applies also to this richer model.

Remark 2. Although we have not yet stated the principal's problem, it is fairly clear that her first-best is the mechanism $\left(x^0, X^1\right) \equiv \left(u^0, u^1\right)$, which fails to be incentive-compatible due to the conflict of interest $\left(u^1 < u^0\right)$. If there were no conflict of interest $\left(u^1 \geq u^0\right)$, then the first-best would be IC.

In the sequel, we equip the set $\mathbf{R}_+$ of times with the Lebesgue measure, so that a 'null set of times' means a set of Lebesgue measure zero, and 'almost everywhere (a.e.)' means 'except possibly on a null set of times'.

Observe that two IC mechanisms $\left(x^0, X^1\right)$ and $\left(x^{0\dagger}, X^1\right)$ which differ only in that $x^0 \neq x^{0\dagger}$ on a null set are payoff-equivalent.[17] For this reason, we shall not distinguish between such mechanisms in the sequel, instead treating them as identical.[18]

[16] This is because an agent who has delayed from t to $t+d$ (for $d < \varepsilon$) finds herself in the same shoes as an agent who observed the breakthrough only at $t+d$, and IC requires that such an agent not wish to delay until $t+2d$, etc. Similar reasoning applies if ε varies with τ and/or is random.

[17] x^0 enters payoffs as $\mathbf{E}_G\left(\int_0^\tau e^{-rt} x_t^0 \mathrm{d}t\right)$ and $\mathbf{E}_G\left(\int_0^\tau e^{-rt} F^0\left(x_t^0\right) \mathrm{d}t\right)$, respectively. Modifying x^0 on a null set has no effect on the integrals, and thus leaves both players' payoffs unchanged, no matter what the breakthrough distribution G.

[18] We term such $\left(x^0, X^1\right)$ and $\left(x^{0\dagger}, X^1\right)$ *versions* of each other. A mechanism is really an equivalence class of versions, i.e. a maximal set whose every element is a version of every other.

3 The principal's problem

In this section, we formulate the principal's problem, and define undominated and optimal mechanisms. We then derive an upper bound on the agent's utility in undominated mechanisms.

3.1 After disclosure

To determine the principal's payoff, we must fill in the gaps in the definition of a mechanism. So fix a mechanism (x^0, X^1), and suppose that the agent discloses at time t. For each of the remaining periods $s \in [t, \infty)$, the principal must determine

(1) which technology (F^0 or F^1) will be used, and

(2) what flow utility $x_s^{1,t}$ the agent will enjoy.

Part (1) is straightforward: the principal is always (weakly) better off using the new technology.

For (2), the principal must choose a (measurable) utility flow $x^{1,t} : [t, \infty) \to [0, \infty)$ subject to providing the agent with the continuation utility specified by the mechanism:

$$r \int_t^\infty e^{-r(s-t)} x_s^{1,t} \mathrm{d}s = X_t^1.$$

She chooses so as to maximise her post-disclosure payoff

$$r \int_t^\infty e^{-r(s-t)} F^1\left(x_s^{1,t}\right) \mathrm{d}s.$$

Since the frontier F^1 is concave, the constant flow $x^{1,t} \equiv X_t^1$ is optimal.

By putting together parts (1) and (2), we conclude that under a mechanism (x^0, X^1), the principal earns a flow payoff of $F^1(X_t^1)$ forever following disclosure at time t.

3.2 Undominated and optimal mechanisms

The principal's payoff from an incentive-compatible mechanism (x^0, X^1) is

$$\Pi_G\left(x^0, X^1\right) := \mathbf{E}_G\left(r \int_0^\tau e^{-rt} F^0\left(x_t^0\right) \mathrm{d}t + e^{-r\tau} F^1\left(X_\tau^1\right)\right),$$

where the expectation is over the random breakthrough time $\tau \sim G$.[19] Her problem is to maximise her payoff by choosing among incentive-compatible mechanisms.

A basic adequacy criterion for a mechanism is that it not be *dominated* by another mechanism, by which we mean that the alternative mechanism is weakly better under every distribution and strictly better under at least one distribution. Formally:

Definition 2. Let (x^0, X^1) and $(x^{0\dagger}, X^{1\dagger})$ be incentive-compatible mechanisms. The former *dominates* the latter iff

$$\Pi_G\left(x^0, X^1\right) \geq (>)\, \Pi_G\left(x^{0\dagger}, X^{1\dagger}\right)$$

for every (some) CDF G with $G(0) = 0$. An IC mechanism is *undominated* iff no IC mechanism dominates it.

Domination is a distribution-free concept: the principal (weakly) prefers a dominating mechanism no matter what her belief G about the likely time of the breakthrough.

Definition 3. An incentive-compatible mechanism is *optimal* for a distribution G iff it maximises Π_G and is undominated.

Remark 3. We show in supplemental appendix K that undominated and optimal mechanisms exist, and that our results remain valid if dominance is strengthened to require strict inequality for some *full-support* distribution G, or even for *every* full-support G.[20]

3.3 An upper bound on agent utility

Absent incentive concerns, the principal never wishes to give the agent utility strictly exceeding u^0, since both frontiers are downward-sloping to the right of u^0. The principal could use utility promises in excess of u^0 as an incentive tool, however. This is never worthwhile:

Lemma 1. Any undominated incentive-compatible mechanism (x^0, X^1) satisfies $x^0 \leq u^0$ almost everywhere.

Proof. Let (x^0, X^1) be an incentive-compatible mechanism in which $x^0 > u^0$ on a non-null set of times. Consider the alternative mechanism

$$\left(\min\left\{x^0, u^0\right\}, X^1\right)$$

[19] To allow for the possibility that $X^1_\tau = \infty$, define $F^1(\infty) := -\infty$.

[20] As usual, 'full-support' means that every Lebesgue non-null set has positive probability.

in which the agent's pre-disclosure flow is capped at u^0. This mechanism dominates the original one: its pre-disclosure flow is lower, strictly on a non-null set, and the frontier F^0 is strictly decreasing on $[u^0, \infty)$.[21] And it is incentive-compatible: in each period t, prompt disclosure is as attractive as in the original (incentive-compatible) mechanism, and disclosing with delay (or never disclosing) is weakly less attractive since the agent earns a lower flow payoff $\min\{x^0, u^0\} \leq x^0$ while delaying. ■

4 Keeping the agent indifferent

In this section, we fully characterise how undominated mechanisms incentivise the agent. The agent's period-t choice in a mechanism (x^0, X^1), if she has observed the breakthrough and not yet disclosed it, is between

- disclosing promptly (payoff X_t^1),
- disclosing with any delay $d > 0$ (payoff $X_t^0 + e^{-rd}(X_{t+d}^1 - X_{t+d}^0)$), and
- never disclosing (payoff X_t^0).

Our first theorem asserts that in undominated mechanisms, the agent must always be indifferent between all three courses of action:

Theorem 1. Any undominated incentive-compatible mechanism (x^0, X^1) satisfies $X^0 = X^1$.

A tempting but incorrect intuition for Theorem 1 is that if the agent were to strictly prefer prompt disclosure in period t, then the principal could reduce her disclosure reward X_t^1 without violating IC. The trouble with this intuition is that if $X_t^1 \leq u^1$, then lowering it would *hurt* the principal (refer to Figure 1 on p. 7). This is no mere quibble, for (as we shall see) undominated mechanisms will spend time in $[0, u^1]$. More broadly, in a general dynamic environment, it is not clear that IC ought to bind everywhere.

The proof is in appendix A. Below, we outline the main idea in discrete time, then highlight the additional difficulties posed by continuous time.

Partial sketch proof. Let time $t \in \{0, 1, 2, \dots\}$ be discrete, and write $\beta \coloneqq e^{-r}$ for the discount factor. A mechanism (x^0, X^1) is incentive-compatible iff in each period s,

[21] F^0 is strictly decreasing above u^0 because it is concave and uniquely maximised at u^0.

(a) the agent does not prefer to delay disclosure by one period:

$$X_s^1 \geq (1-\beta)x_s^0 + \beta X_{s+1}^1, \quad \text{and}$$

(b) the agent does not prefer to *never* disclose: $X_s^1 \geq X_s^0$.

(Part (a) suffices to ensure that the agent does not wish to delay disclosure by two or more periods.) We shall show that undominatedness requires that the *delay IC* inequalities in (a) be equalities. We omit the argument showing that the *non-disclosure IC* inequalities in part (b) must also be equalities.

Let (x^0, X^1) be an incentive-compatible mechanism in which the agent strictly prefers prompt disclosure to delay at time t:

$$X_t^1 > (1-\beta)x_t^0 + \beta X_{t+1}^1. \tag{>}$$

We shall show that this mechanism is dominated by another IC mechanism. Observe first that if the terms x_t^0 and X_{t+1}^1 on the right-hand side of (>) are $\geq u^1$, then the left-hand side X_t^1 must strictly exceed u^1. Equivalently, it must be that either

$$\text{(i) } X_t^1 > u^1, \quad \text{(ii) } x_t^0 < u^1, \quad \text{or} \quad \text{(iii) } X_{t+1}^1 < u^1.$$

We consider each case separately, in reverse order.

If (iii) holds, consider raising X_{t+1}^1 by an amount $\varepsilon > 0$ small enough to preserve time-t delay IC and to keep X_{t+1}^1 below u^1:

$$X_t^1 \geq (1-\beta)x_t^0 + \beta\left(X_{t+1}^1 + \varepsilon\right) \quad \text{and} \quad X_{t+1}^1 + \varepsilon \leq u^1.$$

This modification slackens (and thus preserves) time-$(t+1)$ delay IC, leaves all other periods' delay IC unaffected, and slackens (and thus preserves) part (b) of IC. So the resulting mechanism is incentive-compatible, and it dominates the original mechanism because the principal's payoff is the same except if the breakthrough occurs in period $t+1$, in which case her payoff is strictly better since F^1 is strictly increasing on on $[0, u^1]$.[22]

In case (ii), we may similarly raise x_t^0 while keeping it below u^1 and preserving time-t delay IC. Delay IC is unaffected in all other periods, and it is easily verified that part (b) of IC is preserved. Thus the modified mechanism is incentive-compatible, and it dominates the original mechanism because F^0 is strictly increasing on $[0, u^1] \subseteq [0, u^0]$—in particular, the principal is strictly better off if the breakthrough occurs in period $t+1$ or later.

[22] F^1 is strictly increasing on $[0, u^1]$ because it is concave and uniquely maximised at u^1.

Finally, if (i) holds, consider lowering X^1_t by an amount small enough to preserve time-t delay IC and to keep X^1_t above u^1. This modification slackens (and thus preserves) time-$(t-1)$ delay IC, leaves delay IC unaffected in all other periods, and can easily be shown to preserve part (b) of IC. The resulting mechanism is therefore incentive-compatible, and it dominates the original mechanism since F^1 is strictly decreasing on $[u^1, \infty)$, so that the principal earns a strictly higher payoff if the breakthrough occurs at t. ■

The proof in appendix A is based on the logic of the sketch above, but must handle two issues that arise in continuous time. First, in case (ii), x^0 must be increased on a *non-null* set of times if the principal's payoff is to increase strictly under some distribution. Secondly, in cases (i) and (iii), it is typically not possible to modify X^1 in a single period while preserving IC.

In light of Theorem 1, an undominated incentive-compatible mechanism (x^0, X^1) is pinned down by the pre-disclosure flow x^0, since the disclosure reward X^1 must be the one that makes the agent indifferent:

$$X^1_t = X^0_t = r \int_t^\infty e^{-r(s-t)} x^0_s \mathrm{d}s \quad \text{for each } t \in \mathbf{R}_+.$$

We therefore drop superscripts in the sequel, writing an IC mechanism simply as (x, X). (Such a mechanisms are automatically incentive-compatible, so we refer to them simply as a 'mechanisms'.) By Lemma 1, we need only consider mechanisms (x, X) that satisfy $x \leq u^0$ a.e.

5 Deadline mechanisms

In this section, we provide a condition under which every undominated mechanism is a simple *deadline mechanism.* The condition is that the old utility possibility frontier F^0 be affine on $[0, u^0]$, as depicted in Figure 3. This is a substantial assumption, but one which holds (at least approximately) in some applications, such as unemployment insurance (§8). We further characterise the optimal choice of deadline, given the breakthrough distribution.

Write $u^\star$ for the unique $u \in [0, u^0]$ at which the frontiers are furthest apart, as depicted in Figure 3.[23]

Definition 4. A mechanism (x, X) is a *deadline mechanism* iff

$$x_t = \begin{cases} u^0 & \text{for } t \leq T \\ u^\star & \text{for } t > T \end{cases} \quad \text{for some } T \in [0, \infty].$$

[23] It is unique because $F^1 - F^0$ has a strict local maximum by assumption, which must be global since $F^1 - F^0$ is concave (because F^1 is concave and F^0 is affine).

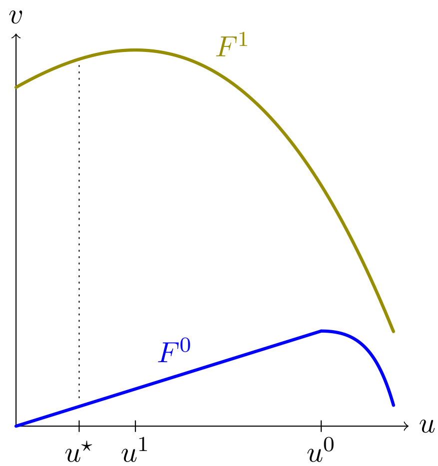


Figure 3: Utility possibility frontiers. $u^\star$ is the unique point at which the frontiers are furthest apart.

A deadline mechanism gives an agent who has not yet disclosed the high, efficient pre-disclosure flow u^0 until a deterministic deadline T, and the inefficiently low pre-disclosure flow $u^\star$ afterwards. Note that the values of u^0 and $u^\star$ are pinned down by the technologies F^0 and F^1, leaving the deadline T as the only free parameter. The agent's reward X upon disclosure in a deadline mechanism (given by Theorem 1) is decreasing until the deadline, and is then constant at $u^\star$:

$$X_t = \begin{cases} \left(1 - e^{-r(T-t)}\right) u^0 + e^{-r(T-t)} u^\star & \text{for } t \leq T \\ u^\star & \text{for } t > T. \end{cases} \tag{R}$$

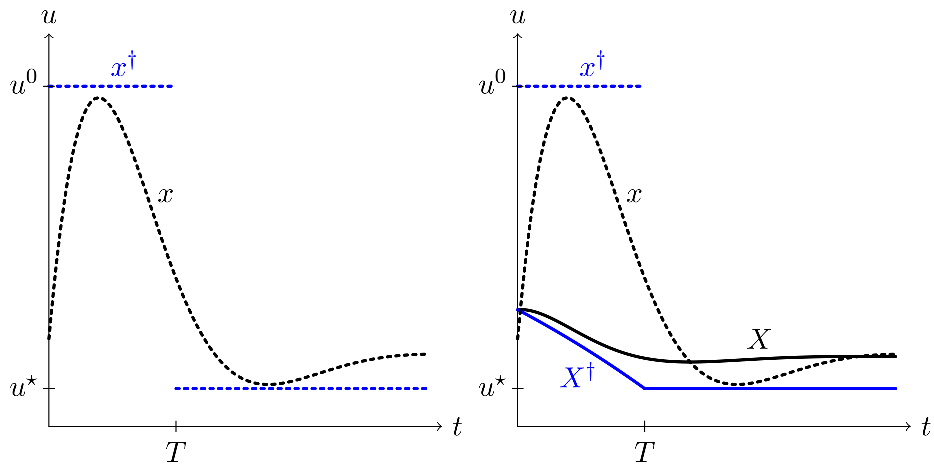
5.1 Only deadline mechanisms are undominated

In the affine case, only deadline mechanisms are admissible:

Theorem 2. If the old frontier F^0 is affine on $[0, u^0]$, then any undominated mechanism is a deadline mechanism.

Aside from being strikingly simple, deadline mechanisms have stark welfare properties. Absent a breakthrough, the old technology is operated without any distortion before the deadline, and with a distortion afterwards that hurts both the principal and the agent. If a breakthrough occurs early on (while $X \geq u^1$), then the new technology is deployed efficiently: payoffs are on the downward-sloping part of the frontier F^1.[24] By contrast, a late

[24] A detail: $X \geq u^1$ holds early on only if the deadline is sufficiently late. We show in the next section that this must be the case in undominated mechanisms.



(a) $x^\dagger$ is higher early and lower late.

(b) $X^\dagger \leq X$, with equality at 0.

Figure 4: Sketch proof of Theorem 2: front-loading by a deadline mechanism.

breakthrough leads the new technology to be operated inefficiently.[25] In short, deadline mechanisms are efficient if the breakthrough occurs early, and inefficient otherwise.

We prove Theorem 2 in appendix B. Below, we give an intuitive sketch proof for a special case.

Sketch proof of Theorem 2. Fix a mechanism (x, X) with $x \leq u^0$ that is not a deadline mechanism. Assume for simplicity that $x \geq u^\star$ and that the frontiers F^0, F^1 are differentiable. We will argue that the deadline mechanism $(x^\dagger, X^\dagger)$ whose deadline T satisfies

$$\underbrace{\left(1 - e^{-rT}\right) u^0 + e^{-rT} u^\star}_{\equiv X_0^\dagger} = X_0$$

gives the principal a weakly higher payoff than (x, X) under any breakthrough distribution G. (We skip the final step of showing that her payoff is strictly higher under some G.)

This deadline mechanism is a *front-loading* of the original mechanism: the flow has the same present value at the outset, but is higher early (before the deadline) and lower late (after the deadline), as depicted in Figure 4a.

[25] Provided $u^\star < u^1$, a sufficient condition for which is that F^1 have no kink at u^1.

Since the deadline mechanism concentrates the high flow at the beginning, the present value $X_t^\dagger$ of the remaining deadline flow at any time t is lower than the present value X_t of the remainder of the original flow. By our choice of the deadline T, the two are equal at $t = 0$. This is depicted in Figure 4b.

The principal's payoff may be written

$$\Pi_G(x, X) = \mathbf{E}_G\Big(\underbrace{Y_0 - e^{-r\tau}Y_\tau}_{\text{pre-disclosure}} + \underbrace{e^{-r\tau}F^1(X_\tau)}_{\text{post-disclosure}}\Big),$$

where

$$Y_t \coloneqq r\int_t^\infty e^{-r(s-t)}F^0(x_s)\mathrm{d}s$$

is her period-t continuation payoff if the agent never discloses. For a given $t > 0$, there are two effects of slightly lowering X_t:

- Post-disclosure: if the breakthrough occurs at $\tau = t$, then the principal's continuation payoff changes by $-F^{1\prime}(X_t)$. This is beneficial if $X_t > u^1$ (where $F^{1\prime} < 0$), and is otherwise costly.

- Pre-disclosure: since higher flows now occur earlier, the principal earns a better payoff before the breakthrough. To quantify this benefit, observe that since F^0 is affine on $[0, u^0]$, we have

  $$Y_t = r\int_t^\infty e^{-r(s-t)}F^0(x_s)\mathrm{d}s = F^0\left(r\int_t^\infty e^{-r(s-t)}x_s\mathrm{d}s\right) = F^0(X_t),$$

  and thus the principal's pre-disclosure payoff reads

  $$Y_0 - e^{-r\tau}Y_\tau = F^0(X_0) - e^{-r\tau}F^0(X_\tau).$$

  Thus the benefit of lowering X_t is proportional to $F^{0\prime}(X_t)$. (Recall that front-loading leaves X_0 unchanged.)

Comparing marginal cost with marginal benefit, we see that lowering X pointwise is beneficial so long as $F^{1\prime} < F^{0\prime}$. And this holds to the right of $u^\star$, since $F^1 - F^0$ is concave and uniquely maximised at $u^\star$. ■

Theorem 2 provides a rationale for deadline mechanisms even when F^0 is not exactly affine. As we show in supplemental appendix J, the principal loses little by restricting attention to deadline mechanisms provided F^0 has only moderate curvature.

5.2 Undominated deadlines

Theorem 2 asserts that only deadline mechanisms are undominated when F^0 is affine, but does not adjudicate between deadlines. In fact, not every deadline mechanism is undominated. Consider a deadline T so early that $X_0 < u^1$. Since the disclosure reward X decreases over time in a deadline mechanism, we have $X_\tau < u^1$ whatever the time τ of the breakthrough.

The principal can do better by using the later deadline $\underline{T}$ that satisfies $X_0 = u^1$, or explicitly (using equation (R) on p. 17)

$$\left(1 - e^{-r\underline{T}}\right)u^0 + e^{-r\underline{T}}u^\star = u^1.$$

This deadline provides the agent with a higher disclosure reward X in every period (and strictly before $\underline{T}$). Since F^1 is strictly increasing on $\left[0, u^1\right]$, the principal's post-disclosure payoff $F^1(X_\tau)$ is thus larger no matter what the breakthrough time τ (strictly if $\tau < \underline{T}$). In addition, the principal enjoys the high pre-disclosure flow payoff $F^0\left(u^0\right) > F^0(u^\star)$ for longer, which benefits her in case of a late breakthrough.

Undominatedness thus requires a deadline no earlier than $\underline{T}$. This condition is not only necessary, but also sufficient:

Proposition 1. If the old frontier F^0 is affine on $\left[0, u^0\right]$, then a mechanism is undominated exactly if it is a deadline mechanism with deadline $T \in [\underline{T}, \infty]$.

The proof is in appendix C.

5.3 Optimal deadline

Proposition 1 narrows the search for an optimal mechanism to deadline mechanisms with a sufficiently late deadline. The optimal choice among these depends on the breakthrough distribution G.

A late deadline is beneficial if the breakthrough occurs late, as the efficient high utility u^0 is then provided for a long time. The cost is that in case of an early breakthrough, the agent must be provided with a utility of $X > u^1$ forever, giving the principal a low payoff. A first-order condition balances this trade-off:

Proposition 2. Assume that the old frontier F^0 is affine on $\left[0, u^0\right]$, that F^1 is differentiable on $\left(0, u^0\right)$, and that $u^\star > 0$. A mechanism is optimal for G iff it is a deadline mechanism and satisfies $\mathbf{E}_G\left(F^{1\prime}(X_\tau)\right) = 0$.

In other words, the new technology should be operated optimally *on average*. This is a restriction on the deadline T because X is a function of it, as described by equation (R) on p. 17.

The proof is in appendix D, where we derive a general first-order condition that is valid without any auxiliary assumptions, then show that it can be written as $\mathbf{E}_G(F^{1\prime}(X_\tau)) = 0$ when F^0 is affine, F^1 is differentiable and $u^\star$ is interior.

In the same appendix, we derive comparative statics for optimal deadlines: they become later as the breakthrough distribution G becomes later in the sense of first-order stochastic dominance. By (R), a later deadline provides the agent with a higher ex-ante payoff X_0.

6 Optimal mechanisms in general

In this section, we depart from the affine case of the previous section. We show that optimal mechanisms share all of the salient features of deadline mechanisms, except for their abruptness: the agent's utility may instead decline gradually. Given the breakthrough distribution G, we describe the optimal transition path.

6.1 Qualitative features of optimal mechanisms

Let $u^\star$ denote the rightmost $u \in [0, u^0]$ at which the old and new frontiers F^0, F^1 have equal slopes,[26] as depicted in Figure 5. This definition agrees with the one in the previous section when F^0 is affine. Note that F^0 is steeper than F^1 on $(u^\star, u^0]$.[27]

Theorem 3. If F^0 is strictly concave on $[0, u^0]$ and F^0, F^1 possess uniformly continuous derivatives on $(0, u^0)$, then any mechanism (x, X) that is optimal for some distribution G with $G(0) = 0$ and unbounded support has x decreasing

$$\text{from} \quad \lim_{t \to 0} x_t = u^0 \quad \text{toward} \quad \lim_{t \to \infty} x_t = u^\star.$$[28]

As in the affine case, optimal mechanisms are efficient early and inefficient late. The agent still receives the efficient high utility u^0 at the outset. As

[26] 'Equal slopes' formally means that F^0, F^1 share a supergradient (see Rockafellar, 1970, part V). $u^\star$ is well-defined because at $u = 0$, both F^0 and F^1 admit ∞ as a supergradient.

[27] 'Steeper' means that any supergradient of F^0 strictly exceeds any supergradient of F^1. Clearly $u^\star$ is the smallest $u \in [0, u^0]$ such that F^0 is steeper than F^1 on $(u, u^0]$.

[28] Recall that a mechanism has multiple *versions* (footnote 18, p. 11). Theorem 3 asserts that any optimal mechanism has a version with the stated properties.

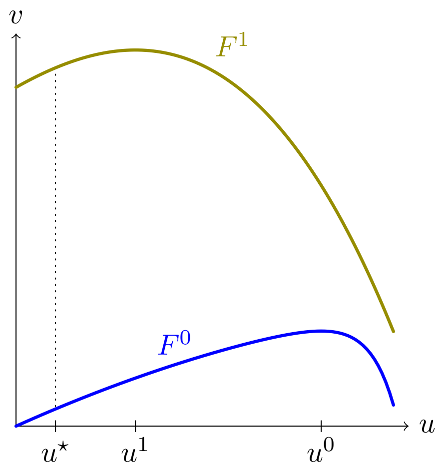


Figure 5: Utility possibility frontiers. $u^\star$ is the rightmost point at which F^0 and F^1 have equal slopes.

time passes, her utility if she has not disclosed declines, destroying surplus. By Theorem 1, the agent's post-disclosure utility is also lower the later the breakthrough occurs. If the breakthrough is early enough, then she receives a utility exceeding u^1,[29] which is efficient since the frontier F^1 slopes downward there; otherwise, the agent earns an inefficiently low post-disclosure utility.

The proof is in appendix F. It establishes a slightly stronger conclusion: any *undominated* mechanism must have x decreasing, and optimality for some breakthrough distribution (with unbounded support) requires that x start at u^0 (and converge to $u^\star$). We believe that our proof can be extended to drop the strict concavity and differentiability hypotheses.

6.2 Optimal transition

Theorem 3 pins down the qualitative features of optimal mechanisms, but does not specify the precise manner in which the agent's utility ought to decline from u^0 toward $u^\star$. The optimal path is described by an Euler equation that depends on the breakthrough distribution:

Proposition 3. Assume that $u^\star > 0$ and that F^0, F^1 possess bounded derivatives on $(0, u^0)$. Then any mechanism (x, X) that is optimal for G satisfies the initial condition $\mathbf{E}_G(F^{1\prime}(X_\tau)) = 0$ and the Euler equation

$$F^{0\prime}(x_t) \geq \mathbf{E}_G\Big(F^{1\prime}(X_\tau)\Big|\tau > t\Big) \quad \text{for every } t \in \mathbf{R}_+ \text{ at which } G(t) < 1, \text{ with equality if } x_t < u^0.^{30}$$

[29] The proof of Theorem 3 shows that $X_t^1 \geq u^1$ in early periods t. This parallels the affine case, where the deadline must be late enough that $X_0^1 \geq u^1$ (Proposition 1, p. 20).

The initial condition $\mathbf{E}_G\big(F^{1\prime}(X_\tau)\big) = 0$ demands that the new technology be used optimally on average, just like the first-order condition for the optimal deadline in the affine case (Proposition 2). The Euler equation describes a decreasing path from u^0 toward $u^\star$ that is gradual if F^0 is strictly concave on $[0, u^0]$, and abrupt if it is affine.

Without the interiority ($u^\star > 0$) and differentiability hypotheses, a more general (superdifferential) Euler equation characterises the optimal transition path. We prove in appendix G that this equation is necessary for optimality, whence Proposition 3 follows, and furthermore provide conditions under which it is sufficient.

As for comparative statics, it can be shown that as the breakthrough distribution G becomes later in the sense of monotone likelihood ratio, the disclosure reward X increases in every period. (The pre-disclosure flow x need not increase pointwise.) It follows in particular that the agent's ex-ante payoff X_0 improves. The (long) proof may be found on the authors' websites.

7 The limited role of transfers

Monetary transfers are typically a powerful tool in mechanism design. In this section, we show that they have limited value in our environment: undominated mechanisms may never pay the agent, and when they do, payments occur only after disclosure.

Our model (§2) nests applications both with and without monetary transfers: allowing payments merely changes the shapes of the frontiers F^0, F^1. To see how, suppose that the frontiers capture non-monetary utility possibilities alone, and that in addition to setting the agent's gross utility $u \in [0, \infty)$, the principal can pay her $w \geq 0$ (in an arbitrary history-dependent fashion). Net flow utilities are then $u + w$ for the agent and $F^j(u) - w$ for the principal, where $j \in \{0, 1\}$ is the technology used.[31]

Our assumption that the principal cannot charge the agent reflects a limited-liability constraint. Such constraints are common in the delegation literature,[32] as well as elsewhere in contract theory.[33] Without limited liability,

[30] $F^{j\prime}(0)$ ($F^{j\prime}(u^0)$) for $j \in \{0, 1\}$ denotes the right-hand (left-hand) derivative, which is well-defined by concavity.

[31] The timing of payments does not matter: payoffs are unchanged if a payment is delivered late, but with interest accrued at rate r (the common discount rate).

[32] For example, Aghion and Tirole (1997), Berkovitch and Israel (2004), Alonso and Matouschek (2008) and Armstrong and Vickers (2010).

[33] See the textbook treatment by Laffont and Martimort (2002, §3.5) and the seminal papers of Sappington (1983) and Lewis and Sappington (2000).

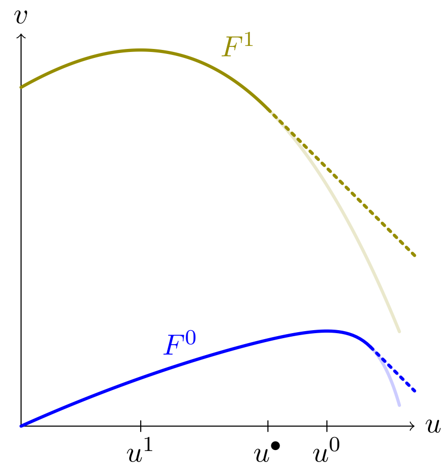


Figure 6: Utility possibility frontiers. The frontier F^1 has slope -1 at $u^\bullet$. Monetary transfers expand a frontier whenever its slope is < -1.

the incentive problem is trivial because the principal can attain first-best.[34]

To understand the implications of transfers, begin by considering utility possibilities in isolation, neglecting incentives and dynamics. (Refer to Figure 6.) Let $u^\bullet$ denote the rightmost agent utility at which F^1 has slope ≥ -1.[35] If the principal wishes to provide the agent with a high utility $u > u^\bullet$ using the new technology, she could set gross utility at u and make no payment, giving her payoff $F^1(u)$. An alternative is to provide a lower gross utility of $u^\bullet$ and to make up the shortfall by paying the agent $w = u - u^\bullet$. This earns the principal a payoff of $F^1(u^\bullet) - u + u^\bullet$, which exceeds $F^1(u)$ since F^1 has slope < -1 on $[u^\bullet, u]$.

In short, allowing for transfers expands the utility possibility frontiers where they are steeply downward-sloping: F^j is replaced by the pointwise smallest function that exceeds F^j and has slope ≥ -1 everywhere.[36] The expanded frontiers satisfy our model assumptions (§2), and thus the preceding results remain applicable, with transfers used only when the agent is provided with a utility at which the expanded frontier differs from the original. Therefore:

Proposition 4. Suppose that monetary transfers are permitted. Undominated mechanisms do not utilise transfers prior to disclosure, and use them

[34]The effective utility possibility frontiers (inclusive of transfers) are downward-sloping in this case, so that $u^0 = u^1 = 0$. Since there is no conflict of interest ($u^1 \not< u^0$), the principal's first-best is incentive-compatible (Remark 2, p. 11).

[35]Formally, $u^\bullet$ is the largest $u \in [0, \infty)$ at which F^1 admits -1 as a supergradient, and $u^\bullet := \infty$ if there is no such u.

[36]'Slope ≥ -1 everywhere' means 'admits a supergradient ≥ -1 at every $u \in [0, \infty)$'.

after disclosure iff the agent is to be provided with utility strictly exceeding $u^\bullet$. If $u^0 \leq u^\bullet$, then undominated mechanisms do not use transfers.

Proof. By Lemma 1 (p. 13), the agent is never provided with utility exceeding u^0 in an undominated mechanism, so receives pre-disclosure utility at which F^0 and its transfers-expansion coincide, meaning that transfers are not used. After disclosure, undominated mechanisms use transfers exactly if they provide the agent with utility $> u^\bullet$, since F^1 and its expansion by transfers differ only there. If $u^0 \leq u^\bullet$, then by Lemma 1, no undominated mechanism gives the agent utility $> u^\bullet$. ■

In a static example with F^1 affine on $[u^1, u^0]$, Armstrong and Vickers (2010, §3.2) showed that paying the agent is suboptimal if F^1 is sufficiently flat.[37] Proposition 4 extends this insight to a dynamic setting with non-linear technologies.

8 Application to unemployment insurance

In this section, we use our results to address a key tension in the design of unemployment insurance schemes: that between providing unemployed workers with a comfortable living while incentivising them to accept work if offered it. In our model, an unemployed worker privately observes job offers, and the state controls the worker's consumption and labour supply.

We show that a simple deadline scheme is approximately optimal: a high unemployment benefit is provided to the short-term unemployed, while only a subsistence benefit is paid to those remaining unemployed after a deadline. This rationalises the German *Hartz IV* system, which has exactly this structure.[38] We further show that the particular deadlines used in Hartz IV are consistent with optimal deadline choice according to our model.

Related literature. The literature on optimal unemployment insurance has two main strands. The first studies the moral-hazard problem of incentivising job-search effort (Shavell & Weiss, 1979; Hopenhayn & Nicolini, 1997). We contribute to the second strand, which is concerned with the adverse-selection problem arising from the fact that only the worker observes job offers (Atkeson & Lucas, 1995).[39] We depart from existing work by allowing for job offers that do not expire immediately, so that workers can

[37] They credit Berkovitch and Israel (2004) with a similar observation.
[38] See Price (2019, §2) for a nice English-language overview of Hartz IV.
[39] See also Thomas and Worrall (1990), Atkeson and Lucas (1992), and Hansen and İmrohoroğlu (1992).

delay acceptance (at least by a little). Whereas optimal mechanisms in the literature typically feature rich history-dependence, our results recommend a simple and practical deadline scheme similar to the German Hartz IV.

8.1 Model

A worker (agent) is unemployed. At a random time $\tau \sim G$, she receives a job offer, which she can accept immediately or with a delay. The offer may eventually expire, in which case the worker may subsequently receive further offers.[40]

The worker's utility from consuming $C \geq 0$ and (if employed) working $L \geq 0$ hours is $u = \phi(C) - \kappa(L)$. We assume that ϕ and κ are respectively strictly concave and strictly convex, that both are strictly increasing and continuous at zero with $\phi(0) = \kappa(0) = 0$, and that

$$\lim_{C\to\infty} \phi'(C) = 0, \quad \lim_{C\to 0} \phi'(C) = \infty \quad \text{and} \quad \lim_{L\to 0} \kappa'(L) = 0.$$

We interpret $C = 0$ as a subsistence level of consumption.

As is standard in the literature (e.g. Atkeson & Lucas, 1995), we assume that the state (principal) controls the worker's consumption C and labour supply L.[41] The state is the custodian of the social interest, caring about both the worker's welfare and net tax revenue. Writing $w > 0$ for the worker's hourly wage and $\lambda > 0$ for the shadow value of public funds, flow social welfare is

$$v = u + \lambda(jwL - C),$$

where $j = 1$ if the worker is employed and $j = 0$ if not.

The utility possibility frontier when the worker is unemployed is

$$F^0(u) = u + \lambda \max_{C\geq 0}\{-C : \phi(C) = u\} = u - \lambda\phi^{-1}(u),$$

while for employed workers it is

$$\begin{aligned} F^1(u) &= u + \lambda \max_{C,L\geq 0}\{wL - C : \phi(C) - \kappa(L) = u\} \\ &= u + \lambda \max_{L\geq 0}\Big\{wL - \phi^{-1}(u + \kappa(L))\Big\}. \end{aligned}$$

The frontiers are depicted in Figure 7. As the figure suggests, they satisfy our model assumptions (§2):

[40] Recall from Remark 1 (p. 11) that incentive-compatibility is the same whether or not offers eventually expire.

[41] The idea is that consumption is steered using taxes and benefits, while labour supply is controlled by setting work requirements enforced by harsh sanctions (such as fines).

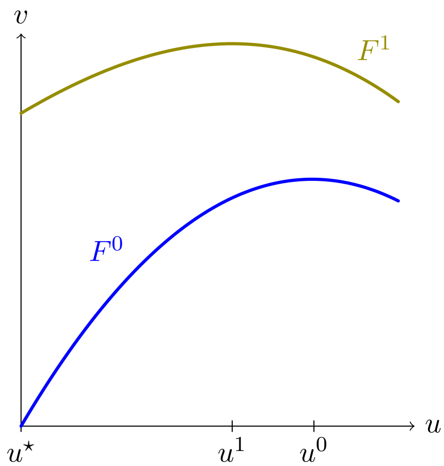


Figure 7: Utility possibility frontiers for unemployment insurance.

Lemma 2. In the application to unemployment insurance, the frontiers F^0, F^1 are strictly concave and continuous, with unique peaks u^0, u^1 that satisfy $u^0 > u^1$. The gap $F^1 - F^0$ is strictly decreasing, so that $u^\star = 0$.

We prove Lemma 2 in appendix H. The conflict of interest $u^0 > u^1$ arises because the marginal social benefit of an additional hour worked exceeds the marginal private benefit, since the worker fails to internalise the social benefit of additional tax revenue.

We shall use the term 'unemployment insurance (UI) scheme' for a mechanism. By Theorem 1 (p. 14), undominated schemes keep the worker only just willing promptly to accept an offer, so have the form (x, X).

A UI scheme (x, X) may be described in terms of benefits and taxes, as follows. The utility of a worker who remains unemployed at t is $x_t = \phi(b_t)$, where b_t is her unemployment benefit. If the worker accepts a job at t, then she earns utility

$$X_t = \phi(wL_t - \theta_t) - \kappa(L_t)$$

in every subsequent period, where $\theta_t \coloneqq wL_t - C_t$ is her per-period tax bill.

8.2 Optimal unemployment insurance

Undominated UI schemes are described by Theorem 3 (p. 21): unemployment benefits $b_t = \phi^{-1}(x_t)$ decrease gradually from $b^0 \coloneqq \phi^{-1}(u^0)$ toward $0 = \phi^{-1}(u^\star)$. Thus the unemployed enjoy socially efficient consumption at the outset, but their benefits are tapered over time. The benefits paid to the long-term unemployed are just enough to cover their basic needs: their consumption is 'zero', which (recall) we interpret to mean bare subsistence. Given the distribution G of job offers, the precise optimal path of unemployment benefits from b^0 toward subsistence is described by Proposition 3 (p. 22).

Employed workers are rewarded with a higher continuation utility X_t the earlier they accept a job. This corresponds both to fewer hours worked and to more generous tax treatment of their earnings (and thus higher consumption).

Suppose that the worker's consumption utility function ϕ has little curvature. (This may be interpreted as limited risk-aversion.) In this case, the utility possibility frontier F^0 for the unemployed is close to affine. It follows by Theorem 2 (p. 17), or more precisely by its approximate variant in supplemental appendix J, that a *deadline scheme* is approximately optimal.

A deadline UI scheme is parametrised by a deadline T, and we call an unemployed worker *short-term unemployed* before this deadline and *long-term unemployed* afterwards. The short-term unemployed are paid the socially efficient benefit $b^0 = \phi^{-1}(u^0)$, whereas the long-term unemployed earn only a subsistence benefit.

The German Hartz IV system has this deadline form. An unemployed worker earns the high *Arbeitslosengeld I* benefit before a deadline. If she remains unemployed after the deadline, then her benefits are reduced to the much lower *Arbeitslosengeld II* level.[42]

Under Hartz IV, older workers are set a later deadline, allowing them to collect *Arbeitslosengeld I* for longer. To rationalise this, observe that older workers plausibly have a harder time finding a new job, so face a job offer distribution G that is later in the sense of first-order stochastic dominance. It is then optimal to set a later deadline for older workers, as Hartz IV does.[43]

Appendices

A Proof of Theorem 1 (p. 14)

Given a mechanism (x^0, X^1), define Δ by $\Delta_t \coloneqq X^1_t - X^0_t$.

Observation 1. A mechanism (x^0, X^1) is incentive-compatible exactly if $t \mapsto e^{-rt}\Delta_t$ is (a) decreasing and (b) non-negative.

Proof. Upon rearrangement, part (a) (part (b)) of the definition of incentive-compatibility (p. 11) requires precisely that $t \mapsto e^{-rt}(X^1_t - X^0_t)$ be decreasing

[42] In 2020, *Arbeitslosengeld I* amounted to 60% of the worker's net salary in her previous job, while *Arbeitslosengeld II* was a mere €432 per month (€5184 per annum).

[43] In particular, the optimal deadline described by Proposition 2 (p. 20) is later for older workers, as noted at the end of §5.3 and proved in appendix D (p. 40).

(non-negative). ■

We shall prove the following strengthening of Theorem 1:

Theorem 1′. If an IC mechanism (x^0, X^1) does not satisfy $\Delta \equiv 0$, then there is another IC mechanism $(x^{0\dagger}, X^{1\dagger})$ such that

$$\Pi_G\Big(x^{0\dagger}, X^{1\dagger}\Big) \geq (>) \, \Pi_G\Big(x^0, X^1\Big) \quad \text{for every (full-support) } G.$$

This implies that undominated mechanisms must satisfy $\Delta = 0$ even if dominance is strengthened (and thus undominatedness weakened) to require strictness for every full-support G.

The proof relies on the following lemma, proved in §A.1 below.

Lipschitz lemma. If the Δ of an IC mechanism (x^0, X^1) is not Lipschitz continuous on $[0, T]$ for some $T \in \mathbf{R}_+$, then there is another IC mechanism $(x^{0\dagger}, X^{1\dagger})$ such that

$$\Pi_G\Big(x^{0\dagger}, X^{1\dagger}\Big) \geq (>) \, \Pi_G\Big(x^0, X^1\Big) \quad \text{for every (full-support) } G.$$

Proof of Theorem 1′. Let (x^0, X^1) be an incentive-compatible mechanism, and suppose that Δ is not identically zero; we seek another IC mechanism that yields a weakly (strictly) higher payoff under every (full-support) distribution G. If Δ fails to be Lipschitz continuous on some interval $[0, T]$ where $T \in \mathbf{R}_+$, then we are done by Lipschitz lemma. Suppose instead that Δ is Lipschitz continuous on every such interval.

Define

$$A \coloneqq \{t > 0 : \Delta \text{ is differentiable at } t \text{ and } \Delta'_t < r\Delta_t\} \subseteq \mathbf{R}_+.$$

Claim. The set

$$\Big\{t \in A : x^0_t < u^0 \text{ or } X^1_t \neq u^1\Big\}$$

is non-null.

Proof. Suppose not; we seek a contradiction. Let $A' \subseteq A$ be the set of non-isolated elements of

$$A'' \coloneqq \Big\{t \in A : x^0_t = u^0 \text{ and } X^1_t = u^1\Big\},$$

and note that $A \setminus A'$ is null since $A \setminus A''$ is null by hypothesis and $A'' \setminus A'$ is at most countable. Define

$$B \coloneqq \{t > 0 : \Delta \text{ is differentiable at } t \text{ and } \Delta'_t = r\Delta_t\}.$$

The set $A \cup B \subseteq \mathbf{R}_+$ has full measure since Δ is differentiable a.e. (being Lipschitz continuous), and thus $A' \cup B$ has full measure since $A \setminus A'$ is null.

Since Δ and X^0 are differentiable on A', so is $X^1 = \Delta - X^0$, and its derivative is zero.[44] Thus

$$\Delta'_t = r\left(u^0 - X^0_t\right) \geq 0 \quad \text{for every } t \in A'. \tag{1}$$

Since $\Delta' = r\Delta \geq 0$ on B by incentive-compatibility, and $A' \cup B$ has full measure, we conclude that $\Delta' \geq 0$ on a set of full measure. Since Δ is Lipschitz continuous, it follows that Δ is increasing.

Since Δ is non-negative by incentive-compatibility and by hypothesis not identically zero, we have $\Delta_{t'} > 0$ for some $t' \in \mathbf{R}_+$. Since Δ is increasing, it follows that Δ' is bounded away from zero on $B \cap (t', \infty)$.[45] Because Δ is Lipschitz continuous and bounded above and $A' \cup B$ has full measure, it follows that A' is non-null and that $\inf_{A'} \Delta' = 0$.[46] It follows by (1) that $\sup_{A'} X^0 = u^0$, and thus $\inf_{A'} \Delta \leq 0$, which implies that $\Delta_0 \leq 0$ since Δ is increasing. But then $e^{-rt'}\Delta_{t'} > 0 \geq e^{-r\times 0}\Delta_0$, a contradiction with the incentive-compatibility of (x^0, X^1). ☐

By the claim, there are two cases: either $x^0 < u^0$ on a non-null subset of A, or else $X^1 \neq u^1$ on a non-null subset of A. In each case, we shall construct an incentive-compatible mechanism that does weakly (strictly) better than (x^0, X^1) for every (full-support) distribution G.

Case 1: $x^0 < u^0$ on a non-null subset of A. Since Δ is non-negative by incentive-compatibility and by hypothesis not identically zero, it is strictly positive at some $t' \in \mathbf{R}_+$, and thus $\Delta_0 \geq e^{-rt'}\Delta_{t'} > 0$ by incentive-compatibility. IC also implies that if $\Delta_t = 0$ then $\Delta = 0$ on $[t, \infty)$, and thus there is a $t_0 \in (t', \infty]$ such that $\Delta > 0$ on $[0, t_0)$ and $\Delta = 0$ on $[t_0, \infty)$. Since $\Delta' \leq r\Delta = 0$ on $[t_0, \infty)$ by incentive-compatibility, it follows that $A \subseteq [0, t_0)$, so that $\Delta > 0$ on A. It follows that there is an $\varepsilon > 0$ such that the set

$$A_\varepsilon \coloneqq \left\{t \in A : x^0_t + \varepsilon < u^0,\ \Delta_t > \varepsilon \text{ and } \Delta'_t \leq r(\Delta_t - \varepsilon)\right\}$$

[44] The derivative is zero because for any $t \in A'$, we may (since t is not isolated in A'') choose a sequence $(t_n)_{n\in\mathbf{N}}$ in $A'' \setminus \{t\}$ converging to t, along which $(X^1_{t_n} - X^1_t)/(t_n - t) = 0$.

[45] In particular, $\Delta' = r\Delta \geq r\Delta_{t'} > 0$ on $B \cap (t', \infty)$.

[46] Otherwise Δ' would be bounded away from zero on the entire full-measure set $A' \cup B \subseteq \mathbf{R}_+$, which is impossible given that Δ is Lipschitz and bounded above.

is non-null.[47]

Now, define $x^{0\dagger} : \mathbf{R}_+ \to \mathbf{R}_+$ by

$$x_t^{0\dagger} \coloneqq \begin{cases} x_t^0 + \varepsilon & \text{for } t \in A_\varepsilon \\ x_t^0 & \text{for } t \notin A_\varepsilon. \end{cases}$$

Clearly $x^{0\dagger} \geq x^0$ and $x^{0\dagger} \neq x^0$ on a non-null set, so that

$$\Pi_G\left(x^{0\dagger}, X^1\right) \geq (>)\ \Pi_G\left(x^0, X^1\right) \quad \text{for every (full-support) } G$$

by the strict monotonicity of F^0 on $[0, u^0]$.

It remains to verify that $(x^{0\dagger}, X^1)$ is incentive-compatible. $\Delta^\dagger$ is non-negative since $\Delta^\dagger \geq \Delta - \varepsilon > 0$ on A_ε and $\Delta^\dagger = \Delta \geq 0$ off A_ε. To see that the Lipschitz continuous map $t \mapsto e^{-rt}\Delta_t^\dagger$ is decreasing, it suffices to show that its derivative is non-positive:

$$\begin{aligned}
\Delta_t^{\dagger\prime} - r\Delta_t^\dagger &= \Delta_t' + \frac{\mathrm{d}}{\mathrm{d}t}\left(X_t^0 - X_t^{0\dagger}\right) - r\Delta_t^\dagger \\
&= \Delta_t' - r\left(x_t^0 - x_t^{0\dagger}\right) + r\left(X_t^0 - X_t^{0\dagger}\right) - r\Delta_t^\dagger \\
&= (\Delta_t' - r\Delta_t) + r\left(x_t^{0\dagger} - x_t^0\right) \\
&\begin{cases} \leq -r\varepsilon + r\varepsilon = 0 & \text{for } t \in A_\varepsilon \\ \leq 0 + 0 = 0 & \text{for } t \notin A_\varepsilon. \end{cases}
\end{aligned}$$

Case 2: $X^1 \neq u^1$ on a non-null subset of A. In this case, either

$$C \coloneqq \left\{t \in A : X_t^1 > u^1\right\} \quad \text{or} \quad \left\{t \in A : X_t^1 < u^1\right\}$$

is non-null. We shall assume that it is the former, omitting the similar argument for the other case.

There must be a $t' \in C$ such that $(t', t'') \cap C$ is non-null for every $t'' > t'$.[48] Since Δ and X^0 are Lipschitz continuous, so is $X^1 = \Delta - X^0$.

[47] Clearly A_0 is non-null, and thus by continuity of measures

$$0 < \lambda(A_0) = \lambda\left(\bigcup_{n \in \mathbf{N}} A_{1/n}\right) = \lim_{n \to \infty} \lambda\left(A_{1/n}\right)$$

(where λ denotes the Lebesgue measure), so that $\lambda(A_{1/n}) > 0$ for some $n \in \mathbf{N}$.

[48] If not, then the absolutely continuous function $f(t) \coloneqq \lambda(C \cap [0, t])$ (where λ denotes the Lebesgue measure) would be locally constant, so that $f' = 0$ a.e., which is impossible since $f' = \mathbf{1}_C$ and C is non-null.

We may therefore choose $\underline{t}, \bar{t} \in \mathbf{R}_+$ and an $\varepsilon > 0$ such that $\underline{t} < t' < \bar{t}$ and $X^1 > u^1 + \varepsilon$ on $[\underline{t}, \bar{t}]$. Notice that

$$\int_t^{\bar{t}} (\Delta' - r\Delta) < 0 \quad \text{for any } t \in [\underline{t}, t']$$

since $\Delta' - r\Delta \leq 0$ by incentive-compatibility, with strict inequality on the non-null subset $(t', \bar{t}) \cap C \subseteq A$ of $[t, \bar{t}]$. This together with the fundamental theorem of calculus yields for any $t \in [\underline{t}, t']$ that

$$\Delta_{\bar{t}} = \Delta_t + \int_t^{\bar{t}} \Delta' < \Delta_t + r \int_t^{\bar{t}} \Delta \leq \Delta_t \left(1 + r \int_t^{\bar{t}} e^{r(s-t)} \mathrm{d}s \right) = e^{r(\bar{t}-t)} \Delta_t,$$

where the weak inequality holds by incentive-compatibility.

Now, choose a $\gamma \in (0,1)$ large enough that

$$\gamma X_t^1 + (1-\gamma)\left(X_t^0 + e^{-r(\bar{t}-t)} \Delta_{\bar{t}} \right) \geq u^1 \quad \text{for every } t \in [\underline{t}, \bar{t}],$$

and define $X^{1\dagger} : \mathbf{R}_+ \to [0, \infty]$ by

$$X_t^{1\dagger} := \begin{cases} \gamma X_t^1 + (1-\gamma)\left(X_t^0 + e^{-r(\bar{t}-t)} \Delta_{\bar{t}} \right) & \text{for } t \in [\underline{t}, \bar{t}] \\ X_t^1 & \text{for } t \notin [\underline{t}, \bar{t}]. \end{cases}$$

Then $u^1 \leq X^{1\dagger} \leq X^1$ and $X^{1\dagger} \neq X^1$ on a non-null set (namely, $[\underline{t}, t']$), so

$$\Pi_G\left(x^0, X^{1\dagger}\right) \geq\!(>)\, \Pi_G\left(x^0, X^1\right) \quad \text{for every (full-support) } G$$

since F^1 is strictly increasing on $[0, u^1]$ and strictly decreasing on $[u^1, \infty)$. To see that $(x^0, X^{1\dagger})$ is incentive-compatible, observe that

$$\Delta_t^\dagger := X_t^{1\dagger} - X_t^0 = \begin{cases} \gamma \Delta_t + (1-\gamma) e^{-r(\bar{t}-t)} \Delta_{\bar{t}} & \text{for } t \in [\underline{t}, \bar{t}] \\ \Delta_t & \text{for } t \notin [\underline{t}, \bar{t}]. \end{cases}$$

Clearly $\Delta \geq 0$, and

$$\Delta^{\dagger\prime} - r\Delta^\dagger = \gamma(\Delta' - r\Delta) \geq 0$$

by incentive-compatibility of (x^0, X^1). This implies $t \mapsto e^{-rt}\Delta_t^\dagger$ is decreasing on $[0, \underline{t})$ and on $[\underline{t}, \infty)$ since $\Delta^\dagger$ is Lipschitz continuous on these intervals, and for $s \in [0, \underline{t})$ and $s' \in [\underline{t}, \infty)$ we have

$$\begin{aligned} e^{-rs}\Delta_s^\dagger &= e^{-rs}\Delta_s \\ &\geq \min\left\{ \gamma e^{-rs'} \Delta_{s'} + (1-\gamma) e^{-r\bar{t}} \Delta_{\bar{t}}, e^{-rs'} \Delta_{s'} \right\} = e^{-rs'} \Delta_{s'}^\dagger. \end{aligned}$$ ■

A.1 Proof of the Lipschitz lemma (p. 29)

Consider an IC mechanism (x^0, X^1) whose Δ is not Lipschitz continuous on $[0,T]$ for some $T \in (0,\infty)$. We may assume that $x^0 \leq u^0$ a.e., since otherwise (recalling the proof of Lemma 1 on p. 13) the IC mechanism $(\min\{x^0, u^0\}, X^1)$ is weakly (strictly) better for every (full-support) distribution G.

Define $h : \mathbf{R}_+ \to \mathbf{R}$ by

$$h(t) \coloneqq e^{-rt}\left(X_t^1 - X_t^0\right) \quad \text{for each } t \in \mathbf{R}_+.$$

Since the mechanism is IC, h is decreasing and non-negative by Observation 1 (p. 28). Because X^0 is Lipschitz continuous on $[0,T]$, it must be that X^1 is not Lipschitz on $[0,T]$, and this must be because it *decreases* abruptly.[49]

Suppose first that X^1 has a discontinuity at some $t \in [0,T]$. Then we may modify X^1 on a small interval $[t, t+\varepsilon)$ to be pointwise strictly closer to u^1 while keeping h decreasing, and thus preserving IC. This mechanism is weakly (strictly) better for any (full-support) distribution G since F^1 is strictly increasing on $[0, u^1]$ and strictly decreasing on $[u^1, \infty)$.

It remains to consider the case in which X^1 is continuous on $[0,T]$. As in the discontinuous case, we shall show that X^1 may be brought pointwise closer to u^1 while preserving IC. Since X^0 and $t \mapsto e^{rt}$ are Lipschitz continuous on $[0,T]$, there is an $L > 0$ such that for any $t' < t''$ in $[0,T]$ satisfying

$$X_{t'}^1 - X_{t''}^1 > L(t'' - t'), \tag{2}$$

we have $h(t') > h(t'')$. Since X^1 is continuous, we may partition $[0,T)$ into countably many intervals of the form $[a,b)$ over which $X^1 - u^1$ does not change sign, and either

(i) X^1 is constant at u^1 on $[a,b)$, or

(ii) $X_t^1 \neq u^1$ for every $t \in (a,b)$.

Since X^1 is not L-Lipschitz on $[0,T]$, it fails to be L-Lipschitz on one of the intervals $[a,b)$ satisfying (ii). We may therefore choose $t' < t''$ in (a,b) for which (2) holds. By (ii) and continuity of X^1, we have either $X^1 > u^1$ on $[t', t''] \subseteq (a,b)$, or else $X^1 < u^1$ on $[t', t'']$. We assume the former, omitting the analogous argument for the latter case.

[49] Since h is decreasing and X^0 is Lipschitz, the rate of change of X^1 is bounded above in the sense that for some $K > 0$ we have $X_{t''}^1 - X_{t'}^1 \leq K(t'' - t')$ for any $t' < t''$ in $[0,T]$. So its failure to be Lipschitz on $[0,T]$ must be on account of its decreasing unboundedly quickly somewhere on $[0,T]$.

Since $s \mapsto e^{rs}h(t'') + X^0_s$ is continuous and is strictly positive at $s = t''$,

$$t^\star := \inf\Big\{t \in [t', t''] : e^{rs}h(t'') + X^0_s \geq u^1 \text{ for all } s \in [t, t'']\Big\}$$

is well-defined and $t^\star < t''$. Define

$$X^{1\dagger}_t := \begin{cases} e^{rt}h(t'') + X^0_t & \text{for } t \in [t^\star, t'') \\ X^1_t & \text{for } t \notin [t^\star, t''), \end{cases}$$

and consider the mechanism $(x^0, X^{1\dagger})$ and its associated $h^\dagger$. This mechanism is incentive-compatible by Observation 1 since

$$h^\dagger(t) = e^{-rt}\Big(X^{1\dagger}_t - X^0_t\Big) = \begin{cases} h(t'') & \text{for } t \in [t^\star, t''] \\ h(t) & \text{for } t \notin [t^\star, t''] \end{cases}$$

is decreasing and non-negative.

It remains to show that $(x^0, X^{1\dagger})$ yields a weakly (strictly) higher payoff than (x^0, X^1) under every (full-support) distribution G. Since X^1 and $X^{1\dagger}$ differ only on $[t^\star, t'')$ and F^1 is strictly decreasing on $[u^1, \infty)$, it suffices to prove that $X^{1\dagger}$ is pointwise closer to u^1:

$$u^1 \leq X^{1\dagger}_t \leq (<)\ X^1_t \quad \text{for every (some) } t \in [t^\star, t'').$$[50]

The first inequality holds by definition of $t^\star$. For the second, observe that

$$X^{1\dagger}_t - X^1_t = e^{rt}\Big[h^\dagger(t) - h(t)\Big] = e^{rt}\big[h(t'') - h(t)\big] \leq 0 \quad \text{for } t \in [t^\star, t'')$$

since h is decreasing. We claim that the inequality is strict at $t = t^\star$. If $t^\star = t'$, then this is true because $h(t') > h(t'')$. And if not, then $t^\star \in (t', t'')$, in which case

$$X^{1\dagger}_{t^\star} = u^1 < X^1_{t^\star}$$

by continuity of X^0 and $X^1 > u^1$. ■

B Proof of Theorem 2 (p. 17)

Observation 2. If the old frontier F^0 is affine on $[0, u^0]$, then the principal's payoff may be written

$$\Pi_G(x, X) = F^0(X_0) + \mathbf{E}_G\Big(e^{-r\tau}\Big[F^1 - F^0\Big](X_\tau)\Big).$$

[50]It is enough for the inequality to be strict at a single time $t \in [t^\star, t'')$, since it then holds strictly on an interval by the continuity of X^1 and $X^{1\dagger}$ on $[t^\star, t'')$.

Proof. The principal's payoff is

$$\Pi_G(x, X) = \mathbf{E}_G\Big(Y_0 + e^{-r\tau}\Big[F^1(X_\tau) - Y_\tau\Big]\Big),$$

where

$$Y_t \coloneqq r\int_t^\infty e^{-r(s-t)}F^0(x_s)\mathrm{d}s$$

is her payoff from the perspective of period t if the agent never discloses. If F^0 is affine on $\big[0, u^0\big]$, then for every period t,

$$Y_t = r\int_t^\infty e^{-r(s-t)}F^0(x_s)\mathrm{d}s = F^0\Big(r\int_t^\infty e^{-r(s-t)}x_s\mathrm{d}s\Big) = F^0(X_t). \qquad \blacksquare$$

Proof of Theorem 2. Fix a mechanism (x, X) with $x \leq u^0$ a.e. that is not a deadline mechanism;[51] we will show that it is dominated by the deadline mechanism $\big(x^\dagger, X^\dagger\big)$ whose deadline T satisfies

$$\Big(1 - e^{-rT}\Big)u^0 + e^{-rT}u^\star \equiv X_0^\dagger = X_0 \vee u^\star,$$

where '$\vee$' denotes the pointwise maximum.

Claim. $X^\dagger \leq X \vee u^\star$.

Proof. For $t \geq T$, we have $X^\dagger = u^\star \leq X \vee u^\star$. For $t < T$, suppose first that $X_0^\dagger = X_0$; then since $x^\dagger = u^0 \geq x$ on $[0, t] \subseteq [0, T]$, we have

$$\begin{aligned} e^{-rt}X_t^\dagger &= X_0^\dagger - r\int_0^t e^{-rs}x_s^\dagger \mathrm{d}s \\ &\leq X_0 - r\int_0^t e^{-rs}x_s\mathrm{d}s = e^{-rt}X_t \leq e^{-rt}(X_t \vee u^\star). \end{aligned}$$

If instead $X_0^\dagger = u^\star$, then the fact that $x^\dagger \geq u^\star$ yields

$$\begin{aligned} e^{-rt}X_t^\dagger &= X_0^\dagger - r\int_0^t e^{-rs}x_s^\dagger \mathrm{d}s \\ &\leq u^\star - r\int_0^t e^{-rs}u^\star\mathrm{d}s = e^{-rt}u^\star \leq e^{-rt}(X_t \vee u^\star). \qquad \square \end{aligned}$$

The concave function $F^1 - F^0$ is uniquely maximised at $u^\star$, so is strictly increasing on $[0, u^\star]$ and strictly decreasing on $\big[u^\star, u^0\big]$. Since $u^\star \leq X^\dagger \leq X \vee u^\star$ by the claim, it follows that

$$\Big[F^1 - F^0\Big]\Big(X^\dagger\Big) \geq \Big[F^1 - F^0\Big](X \vee u^\star). \tag{3}$$

[51] Any IC mechanism not of this form is dominated by Lemma 1 and Theorem 1 (pages 13 and 14).

Since $X \vee u^\star \geq X$, and the two differ only when both are in $[0, u^\star]$, we have

$$\Big[F^1 - F^0\Big](X \vee u^\star) \geq \Big[F^1 - F^0\Big](X), \tag{4}$$

which chained together with the preceding inequality yields

$$\Big[F^1 - F^0\Big]\Big(X^\dagger\Big) \geq \Big[F^1 - F^0\Big](X). \tag{5}$$

The facts that $X_0^\dagger = X_0 \vee u^\star \geq X_0$ and that F^0 is increasing on $[0, u^0]$ together imply

$$F^0\Big(X_0^\dagger\Big) \geq F^0(X_0). \tag{6}$$

Thus for any distribution G, using the expression in Observation 2 for the principal's payoff, we have

$$\begin{aligned}
\Pi_G\Big(x^\dagger, X^\dagger\Big) &= F^0\Big(X_0^\dagger\Big) + \mathbf{E}_G\Big(e^{-r\tau}\Big[F^1 - F^0\Big]\Big(X_\tau^\dagger\Big)\Big) \\
&\geq F^0\Big(X_0^\dagger\Big) + \mathbf{E}_G\Big(e^{-r\tau}\Big[F^1 - F^0\Big](X_\tau)\Big) && \text{by (5)} \\
&\geq F^0(X_0) + \mathbf{E}_G\Big(e^{-r\tau}\Big[F^1 - F^0\Big](X_\tau)\Big) && \text{by (6)} \\
&= \Pi_G(x, X).
\end{aligned}$$

It remains show that $(x^\dagger, X^\dagger)$ delivers a *strict* improvement for some G. We shall accomplish this by showing that the inequality (5) holds strictly on a non-null set of times, so that the second inequality in the above display is strict for any G with full support. Since $X^\dagger \leq X \vee u^\star$ by the claim and $X, X^\dagger$ are continuous, there are two cases: either (a) $X^\dagger < X \vee u^\star$ on a non-null set of times, or (b) $X^\dagger = X \vee u^\star$.

Case (a): $X^\dagger < X \vee u^\star$ on a non-null set $\mathcal{T}$. In this case, the inequality (3) holds strictly on $\mathcal{T}$, and thus so does (5).

Case (b): $X^\dagger = X \vee u^\star$. Since the original mechanism (x, X) is not a deadline mechanism, there must be a non-null set of times on which $x \neq x^\dagger$, and thus $X \neq X^\dagger = X \vee u^\star$ on some non-null set $\mathcal{T}$, so that $X < X \vee u^\star$ on $\mathcal{T}$. Then (4) is strict on $\mathcal{T}$, and thus so is (5). ■

C Proof of Proposition 1 (p. 20)

By Theorem 2, any undominated mechanism is a deadline mechanism. We showed in the text (§5.2, p. 20) that those with deadline $T < \underline{T}$ are dominated, so it remains only to show that those with deadline $T \geq \underline{T}$ are not. Write (x^T, X^T) for the deadline mechanism with deadline T, and write $\pi_G(T)$ for its payoff under a distribution G.

Fix any deadline $T \in [\underline{T}, \infty]$. By Theorem 2, it suffices to show that (x^T, X^T) is not dominated by another deadline mechanism.[52] We consider the cases $T \in (\underline{T}, \infty)$, $T = \infty$ and $T = \underline{T}$ separately.

Case 1: finite deadline exceeding $\underline{T}$. Fix a deadline $T \in (\underline{T}, \infty)$; we shall identify a distribution G with $G(0) = 0$ under which the deadline T yields a strictly higher payoff than any other deadline. In particular, consider the point mass at $T - \underline{T}$. The mechanism (x^T, X^T) has $x = u^0$ on $[0, T - \underline{T}] \subseteq [0, T]$ and (by definition of $\underline{T}$)

$$X^T_{T-\underline{T}} = \left(1 - e^{-r\underline{T}}\right)u^0 + e^{-r\underline{T}}u^\star = u^1.$$

Thus (x^T, X^T) provides flow payoff $F^0(u^0)$ before the breakthrough and $F^1(u^1)$ afterwards, which is the first-best. Any other deadline T' has $X^{T'}_{T-\underline{T}} \neq u^1$, so provides a strictly lower post-disclosure payoff and a no higher pre-disclosure payoff.

Case 2: infinite deadline. Fix an arbitrary finite deadline $T < \infty$; we must show that the mechanism (x^T, X^T) does not dominate (x^∞, X^∞). To that end, we shall identify a distribution G with $G(0) = 0$ under which the former mechanism has a strictly lower payoff. In particular, let G^t denote the point mass at $t \geq T$. Under this distribution, the payoff difference between the two mechanisms is

$$\begin{aligned}
&\pi_{G^t}(T) - \pi_{G^t}(\infty) \\
&\qquad = \left[\left(1 - e^{-rT}\right)F^0\left(u^0\right) + \left(e^{-rT} - e^{-rt}\right)F^0(u^\star) + e^{-rt}F^1(u^\star)\right] \\
&\qquad\quad - \left[\left(1 - e^{-rt}\right)F^0\left(u^0\right) + e^{-rt}F^1\left(u^0\right)\right] \\
&\qquad = e^{-rt}\left\{\left[F^1(u^\star) - F^1\left(u^0\right)\right] - \left[F^0(u^\star) - F^0\left(u^0\right)\right]\right\} \\
&\qquad\quad + e^{-rT}\left[F^0(u^\star) - F^0\left(u^0\right)\right].
\end{aligned}$$

The second term is strictly negative since F^0 is uniquely maximised at u^0 and $u^\star \leq u^1 < u^0$. By choosing $t \geq T$ large enough, we can make the first term as small as we wish, so that the payoff difference is strictly negative.

Case 3: deadline $\underline{T}$. We must show that (x^T, X^T) does not dominate $(x^{\underline{T}}, X^{\underline{T}})$ for any $T \neq \underline{T}$. We already showed this for $T < \underline{T}$.[53] For the

[52] More fully: if (x^T, X^T) were dominated, then by Corollary 6 in supplemental appendix K, it would be dominated by an undominated mechanism, which by Theorem 2 must be a deadline mechanism.

[53] We proved in the text (§5.2, p. 20) that $(x^{\underline{T}}, X^{\underline{T}})$ dominates (x^T, X^T) for any $T < \underline{T}$.

remainder, fix a deadline $T > \underline{T}$; we shall find a $t > 0$ such that

$$\pi_{G^t}(T) - \pi_{G^t}(\underline{T}) = e^{-rt}\Big[F^1\Big(X_t^T\Big) - F^1\Big(X_t^{\underline{T}}\Big)\Big] < 0,$$

where G^t is the point mass at t. Fix an

$$\varepsilon \in \Big(0, F^1\Big(u^1\Big) - F^1(X_0^T)\Big).$$

As $t \to 0$, by definition of $\underline{T}$, $X_t^{\underline{T}}$ and X_t^T converge monotonically to u^1 and to $X_0^T < u^1$, respectively. Since F^1 is continuous and strictly monotone on either side of its maximum u^1, we may choose $t > 0$ small enough that

$$F^1\Big(u^1\Big) - F^1\Big(X_t^{\underline{T}}\Big) < \varepsilon/2 \quad \text{and} \quad F^1\Big(X_t^T\Big) - F^1\Big(X_0^T\Big) < \varepsilon/2.$$

Summing these inequalities and rearranging yields

$$F^1\Big(X_t^T\Big) - F^1\Big(X_t^{\underline{T}}\Big) < \varepsilon - \Big[F^1\Big(u^1\Big) - F^1\Big(X_0^T\Big)\Big] < 0$$

by our choice of ε, and thus $\pi_{G^t}(T) - \pi_{G^t}(\underline{T}) < 0$. ■

D Generalisation and proof of Proposition 2 (p. 20)

In this appendix, we state and prove a general characterisation of optimal deadlines which entails Proposition 2. Write (x^T, X^T) for the deadline mechanism with deadline $T \in [0, \infty]$. We shall show that optimal deadlines are characterised by the first-order condition

$$-\int_{[0,T]} F^{1-}\Big(X_t^T\Big)G(\mathrm{d}t) \leq [1 - G(T)]\alpha \leq -\int_{[0,T]} F^{1+}\Big(X_t^T\Big)G(\mathrm{d}t), \qquad (\partial)$$

where F^{1-} (F^{1+}) is the left-hand (right-hand) derivative of F^1,[54] and

$$\alpha := \frac{F^0(u^0) - F^0(u^\star)}{u^0 - u^\star}.$$

Remark 4. If F^1 is differentiable on $(0, u^0)$, then (∂) reads

$$[1 - G(T)]\alpha + \int_{[0,T]} F^{1\prime}(X_t)G(\mathrm{d}t) = 0.$$

If in addition F^0 is affine on $[0, u^0]$ and $u^\star$ strictly exceeds zero, then

$$\alpha = F^{0\prime}(u^\star) = F^{1\prime}(u^\star) = F^{1\prime}(X_t) \quad \text{for any } t \geq T,$$

[54]These are well-defined since F^1 is concave.

and thus the first-order condition may be written as

$$\int_{(T,\infty)} F^{1\prime}(X_t)G(\mathrm{d}t) + \int_{[0,T]} F^{1\prime}(X_t)G(\mathrm{d}t) = 0,$$

which is equivalent to $\mathbf{E}_G\big(F^{1\prime}(X_\tau)\big) = 0$, the version given in Proposition 2.

We begin by showing that (∂) is necessary for optimality. This result does not rely on affineness of F^0, so delivers the best deadline mechanisms even when it is not exactly optimal to use a deadline mechanism.

Lemma 3. Among deadline mechanisms, the best for distribution G satisfy (∂). Any such deadline is finite.

The proof relies on the following observation.

Observation 3. For any distribution G, define $\pi_G : [0,\infty] \to \mathbf{R}$ by

$$\pi_G(T) \coloneqq \int_{\mathbf{R}_+} \Bigg[r \int_0^{\min\{t,T\}} e^{-rs} F^0\Big(u^0\Big)\mathrm{d}s + r\int_{\min\{t,T\}}^t e^{-rs} F^0(u^\star)\mathrm{d}s \\ + e^{-rt}F^1\Big(X_t^T\Big)\Bigg] G(\mathrm{d}t).$$

This is the principal's payoff under G from deadline T. Since

$$\frac{\mathrm{d}}{\mathrm{d}T} X_t^T = \begin{cases} re^{-r(T-t)}\big(u^0 - u^\star\big) & \text{for } t \leq T \\ 0 & \text{for } t > T, \end{cases}$$

its right- and left-hand derivatives are

$$\pi_G^+(T) = K\left([1-G(T)]\alpha + \int_{[0,T]} F^{1+}\Big(X_t^T\Big)G(\mathrm{d}t)\right) \qquad \text{for } T \in [0,\infty)$$

$$\pi_G^-(T) = -K\left([1-G(T)]\alpha + \int_{[0,T]} F^{1-}\Big(X_t^T\Big)G(\mathrm{d}t)\right) \qquad \text{for } T \in (0,\infty)$$

where $K > 0$ is a constant.

Proof of Lemma 3. $\pi_G^+(T) \leq 0$ is necessary for $T \in [0,\infty)$ to be optimal, and this rules out $T = 0$ since $\pi_G^+(0) > 0$. Given $T > 0$, $\pi_G^-(T) \leq 0$ is also necessary for $T \in (0,\infty)$ to be optimal. Finally, observe that $\pi_G^+(T) < 0$ for all sufficiently large T since

$$\int_{[0,T]} F^{1+}\Big(X_t^T\Big)G(\mathrm{d}t) \to F^{1+}\Big(u^0\Big) < 0 \quad \text{as } T \to \infty.$$

Thus $T = \infty$ violates (∂), and therefore cannot be optimal since

$$\pi_G(\infty) = \lim_{T\to\infty} \pi_G(T). \qquad \blacksquare$$

Remark 5. Not every undominated deadline mechanism can be optimal. In particular, deadlines $\underline{T}$ and ∞ violate the first-order condition (∂) for any distribution G with $G(0) = 0$, so are never optimal by Lemma 3. By contrast, the proof of Proposition 1 in appendix C shows that every deadline $T \in (\underline{T}, \infty)$ is optimal for some distribution G with $G(0) = 0$.

When F^0 is affine, (∂) is both necessary and sufficient for optimality:

Proposition 2′. If the old frontier F^0 is affine on $[0, u^0]$, then a mechanism is optimal for G iff it is a deadline mechanism with deadline satisfying (∂).

In light of Remark 4, this result immediately implies Proposition 2.

Proof. All optimal mechanisms are deadline mechanisms by Theorem 2 (p. 17), and satisfy (∂) by Lemma 3. For the converse, consider a deadline mechanism (x^T, X^T) that satisfies (∂). Then $T > \underline{T}$ by Remark 5, so that (x^T, X^T) is undominated by Proposition 1 (p. 20). It remains to show that (x^T, X^T) is optimal for G. It suffices to demonstrate that T maximises π_G.[55]

It suffices to prove that π_G is concave, or equivalently that π_G^+ is decreasing. To that end, take $T < T'$, and compute

$$\begin{aligned} \frac{\pi_G^+(T') - \pi_G^+(T)}{K} &= [-G(T') - G(T)]\alpha + \int_{(T,T']} F^{1+}\left(X_t^{T'}\right) G(\mathrm{d}t) \\ &\quad + \int_{[0,T]} \left[F^{1+}\left(X_t^{T'}\right) - F^{1+}\left(X_t^{T}\right)\right] G(\mathrm{d}t) \\ &= \int_{(T,T']} \left[F^{1+}\left(X_t^{T'}\right) - \alpha\right] G(\mathrm{d}t) + \int_{[0,T]} \left[F^{1+}\left(X_t^{T'}\right) - F^{1+}\left(X_t^{T}\right)\right] G(\mathrm{d}t). \end{aligned}$$

The first term is non-positive since $F^{1+} \leq \alpha$ on $[u^\star, u^0] \ni X^{T'}$, and the second is non-positive since F^{1+} is decreasing and $X^{T'} \geq X^T$. $\blacksquare$

Corollary 1 (comparative statics)**.** If G first-order stochastically dominates $G^\dagger$, then $T \geq T^\dagger$ for some deadline T ($T^\dagger$) optimal for G (for $G^\dagger$).

[55] (x^T, X^T) is then better under G than any *non*-deadline mechanism (x, X), since any such is dominated by some deadline mechanism $\left(x^{T^\dagger}, X^{T^\dagger}\right)$ by Corollary 6 in supplemental appendix K (p. 71), so that $\Pi_G\left(x^T, X^T\right) \geq \Pi_G\left(x^{T^\dagger}, X^{T^\dagger}\right) \geq \Pi_G(x, X)$.

Proof. By Topkis's theorem,[56] it suffices to show that $\pi_G^+ \geq \pi_{G^\dagger}^+$ ('increasing differences'). And indeed, we have for any $T \in \mathbf{R}_+$ that

$$\begin{aligned}\frac{\pi_G^+(T)}{K} &= \mathbf{E}_G\left(\mathbf{1}_{[0,T]}(\tau) \times F^{1+}\left(X_\tau^T\right) + \mathbf{1}_{(T,\infty)}(\tau) \times \alpha\right) \\ &\geq \mathbf{E}_{G^\dagger}\left(\mathbf{1}_{[0,T]}(\tau) \times F^{1+}\left(X_\tau^T\right) + \mathbf{1}_{(T,\infty)}(\tau) \times \alpha\right) = \frac{\pi_{G^\dagger}^+(T)}{K},\end{aligned}$$

where the equalities hold by Observation 3, and the inequality holds because G first-order stochastically dominates $G^\dagger$ and the map

$$t \mapsto \mathbf{1}_{[0,T]}(t) \times F^{1+}\left(X_t^T\right) + \mathbf{1}_{(T,\infty)}(t) \times \alpha$$

is increasing since F^{1+} and X^T are decreasing and we have $F^{1+} \leq \alpha$ on $\left[u^\star, u^0\right] \ni X^T$. ∎

E A superdifferential Euler equation

In this appendix, we define a superdifferential Euler equation for the principal's problem, and give conditions under which it is necessary and sufficient for a mechanism to be optimal. We shall use this result in the next two appendices to prove Theorem 3 and Proposition 3 (pages 21 and 22).

Definition 5. A mechanism (x, X) *satisfies the Euler equation* iff there are measurable $\phi^0, \phi^1 : \mathbf{R}_+ \to (-\infty, \infty]$ such that for every $t \in \mathbf{R}_+$, $\phi^0(t)$ is a supergradient of F^0 at x_t,[57] $\phi^1(t)$ is a supergradient of F^1 at X_t, and

$$[1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G \begin{cases} \geq 0 & \text{if } x_t > 0 \\ \leq 0 & \text{if } x_t < u^0. \end{cases} \tag{E}$$

Euler lemma. Let (x, X) be an undominated mechanism, and let G be a distribution with $G(0) = 0$. If the right-derivatives of F^0, F^1 at $u = 0$ are finite, then (x, X) is optimal for G if it satisfies the Euler equation. If in addition F^0, F^1 are differentiable on $(0, u^0)$, then (x, X) is optimal for G only if it satisfies the Euler equation.

We give the (reasonably standard) proof in supplemental appendix N.

[56] See e.g. Theorem 2.8.1 in Topkis (1998, p. 76).
[57] See Rockafellar (1970, part V) for definitions.

F Strengthening and proof of Theorem 3 (p. 21)

In this appendix, we refine and prove Theorem 3. In particular, we show (a) that if a mechanism is undominated, then its pre-disclosure flow must be decreasing, and (b) that if in addition the mechanism is optimal for some distribution G with $G(0) = 0$ (and unbounded support), then the pre-disclosure flow must start at u^0 (and converge to $u^\star$).

Theorem 3(a). Suppose that F^0 is strictly concave on $\left[0, u^0\right]$ and that F^0, F^1 possess uniformly continuous derivatives on $\left(0, u^0\right)$. Then any undominated mechanism (x, X) has $X_0 \geq u^1$ and $X \geq u^\star$, and (a version of) x is decreasing.

Theorem 3(b). Suppose that F^0 is strictly concave on $\left[0, u^0\right]$ and that F^0, F^1 possess uniformly continuous derivatives on $\left(0, u^0\right)$, and let (x, X) be optimal for some distribution G with $G(0) = 0$. Then (a version of) x satisfies $\lim_{t \to 0} x_t = u^0$, and if G has unbounded support, then also $\lim_{t \to \infty} x_t = u^\star$.

We prove Theorem 3(b) in the next section, relying on Theorem 3(a) and on the Euler lemma from the preceding appendix. We then prove Theorem 3(a) in several steps: we show in §F.2 that $X_0 \geq u^1$, obtain some necessary lemmata in §F.3–§F.5, then prove in §F.6 that $X \geq u^\star$ and in §F.7 that x is decreasing.

In light of Lemma 1 and Theorem 1 (pages 13 and 14), we need only consider mechanisms of the form (x, X) with $x \leq u^0$ a.e. Such a mechanism may be equated with its pre-disclosure flow x. Throughout, we shall write

$$\Pi_t(x) \coloneqq r \int_0^t e^{-rs} F^0(x_s) \mathrm{d}s + e^{-rt} F^1(X_t)$$

for the principal's payoff from such a mechanism if the breakthrough occurs at $t \in \mathbf{R}_+$.

F.1 Proof of Theorem 3(b)

Assume that F^0, F^1 satisfy the hypothesis, and let (x, X) be optimal for a distribution G with $G(0) = 0$. Then (x, X) is undominated, so $X \geq u^\star$ and x is decreasing by Theorem 3(a). It follows that $u^\star \leq X \leq x \leq u^0$, where the last inequality holds by Lemma 1. The limits

$$\bar{u} \coloneqq \lim_{t \to 0} x_t \quad \text{and} \quad \underline{u} \coloneqq \lim_{t \to \infty} x_t$$

exist since x is monotone and bounded.

By the Euler lemma in appendix E (p. 41), (x, X) must satisfy the Euler equation (E). Let $\phi^0, \phi^1 : \mathbf{R}_+ \to [0, \infty]$ be supergradients for which (E) (p. 41) holds. Write F^{j-} (F^{j+}) denote the left-hand (right-hand) derivative of F^j for $j \in \{0, 1\}$.

To show that $\bar{u} = u^0$, suppose toward a contradiction that $\bar{u} < u^0$. Then for $t > 0$ sufficiently small, we have $x_t < u^0$, and thus

$$\begin{aligned} 0 &\geq [1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G && \text{by (E)} \\ &\geq [1 - G(t)]F^{0+}(x_t) + \int_{[0,t]} \phi^1 \mathrm{d}G && \text{by definition of } \phi^0. \end{aligned} \tag{7}$$

Since F^{0+} and x are decreasing, $t \mapsto F^{0+}(x_t)$ is convergent with limit

$$\lim_{t \to 0} F^{0+}(x_t) \geq F^{0+}(\bar{u}).$$

Thus taking the limit as $t \to 0$ on the right-hand side of (7) yields $0 \geq F^{0+}(\bar{u})$, which contradicts the supposition that $\bar{u} < u^0$.

To show that $\underline{u} = u^\star$, assume that G has unbounded support, and suppose toward a contradiction that $\underline{u} > u^\star$. Since x is decreasing, we have $x_t > u^\star \geq 0$ for every $t \in \mathbf{R}_+$. It must be that $x_T < u^0$ for some $T \in \mathbf{R}_+$, because otherwise (E) would fail for t large enough. For every $t \geq T$, we have $0 < x_t < u^0$ since x is decreasing, and thus

$$[1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G = 0. \tag{8}$$

Since ϕ^0 is bounded (below by $F^{0+}(u^0)$, and above by $F^{0+}(0)$), taking the limit as $t \to \infty$ yields

$$\int_{[0,\infty)} \phi^1 \mathrm{d}G = 0.$$

Subtracting this from (8) and dividing by $1 - G(t)$ (strictly positive since G has unbounded support) yields

$$0 = \phi^0(t) - \mathbf{E}_G\left(\phi^1(t) \middle| t > t\right) \geq F^{0+}(x_t) - F^{1-}(\underline{u}),$$

where the inequality holds since $X \geq \underline{u} > 0$. Since F^{0+} is right-continuous,[58] and x is decreasing, taking the limit as $t \to \infty$ yields $F^{0+}(\underline{u}) \leq F^{1-}(\underline{u})$, which contradicts the supposition that $\underline{u} > u^\star$. ■

[58] See Theorem 24.1 in Rockafellar (1970, p. 227).

F.2 Proof of Theorem 3(a): $X_0 \geq u^1$

Let F^0, F^1 satisfy the hypotheses of Theorem 3(a), and fix a mechanism (x, X) with $X_0 < u^1$; we shall construct a mechanism that dominates it. Let

$$t' := \sup\Big\{t \in \mathbf{R}_+ : x = u^0 \text{ a.e. on } [0,t]\Big\},$$

and observe that it is finite since $X_0 < u^1 < u^0$. Given $T > t'$, define

$$A := \Big\{t \in [t', T] : x_t < u^0\Big\},$$

and note that it is non-null by definition of t'. For $\varepsilon \in (0,1)$, consider the mechanism $x^{T,\varepsilon}$ given by

$$x_t^{T,\varepsilon} := \begin{cases} (1-\varepsilon)x_t + \varepsilon u^0 & \text{for } t \in A \\ x_t & \text{for } t \notin A. \end{cases}$$

We shall show that this mechanism dominates x for some choice of T and ε.

Since $X^{T,\varepsilon} = X$ on $[T, \infty)$, it holds for any $t \in \mathbf{R}_+$ that

$$\begin{aligned} \Pi_t\Big(x^{T,\varepsilon}\Big) - \Pi_t(x) = &\int_{A \cap (0,t)} r e^{-rs} \Big[F^0\Big(x_s^{T,\varepsilon}\Big) - F^0(x_s)\Big] \mathrm{d}s \\ &+ \mathbf{1}_{[0,T)}(t) e^{-rt} \Big[F^1\Big(X_t^{T,\varepsilon}\Big) - F^1(X_t)\Big]. \end{aligned}$$

The integrand in the first term is non-negative and strictly positive on A since $x \leq (<)\, x^{T,\varepsilon} < u^0$ (on A) and F^0 is strictly increasing on $[0, u^0]$. Since A is non-null, it follows that the first term is non-negative for every $t \in \mathbf{R}_+$, and strictly positive for $t \geq T$. It remains only to show that T and ε may be chosen so that the second term is non-negative.

Since $X_0 < u^1$, X is decreasing on $[0, t']$, so that $X_{t'} < u^1$ in particular. Since X is continuous, $T > t'$ may be chosen so that X is below and bounded away from u^1 on $[0, T]$. Given this T, we may choose $\varepsilon \in (0,1)$ small enough that the same holds for $X^{T,\varepsilon}$. We then have $X \leq X^{T,\varepsilon} < u^1$ on $[0, T]$, which since F^1 is increasing on $[0, u^1]$ implies that $F^1(X_t) \leq F^1(X_t^{T,\varepsilon})$ for every $t \leq T$, as desired. ■

F.3 Equivalence of local and global monotonicity

For the proof of the remainder of Theorem 3(a), we shall rely an equivalence between monotonicity of a version of x and a notion of local monotonicity.

Given a mechanism x and a non-null set $A \subseteq \mathbf{R}_+$ of times, recall that the *essential infimum and supremum* are defined by

$$\operatorname*{ess\,inf}_A x := \sup\{b \in \mathbf{R} : x_t \geq b \text{ a.e. on } A\} \quad \text{and}$$
$$\operatorname*{ess\,sup}_A x := \inf\{b \in \mathbf{R} : x_t \leq b \text{ a.e. on } A\}.$$

Definition 6. For a mechanism x and a period $t \in (0, \infty)$, we write

$$\inf x_{t-} := \lim_{t' \uparrow t} \operatorname*{ess\,inf}_{(t',t)} x, \qquad \inf x_{t+} := \lim_{t' \downarrow t} \operatorname*{ess\,inf}_{(t,t')} x$$
$$\sup x_{t-} := \lim_{t' \uparrow t} \operatorname*{ess\,sup}_{(t',t)} x, \quad \text{and} \quad \sup x_{t+} := \lim_{t' \downarrow t} \operatorname*{ess\,sup}_{(t,t')} x.$$

The following characterises monotonicity in terms of local monotonicity.

Monotonicity lemma. Let $x : \mathbf{R}_+ \to [0, u^0]$ be measurable. The following are equivalent:

(I) Some version of x is decreasing.

(II) For all $t \in (0, \infty)$, there are $t' < t < t''$ such that $\operatorname{ess\,inf}_{(t',t)} x \geq \operatorname{ess\,sup}_{(t,t'')} x$.

Furthermore, following are equivalent:

(I′) Some version of x is increasing.

(II′) For all $t \in (0, \infty)$, there are $t' < t < t''$ such that $\operatorname{ess\,sup}_{(t',t)} x \leq \operatorname{ess\,inf}_{(t,t'')} x$.

To prove the monotonicity lemma, we rely on an observation and a corollary thereof. The corollary will also be used directly in the next section.

Observation 4. If a measurable set $A \subseteq \mathbf{R}_+$ is such that for a.e. $t \in A$, either $A \cap (t, t + \delta)$ or $A \cap (t - \delta, t)$ is null for some $\delta > 0$, then A is itself null.

Proof. Suppose that A satisfies the hypothesis; we shall show that it is null. Write λ for the Lebesgue measure, and define $\psi : [0, \infty) \to \mathbf{R}$ by

$$\psi(t) := \lambda(A \cap (0, t)) = \int_0^t \mathbf{1}_A \mathrm{d}\lambda.$$

By Lebesgue's fundamental theorem of calculus, ψ is differentiable a.e. with derivative $\psi' = \mathbf{1}_A$ a.e. By the property of A, it must be that $\psi' = 0$ a.e. Thus $\mathbf{1}_A = 0$ a.e., which is to say that A is null. ■

Corollary 2. For all $0 \le t' < t'' \le \infty$ and $\varepsilon > 0$, there are $T, T' \in (t', t'')$ such that

$$\max\{\inf x_{T-}, \inf x_{T+}\} \le \operatorname*{ess\,inf}_{(t',t'')} x + \varepsilon \quad \text{and}$$
$$\min\{\sup x_{T'-}, \sup x_{T'+}\} \ge \operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon.$$

Proof. Fix $0 \le t' < t'' \le \infty$ and $\varepsilon > 0$; we shall construct T, omitting the analogous construction of T'. Observe that

$$A \coloneqq \left\{ s \in (t', t'') : x_s \le \operatorname*{ess\,inf}_{(t',t'')} x + \varepsilon \right\}$$

is non-null. Thus by Observation 4, there is a period $T \in (t', t'')$ such that $(T - \delta, T) \cap A$ and $(T, T + \delta) \cap A$ are non-null for every $\delta > 0$. Therefore

$$\max\left\{ \operatorname*{ess\,inf}_{(T-\delta,T)} x, \operatorname*{ess\,inf}_{(T,T+\delta)} x \right\} \le \operatorname*{ess\,inf}_{(t',t'')} x + \varepsilon,$$

which upon letting $\delta \downarrow 0$ yields the desired result. ■

Proof of the monotonicity lemma. It suffices to prove the equivalence of (I) and (II), since x satisfies (I′) (resp. (II′)) exactly if $u^0 - x$ satisfies (I) (resp. (II)). It is immediate that (I) implies (II). To establish the converse, consider a third property:

(III) $\operatorname{ess\,inf}_{(0,t)} x \ge \operatorname{ess\,sup}_{(t,\infty)} x$ for all $t \in (0, \infty)$.

We shall show first that (III) implies (I), and then that (II) implies (III).

(III) implies (I). Suppose that x satisfies (III). Define $x^- : (0, t) \to \mathbf{R}$ by

$$x^-_t \coloneqq \begin{cases} u^0 & \text{if } t = 0 \\ \operatorname{ess\,inf}_{(0,t)} x & \text{if } t > 0. \end{cases}$$

Clearly x^- is decreasing; we will show that it is a version of x. Let $x^+ : (0, t) \to \mathbf{R}$ be given by $x^+_t \coloneqq \operatorname{ess\,sup}_{(t,\infty)} x$. Since (III) holds, we have $x^- \ge x^+$. It thus suffices to show that the sets

$$A \coloneqq \left\{ t > 0 : x^-_t > x^+_t \right\},$$
$$B \coloneqq \left\{ t > 0 : x_t > x^-_t \right\}, \quad \text{and}$$
$$C \coloneqq \left\{ t > 0 : x_t < x^+_t \right\}$$

are null.

We begin with A. Fix $t \in A$, and note that for all $t' > t$, we have

$$x^-_{t'} \leq \operatorname*{ess\,inf}_{(t,t')} x \leq \operatorname*{ess\,sup}_{(t,t')} x \leq x^+_t.$$

Since $x^+_t < x^-_t$, it follows that x^- is discontinuous at t. Because x^- is decreasing, it follows that A is at most countable, hence null.

We next show that B is null; we omit the similar argument for C. Fix an $\varepsilon > 0$; we shall prove that

$$B_\varepsilon \coloneqq \left\{ t > 0 : x_t > x^-_t + \varepsilon \right\}$$

is null. This is sufficient as $B_{1/k} \subseteq B_{1/(k+1)}$ for $k \in \mathbf{N}$ and $B = \bigcup_{k \in \mathbf{N}} B_{1/k}$, so that $\lambda(B) = \lim_{k \to \infty} \lambda(B_{1/k})$ by continuity of measures.

We shall show that for every continuity point t of x^-, there is a $\delta > 0$ such that $B_\varepsilon \cap (t, t+\delta)$ is null. Since a.e. $t \in \mathbf{R}_+$ is a continuity point of x^- (its discontinuities being at most countable since it is decreasing), this yields that B_ε is null by Observation 4.

So fix a $t > 0$ at which x^- is continuous. Then there is a $\delta > 0$ such that $x^-_{t'} \geq x^-_t - \varepsilon/2$ for all $t' \in [t, t+\delta]$. Thus for $t' \in B_\varepsilon \cap (t, t+\delta)$, we have

$$x_{t'} > \ x^-_{t'} + \varepsilon \geq x^-_t - \varepsilon/2 + \varepsilon = x^-_t + \varepsilon/2.$$

It must be that $B_\varepsilon \cap (t, t+\delta)$ is null, since otherwise

$$x^-_t < \operatorname*{ess\,sup}_{B_\varepsilon \cap (t, t+\delta]} x \leq x^+_t,$$

a contradiction with our hypothesis (III).

(II) implies (III). Suppose that x satisfies (II). Fix $t \in (0, \infty)$, and let $t' < t < t''$ satisfy (II). Define

$$T \coloneqq \inf \left\{ s' \in \mathbf{R}_+ : \operatorname*{ess\,inf}_{(s',t)} x \geq \operatorname*{ess\,sup}_{(t,t'')} x \right\}.$$

Note that $T \leq t' < t$, and further that

$$\operatorname*{ess\,inf}_{(T,t)} x \geq \operatorname*{ess\,sup}_{(t,t'')} x. \tag{9}$$

We shall prove that $T = 0$; a similar argument may be used to show that

$$\sup \left\{ s'' \in \mathbf{R}_+ : \operatorname*{ess\,inf}_{(0,t)} x \geq \operatorname*{ess\,sup}_{(t,s'')} x \right\} = \infty.$$

Suppose toward a contradiction that $T > 0$. Since (II) holds, there are $s' < T < s''$ such that $\operatorname{ess\,inf}_{(s',T)} x \geq \operatorname{ess\,sup}_{(T,s'')} x$. Then

$$\operatorname*{ess\,inf}_{(s',T)} x \geq \operatorname*{ess\,sup}_{(T,s'')} x \geq \operatorname*{ess\,sup}_{(T,\min\{s'',t\})} x \geq \operatorname*{ess\,inf}_{(T,\min\{s'',t\})} x \geq \operatorname*{ess\,inf}_{(T,t)},$$

and thus

$$\operatorname*{ess\,inf}_{(T,t)} x = \min\left\{\operatorname*{ess\,inf}_{(s',T)} x, \operatorname*{ess\,inf}_{(T,t)} x\right\} = \operatorname*{ess\,inf}_{(s',t)} x.$$

But then $\operatorname{ess\,inf}_{(s',t)} x \geq \operatorname{ess\,sup}_{(t,t'')} x$ by (9), which by definition of T implies that $T \leq s'$—a contradiction. ■

F.4 Local non-decrease and non-increase

In this section, we introduce a local notion of 'does not decrease/increase' and deduce a variety of results that will be used to prove the remainder of Theorem 3(a): Observations 5 and 6, Corollary 3, Lemma 4 and Corollary 4.

Definition 7. Given a measurable $x : \mathbf{R}_+ \to [0, u^0]$, we say that x *does not decrease* at some $t \in (0, \infty)$ iff $\operatorname{ess\,inf}_{(t',t)} x < \operatorname{ess\,sup}_{(t,t'')} x$ for all $t' < t < t''$. Similarly, x *does not increase* at t iff $\operatorname{ess\,sup}_{(t',t)} x > \operatorname{ess\,inf}_{(t,t'')} x$ for all $t' < t < t''$.

Observation 5. Let $0 \leq t' < t'' \leq \infty$. If $\operatorname{ess\,sup}_{(t',t'')} x \leq X_{t'}$, then X is increasing on $[t', t'')$. Similarly, if $\operatorname{ess\,inf}_{(t',t'')} x \geq X_{t'}$, then X is decreasing on $[t', t'')$.

Proof. We prove the first part; the second is analogous. Suppose that $\operatorname{ess\,sup}_{(t',t'')} x \leq X_{t'}$, and fix any $T < T'$ in $[t', t'')$; we must show that $X_T \leq X_{T'}$. Note first that

$$\begin{aligned} X_{t'} &= \int_{t'}^{T'} r e^{-r(s-t')} x_s \mathrm{d}s + e^{-r(T'-t')} X_{T'} \\ &\leq \left(1 - e^{-r(T'-t')}\right) X_{t'} + e^{-r(T'-t')} X_{T'}, \end{aligned}$$

which is to say that $X_{t'} \leq X_{T'}$. It follows that

$$\begin{aligned} X_T &= \int_{T}^{T'} r e^{-r(s-T)} x_s \mathrm{d}s + e^{-r(T'-T)} X_{T'} \\ &\leq \left(1 - e^{-r(T'-T)}\right) X_{t'} + e^{-r(T'-T)} X_{T'} \\ &\leq X_{T'}. \end{aligned}$$

■

Observation 6. Let $t' > 0$. If $\operatorname{ess\,inf}_{(t,t')} x < \inf x_{t'-}$ for all $t < t'$, then for all $t'' < t'$, there is a $T \in (t'', t')$ such that x does not decrease at T and $\inf x_{T-} \leq \operatorname{ess\,inf}_{(T,t')} x$.

Proof. Fix $t'' < t'$, and note that $\operatorname{ess\,inf}_{(t'',t')} x < \inf x_{t'-}$. Clearly there is a $T' \in (t'', t')$ such that $\operatorname{ess\,inf}_{(t'',t')} x < \operatorname{ess\,inf}_{(T',t')} x$. Note that the map $t \mapsto \operatorname{ess\,inf}_{(t,t')} x$ is right-continuous on $(0, t')$. Let

$$T \coloneqq \min\left\{ t \in [t'', T'] : \operatorname*{ess\,inf}_{(t,t')} x = \operatorname*{ess\,inf}_{(T',t')} x \right\}.$$

Then $t'' < T \leq T' < t'$, and $\operatorname{ess\,inf}_{(t,t')} x < \operatorname{ess\,inf}_{(T,t')} x$ for all $t < T$. As

$$\operatorname*{ess\,inf}_{(t,t')} x = \min\left\{ \operatorname*{ess\,inf}_{(t,T)} x, \operatorname*{ess\,inf}_{(T,t')} x \right\} \quad \text{for any } t < T,$$

we have

$$\operatorname*{ess\,inf}_{(t,T)} x = \operatorname*{ess\,inf}_{(t,t')} x < \operatorname*{ess\,inf}_{(T,t')} x \quad \text{for any } t < T.$$

Letting $t \uparrow T$ yields $\inf x_{T-} \leq \operatorname{ess\,inf}_{(T,t')} x$. Further, for all $t < T < s$,

$$\operatorname*{ess\,inf}_{(t,T)} x < \operatorname*{ess\,inf}_{(T,t')} x \leq \operatorname*{ess\,inf}_{(T,\min\{t',s\})} x \leq \operatorname*{ess\,sup}_{(T,\min\{t',s\})} x \leq \operatorname*{ess\,sup}_{(T,s)} x$$

so that x does not decrease at T. ■

To prove the next two results (Corollary 3 and Lemma 4), we rely on the following observation.

Observation 7. Let $t > 0$. If

$$t' \coloneqq \sup\{s \geq t : \text{some version of } x \text{ is increasing (decreasing) on } [t, s]\}$$

is finite, then for any $\varepsilon > 0$, there there is a $T \in [t', t' + \varepsilon)$ such that x does not increase (decrease) at T and

$$\sup x_{T-} \geq \min\{\sup x_{t-}, \sup x_{t+}\} \quad \Big(\inf x_{T-} \leq \max\{\inf x_{t-}, \inf x_{t+}\}\Big).$$

Proof. We prove the first part; the argument for the second is analogous. Fix a $t > 0$, and suppose that

$$t' \coloneqq \sup\{s \geq t : \text{some version of } x \text{ is increasing on } [t, s]\} < \infty.$$

Further fix an $\varepsilon > 0$. Note that

$$\sup x_{t'-} \geq \min\{\sup x_{t-}, \sup x_{t+}\},$$

since if $t' > t$ then $\sup x_{t'-} \geq \sup x_{t+}$ because some version of x is decreasing on $[t, t']$, and if $t' = t$ then $\sup x_{t'-} = \sup x_{t-}$. It therefore suffices to find a $T \in [t', t' + \varepsilon)$ such that x does not increase at T and $\sup x_{T-} \geq \sup x_{t'-}$.

If x does not increase at t', then let $T := t'$. Otherwise, there is an $s > t'$ such that $\sup x_{t'-} \leq \operatorname{ess\,inf}_{(t',s)} x$. By the definition of t' and the monotonicity lemma (in particular, the fact that (II′) implies (I′)), there must be a $T \in (t', \min\{s, t' + \varepsilon\})$ at which x does not increase. Then

$$\sup x_{T-} \geq \inf x_{T-} \geq \operatorname{ess\,inf} x_{(t',s)} \geq \sup x_{t'-},$$

where the middle inequality holds since $T \in (t', s)$. ■

Corollary 3. Let $t' \in \mathbf{R}_+$ be such that $\sup x_{t'+} > \inf x_{t'+}$. Then for any $\varepsilon > 0$, there is a $T \in [t', t' + \varepsilon)$ such that x does not decrease at T and $\inf x_{T-} \leq \inf x_{t'+} + \varepsilon$.

Proof. Fix $\varepsilon > 0$. Note that no version of x is decreasing on $(t', t' + \varepsilon)$, since this would imply that $\sup x_{t'+} = \inf x_{t'+}$. Thus by the monotonicity lemma (p. 45), there is a $t'' \in (t', t' + \varepsilon)$ at which x does not decrease. Moreover, by Corollary 2 (p. 46), there is a $T' \in (t', t'')$ such that

$$\max\{\inf x_{T'-}, \inf x_{T'+}\} \leq \operatorname*{ess\,inf}_{(t',t'')} x + \varepsilon \leq \inf x_{t'+} + \varepsilon.$$

Since x does not decrease at t'', the monotonicity lemma yields that

$$\sup\{s \geq T' : \text{some version of } x \text{ is decreasing on } [T', s]\} \leq t''.$$

Thus by Observation 7, there is a $T \in [T', t'' + \varepsilon)$ such that x does not decrease at T and

$$\inf x_{T-} \leq \max\{\inf x_{T'-}, \inf x_{T'+}\} \leq \inf x_{t'+} + \varepsilon. \qquad ■$$

Lemma 4. Let $t' > 0$. If $\operatorname{ess\,sup}_{(t',t)} x > X_{t'}$ for all $t > t'$, then for every $\varepsilon > 0$, there is a $T > 0$ such that x does not increase at T,

$$\sup x_{T-} \geq \min\{\sup x_{t'-}, \sup x_{t'+}\}, \quad \text{and} \tag{10}$$
$$X_{t'} + \varepsilon \geq X_T. \tag{11}$$

Similarly, if $\operatorname{ess\,inf}_{(t',t)} x < X_{t'}$ for all $t > t'$, then for every $\varepsilon > 0$, there is a $T > 0$ such that x does not decrease at T,

$$\inf x_{T-} \leq \max\{\inf x_{t'-}, \inf x_{t'+}\}, \quad \text{and}$$
$$X_{t'} - \varepsilon \leq X_T.$$

Proof. We prove the first part; the second is analogous. Suppose that $\operatorname{ess\,sup}_{(t',t)} x > X_{t'}$ for all $t > t'$, and an fix $\varepsilon > 0$; we seek a $T > 0$ such that x does not increase at T and (10) and (11) hold.

Let

$$T' := \sup\{s \geq t' : \text{some version of } x \text{ is increasing on } [t', s]\}.$$

We claim that X is decreasing on (t', T'). If $T' = t'$, there is nothing to prove; so assume that $T' > t'$. Then

$$\operatorname*{ess\,inf}_{(t',T')} x = \sup x_{t'+} \geq X_{t'},$$

where the inequality holds by hypothesis. It follows by Observation 5 that X is decreasing on (t', T').

We claim that T' is finite. Suppose toward a contradiction that $T' = \infty$, so that X is decreasing on $[t', \infty)$. Then for $t > t'$, we have

$$\lim_{s \to \infty} x_s \geq \operatorname*{ess\,sup}_{(t',t)} x > X_{t'} \geq \lim_{s \to \infty} X_s,$$

which is impossible since X must have the same limit as x is convergent.

Since X is decreasing on (t', T'), we have $X_{t'} \geq X_{T'}$, which since X is continuous implies that (11) holds for every $T \geq T'$ sufficiently close to T'. Because $T' < \infty$, Observation 7 ensures that T may be chosen so that x does not increase at T and (10) holds. ■

Corollary 4. Let $0 \leq t' < t'' \leq \infty$. If $\operatorname{ess\,sup}_{(t',t'')} x > \sup_{(t',t'')} X$, then for every $\varepsilon > 0$, there is a $T \in (0, \infty)$ such that x does not increase at T,

$$\sup x_{T-} \geq \operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon, \quad \text{and} \tag{12}$$

$$\sup_{(t',t'')} X + \varepsilon \geq X_T. \tag{13}$$

Similarly, if $\operatorname{ess\,inf}_{(t',t'')} x < \inf_{(t',t'')} X$, then for every $\varepsilon > 0$, there is a $T \in (0, \infty)$ such that x does not decrease at T,

$$\inf x_{T-} \leq \operatorname*{ess\,inf}_{(t',t'')} x + \varepsilon, \quad \text{and} \tag{14}$$

$$\inf_{(t',t'')} X - \varepsilon \leq X_T. \tag{15}$$

Proof. We prove the first part; the second is analogous. Suppose that $\operatorname{ess\,sup}_{(t',t'')} x > \sup_{(t',t'')} X$. Fix $\varepsilon > 0$, and assume without loss of generality that

$$\operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon > \sup_{(t',t'')} X.$$

From Corollary 2 (p. 46), there is a $T' \in (t', t'')$ such that

$$\min\{\sup x_{T'-}, \sup x_{T'+}\} \geq \operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon.$$

Thus for every $s > T'$, we have

$$\operatorname*{ess\,sup}_{(T',s)} x \geq \sup x_{T'+} \geq \operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon > \sup_{(t',t'')} X \geq X_{T'},$$

where the final inequality holds since $T' \in (t', t'')$. Then Lemma 4 implies the existence of a $T \in (0, \infty)$ such that x does not increase at T,

$$\sup x_{T-} \geq \min\{\sup x_{T'-}, \sup x_{T'+}\} \geq \operatorname*{ess\,sup}_{(t',t'')} x - \varepsilon, \quad \text{and}$$
$$X_T \leq X_{T'} + \varepsilon \leq \sup_{(t',t'')} X + \varepsilon. \qquad \blacksquare$$

F.5 Local mean-preserving contractions

In this section, we provide sufficient conditions for a *local mean-preserving contraction* of the pre-disclosure flow x to improve the principal's payoff under any breakthrough distribution. This modification of a mechanism will be used in proving the remaining parts of Theorem 3(a).

Local contraction lemma. Let (x, X) be a mechanism such that $X_0 \geq u^1$ and $x \leq u^0$. Given a measurable and non-null $A \subseteq \mathbf{R}_+$, define

$$\overline{x}^A \coloneqq \frac{\int_A re^{-rs} x_s \mathrm{d}s}{\int_A re^{-rs} \mathrm{d}s} \in \mathbf{R}_+.$$

Given $\varepsilon \in (0, 1)$, consider the mechanism $x^{A,\varepsilon}$ given by

$$x_t^{A,\varepsilon} \coloneqq \begin{cases} (1-\varepsilon)x_t + \varepsilon \overline{x}^A & \text{if } t \in A \\ x_t & \text{otherwise.} \end{cases}$$

$x^{A,\varepsilon}$ dominates x for some A and ε if one of the following is true:

(a) for some $T > 0$, x does not decrease at T and

$$F^{0\prime}(\inf x_{T-}) > F^{1\prime}(X_T), \quad \text{or}$$

(b) for some $T > 0$, x does not increase at T and

$$F^{0\prime}(\sup x_{T-}) < F^{1\prime}(X_T).$$

Proof. Fix $T \in (0,\infty)$ and suppose that (a) holds; we omit the similar argument when (b) holds. We shall find a measurable, non-null and bounded $A \subseteq \mathbf{R}_+$ and an $\varepsilon \in (0,1)$ such that $\Pi_t(x^{A,\varepsilon}) \geq \Pi_t(x)$ for every $t \in \mathbf{R}_+$, with strict inequality for $t > \sup A$. (Π_t was defined on p. 42.)

Since $F^{0\prime}(\inf x_{T-}) > F^{1\prime}(X_T)$, $F^{0\prime}$ and $F^{1\prime}$ are continuous, and $F^{0\prime}$ is decreasing, there is an $\varepsilon' \in (0,T)$ such that

$$F^{0\prime}(u) \geq F^{1\prime}(X_t) + \varepsilon' \quad \text{for all } u \leq \inf x_{T-} + \varepsilon' \text{ and } t \in (T-\varepsilon', T+\varepsilon').$$

Since x does not decrease at T, we have $\operatorname{ess\,inf}_{(T-\varepsilon',T)} x < \operatorname{ess\,sup}_{(T,T+\varepsilon')} x$. Then there is a measurable and non-null $A \subseteq (T-\varepsilon', T+\varepsilon')$ such that[59]

$$\begin{aligned} &\bar{x}^A \leq \inf x_{T-} + \varepsilon', \\ &x_t < \bar{x}^A \qquad \text{for all } t \in A \cap (T-\varepsilon', T), \text{ and} \qquad (16)\\ &x_t > \bar{x}^A \qquad \text{for all } t \in A \cap (T, T+\varepsilon'). \qquad (17) \end{aligned}$$

[59]To construct such an A, fix a

$$u \in \left(\operatorname*{ess\,inf}_{(T-\varepsilon',T)} x, \min\left\{ \operatorname*{ess\,inf}_{(T-\varepsilon',T)} x + \varepsilon', \operatorname*{ess\,sup}_{(T,T+\varepsilon')} x \right\} \right),$$

and let

$$A_- \coloneqq \left\{ t \in \left(T-\varepsilon', T\right) : x_t < u \right\} \quad \text{and} \quad A_+ \coloneqq \left\{ t \in \left(T, T+\varepsilon'\right) : x_t > u \right\}.$$

It suffices to find a measurable and non-null $A \subseteq A_- \cup A_+$ such that $\bar{x}^A = u$. Clearly A_- and A_+ are measurable and non-null. Let

$$a_- \coloneqq \inf\{t \in \mathbf{R}_+ : \lambda(A_- \cap (0,t)) > 0\} \quad \text{and} \quad a_+ \coloneqq \sup\{t \in \mathbf{R}_+ : \lambda(A_+ \cap (t,\infty)) > 0\},$$

where λ denotes the Lebesgue measure. Then $a_- < \sup A_- \leq T \leq \inf A_+ < a_+$. Given $z \in [0,1]$, let

$$A_z \coloneqq (A_- \cap [0, T + z(a_- - T)]) \cup (A_+ \cap [a_+ + z(T - a_+), \infty)).$$

Then, A_z is measurable and non-null, and $A_z \subseteq A_- \cup A_+ \subseteq (T-\varepsilon', T+\varepsilon')$. Moreover, $z \mapsto \bar{x}^{A_z}$ is continuous, with $\bar{x}^{A_0} = \bar{x}^{A_-} < u$ and $\bar{x}^{A_1} = \bar{x}^{A_+} > u$, so there is a $z \in (0,1)$ at which $\bar{x}^{A_z} = u$ by the intermediate value theorem. The set $A \coloneqq A_z$ has the desired properties.

Note that

$$\int_A re^{-rs} x_s^{A,\varepsilon} \mathrm{d}s = \int_A re^{-rs} x_s \mathrm{d}s, \tag{18}$$

so that $X_t^{A,\varepsilon} = X_t$ for all $t \notin (\inf A, \sup A)$. Thus $\Pi_t(x^{T,\varepsilon}) = \Pi_t(x)$ for every $t \leq \inf A$, while for $t \geq \sup A$ we have by strict concavity of F^0 that

$$\begin{aligned}
\Pi_t\left(x^{A,\varepsilon}\right) - \Pi_t(x) &= \int_A re^{-rs}\left[F^0\left(x_s^{A,\varepsilon}\right) - F^0(x_s)\right]\mathrm{d}s \\
&\geq \varepsilon \int_A re^{-rs}\left[F^0\left(\overline{x}^A\right) - F^0(x_s)\right]\mathrm{d}s \\
&> 0
\end{aligned}$$

where the second inequality follows from Jensen's inequality because A is non-null and $x_t \neq \overline{x}^A$ for all $t \in A \setminus \{T\}$.[60] It remains to find an $\varepsilon \in (0,1)$ such that $\Pi_t(x^{A,\varepsilon}) \geq \Pi_t(x)$ for all $t \in (\inf A, \sup A)$.

Let $\kappa : A \times (\inf A, \sup A) \to \mathbf{R}$ and $\psi : (\inf A, \sup A) \to \mathbf{R}$ be given by

$$\begin{aligned}
\kappa(s,t) &:= \frac{F^0\left(x_s^{A,\varepsilon}\right) - F^0(x_s)}{x_s^{A,\varepsilon} - x_s} - F^{0\prime}(x_s) \\
&\quad - \frac{F^1\left(X_t^{A,\varepsilon}\right) - F^1(X_t)}{X_t^{A,\varepsilon} - X_t} + F^{1\prime}(X_t) \\
\text{and} \quad \psi(t) &:= \int_A \mathbf{1}_{[0,t)}(s) re^{-rs}\left(\overline{x}^A - x_s\right)\left[F^{0\prime}(x_s) - F^{0\prime}\left(\overline{x}^A\right)\right]\mathrm{d}s.
\end{aligned}$$

Fix $t \in (\inf A, \sup A)$ and note that

$$\begin{aligned}
X_t^{A,\varepsilon} - X_t &= \varepsilon \int_A \mathbf{1}_{[t,\infty)}(s) re^{-r(s-t)}\left(\overline{x}^A - x_s\right)\mathrm{d}s \\
&= -\varepsilon \int_A \mathbf{1}_{[0,t)}(s) re^{-r(s-t)}\left(\overline{x}^A - x_s\right)\mathrm{d}s
\end{aligned}$$

[60] The second inequality is equivalent to $F^0(\overline{x}^A) > \int_A re^{-rs} F^0(x_s)\mathrm{d}s \big/ \int_A re^{-rs}\mathrm{d}s$, which is in turn equivalent to $F^0(\mathbf{E}_{Z\sim H}(Z)) > \mathbf{E}_{Z\sim H}(F^0(Z))$, where H is the CDF $t \mapsto 1 - e^{-rt}$, conditioned on the event '$t \in A$' and pushed-forward by x. This holds by Jensen's inequality since F^0 is strictly concave and H is non-degenerate.

where the last equality follows from (18). Thus

$$
\begin{aligned}
&\Pi_t\left(x^{A,\varepsilon}\right) - \Pi_t(x) \\
&= \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\left[F^0\left(x_s^{A,\varepsilon}\right) - F^0(x_s)\right]\mathrm{d}s + e^{-rt}\left[F^1\left(X_t^{A,\varepsilon}\right) - F^1(X_t)\right] \\
&= \varepsilon \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\left(\bar{x}^A - x_s\right)\left[F^{0\prime}(x_s) - F^{1\prime}(X_t) + \kappa(s,t)\right]\mathrm{d}s \\
&= \varepsilon\Big\{\psi(t) \\
&\qquad + \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\left(\bar{x}^A - x_s\right)\left[F^{0\prime}\left(\bar{x}^A\right) - F^{1\prime}(X_t) + \kappa(s,t)\right]\mathrm{d}s\Big\}.
\end{aligned}
$$

By (16) and (17), we have $x \neq \bar{x}^A$ on $A \setminus \{T\}$, so that

$$\left(\bar{x}^A - x_s\right)\left[F^{0\prime}(x_s) - F^{0\prime}\left(\bar{x}^A\right)\right] > 0$$

since $F^{0\prime}$ is strictly decreasing. Thus ψ is non-negative and increasing, and $\psi(T) > 0$ since $A \cap (0,T)$ is non-null by (17) and (18).

Since $F^{0\prime}$ and $F^{1\prime}$ are monotone, the mean-value theorem yields

$$|\kappa(s,t)| \leq \left|F^{0\prime}(x_s) - F^{0\prime}\left(x_s^{A,\varepsilon}\right)\right| + \left|F^{1\prime}(X_t) - F^{1\prime}\left(X_t^{A,\varepsilon}\right)\right|.$$

Further, since x and X take values in $\left[0, u^0\right]$, $|x^{A,\varepsilon} - x|$ and $|X^{A,\varepsilon} - X|$ are bounded above by εu^0. Because $F^{0\prime}$ and $F^{1\prime}$ are uniformly continuous on $\left[0, u^0\right]$, it follows that we can choose $\varepsilon \in (0,1)$ such that

$$\sup|\kappa| \leq \min\left\{\varepsilon', \frac{\psi(T)}{2\varepsilon' r u_0}\right\}.$$

(ε' was introduced on p. 53.)

Consider $t \in (\inf A, T]$. We have

$$
\begin{aligned}
&\Pi_t\left(x^{A,\varepsilon}\right) - \Pi_t(x) \\
&\qquad \geq \varepsilon \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\left(\bar{x}^A - x_s\right)\left[F^{0\prime}\left(\bar{x}^A\right) - F^{1\prime}(X_t) + \kappa(s,t)\right]\mathrm{d}s \\
&\qquad \geq \varepsilon \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\left(\bar{x}^A - x_s\right)\left[F^{0\prime}\left(\bar{x}^A\right) - F^{1\prime}(X_t) - \varepsilon'\right]\mathrm{d}s \\
&\qquad \geq 0
\end{aligned}
$$

where the first inequality holds since $\psi \geq 0$, the second holds by (16) and our choice of ε, and the last holds by definition of ε'.

To conclude, consider $t \in [T, \sup A)$. Observe that

$$\int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\Big(\bar{x}^A - x_s\Big)\mathrm{d}s \geq \int_A r e^{-rs}\Big(\bar{x}^A - x_s\Big)\mathrm{d}s = 0,$$

where the inequality follows from (17). Thus

$$\begin{aligned}
\Pi_t\Big(x^{A,\varepsilon}\Big) - \Pi_t(x) &\geq \varepsilon\bigg(\psi(t) + \int_A \mathbf{1}_{[0,t)}(s) r e^{-rs}\Big(\bar{x}^A - x_s\Big)\kappa(s,t)\mathrm{d}s\bigg) \\
&\geq \varepsilon\Big(\psi(t) - 2\varepsilon' r u_0 \sup|\kappa|\Big) \\
&\geq 0,
\end{aligned}$$

where the first inequality holds since $F^{0\prime}(\bar{x}^A) \geq F^{1\prime}(X_t)$, the second since $A \subseteq (T-\varepsilon', T+\varepsilon')$ and $|\bar{x}^A - x|$ is bounded above by u^0, and the last holds by our choice of ε and the fact that ψ is increasing. ■

F.6 Proof of Theorem 3(a): $X \geq u^\star$

Let x be a mechanism with $x \leq u^0$, and assume without loss of generality that $X_0 \geq u^1$. Suppose that $X_{t''} < u^\star$ for some $t'' \in \mathbf{R}_+$; we must show that x is dominated. By the local contraction lemma (p. 52), it suffices to find a period $T \in (0, \infty)$ in which (b) holds.

We have $X_0 \geq u^1 > u^\star > X_{t''}$. Let

$$t' \coloneqq \max\{s \in [0, t''] : X_s \geq u^\star\},$$

noting that $t' < t''$ and $X_{t'} = u^\star$. It follows that

$$\operatorname*{ess\,sup}_{(t',t'')} x > X_{t'},$$

since otherwise X would be increasing on $[t', t'')$ by Observation 5 (p. 48), which would contradict $X_{t''} < u^\star = X_{t'}$ since X is continuous. Thus since F^0 is strictly concave and $X_{t'} = u^\star$, we have

$$F^{0\prime}\Bigg(\operatorname*{ess\,sup}_{(t',t'')} x\Bigg) < F^{0\prime}(X_{t'}) = F^{1\prime}(X_{t'}) = F^{1\prime}\Bigg(\sup_{(t',t'')} X\Bigg)$$

where the last equality holds since $\sup_{(t',t'')} X = u^\star = X_{t'}$ by definition of t'.

Since

$$\operatorname*{ess\,sup}_{(t',t'')} x > X_{t'} = \sup_{(t',t'')} X,$$

Corollary 4 (p. 51) yields that there is for every $\varepsilon > 0$ a $T \in (0, \infty)$ such x does not increase at T and (12) and (13) hold. Since $F^{0\prime}$ and $F^{1\prime}$ are continuous and decreasing, we may choose $\varepsilon > 0$ small enough that (12) and (13) ensure

$$F^{0\prime}(\sup x_{T-}) < F^{1\prime}(X_T).$$

We have found at $T > 0$ at which (b) holds. ■

F.7 Proof of Theorem 3(a): x is decreasing

Let x be a mechanism with $x \leq u^0$, and assume without loss of generality that $X \geq u^\star$. Suppose that no version of x is decreasing; we shall show that x is dominated. By the local contraction lemma (p. 52), it suffices to find a period $T \in (0, \infty)$ in which either (a) or (b) holds.

Since $X \geq u^\star$, we have

$$F^{0\prime}(X_t) \geq F^{1\prime}(X_t) \quad \text{for every } t \in \mathbf{R}_+. \tag{19}$$

Since no version of x is decreasing, the monotonicity lemma implies that there is a $t' \in (0, \infty)$ at which x does not decrease. If (a) holds at $T = t'$, then we are done. So suppose not:

$$F^{0\prime}(\inf x_{t'-}) \leq F^{1\prime}(X_{t'}). \tag{20}$$

We consider three cases. Case 1 is when $\operatorname{ess\,inf}_{(t,t')} x < X_{t'}$ for every $t \in (0, t')$, while in the remaining two cases we instead have $\operatorname{ess\,inf}_{(s',t')} x \geq X_{t'}$ for some $s' \in (0, t')$. In Case 2, the inequality (20) is strict, while in Case 3 it is an equality. In each case, we find a period in which either (a) or (b) holds.

Case 1: $\operatorname{ess\,inf}_{(t,t')} x < X_{t'}$ *for all* $t \in (0, t')$. Note that (20) implies that $\inf x_{t'-} \geq X_{t'}$, since otherwise we'd have

$$F^{0\prime}(\inf x_{t'-}) > F^{0\prime}(X_{t'}) \geq F^{1\prime}(X_{t'})$$

by strict concavity of F^0 and (19), which would contradict (20). Thus

$$\operatorname*{ess\,inf}_{(t,t')} x < X_{t'} \leq \inf x_{t'-} \quad \text{for all } t < t',$$

which by Observation 6 (p. 49) implies the existence of a $T < t'$ such that x does not decrease at T and $\inf x_{T-} \leq \operatorname{ess\,inf}_{(T,t')} x$. We shall show that $F^{0\prime}(\inf x_{T-}) > F^{1\prime}(X_T)$, so that (a) holds at T.

It must be that $X_T > \inf x_{T-}$. For if not, then we have

$$X_T \leq \inf x_{T-} \leq \operatorname*{ess\,inf}_{(T,t')} x,$$

so that that X is decreasing on $[T, t')$ by Observation 5 (p. 48). But then (by continuity of X)

$$X_{t'} \leq X_T \leq \operatorname*{ess\,inf}_{(T,t')} x,$$

a contradiction with the Case-1 hypothesis.

Since $X_T > \inf x_{T-}$, we have by strict concavity of F^0 and (19) that

$$F^{0\prime}(\inf x_{T-}) > F^{0\prime}(X_T) \geq F^{1\prime}(X_T),$$

as desired.

Case 2: $\operatorname{ess\,inf}_{(s',t')} x \geq X_{t'}$ *for some* $s' \in (0, t')$*, and* $F^{0\prime}(\inf x_{t'-}) < F^{1\prime}(X_{t'})$. Fix an s' satisfying the hypothesis. Since x does not decrease at t', we have for every $t > t'$ that

$$\operatorname*{ess\,sup}_{(t',t)} x > \operatorname*{ess\,inf}_{(s',t')} x \geq X_{t'}.$$

Thus by Lemma 4 (p. 50), there is for every $\varepsilon > 0$ a $T > 0$ such that x does not increase at T and (10) and (11) hold. We shall show that by choosing $\varepsilon > 0$ small enough, we may ensure that $F^{0\prime}(\sup x_{T-}) < F^{1\prime}(X_T)$, so that T satisfies (b).

Note that $\inf x_{t'-} \leq \sup x_{t'+}$ since x does not decrease at t', so that

$$\inf x_{t'-} \leq \min\{\sup x_{t'-}, \sup x_{t'+}\} \leq \sup x_{T-},$$

where the second inequality holds by (10). This together with the strict concavity of F^0 yields

$$F^{0\prime}(\sup x_{T-}) \leq F^{0\prime}(\inf x_{t'-}) < F^{1\prime}(X_{t'})$$

where the strict inequality is from the Case-2 hypothesis. Since $X_T \leq X_{t'} + \varepsilon$ by (11) and $F^{1\prime}$ is continuous and decreasing, we may choose $\varepsilon > 0$ small enough that the strict inequality is preserved when the $X_{t'}$ on the right-hand side is replaced by X_T.

Case 3: $\operatorname{ess\,inf}_{(s',t')} x \geq X_{t'}$ *for some* $s' \in (0, t')$*, and* $F^{0\prime}(\inf x_{t'-}) = F^{1\prime}(X_{t'})$. Fix an s' satisfying the hypothesis. Observe that $\inf x_{t'-} \geq X_{t'}$, since otherwise the strict concavity of F^0 would imply

$$F^{0\prime}(\inf x_{t'-}) > F^{0\prime}(X_{t'}) \geq F^{1\prime}(X_{t'}),$$

where the weak inequality holds by (19) (p. 57). We consider separately the cases (a) $\inf x_{t'-} = X_{t'}$ and (b) $\inf x_{t'-} > X_{t'}$.

Case 3(a): $\inf x_{t'-} = X_{t'}$. In this case, since $X \geq u^\star$, the Case-3 hypothesis implies that $X_{t'} = u^\star \leq X$, and thus $X_{t'} = \inf_{(t',\infty)} X$.

It must be that

$$\operatorname*{ess\,inf}_{(t',\infty)} x < X_{t'}.$$

For if not, then X is decreasing on $[t', \infty)$ by Observation 5 (p. 48), implying that X is constant on this interval. But then

$$\operatorname*{ess\,sup}_{(t',\infty)} x = X_{t'} \leq \operatorname*{ess\,inf}_{(s',t')} x,$$

by the Case-3 hypothesis, a contradiction with the fact that x does not decrease at t'.

It follows by Corollary 4 (p. 51), that for any $\varepsilon > 0$, there is a $T > 0$ such that x does not decrease at T and (14) and (15) hold. By strict concavity of F^0, we have

$$F^{0\prime}\left(\operatorname*{ess\,inf}_{(t',\infty)} x\right) > F^{0\prime}(X_{t'}) = F^{0\prime}(\inf x_{t'-}) = F^{1\prime}(X_{t'}) = F^{1\prime}\left(\inf_{(t',\infty)} X\right).$$

By (14) and (15), since $F^{0\prime}, F^{1\prime}$ are continuous and decreasing, we may choose $\varepsilon > 0$ small enough that

$$F^{0\prime}(\inf x_{T-}) > F^{1\prime}(X_T),$$

so that (a) holds at T.

Case 3(b): $\inf x_{t'-} > X_{t'}$. Clearly $\operatorname{ess\,inf}_{(t,t')} x \leq \inf x_{t'-}$ for every $t < t'$. We consider three sub-cases. In the first, the inequality is strict at every $t < t'$. The latter two have equality at some $t < t'$, and differ in whether $\inf x_{t'+} < \inf x_{t'-}$ or $\inf x_{t'+} \geq \inf x_{t'-}$.

Case 3(b)i: $\operatorname{ess\,inf}_{(t,t')} x < \inf x_{t'-}$ *for every* $t < t'$. By the Case-3(b) hypothesis and continuity of X, there is a $t < t'$ such that $\operatorname{ess\,inf}_{(t,t')} x > X_t$, so that X is decreasing on $[t, t']$ by Observation 5 (p. 48). Observation 6 (p. 49) implies the existence of $T \in (t, t')$ such that x does not decrease at T and

$$\inf x_{T-} \leq \operatorname*{ess\,inf}_{(T,t')} x < \inf x_{t'-}.$$

Since F^0 is strictly concave, it follows that

$$F^{0\prime}(\inf x_{T-}) > F^{0\prime}(\inf x_{t'-}) = F^{1\prime}(X_{t'}) \geq F^{1\prime}(X_T),$$

where equality holds by the Case-3 hypothesis, and the last inequality holds since $X_T \geq X_{t'}$ and F^1 is concave. Thus T satisfies (a).

Case 3(b)ii: $\operatorname{ess\,inf}_{(s'',t')} x = \inf x_{t'-}$ *for some* $s'' < t'$, *and* $\inf x_{t'+} < \inf x_{t'-}$. Since F^0 is strictly concave, we have

$$F^{0\prime}(\inf x_{t'+}) > F^{0\prime}(\inf x_{t'-}) = F^{1\prime}(X_{t'}),$$

where the equality holds by the Case-3 hypothesis. Since x does not decrease at t', we have

$$\sup x_{t'+} \geq \inf x_{t'-} > \inf x_{t'+},$$

so that by Corollary 3 (p. 50), for any $\varepsilon > 0$, there is a $T \in [t', t' + \varepsilon)$ such that x does not decrease at T and $\inf x_{T-} \leq \inf x_{t'+} + \varepsilon$. Since $F^{0\prime}$ is decreasing and continuous and X and $F^{1\prime}$ are continuous, we may choose $\varepsilon > 0$ small enough that the inequality is preserved:

$$F^{0\prime}(\inf x_{T-}) > F^{1\prime}(X_T).$$

So (a) holds at T.

Case 3(b)iii: $\operatorname{ess\,inf}_{(s'',t')} x = \inf x_{t'-}$ *for some* $s'' < t'$, *and* $\inf x_{t'+} \geq \inf x_{t'-}$. We have

$$\inf x_{t'+} \geq \inf x_{t'-} > X_{t'},$$

where the strict inequality holds by the Case-3(b) hypothesis. Thus there must be a $t'' > t'$ such that $\operatorname{ess\,inf}_{(t',t'')} x \geq X_{t'}$, so that X is decreasing on $[t', t'')$ by Observation 5 (p. 48). This implies in particular that $\sup_{(t',t'')} X = X_{t'}$.

Furthermore, since x does not decrease at t', we have

$$\operatorname*{ess\,sup}_{(t',t'')} x > \operatorname*{ess\,inf}_{(s'',t')} x = \inf x_{t'-} > X_{t'} = \sup_{(t',t'')} X,$$

where the first equality holds by the Case-3(b)iii hypothesis, and the second inequality is the Case-3(b) hypothesis. Thus since F^0 is strictly concave, we have

$$F^{0\prime}\left(\operatorname*{ess\,sup}_{(t',t'')} x\right) < F^{0\prime}(\inf x_{t'-}) = F^{1\prime}(X_{t'}) = F^{1\prime}\left(\sup_{(t',t'')} X\right),$$

where the first equality holds by the Case-3 hypothesis. By Corollary 4 (p. 51), there is for any $\varepsilon > 0$ a period $T > 0$ in which x does not increase and (12) and (13) hold. Since $F^{0\prime}$ and $F^{1\prime}$ are continuous and decreasing, we may choose $\varepsilon > 0$ small enough that (12) and (13) yield

$$F^{0\prime}(\sup x_{T-}) < F^{1\prime}(X_T),$$

so that (b) holds at T. ■

G Generalisation and proof of Proposition 3 (p. 22)

In this appendix, we characterise the optimal transition path from u^0 toward $u^\star$ in the general (non-affine F^0) case.

The Euler lemma in appendix E (p. 41) provides that if F^0, F^1 possess bounded derivatives on $(0, u^0)$, then an undominated mechanism is optimal for G exactly if it satisfies the superdifferential Euler equation in appendix E (p. 41). This result strengthens Proposition 3: it does not rely on the assumption that $u^\star > 0$, and it asserts that the Euler equation is equivalent to (not merely necessary for) optimality.

It remains only to show that when $u^\star > 0$, the superdifferential Euler equation may be expressed as in Proposition 3. We shall make use of Theorem 3(b) from the previous appendix (p. 42).

Proof of Proposition 3. Assume that $u^\star > 0$ and that F^0, F^1 have bounded derivatives on $(0, u^0)$, and let (x, X) satisfy the superdifferential Euler equation in appendix E (p. 41); we must show that $\mathbf{E}_G(F^{1\prime}(X_\tau)) = 0$ and

$$F^{0\prime}(x_t) \geq \mathbf{E}_G\left(F^{1\prime}(X_\tau)\middle|\tau > t\right) \quad \begin{array}{l}\text{for every } t \in \mathbf{R}_+ \text{ at which } G(t) < 1,\\ \text{with equality if } x_t < u^0.\end{array}$$

By the Euler lemma in appendix E (p. 41), (x, X) is optimal for G. Thus by Theorem 3(b) from the previous appendix, x must be decreasing with $x \geq X \geq u^\star$. Since x is monotone and bounded, it converges as $t \to \infty$.

Claim. $\lim_{t\to\infty} x_t < u^0$.

Proof. Suppose toward a contradiction that $\lim_{t\to\infty} x_t = u^0$, so that (since x is decreasing) $x = X = u^0$. Then by the superdifferential Euler equation (E) (p. 41), we have

$$[1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G \geq 0 \quad \text{for every } t \in \mathbf{R}_+,$$

where for each $t \in \mathbf{R}_+$, $\phi^0(t)$ ($\phi^1(t)$) is a supergradient of F^0 (of F^1) at u^0. Since ϕ^0 is bounded above by $F^{0-}(u^0)$, we have

$$[1 - G(t)]F^{0-}\left(u^0\right) + \int_{[0,t]} \phi^1 \mathrm{d}G \geq [1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G \geq 0.$$

Letting $t \to \infty$ yields

$$\int_{[0,\infty)} \phi^1 \mathrm{d}G \geq 0,$$

which is impossible since ϕ^1 is bounded above by $F^{1-}(u^0) < 0$. □

Since $x \geq u^\star > 0$, and F^0, F^1 are differentiable on $(0, u^0)$, the superdifferential Euler equation (E) reads

$$[1 - G(t)]F^{0\prime}(x_t) + \int_{[0,t]} F^{1\prime}(X_s)G(\mathrm{d}s) \begin{cases} = 0 & \text{if } x_t < u^0 \\ \geq 0 & \text{if } x_t = u^0, \end{cases} \tag{E$'$}$$

where $F^{0\prime}(u^0)$ denotes the left-hand derivative.[61] By the claim, the equality case must hold for all sufficiently late times t, so that letting $t \to \infty$ yields

$$\mathbf{E}_G\left(F^{1\prime}(X_\tau)\right) = 0.$$

Subtracting $\mathbf{E}_G(F^{1\prime}(X_\tau))$ from (E$'$) therefore yields

$$[1 - G(t)]F^{0\prime}(x_t) - \int_{(t,\infty)} F^{1\prime}(X_s)G(\mathrm{d}s) \begin{cases} = 0 & \text{if } x_t < u^0 \\ \geq 0 & \text{if } x_t = u^0, \end{cases}$$

which for periods t with $G(t) < 1$ may be written as

$$F^{0\prime}(x_t) \geq \mathbf{E}_G\left(F^{1\prime}(X_\tau)\middle|\tau > t\right), \quad \text{with equality if } x_t < u^0. \qquad \blacksquare$$

H Proof of Lemma 2 (p. 27)

Clearly F^0, F^1 are well-defined and continuous on $[0, \infty)$. It remains to show that they are strictly concave with peaks u^0, u^1 satisfying $u^0 > u^1$, and that the gap $F^1 - F^0$ is strictly decreasing.

Claim. For each $u > 0$, the maximisation problem in the expression for $F^1(u)$ has a unique solution $L^\star(u) \in (0, \infty)$. Furthermore, $\lim_{u \to \infty} L^\star(u) = 0$.

[61] F^1 is differentiable at X_s for each $s \in \mathbf{R}_+$ since $0 < u^\star \leq X < u^0$ by the claim.

Proof. Fix $u > 0$, and write $f : \mathbf{R}_+ \to \mathbf{R}$ for the objective. Its (right-hand) derivative is

$$f'(L) = w - \frac{\kappa'(L)}{\phi'(\phi^{-1}(u + \kappa(L)))}.$$

Clearly f' is strictly decreasing, so there is at most one maximiser. We have $f'(0) = w > 0$, whereas $f'(L) < 0$ for any large enough $L > 0$ since

$$\lim_{L\to\infty} f'(L) \leq w - \lim_{L\to\infty} \frac{\kappa'(L)}{\phi'(\phi^{-1}(\kappa(L)))} = -\infty,$$

where the equality holds since the numerator is bounded away from zero as $L \to \infty$ while the denominator vanishes. Thus the unique maximiser $L^\star(u)$ of f is interior, and therefore satisfies the first-order condition $f'(L^\star(u)) = 0$. As $u \to \infty$ in the first-order condition, the denominator in the fraction vanishes since $\lim_{C\to\infty} \phi'(C) = 0$, requiring the numerator also to vanish, which since $\kappa' > 0$ on $(0, \infty)$ demands that $L^\star(u) \to 0$. $\square$

F^0 is strictly concave since ϕ is. Its (right-hand) derivative

$$F^{0\prime}(u) = 1 - \frac{\lambda}{\phi'(\phi^{-1}(u))}$$

is strictly positive at $u = 0$ since $\lim_{C\to 0} \phi'(C) = \infty$, and is strictly negative for u large enough since $\phi^{-1}(u) \to \infty$ as $u \to \infty$ and $\lim_{C\to\infty} \phi'(C) = 0$. Thus F^0 is uniquely maximised at $u^0 \in (0, \infty)$ satisfying the first-order condition

$$\phi'\Big(\phi^{-1}\Big(u^0\Big)\Big) = \lambda.$$

F^1 is also strictly concave: for $u \neq u'$ in $[0, \infty)$ and $\eta \in (0, 1)$, we have

$$\begin{aligned}
&F^1(\eta u + (1-\eta)u') \\
&\quad = \eta u + (1-\eta)u' + \lambda \max_{L\geq 0}\Big\{wL - \phi^{-1}(\eta u + (1-\eta)u' + \kappa(L))\Big\} \\
&\quad > \eta u + (1-\eta)u' \\
&\qquad + \lambda \max_{L\geq 0}\Big\{wL - \eta\phi^{-1}(u + \kappa(L)) - (1-\eta)\phi^{-1}(u' + \kappa(L))\Big\} \\
&\quad \geq \eta\Big[u + \lambda \max_{L\geq 0}\Big\{wL - \phi^{-1}(u + \kappa(L))\Big\}\Big] \\
&\qquad + (1-\eta)\Big[u' + \lambda \max_{L\geq 0}\Big\{wL - \phi^{-1}(u' + \kappa(L))\Big\}\Big] \\
&\quad = \eta F^1(u) + (1-\eta)F^1(u'),
\end{aligned}$$

where the strict inequality holds since $-\phi^{-1}$ is strictly concave. By the envelope theorem, we have

$$F^{1\prime}(u) = 1 - \frac{\lambda}{\phi'(\phi^{-1}(u + \kappa(L^\star(u))))}.$$

This expression is strictly negative for u large enough because the denominator in the fraction vanishes as $u \to \infty$ since $L^\star(u) \to 0$ by the claim. Thus F^1 has a unique maximiser $u^1 \in [0, \infty)$, which satisfies the first-order condition

$$\phi'\Big(\phi^{-1}\Big(u^1 + \kappa\Big(L^\star\Big(u^1\Big)\Big)\Big)\Big) \le \lambda, \quad \text{with equality if } u^1 > 0.$$

To show that $u^0 > u^1$, consider two cases. If $u^1 = 0$, then $u^0 > 0 = u^1$. If instead $u^1 > 0$, then the first-order conditions for u^0 and u^1 together yield

$$\phi'\Big(\phi^{-1}\Big(u^0\Big)\Big) = \lambda = \phi'\Big(\phi^{-1}\Big(u^1 + \kappa\Big(L^\star\Big(u^1\Big)\Big)\Big)\Big).$$

Thus

$$u^0 = u^1 + \kappa\Big(L^\star\Big(u^1\Big)\Big) > u^1$$

since $L^\star(u^1) > 0$ by the claim and $\kappa > 0$ on $(0, \infty)$.

It remains only to show that $F^1 - F^0$ is strictly decreasing, so that $u^\star = 0$. It suffices to show that $F^{1\prime} < F^{0\prime}$ on $(0, \infty)$. So fix any $u > 0$. Since $L^\star(u) > 0$ by the claim, and $\kappa > 0$ on $(0, \infty)$, we have

$$F^{0\prime}(u) = 1 - \frac{\lambda}{\phi'(\phi^{-1}(u))} > 1 - \frac{\lambda}{\phi'(\phi^{-1}(u + \kappa(L^\star(u))))} = F^{1\prime}(u). \qquad \blacksquare$$

Supplemental appendices

I Extensions

We claimed in §2.1 that our results remain true in a variety of extensions of our model. In this appendix, we validate these assertions.

I.1 Breakthrough at the outset

Our analysis considers only breakthrough distributions G that satisfy $G(0) = 0$, so that the new technology is certainly unavailable at the outset. In this appendix, we examine the implications of relaxing this assumption.

The alternative notion of dominance that quantifies over *all* distributions G is equivalent to the one we study in the remainder of the paper: in either case, one mechanism dominates another exactly if the former yields a weakly (strictly) higher ex-post payoff for any (some) breakthrough time. Thus Lemma 1 (p. 13) and Theorem 1 (p. 14) remain valid, so that any undominated mechanism (x^0, X^1) must satisfy $x^0 \leq u^0$ a.e. and $X^0 = X^1$. We may thus write (x, X) for an undominated mechanism, as in the text.

Now fix a distribution G with $G(0) > 0$. Proposition 3 (p. 22) remains valid, with the same proof (whether or not F^0 is affine on $[0, u^0]$), so that any optimal mechanism (x, X) must satisfy the initial condition and Euler equation. This implies that x must be decreasing and converge to $u^\star$.

The difference is that $x_{0+} := \lim_{t \to 0} x_t$ is no longer equal to u^0, but rather satisfies

$$[1 - G(0)]F^{0\prime}(x_{0+}) + G(0)F^{1\prime}(X_0) = 0.$$

As in the text, the pre-disclosure flow x is undistorted at the outset. 'Undistorted' has a different meaning, however: since disclosure might occur immediately, it is optimal to lower x_{0+} from u^0 so as to lower X_0 toward u^1.

When F^0 is affine on $[0, u^0]$, the front-loading argument leading to Theorem 2 (p. 17) remains applicable, so that optimal mechanisms have a generalised deadline form: the pre-disclosure flow x is equal to a high value in $(u^1, u^0]$ before a deadline, and equal to $u^\star$ afterwards.

I.2 Random technology

In our model, the new technology F^1 is known in advance—only its date of arrival is uncertain. In this appendix, we show that all of our results remain valid if the new technology is uncertain, provided the agent is not privately informed about its realisation.

Let $\mathcal{F}$ be a finite set of of concave and upper semi-continuous functions $[0, \infty) \to [-\infty, \infty)$ with unique peaks. The new frontier $\boldsymbol{F}$ is a random element of $\mathcal{F}$, assumed for simplicity to be independent of the breakthrough time τ. Write $U^1(F)$ for the unique peak of $F \in \mathcal{F}$, and $u^1 := \mathbf{E}(U^1(\boldsymbol{F}))$ for its expectation. We assume that there is a conflict of interest: $u^1 < u^0$.

The agent privately observes when the breakthrough occurs, but she does not learn the realised value of the new technology $\boldsymbol{F}$. This means that the agent cannot easily determine the payoff consequences for the principal of the new technology, which is natural in many (but not all) applications.

A mechanism specifies, for each period t, the agent's utility x_t^0 if she has not already disclosed, as well as the continuation utility $\widehat{X}_t(F)$ with which she is rewarded for disclosing at time t if the new technology is $F \in \mathcal{F}$. Since the agent does not know F prior to disclosure, only the expectation

$$X_t^1 \coloneqq \mathbf{E}\Big(\widehat{X}_t(\boldsymbol{F})\Big)$$

matters for her incentives.

For a given value $X_t^1 = u$ of the expectation, the principal chooses $\widehat{X}_t : \mathcal{F} \to [0,\infty)$ to maximise

$$\mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}_t(\boldsymbol{F})\Big)\Big) \quad \text{subject to} \quad \mathbf{E}\Big(\widehat{X}_t(\boldsymbol{F})\Big) = u.$$

We write

$$F^1(u) \coloneqq \max_{\widehat{X} \in M_u} \mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}(\boldsymbol{F})\Big)\Big)$$

for the value of this problem, where M_u denotes the space of maps $\mathcal{F} \to [0,\infty)$ that have expected value u under $\boldsymbol{F}$.[62]

To characterise the pre-disclosure flow x^0 and expected disclosure reward X^1 in undominated mechanisms, we may study the deterministic model in which the new technology is F^1. (The technology-dependent disclosure reward $\widehat{X}$ may be backed out from the above maximisation problem.) This deterministic model satisfies our model assumptions:

Lemma 5. *F^1 is concave and upper semi-continuous, with unique peak at $u^1 = \mathbf{E}(U^1(\boldsymbol{F}))$.*

Our results therefore remain valid, characterising the x^0 and X^1 of undominated mechanisms in the random-technology model.

Proof. For concavity, take $u, u^\dagger \in [0,\infty)$ and $\lambda \in (0,1)$. Let $\widehat{X} \in M_u$ and $\widehat{X}^\dagger \in M_{u^\dagger}$ be maximisers:

$$\mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}(\boldsymbol{F})\Big)\Big) = F^1(u) \quad \text{and} \quad \mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}^\dagger(\boldsymbol{F})\Big)\Big) = F^1\Big(u^\dagger\Big).$$

Then $\lambda\widehat{X} + (1-\lambda)\widehat{X}^\dagger$ belongs to $M_{\lambda u + (1-\lambda)u^\dagger}$, so that

$$\begin{aligned} &F^1\Big(\lambda u + (1-\lambda)u^\dagger\Big) \\ &\qquad \geq \mathbf{E}\Big(\boldsymbol{F}\Big(\lambda\widehat{X}(\boldsymbol{F}) + (1-\lambda)\widehat{X}^\dagger(\boldsymbol{F})\Big)\Big) \\ &\qquad \geq \mathbf{E}\Big(\lambda\boldsymbol{F}\Big(\widehat{X}(\boldsymbol{F})\Big) + (1-\lambda)\boldsymbol{F}\Big(\widehat{X}^\dagger(\boldsymbol{F})\Big)\Big) \quad \text{since } \boldsymbol{F} \text{ is concave a.s.} \\ &\qquad = \lambda F^1(u) + (1-\lambda)F^1\Big(u^\dagger\Big). \end{aligned}$$

[62]The maximum exists (so that F^1 is well-defined) because M_u with the pointwise topology is compact (being a closed and bounded subset of the Euclidean space $[0,\infty)^{|\mathcal{F}|}$) and the maximand is upper semi-continuous since every element of $\mathcal{F}$ is.

To see that F^1 attains a maximum at u^1, observe that F^1 is bounded above by $\mathbf{E}(\boldsymbol{F}(U^1(\boldsymbol{F})))$, and that it attains this value at u^1. The peak is unique because for $u \neq u^1$, any $\widehat{X} \in M_u$ must have $\widehat{X} \neq U^1$ on a non-null set, and thus

$$\boldsymbol{F}\Big(\widehat{X}(\boldsymbol{F})\Big) \leq (<) \; \boldsymbol{F}\Big(U^1(\boldsymbol{F})\Big) \quad \text{a.s. (with positive probability).}$$

For upper semi-continuity, observe that since F^1 is concave, it is continuous on the interior of its effective domain, which is a convex set that contains u^1. F^1 is trivially continuous off the closure of its effective domain, where it is constant and equal to $-\infty$. It remains only to establish that

$$\limsup_{u' \to u} F^1(u') \leq F^1(u)$$

for u on the boundary of the effective domain (there are at most two such us). It suffices to show for an arbitrary decreasing sequence $(u_n)_{n\in\mathbf{N}}$ in the interior of the effective domain converging to some $u \in \big[0, u^1\big]$ that

$$\lim_{n\to\infty} F^1(u_n) \leq F^1(u)$$

(where the limit exists since $\big(F^1(u_n)\big)_{n\in\mathbf{N}}$ is eventually monotone), and similarly for increasing sequences converging to $u \geq u^1$. We show the former, omitting the analogous argument for the latter.

For each $n \in \mathbf{N}$, let $\widehat{X}_n \in M_{u_n}$ be a maximiser at u_n:

$$\mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}_n(\boldsymbol{F})\Big)\Big) = F^1(u_n).$$

Since $\bigcup_{u' \in [0,u^1]} M_{u'}$ is compact (because bounded), the sequence $\big(\widehat{X}_n\big)_{n\in\mathbf{N}}$ admits a convergent subsequence $\big(\widehat{X}_{n_k}\big)_{k\in\mathbf{N}}$, whose limit we denote by $\widehat{X}$. We have

$$\begin{aligned}
F^1(u) &\geq \mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}(\boldsymbol{F})\Big)\Big) && \text{since } \widehat{X} \in M_u \\
&\geq \mathbf{E}\Big(\lim_{k\to\infty} \boldsymbol{F}\Big(\widehat{X}_{n_k}(\boldsymbol{F})\Big)\Big) && \text{since } \boldsymbol{F} \text{ is upper semi-continuous a.s.} \\
&= \lim_{k\to\infty} \mathbf{E}\Big(\boldsymbol{F}\Big(\widehat{X}_{n_k}(\boldsymbol{F})\Big)\Big) && \text{by bounded convergence} \\
&= \lim_{k\to\infty} F^1(u_{n_k}) \\
&= \lim_{n\to\infty} F^1(u_n) && \text{since } \Big(F^1(u_n)\Big)_{n\in\mathbf{N}} \text{ is convergent.}
\end{aligned}$$

■

I.3 Technologies with restricted domains

In our application unemployment insurance (§8), the agent's utility is bounded above. In this appendix, we show how such domain restrictions may be accommodated within the model presented in §2.

Suppose that technology $j \in \{0,1\}$ can only provide the agent with utility in an interval I^j.[63] The frontier F^j is a concave function defined on this domain, and thus automatically continuous on its interior. It is innocuous to suppose that I^j is closed and that F^j is upper semi-continuous.[64] Assume without loss of generality that $I^0 \subseteq I^1$, meaning that the new technology expands utility possibilities for the agent.[65]

Extend the frontier F^j to all of $[0,\infty)$ by letting $F^j(u) := -\infty$ for every $u \notin I^j$. The extended frontier is upper semi-continuous and concave, with effective domain I^j. Since the principal will never choose any $u \notin I^j$ when using technology j, it is as if these utility levels were not permitted.

All of our results thus remain true as stated. In fact, most can be slightly sharpened. Consider weakening the hypothesis of Theorem 2 (p. 17) to require that F^0 be affine only on $I^0 \cap [0, u^0]$. This permits F^0 to take value $-\infty$ on an interval $[0, \underline{u}) \subseteq [0, u^0]$.[66] Since utility levels below $\underline{u}$ are never used, we may excise them from the model by re-writing the agent's utility as $u' = u - \underline{u}$ and requiring (as before) that $u' \geq 0$. Applying Theorem 2 to this rewritten model delivers the same result under the weaker hypothesis.

For the same reason, we may sharpen Proposition 2 (p. 20), Theorem 3 (p. 21) and Proposition 3 (p. 22) by requiring their (differential) hypotheses to hold on the interior of $I^0 \cap (0, u^0)$ rather than on all of $(0, u^0)$.

I.4 Participation constraint

Our model in §2 assumes that the agent's utility u must be non-negative. In this appendix, we show that our results remain valid if this assumption is replaced by a participation constraint.

Suppose that the agent's utility can take any value $u \in [-K, \infty)$, where

[63] It is without loss of generality for I^j to be convex (i.e. an interval), since any convex combination of feasible utility levels can be attained by rapidly switching back and forth.

[64] The principal can anyway attain utility arbitrarily close to $\lim_{u \downarrow \inf I^j} F^j(u)$ by choosing $u > \inf I^j$ small, and similarly for $\sup I^j$.

[65] If there were a utility level $u \in I^0 \setminus I^1$, then the principal could attain it by rapidly switching back and forth between using the old (new) technology at some $u \in I^0 \setminus I^1$ ($u' \in I^1$). Thus the true new frontier is the concave upper envelope of the function $\widetilde{F}^1 : I^0 \cup I^1 \to [-\infty, \infty)$ defined by $\widetilde{F}^1(u) = F^1(u)$ for $u \in I^1$ and $= F^0(u)$ for $u \in I^0 \setminus I^1$.

[66] It has this form because I^0 is convex and contains u^0.

$K > 0$ is (arbitrarily) large.[67] The agent can quit anytime, earning a continuation payoff worth zero. To avoid trivialities, assume that the principal prefers for the agent to participate. She must then choose a mechanism (x^0, X^1) that satisfies the participation constraint $X^0, X^1 \geq 0$. It suffices to satisfy $X^0 \geq 0$, since incentive-compatibility requires $X^1 \geq X^0$.

Observe that we may re-express the agent's utility as $u + K \in [0, \infty)$. Lemma 1 (p. 13) and Theorem 1 (p. 14) are therefore applicable, ensuring that any undominated mechanism (x^0, X^1) satisfies $x^0 \leq u^0$ a.e. and $X^0 = X^1$. As in the text, we simplify notation by writing (x, X) for such a mechanism.

The argument for Theorem 3 (p. 21) ensures that an optimal mechanism (x, X) must have x decreasing (this does not rely on $u \geq 0$), which implies that $x \geq X$.[68] Thus the binding part of the non-negativity constraint $x, X \geq 0$ is $X \geq 0$, which is exactly the participation constraint.

I.5 When $F^1 - F^0$ admits no strict local maximum

If $F^1 - F^0$ does not admit a strict local maximum, then Theorem 2 changes as follows. Since F^0 is affine in Theorem 2, $F^1 - F^0$ is concave, so it is globally maximised on a non-trivial interval $[\underline{u}^\star, \overline{u}^\star]$.

Definition 8. A mechanism (x, X) is a *generalised deadline mechanism* iff

$$x_t \begin{cases} = u^0 & \text{for } t \leq T \\ \in [\underline{u}^\star, \overline{u}^\star] & \text{for } t > T \end{cases} \quad \text{for some } T \in [0, \infty].$$

With almost no modification, the proof of Theorem 2 in appendix B delivers

Theorem 2′. If the old frontier F^0 is affine on $[0, u^0]$, then any undominated incentive-compatible mechanism is a generalised deadline mechanism.

J Approximate variant of Theorem 2 (p. 17)

Theorem 2 asserts that only deadline mechanisms can be optimal when the old frontier F^0 is affine. In this appendix, we show that approximate affineness suffices for deadline mechanisms to be approximately optimal.

As in §6, let $u^\star$ denote the rightmost $u \in [0, u^0]$ at which F^0, F^1 have equal slopes (i.e. share a supergradient). A deadline mechanism is one that

[67] The lower bound does not bind. We impose it merely to avoid integrability issues.
[68] Since $X_t = r \int_t^\infty e^{-r(s-t)} x_s \mathrm{d}s \leq r \int_t^\infty e^{-r(s-t)} x_t \mathrm{d}s = x_t$ for every $t \in \mathbf{R}_+$.

provides pre-disclosure utility u^0 before a deadline and $u^\star$ afterwards. Write

$$\underline{F}^0(u) \coloneqq F^0(u^\star) + \frac{u}{u^0}\left[F^0\left(u^0\right) - F^0(u^\star)\right] \quad \text{for } u \in \left[u^\star, u^0\right]$$

for the straight line connecting $(u^\star, F^0(u^\star))$ with $(u^0, F^0(u^0))$. Since $\underline{F}^0$ is the pointwise highest affine function everywhere below F^0 on $[u^\star, u^0]$, we call F^0 close to affine iff it is close to $\underline{F}^0$:

Definition 9. For $\varepsilon > 0$, the frontier F^0 is *ε-close to affine* on $[u^\star, u^0]$ iff

$$\max_{u\in[u^\star,u^0]}\left[F^0(u) - \underline{F}^0(u)\right] \leq \varepsilon.$$

Corollary 5. If F^0 is ε-close to affine on $[u^\star, u^0]$, then for any distribution G, the principal loses at most ε by using the best deadline mechanism rather than a fully optimal mechanism.

Proof. Fix a distribution G. By Theorem 2 (p. 17), a best deadline mechanism is precisely an optimal mechanism when F^0 is replaced by $\underline{F}^0$ on $[u^\star, u^0]$;[69] write Π_G^{d} for its value.[70] For any undominated mechanism (x, X), we have $u^\star \leq x \leq u^0$ by Lemma 1 (p. 13) and Theorem 3(a) in appendix F (p. 42), and thus

$$\begin{aligned}\Pi_G(x, X) &= \mathbf{E}_G\left(r\int_0^\tau e^{-rt}\underline{F}^0(x_t)\mathrm{d}t + e^{-r\tau}F^1(X_\tau)\right)\\ &\quad + \mathbf{E}_G\left(r\int_0^\tau e^{-rt}\left[F^0 - \underline{F}^0\right](x_t)\mathrm{d}t\right)\\ &\leq \Pi_G^{\mathrm{d}} + \varepsilon,\end{aligned}$$

which is to say that the principal loses no more than ε by using the best deadline mechanism rather than a fully optimal mechanism. ■

K Properties of undominated and optimal mechanisms

In this appendix, we prove that undominated and optimal mechanisms exist (§K.1) and that our results remain valid under more demanding definitions of dominance (§K.2).

[69] This can be seen from Lemma 3 in appendix D (p. 39).
[70] By inspection, its value is the same whether or not F^0 is replaced by $\underline{F}^0$ on $[u^\star, u^0]$.

K.1 Existence of undominated and optimal mechanisms

We shall assume Lemma 1 and Theorem 1 (pages 13 and 14), neither of whose proofs rely on any existence claim. In light of these, an undominated mechanism may be identified with a measurable map $x : \mathbf{R}_+ \to [0, u^0]$, with the post-disclosure reward X chosen to make the agent indifferent as per Theorem 1.

Write $\mathcal{X}$ for the space of measurable maps $\mathbf{R}_+ \to [0, u^0]$, and endow it with the topology of pointwise convergence. Write $\pi_G : \mathcal{X} \to \mathbf{R}$ for the principal's payoff:

$$\pi_G(x) \coloneqq \mathbf{E}_G\left(r \int_0^\tau e^{-rt} F^0(x_t)\mathrm{d}t + e^{-r\tau} F^1\left(r \int_t^\infty e^{-r(s-t)} x_s \mathrm{d}s \right)\right).$$

The principal's payoff π_G is continuous since F^0, F^1 are continuous and bounded on the relevant domain $[0, u^0]$, so that the bounded convergence theorem is applicable. Furthermore, $\mathcal{X}$ is compact because it is a closed subset of the space of all functions $\mathbf{R}_+ \to [0, u^0]$,[71] which is compact by Tychonoff's theorem.

Further equip (the equivalence classes of) $\mathcal{X}$ with the partial order $\preceq$ defined by $x \preceq x^\dagger$ iff either x is dominated by $x^\dagger$ or $x = x^\dagger$ (a.e.).

Proposition 5. An undominated mechanism exists.

Proof. We must show that $\mathcal{X}$ admits a $\preceq$-maximal element. By Zorn's lemma, it suffices to show that every chain admits an upper bound. So fix a chain $\mathcal{C} \subseteq \mathcal{X}$, wlog one that does not contain an upper bound of itself; we will find an upper bound of $\mathcal{C}$ in $\mathcal{X}$. Let $(x_n)_{n\in\mathbf{N}} \subseteq \mathcal{C}$ be an $\preceq$-increasing sequence with no upper bound in $\mathcal{C}$. Since $\mathcal{X}$ is compact, we may assume (passing to a subsequence if necessary) that $(x_n)_{n\in\mathbf{N}}$ is convergent, denoting the limit by $x^\star \in \mathcal{X}$. It satisfies $x_n \preceq x^\star$ for every $n \in \mathbf{N}$ since π_G is continuous for every CDF G, and thus $x \preceq x^\star$ for every $x \in \mathcal{C}$ since the sequence $(x_n)_{n\in\mathbf{N}}$ has no upper bound in $\mathcal{C}$. Thus $x^\star$ is an upper bound of $\mathcal{C}$ in $\mathcal{X}$. ■

Corollary 6. Any dominated mechanism is dominated by an undominated mechanism.

Proof. Suppose that $x \in \mathcal{X}$ is dominated, and define

$$\mathcal{U} \coloneqq \{x' \in \mathcal{X} : x \prec x'\}.$$

[71] It is closed because the pointwise limit of measurable functions is itself measurable (see e.g. Proposition 2.7 in Folland (1999)).

This set is non-empty since x is dominated. By the argument used to prove Proposition 5, $\mathcal{U}$ has a $\preceq$-maximal element. ∎

Lemma 6. The set of undominated mechanisms is compact.

Proof. Since $\mathcal{X}$ is compact, it suffices to show that the subset of undominated mechanisms is closed. So take a sequence $(x_n)_{n\in\mathbf{N}} \subseteq \mathcal{X}$ of undominated mechanisms converging to $x^\star \in \mathcal{X}$; we will show that $x^\star$ is undominated. Fix an arbitrary $x \in \mathcal{X}$. Undominatedness along the sequence ensures that $x_n \not\prec x$ for every $n \in \mathbf{N}$, which since π_G is continuous implies that $x^\star \not\prec x$. Since x was arbitrary, we have shown that $x^\star$ is undominated. ∎

Corollary 7. For any distribution G, an optimal mechanism exists.

Proof. By Corollary 6, it suffices to show that π_G attains a maximum on the space of undominated mechanisms. This follows immediately from the facts that π_G is continuous and that the space of undominated mechanisms is non-empty (Proposition 5) and compact (Lemma 6). ∎

K.2 Stronger notions of dominance

A potential weakness of our (standard) definition of dominance is that it requires a strict improvement only for some unspecified distribution G. It could thus be that one mechanism dominates another only because the former does strictly better under a strange or implausible distribution. In this appendix, we address this concern by showing that our results remain valid if the definition of dominance is (greatly) strengthened.

Definition 10. Let (x^0, X^1) and $(x^{0\dagger}, X^{1\dagger})$ be incentive-compatible mechanisms. The former *strongly dominates* the latter iff

$$\Pi_G\left(x^0, X^1\right) \geq (>)\, \Pi_G\left(x^{0\dagger}, X^{1\dagger}\right)$$

for every (full-support) CDF G with $G(0) = 0$.

In principle, there could be mechanisms that are dominated, but not strongly dominated. We now argue that this is not the case; in particular, our characterisation of undominated mechanisms remains valid if dominance is replaced by strong dominance. First, Lemma 1 (p. 13) remains valid because the proof strictly improves the principal's flow payoff on a non-null set of times. The strengthening of Theorem 1 stated and proved in appendix A asserts precisely that any mechanism in which the agent is not always indifferent is strongly dominated.

Similarly, the improvements constructed in the proofs of Theorem 2 in appendix B and of Theorem 3(a) in appendix F ensure a strict payoff increase under any full-support distribution, making these results applicable also to strong dominance. Finally, our result Proposition 4 (p. 24) concerning transfers clearly does not hinge on the exact definition of dominance.

Remark 6. An intermediate notion of dominance would require a strict improvement only for *some* full-support distribution. This concept coincides with dominance as defined in the text (p. 13),[72] so admits the same characterisation.

A stronger notion would require a strict improvement only for some distribution close to a salient target distribution $G^\star$. This is natural if the principal is confident in her model $G^\star$ of the world, so willing to contemplate only minor model misspecification. This concept is intermediate between dominance and strong dominance, so yields the same results as they do.

L Revelation principle

In this appendix, we state and prove the revelation principle for our model. We begin with some definitions.

Consider an arbitrary mechanism (x^0, X^1), not necessarily incentive-compatible. Call an agent who observes the breakthrough at time t the *type-t* agent. The type-t agent's payoff from disclosing at $t' \in [t, \infty]$ (where $t' = \infty$ means 'never disclose') is

$$r \int_t^{t'} e^{-r(s-t)} x_s^0 \mathrm{d}s + \mathbf{1}_{\mathbf{R}_+}(t') e^{-r(t'-t)} X_{t'}^1 = X_t^0 + e^{-r(t'-t)} \left(X_{t'}^1 - X_{t'}^0 \right),$$

where by convention $e^{-r\times\infty}(X_\infty^1 - X_\infty^0) \coloneqq 0$. So she chooses $t' \in [t, \infty]$ to maximise $\phi : [0, \infty] \to \mathbf{R}_+$ defined by

$$\phi(t') \coloneqq e^{-rt'} \left(X_{t'}^1 - X_{t'}^0 \right) \quad \text{for every } t' \in [0, \infty].$$

We restrict attention to mechanisms (x^0, X^1) for which the agent's disclosure problem admits a solution. Write $\Phi : \mathbf{R}_+ \to \mathbf{R}_+$ for the value function:

$$\Phi(t) \coloneqq \max_{t' \in [t,\infty]} \phi(t') \quad \text{for every } t \in \mathbf{R}_+.$$

[72] If a mechanism does strictly better than another under some distribution G, then the former does strictly better for some (nearby) full-support distribution since the principal's payoff is continuous in the topology of weak convergence.

Observe that Φ is decreasing.

A *(disclosure) strategy* is a map $\sigma : \mathbf{R}_+ \to \mathbf{R}_+$ such that $\sigma(t) \geq t$ for every $t \in \mathbf{R}_+$. A strategy σ is *optimal* for the mechanism (x^0, X^0) iff

$$\phi(\sigma(t)) = \Phi(t) \quad \text{for every } t \in \mathbf{R}_+.$$

The *prompt strategy* is the identity $\sigma(t) \equiv t$. Observe that (x^0, X^0) is incentive-compatible exactly if the prompt strategy is optimal, i.e. $\Phi = \phi$.

Revelation principle. For any mechanism and optimal strategy for it, there is an incentive-compatible mechanism in which every type of the agent earns the same payoff as under the original mechanism and strategy.

If the principal commits to start using the new technology only at $\sigma(t)$ following disclosure at t, then her payoff is also the same in the two mechanisms. If she instead uses the new technology right away, then she is better off under the incentive-compatible mechanism.

The statement can be generalised in the standard fashion to assert that any equilibrium outcome of any game form is a truthful equilibrium of a 'direct revelation' game form.

Proof of the revelation principle. Fix a mechanism (x^0, X^1), with disclosure payoff ϕ and value Φ, and let σ be an optimal strategy for it. Consider the alternative mechanism $(x^0, X^{1\dagger})$ defined by

$$X_t^{1\dagger} := X_t^0 + e^{-r[\sigma(t)-t]}\left(X^1_{\sigma(t)} - X^0_{\sigma(t)}\right) \quad \text{for every } t \in \mathbf{R}_+,$$

and write $\phi^\dagger$ for its disclosure payoff and $\Phi^\dagger$ for its value. By inspection,

$$\phi^\dagger(t) = e^{-rt}\left(X_t^{1\dagger} - X_t^0\right) = e^{-r\sigma(t)}\left(X^1_{\sigma(t)} - X^0_{\sigma(t)}\right) \quad \text{for every } t \in \mathbf{R}_+.$$

Since σ is an optimal strategy for the original mechanism, we have

$$\phi^\dagger(t) = e^{-r\sigma(t)}\left(X^1_{\sigma(t)} - X^0_{\sigma(t)}\right) = \Phi(t) \quad \text{for every } t \in \mathbf{R}_+,$$

which is to say that every type t of the agent earns the same payoff from the prompt strategy under the new mechanism $(x^0, X^{1\dagger})$ as she earns from the strategy σ under the old mechanism (x^0, X^1). Furthermore, the prompt strategy is optimal for the new mechanism since

$$\Phi^\dagger(t) = \max_{t' \in [t,\infty]} \phi^\dagger(t) = \max_{t' \in [t,\infty]} \Phi(t) = \Phi(t) = \phi^\dagger(t) \quad \text{for every } t \in \mathbf{R}_+,$$

where the third equality holds since Φ is decreasing. ■

M If the agent cannot delay disclosure

The agent's ability to delay disclosure is a key feature of our model, and distinguishes our work from that of Bird and Frug (2019).[73] In this appendix, we show that undominated mechanisms look very different if the new technology is assumed to vanish unless instantly disclosed (as in Bird and Frug (2019)), so that delay is impossible.

In this case, a mechanism (x^0, X^1) is incentive-compatible exactly if the agent always prefers (prompt) disclosure to non-disclosure, i.e. $X^1 \geq X^0$: the 'delay' part (a) of incentive-compatibility (p. 11) is absent. This is a knife-edge case: we saw in Remark 1 (p. 11) that if the agent is able to delay disclosure by even an instant, then part (a) of IC remains in full force.

Lemma 1 (p. 13) remains true (with the same proof), so that undominated mechanisms have $x^0 \leq u^0$. The analogue of Theorem 1 (p. 14) is as follows:

Observation 8. Any undominated IC mechanism (x^0, X^1) satisfies $X^1 = \max\{X^0, u^1\}$.

Proof. Let (x^0, X^1) be an IC mechanism with $X_t^1 \neq \max\{X_t^0, u^1\}$ for some period $t \in \mathbf{R}_+$. Consider the alternative mechanism $(x^0, X^{1\dagger})$, where $X_t^{1\dagger} := \max\{X_t^0, u^1\}$ and $X_s^{1\dagger} := X_s^1$ for $s \neq t$. This mechanism is clearly IC, and the principal earns the same payoff if the breakthrough occurs at $\tau \neq t$. She earns a strictly higher payoff if the breakthrough occurs at t:

– If $X_t^0 > u^1$, then $X_t^1 > X_t^0$ since $X_t^1 \geq X_t^0$ by IC, and

$$X_t^1 \neq \max\left\{X_t^0, u^1\right\} = X_t^0 \quad \text{by hypothesis.}$$

Thus $u^1 < X_t^0 = X_t^{1\dagger} < X_t^1$, so that

$$F^1\left(X_t^{1\dagger}\right) > F^1\left(X_t^1\right) \quad \text{since } F^1 \text{ is strictly decreasing on } \left[u^1, \infty\right).$$

– If $X_t^0 \leq u^1$, then $X_t^{1\dagger} = u^1 = \max\{X_t^0, u^1\} \neq X_t^1$, so that

$$F^1\left(X^{1\dagger}\right) > F^1\left(X_t^1\right) \quad \text{since } F^1 \text{ is uniquely maximised at } u^1. \quad \blacksquare$$

In the affine case, we have the following analogue of Theorem 2 (p. 17):

[73] There are other differences. In their model, the new utility possibility frontier is affine and everywhere downward-sloping, so that $u^1 = u^\star = 0$. The richness of their environment stems from the fact that new technologies continually arrive over time.

Observation 9. If the old frontier F^0 is affine on $[0, u^0]$, then any undominated mechanism (x^0, X^1) has the form

$$x_t^0 = \begin{cases} u^0 & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases} \quad \text{for some deadline } T \in [0, \infty] \tag{21}$$

and $X^1 = \max\{X^0, u^1\}$.

There are two differences from undominated mechanisms in our model. First, the agent's pre-disclosure utility after the deadline T is 0 (the lowest possible) rather than $u^\star$ (where the frontier gap $F^1 - F^0$ is largest). Secondly, the agent is not always indifferent between disclosure and non-disclosure: in particular, she strictly prefers disclosure if the breakthrough occurs late enough that $X^1 < u^1$.

The mechanisms in Observation 9 fail to be incentive-compatible if the agent can delay disclosure. To see why, suppose that the breakthrough occurs in a period t just before the deadline T, so that $X_t^0 \simeq 0 < u^1$. Then $X^1 = u^1$ on $[t, \infty)$, so that the agent earns the same reward no matter when she discloses. The agent then strictly prefers to delay disclosure until T because this permits her to collect the high flow payoff $u^0 > u^1$ in the interim.

Proof of Observation 9. Since F^0 is affine, the principal's payoff from a mechanism (x^0, X^1) under a distribution G with $G(0) = 0$ may be written[74]

$$\Pi_G\left(x^0, X^1\right) = \mathbf{E}_G\left(F^0\left(X_0^0\right) + e^{-r\tau}\phi\left(X_\tau^0\right)\right),$$

where $\phi : [0, \infty) \to \mathbf{R}$ is defined by

$$\phi(u) \coloneqq F^1\left(\max\left\{u, u^1\right\}\right) - F^0(u) \quad \text{for each } u \in [0, \infty).$$

By inspection, ϕ is strictly decreasing on $[0, u^0]$.

Take any mechanism (x^0, X^1), assuming without loss of generality that $x^0 \leq u^0$ and that $X^1 = \max\{X^0, u^1\}$, and suppose that (x^0, X^1) does not satisfy (21); we shall show that it is dominated. Consider the mechanism $(x^{0\dagger}, X^{1\dagger})$ defined by

$$x_t^{0\dagger} \coloneqq \begin{cases} u^0 & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases}$$

[74] To see this in detail, adapt the proof of Observation 2 in appendix B (p. 34).

and $X^{1\dagger} := \max\{X^{0\dagger}, u^1\}$, where T is such that $X_0^{0\dagger} = X_0^0$. It is easy to see that $X^{0\dagger} \leq X^0$,[75] and there must be periods in which $X^{0\dagger} \neq X^0$ since x^0 does not have the form (21). Since ϕ is strictly decreasing, it follows that

$$\Pi_G\left(x^{0\dagger}, X^{1\dagger}\right) \geq \Pi_G\left(x^0, X^1\right) \quad \text{for any distribution } G \text{ with } G(0) = 0,$$

and the inequality is strict for any distribution G that assigns positive probability to those times at which $X^{0\dagger} \neq X^0$. ■

N Proof of the Euler lemma (appendix G, p. 41)

Write $\mathcal{X}$ for the space of measurable functions $\mathbf{R}_+ \to [0, u^0]$, and note that it is convex. For a given breakthrough distribution G, and define $\pi_G : \mathcal{X} \to \mathbf{R}$ by

$$\pi_G(x) := \Pi_G(x, X) = \mathbf{E}_G\left(\int_0^\tau re^{-rs} F^0(x_s)\mathrm{d}s + e^{-r\tau} F^1(X_\tau)\right).$$

For $j \in \{0, 1\}$, let $F^{j\prime}(u, u')$ denote the directional derivative of F^j at u in direction u'.[76] Write

$$\mathrm{D}\pi_G\left(x, x^\dagger - x\right) := \lim_{\alpha \downarrow 0} \frac{\pi_G\left(x + \alpha\left[x^\dagger - x\right]\right) - \pi_G(x)}{\alpha}$$

for the Gateaux derivative of π_G at x in direction $x^\dagger - x$.

The proof of the Euler lemma relies on the following lemma, proved at the end of this section.

Gateaux lemma. For any $x, x^\dagger \in \mathcal{X}$ and bounded measurable $\phi^0, \phi^1 : \mathbf{R}_+ \to \mathbf{R}$, we have

$$\begin{aligned} \mathrm{D}\pi_G(x, x^\dagger - x) = &\int_0^\infty re^{-rt}\left([1 - G(t)]\phi^0(t) + \int_{[0,t]} \phi^1 \mathrm{d}G\right)\left(x_t^\dagger - x_t\right)\mathrm{d}t \\ &+ \mathbf{E}_G\left(\int_0^\tau re^{-rt}\left[F^{0\prime}\left(x_t, x_t^\dagger\right) - \phi^0(t)\right]\left[x_t^\dagger - x_t\right]\mathrm{d}t\right) \\ &+ \mathbf{E}_G\left(e^{-r\tau}\left[F^{1\prime}\left(X_\tau, X_\tau^\dagger\right) - \phi^1(\tau)\right]\left[X_\tau^\dagger - X_\tau\right]\right). \end{aligned} \tag{D}$$

Proof of the Euler lemma. Fix a distribution G with $G(0) = 0$, and assume that the right-derivatives of F^0, F^1 at $u = 0$ are finite. For sufficiency, let

[75] Adapt the argument for the claim in the proof of Theorem 2 (appendix B, p. 35).
[76] Equal to the left-hand (right-hand) derivative for $u' <(>)\, u$, and zero for $u' = u$.

(x, X) be an undominated mechanism that satisfies the Euler equation with supergradients ϕ^0 and ϕ^1. Note that $x \in \mathcal{X}$ by Lemma 1 (p. 13). Fix any $x^\dagger \in \mathcal{X}$; we must show that $\pi_G(x^\dagger) \leq \pi_G(x)$. Note that π_G is concave since F^0, F^1 are and the map $x \mapsto X$ is linear, so that for any $\alpha \in (0,1)$ we have

$$\frac{\pi_G\left(x + \alpha\left(x^\dagger - x\right)\right) - \pi_G(x)}{\alpha} \geq \pi_G\left(x^\dagger\right) - \pi_G(x).$$

Taking the limit as $\alpha \downarrow 0$ yields

$$\mathrm{D}\pi_G\left(x, x^\dagger - x\right) \geq \pi_G\left(x^\dagger\right) - \pi_G(x).$$

It therefore suffices to show that $\mathrm{D}\pi_G(x, x^\dagger - x) \leq 0$. The first term in (D) is non-positive by (E).[77] The second (third) term in (D) is non-positive by definition of ϕ^0 (ϕ^1). Thus $\mathrm{D}\pi_G(x, x^\dagger - x) \leq 0$ by the Gateaux lemma.

For necessity, assume that F^0 and F^1 are differentiable. It is immediate from Lemma 1 (p. 13) that $x \in \mathcal{X}$ is necessary for optimality. For the Euler equation, suppose that (x, X) satisfies $x \in \mathcal{X}$, but not the Euler equation; we will show that it is not optimal for G, i.e. that x does not maximise π_G. To this end, we construct a $x^\dagger \in \mathcal{X}$ such that $\mathrm{D}\pi_G(x, x^\dagger - x) > 0$. This suffices because

$$\mathrm{D}\pi_G\left(x + \alpha\left(x^\dagger - x\right)\right) > \pi_G(x) \quad \text{for } \alpha > 0 \text{ sufficiently small}$$

by definition of $\mathrm{D}\pi_G(x, x^\dagger - x)$.

As (x, X) does not satisfy the Euler equation, it must be that (E) fails on a set $A \subseteq \mathbf{R}_+$ of positive Lebesgue measure, where $\phi^0(t) = F^{0\prime}(x_t)$ and $\phi^1(t) = F^{1\prime}(X_t)$ for each $t \in \mathbf{R}_+$.[78] Let $x^\dagger : \mathbf{R}_+ \to \mathbf{R}_+$ be given by

$$x_t^\dagger = \begin{cases} 0 & \text{if } x_t > 0 \text{ and } [1 - G(t)]F^{0\prime}(x_t) + \int_{[0,t]} F^{1\prime}(X_s)G(\mathrm{d}s) < 0 \\ u^0 & \text{if } x_t < u^0 \text{ and } [1 - G(t)]F^{0\prime}(x_t) + \int_{[0,t]} F^{1\prime}(X_s)G(\mathrm{d}s) > 0 \\ x_t & \text{otherwise.} \end{cases}$$

[77] It is zero whenever $x_t \in (0, u^0)$. When $x_t = u^0$ ($x_t = 0$), the big bracketed term is non-negative (non-positive) by (E) and $x_t^\dagger - x_t \leq (\geq)\ 0$.

[78] Were (E) to hold on a full-measure subset of $\mathbf{R}_+$, then (a version of) (x, X) would satisfy (E) on all of $\mathbf{R}_+$. In particular, x may be altered as follows. Fix $t \in \mathbf{R}_+$ at which (E) fails. Note that, writing $\beta \coloneqq \int_{[0,t]} F^{1\prime}(X_s)G(\mathrm{d}s)$, either $[1 - G(t)]F^{0\prime}(0) + \beta \leq 0$, $[1 - G(t)]F^{0\prime}(u^0) + \beta \geq 0$, or $\beta < \infty$, $G(t) < 1$ and $F^{0\prime}(u^0) < \beta/[1 - G(t)] < F^{0\prime}(0)$. In the first case, let $x_t = 0$. In the second, let $x_t = u^0$. In the last, since F^0 is (continuously) differentiable on $(0, u^0)$, we may pick $x_t \in (0, u^0)$ such that $F^{0\prime}(x_t) = \beta/[1 - G(t)]$.

Clearly $x^\dagger$ is measurable and $\leq u^0$, so belongs to $\mathcal{X}$. Since A is non-null, the first term in (D) is strictly positive, while the second and third terms are zero, so that $\mathrm{D}\pi_G(x, x^\dagger - x) > 0$ by the Gateaux lemma. ■

To prove the Gateaux lemma, we shall use the following standard integration-by-parts result:[79]

IBP lemma. Let ν be a finite measure on $\mathbf{R}_+$, and let L be a ν-integrable function $\mathbf{R}_+ \to \mathbf{R}$ satisfying $L(t) = L(0) + \int_0^t l$ for some integrable $l : \mathbf{R}_+ \to \mathbf{R}$. Then

$$\int_{[0,T]} L \mathrm{d}\nu = L(T)\nu([0,T]) - \int_0^T \nu([0,t]) l(t) \mathrm{d}t \quad \text{for every } T \in \mathbf{R}_+.$$

Proof of the Gateaux lemma. For $\alpha \in (0,1)$, define $h_\alpha^0 : \mathbf{R}_+ \to \mathbf{R}$ by

$$h_\alpha^0(t) := re^{-rt} \frac{F^0\Big(x_t + \alpha\Big(x_t^\dagger - x_t\Big)\Big) - F^0(x_t)}{\alpha}.$$

Note that h_α^0 is measurable, with pointwise limit

$$h^0(t) := re^{-rt} F^{0\prime}\Big(x_t, x_t^\dagger\Big)\Big(x_t^\dagger - x_t\Big)$$

as $\alpha \downarrow 0$. Since F^0 is concave with finite right-derivative at $u = 0$, it is K^0-Lipschitz on $\left[0, u^0\right]$ for some $K^0 > 0$. Thus

$$\left|h_\alpha^0(t)\right| \leq re^{-rt} K^0 \left|x_t - x_t^\dagger\right| \leq re^{-rt} K^0 u^0 \quad \text{for all } \alpha > 0 \text{ and } t \in \mathbf{R}_+, \tag{22}$$

whence

$$\lim_{\alpha \downarrow 0} \int_0^t h_\alpha^0 = \int_0^t h^0 \quad \text{for every } t \in \mathbf{R}_+.$$

by the dominated convergence theorem. (22) further yields that

$$\left| \int_0^t h_\alpha^0 \right| \leq rK^0 u^0 \quad \text{for every } \alpha > 0 \text{ and } t \in \mathbf{R}_+,$$

so that by dominated convergence again,

$$\lim_{\alpha \downarrow 0} \mathbf{E}_G\left(\int_0^\tau h_\alpha^0\right) = \mathbf{E}_G\left(\int_0^\tau h^0\right). \tag{23}$$

[79] See e.g. Theorem 18.4 in Billingsley (1995, p. 236).

Similarly, define $h^1_\alpha : \mathbf{R}_+ \to \mathbf{R}$ for $\alpha \in (0,1)$ by

$$h^1_\alpha(t) \coloneqq e^{-rt} \frac{F^1\Big(X_t + \alpha\Big(X^\dagger_t - X_t\Big)\Big) - F^1(X_t)}{\alpha}.$$

These functions are measurable, and have pointwise limit

$$h^1(t) \coloneqq e^{-rt} F^{1\prime}\Big(X_t, X^\dagger_t\Big)\Big(X^\dagger_t - X_t\Big)$$

as $\alpha \downarrow 0$. F^1 is Lipschitz continuous on $\big[0, u^0\big]$ since it is concave with finite right-derivative at $u = 0$, so the family $(h^1_\alpha)_{\alpha\in(0,1)}$ is uniformly bounded. Thus

$$\lim_{\alpha\downarrow 0} \mathbf{E}_G\Big(h^1_\alpha(\tau)\Big) = \mathbf{E}_G\Big(h^1(\tau)\Big) \tag{24}$$

by dominated convergence.

Using (23) and (24), compute

$$\begin{aligned}
&\mathrm{D}\pi_G\Big(x, x^\dagger - x\Big) \\
&\quad = \lim_{\alpha\downarrow 0}\Big[\mathbf{E}_G\Big(\int_0^\tau h^0_\alpha\Big) + \mathbf{E}_G\Big(h^1_\alpha(\tau)\Big)\Big] \\
&\quad = \mathbf{E}_G\Big(\int_0^\tau h^0\Big) + \mathbf{E}_G\Big(h^1(\tau)\Big) \\
&\quad = \mathbf{E}_G\Big(\int_0^\tau re^{-rs}\phi^0(s)\Big[x^\dagger_s - x_s\Big]\mathrm{d}s\Big) + \mathbf{E}_G\Big(e^{-r\tau}\phi^1(\tau)\Big[X^\dagger_\tau - X_\tau\Big]\Big) \\
&\qquad + \mathbf{E}_G\Big(\int_0^\tau re^{-rt}\Big[F^{0\prime}\Big(x_t, x^\dagger_t\Big) - \phi^0(t)\Big]\Big(x_t - x^\dagger_t\Big)\mathrm{d}t\Big) \\
&\qquad + \mathbf{E}_G\Big(e^{-r\tau}\Big[F^{1\prime}\Big(X_\tau, X^\dagger_\tau\Big) - \phi^1(\tau)\Big]\Big[X^\dagger_\tau - X_\tau\Big]\Big).
\end{aligned}$$

It remains only to show that

$$\begin{aligned}
&\mathbf{E}_G\Big(\int_0^\tau re^{-rs}\phi^0(s)\Big[x^\dagger_s - x_s\Big]\mathrm{d}s\Big) \\
&\qquad\qquad = \int_0^\infty re^{-rt}[1 - G(t)]\phi^0(t)\Big(x^\dagger_t - x_t\Big)\mathrm{d}t
\end{aligned} \tag{25}$$

and that

$$\mathbf{E}_G\Big(e^{-r\tau}\phi^1(\tau)\Big[X^\dagger_\tau - X_\tau\Big]\Big) = \int_0^\infty re^{-rt}\left(\int_{[0,t]}\phi^1\mathrm{d}G\right)\Big(x^\dagger_t - x_t\Big)\mathrm{d}t. \tag{26}$$

To obtain (25), note that $l : \mathbf{R}_+ \to \mathbf{R}$ given by

$$l(t) \coloneqq re^{-rt}\phi^0(t)\Big(x^\dagger_t - x_t\Big) \quad \text{for each } t \in \mathbf{R}_+$$

is integrable since ϕ^0 is bounded and $x_t, x_t^\dagger$ take values in $[0, u^0]$. We may therefore apply IBP lemma to l and the measure ν associated with G to obtain

$$\begin{aligned}
&\int_{[0,T]} \left(\int_0^t re^{-rs} \phi^0(s) \left[x_s^\dagger - x_s \right] \mathrm{d}s \right) G(\mathrm{d}t) \\
&\qquad = \int_0^T re^{-rs} \phi^0(s) \left[x_s^\dagger - x_s \right] \mathrm{d}s \times G(T) \\
&\qquad\quad - \int_0^T G(t) re^{-rt} \phi^0(t) \left[x_t^\dagger - x_t \right] \mathrm{d}t \\
&\qquad = \int_0^T [G(T) - G(t)] re^{-rt} \phi^0(t) \left[x_t^\dagger - x_t \right] \mathrm{d}t \quad \text{for any } T \in \mathbf{R}_+,
\end{aligned}$$

which yields (25) as $T \to \infty$.

To derive (26), define a measure ν_+ on $\mathbf{R}_+$ by

$$\nu_+(A) := \int_A \max\left\{ \phi^1(t), 0 \right\} G(\mathrm{d}t) \quad \text{for any measurable } A \subseteq \mathbf{R}_+.$$

Since ϕ^1 is bounded, ν_+ is absolutely continuous with respect to (the measure associated with) G, with Radon–Nikodým derivative $t \mapsto \max\{\phi^1(t), 0\}$. Furthermore, the function $L : \mathbf{R}_+ \to \mathbf{R}$ defined by

$$L(t) := e^{-rt} \left(X_t^\dagger - X_t \right) \quad \text{for each } t \in \mathbf{R}_+$$

is ν_+-integrable since $X^\dagger$ and X are bounded. Thus for any $T \in \mathbf{R}_+$, we have

$$\int_{[0,T]} L \max\left\{ \phi^1, 0 \right\} \mathrm{d}G = \int_{[0,T]} L \mathrm{d}\nu_+.$$

Furthermore, L satisfies $L(t) = L(0) + \int_0^t l$, where $l : \mathbf{R}_+ \to \mathbf{R}$ is the integrable function given by

$$l(t) := -e^{-rt} \left(x_t^\dagger - x_t \right) \quad \text{for every } t \in \mathbf{R}_+.$$

Thus for every $T \in \mathbf{R}_+$, the IBP lemma yields

$$\begin{aligned}
&\int_{[0,T]} e^{-rt} \left(X_t^\dagger - X_t \right) \nu_+(\mathrm{d}t) \\
&\qquad = e^{-rT} \left(X_T^\dagger - X_T \right) \nu_+([0,T]) + \int_0^T \nu_+([0,t]) re^{-rt} \left(x_t^\dagger - x_t \right) \mathrm{d}t.
\end{aligned}$$

Applying the same argument to the measure

$$\nu_-(A) := -\int_A \min\Big\{\phi^1(t), 0\Big\} G(\mathrm{d}t)$$

and subtracting yields, for each $T \in \mathbf{R}_+$,

$$\begin{aligned}&\int_{[0,T]} e^{-rt}\Big(X_t^\dagger - X_t\Big)\phi^1(t)G(\mathrm{d}t)\\ &\quad = e^{-rT}\Big(X_T^\dagger - X_T\Big)\int_{[0,T]}\phi^1 \mathrm{d}G + \int_0^T\left(\int_{[0,t]}\phi^1\mathrm{d}G\right) re^{-rt}\Big(x_t^\dagger - x_t\Big)\mathrm{d}t.\end{aligned}$$

We obtain (26) upon letting $T \to \infty$. ■

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