

New-Keynesian Trade: Understanding the Employment and Welfare Effects of Trade Shocks*

Andrés Rodríguez-Clare

Mauricio Ulate

José P. Vásquez

March 2021

There is a growing empirical consensus that trade shocks can have important effects on unemployment and nonemployment across local-labor markets within an economy. This paper introduces downward nominal wage rigidity to an otherwise standard quantitative trade model and shows how this framework can generate changes in unemployment and nonemployment that match those uncovered by the empirical literature studying the “China shock.” We also compare the associated welfare effects predicted by this model with those in the model without unemployment. We find that the China shock leads to average welfare increases in most U.S. states, including many that experience unemployment during the transition. However, nominal rigidities reduce the overall U.S. gains from the China shock between one and two thirds. In addition, there are ten states that experience welfare losses in the presence of downward nominal wage rigidity but would have experienced welfare gains without it.

*Andrés Rodríguez-Clare: U.C. Berkeley and NBER, Mauricio Ulate: Federal Reserve Bank of San Francisco, José P. Vásquez: Princeton. We thank Nick Sander, Lorenzo Caliendo, Pete Klenow, Emi Nakamura, Jón Steinsson, Stephanie Schmitt-Grohé, Marc Melitz, Samuel Kortum, Cecile Gaubert, Jonathan Eaton, Oleg itskhoki, Charles Engel, Dave Donalson, Olivier Blanchard, Michael Waugh, Martin Beraja, Ben Faber, Thibault Fally, Yuriy Gorodnichenko, Pierre-Olivier Gourinchas, Mark Bills, David Sraer, Meredith Startz, Matilde Bombardini, Matthew Grant, and Matthias Hoelzlein, as well as seminar participants at Berkeley, Boston College, the FRBSF, NBER, Stanford, UCSC, Toronto, and the West Coast Trade Conference for helpful comments and suggestions. We also thank Marco Badilla for excellent research assistance. This material is based upon work supported by the National Science Foundation under Grant Number 1561854. Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the National Science Foundation, the Federal Reserve Bank of San Francisco, or the Federal Reserve System.

1 Introduction

In their influential paper, [Autor, Dorn, and Hanson \(2013\)](#) (henceforth ADH) show that commuting zones more exposed to the “China shock” experienced significant increases in unemployment and decreases in labor force participation relative to less exposed regions. In contrast, the standard quantitative trade model assumes full employment and a perfectly inelastic labor supply curve (e.g., [Costinot and Rodriguez-Clare, 2014](#)), implying that all the adjustment takes place through wages rather than employment. In this paper, we show how adding downward nominal wage rigidity (DNWR) and home production allows the quantitative trade model to generate changes in unemployment and nonemployment that match those uncovered by ADH during a transition period. We can then use the augmented model with DNWR to study the welfare effects of the China shock for the U.S. on aggregate, as well as the distribution of welfare changes across individual states.

We start from the standard gravity model of trade with multiple sectors and an input-output structure as in [Caliendo and Parro \(2015\)](#). We further assume that there are multiple regions inside the U.S., that there is no labor mobility across regions, and that labor supply is upward sloping because workers decide whether to engage in home production or participate in the labor market. The way in which the China shock affects employment here is clear: regions with positive net exports in sectors experiencing the strongest productivity increases in China (i.e., more exposed to the China shock) suffer a worsening of their terms of trade relative to less exposed regions. This leads to a relative decline in their real wage and employment, as some workers exit the labor force to engage in home production. This model, however, requires an extremely large elasticity of substitution between employment and home production in order to replicate the strong increases in nonemployment in the U.S. regions most exposed to the China shock documented by ADH. Additionally, this model cannot generate any changes in unemployment.

We then add DNWR as in [Schmitt-Grohe and Uribe \(2016\)](#), so that the wage in each region has to be greater or equal that δ times the wage in the previous period (implying that the wage can fall by no more than $100(1 - \delta)$ percent each year).¹ The exact implications of DNWR

¹While the empirical literature studying the presence of DNWR is vast and sometimes conflicting, several recent

depend on our assumptions regarding monetary policy and the exchange rate system, but our results are broadly robust to a range of different assumptions for these features. Our baseline model assumes that all countries have flexible exchange rates vis-à-vis the dollar, and that the world nominal GDP in dollars grows at a constant and exogenous rate which we set equal to zero without loss of generality. A region that is more exposed to China suffers a negative terms-of-trade shock relative to a less exposed region. In turn, a negative terms-of-trade shock implies a contraction in labor demand, and if the DNWR is stringent enough (i.e., δ is high enough) then this leads to a temporary increase in unemployment that subsequently dies out as nominal wages can adjust downwards. If home production is available to workers, then the DNWR leads to even bigger declines in employment as more workers prefer to engage in home production rather than face the possibility of unemployment.

We quantify the effects of the China shock in our model using the “exact-hat algebra” approach to counterfactual analysis popularized by [Dekle et al. \(2007\)](#) and extended to a dynamic context by [Caliendo et al. \(2019\)](#). This methodology ensures that our model perfectly matches the sector-level input-output and trade data at the beginning of the period of analysis (year 2000), and allows for a transparent calibration. Specifically, we only need to calibrate the trade elasticity and the parameters governing the elasticity of labor supply (κ) and the importance of the DNWR (δ). We select a value of the trade elasticity from the literature, and we calibrate κ and δ so that the model matches two key moments from ADH. These moments correspond to the relative increases in unemployment rates and decreases in labor-force participation rates experienced by commuting zones that were more exposed to imports from China.

Our quantitative analysis requires data for sector-level input-output flows as well as trade flows across all pairs of regions in our sample, which includes U.S. states as local labor markets. We construct such a dataset by combining multiple data sources, a set of proportionality assumptions, and implications from the gravity model. The resulting dataset contains 14 sectors (12 of them in manufacturing, one service sector, and one agricultural sector) for 50 U.S. states plus 36 additional countries and an aggregate rest of the world region during the years

papers have found support for its existence (e.g. [Dickens et al., 2007](#); [Daly and Hobijn, 2014](#); [Grigsby et al., 2019](#); [Hazell and Taska, 2019](#)). We fully acknowledge that labor-market frictions in the real world are significantly more complex than our simple DNWR. However, we hope to show that the DNWR in our model is a powerful yet parsimonious way to capture such frictions, which so far have been under-explored in the trade literature.

2000 to 2007, the period used in ADH.

As is common in the literature, we treat the China shock as a productivity improvement that varies across sectors in China, and we follow [Caliendo et al. \(2019\)](#) in calibrating these sector-level productivity shocks so that the model-implied changes in imports from China match those in the data. We do this each year so that we can trace out the dynamic response of the economy to the China shock as it unfolded over the period of analysis. Our calibration leads to a value of $\delta = 0.984$, which implies that wages can fall up to 1.6% annually without the DNWR becoming binding. This value is similar to the one used by [Schmitt-Grohe and Uribe \(2016\)](#). Our calibration also leads to a value of $\kappa = 5.9$, implying a labor supply elasticity of around 2. Despite the fact that labor supply elasticity is the only mechanism in our model to generate the large effects of the China shock on labor-force participation, our estimate still falls within a reasonable range.

The calibrated model generates a significant temporary decline in employment in the regions most exposed to the China shock. For example, Ohio experiences a decline in employment of 3.5% in 2004, but eventually goes back to a level of employment above the one before the shock. This is the typical dynamic response we see for the most exposed states, and it arises from the combination of three forces. First, the China shock that we introduce in the model (calibrated for the 2001-2007 period) is not constant but grows in strength, peaking in 2004. Second, a shock that requires a decline in the nominal wage to maintain full employment increases unemployment in the short run under a DNWR, but then this unemployment erodes quickly as the nominal wage can fall around 1.6% each year. Third, the China shock leads to increases in the real wage for almost all regions, including most of the ones for which full employment would require a decline in the nominal wage.² Since the real wage governs labor supply, and since there is no unemployment in the long run, this implies an increase in employment after the economy fully adjusts to the China shock.

For the U.S. as a whole, the calibrated model implies that the China shock is responsible for around 0.8 percentage points of unemployment in the U.S. in 2004. Combined with a decline in labor supply, this leads to an overall decline in the employment to population ratio of 1.8%.

²This implies that most states experience both an increase in the real wage and an increase in unemployment. This may seem paradoxical, but it is a natural consequence of a shock that implies both an improvement in the terms of trade and a decline in the export price index. In Section 4.3, we come back to this and provide more intuition.

However, in parallel to what happens at the state level, by 2008 the employment to population rate is slightly higher than it was before the shock.

One benefit of our approach is that we can study the effect of the China shock on welfare, and in particular explore how this is affected by DNWR. We compute welfare as the present discounted value of the utility flow in the future, with a discount rate of 0.95 and a utility flow given by the average real wage across all households in an economy (employed, unemployed, and in home production). We find that welfare increases in most regions, including many that experience unemployment during the transition. For the U.S. as a whole, although the China shock remains beneficial in the presence of nominal frictions, those benefits are smaller with these frictions. Specifically, DNWR reduces the average U.S. welfare gain from 0.32% to 0.22% (and the reduction is even larger in some of our extensions). Additionally, there are 10 states that suffer welfare losses with DNWR but would have experienced welfare gains without it. To see how DNWR matters for welfare in some of the most affected states, consider again Ohio. If we compute the welfare effect under the same China shock and the same parameter values except that we switch off the DNWR (i.e., $\delta = 0$), we see that welfare increases by 8 basis points in Ohio, rather than decreasing by 8 basis points as in the model with DNWR.

We study how varying some of the key assumptions in the baseline model affects the above conclusions, as well as the fit of the model with the non-targeted moments in ADH. Specifically, we develop four separate extensions where we: introduce mobility frictions between sectors, consider some of the increases in trade surpluses that occurred in China, use a different exchange rate regime for other countries, and examine a different nominal anchor. The most interesting conclusion that emerges from this analysis is that the model benefits from the addition of frictions to the mobility of workers between manufacturing and non-manufacturing, jointly with the assumption that there is DNWR only in manufacturing. These modifications allow the model to more closely match ADH's results on how exposure to the China shock affects employment and wages in the manufacturing and non-manufacturing sectors. Interestingly, this variant of the model implies that DNWR leads to an even larger reduction in the overall U.S. gains from the China shock, which fall from 0.22% in the baseline model to 0.12%.

Our extensions also show that taking a broader perspective of the China shock so that it also includes an increase in the U.S. trade deficit (instead of solely an increase in Chinese

productivity) makes the DNWR less binding and *weakens* the implied employment effects, although the welfare effects do not change much. In addition, when we assume that other countries have fixed exchange rates vis-a-vis the dollar, the calibration leads to a more stringent DNWR, with δ increasing from 0.984 to 0.989, but the conclusions regarding employment and welfare do not change. In contrast, if we assume that the nominal anchor is a constant growth rate of the U.S. nominal GDP instead of the world's nominal GDP, then the model cannot fully match the ADH findings.³ This last result indicates that taking a narrow view of nominal frictions that focuses entirely on the U.S. might not be enough to properly capture how the economy reacts to real trade shocks (we discuss this in Section 5.3).

We wish to highlight one finding that is relevant for how we think about the exposure measures that have been used in empirical work. ADH measure a region's exposure to the China shock using the region's sectoral employment shares as weights for the sector-level shocks. In the absence of nominal rigidities, however, the theoretically correct exposure measure would weight the sector-level shocks by the region's sector-level net exports rather than employment. We find that the ADH employment-weighted measure becomes relevant in a standard multi-sector gravity model of trade once we add DNWR, since now labor demand has a direct effect on welfare above and beyond those that go through the standard terms-of-trade channel. This is suggestive that DNWR, or other frictions that deliver similar effects, should feature more prominently in trade models, at least when the goal is to understand the adjustment dynamics associated with trade shocks.

Our paper follows in the footsteps of a large literature that analyzes the impacts of trade shocks on different regions or countries. Papers such as ADH, [Caliendo et al. \(2019\)](#), [Galle et al. \(2020\)](#), and [Adao et al. \(2019\)](#) focus on the effect of the China shock on regions of the U.S. Our model incorporates nominal rigidities as a mechanism to deliver involuntary unemployment, which is an uncommon feature in this literature despite its prominence in the empirical papers studying the China shock. To focus on the role of nominal rigidities on employment dynamics, our model does not feature costs of switching sectors, which are explored in [Caliendo et al. \(2019\)](#). We expand on the possible consequences of including such costs in Section 6.5.

³Although this extension cannot perfectly match the ADH findings, the overall pattern of results is broadly similar to the one under the baseline calibration, with the exception that the mean welfare change is smaller.

Another literature explores the effect of trade on unemployment in models with search and matching frictions, see e.g. Davidson and Matusz (2004), Helpman et al. (2010), Hasan et al. (2012) and Heid and Larch (2016). Most recently, Kim and Vogel (2020a,b) introduce search frictions and a labor-leisure choice into a multi-sector trade model where each commuting zone is treated as a small-open economy affected by the China shock. They study how this model can match the ADH findings for the effect of the China shock on income per capita decomposed into the effect on wages, labor supply, and unemployment. We instead focus on DNWR as the friction that generates unemployment, and emphasize the employment and welfare implications of the China shock in a model that allows for intermediate goods and general-equilibrium implications across U.S. states and between these and the rest of the world.⁴

More closely related to our paper is Eaton et al. (2013), which studies the extent to which unmodeled cross-country relative wage rigidities can explain the increases in unemployment and decreases in GDP observed in countries undergoing sudden stops. Relative to this paper, our contribution is to show how DNWR can lead to such relative wage rigidities, to extend the analysis to terms-of-trade shocks in a multi-sector model, and to quantify the effect of the China shock on unemployment and nonemployment across U.S. states between 2000 and 2007.

On the side of open-economy macroeconomics, classic contributions such as Clarida et al. (2002), or various papers by Gali and Monacelli (2005, 2008, 2016), have introduced nominal rigidities in models with a simplified trade structure. Schmitt-Grohe and Uribe (2016) uses a downward nominal wage rigidity to study the effects of trade shocks on a small open economy, Choudhri et al. (2011) studies the implications of nominal rigidities for the gains from trade in a two-country model, and Nakamura and Steinsson (2014), Beraja et al. (2016), or Chodorow-Reich and Wieland (2017) deal with multiple heterogeneous regions in models with nominal rigidities. None of these papers connect to actual sector-level trade flows and hence cannot be used for the quantitative analysis of an event like the China shock.

The rest of the paper proceeds as follows: Section 2 introduces the general framework that incorporates a rich trade structure with dynamic aspects and nominal rigidities. Section

⁴Dix-Carneiro et al. (2020) allow for search and matching frictions in a fully dynamic multi-sector model and explore the effects on workers originally employed in sectors differently exposed to the China shock. The paper does not explore the aggregate effects on employment and unemployment, or how such effects matter for welfare relative to a model without unemployment.

3 describes the data, our calibration of the China shock, and the exposure measure that we use. Section 4 describes the calibration of parameters δ and κ , and the main results under our baseline specification. Section 5 discusses different extensions of the baseline model, such as incorporating mobility frictions between manufacturing and non-manufacturing, incorporating changes in deficits as part of the China shock, or changing the nominal anchor or exchange rate regime. In Section 6, we discuss several miscellaneous topics. Finally, Section 7 concludes.

2 A Quantitative Trade Model with Nominal Rigidities

We present a multi-sector quantitative trade model with an input-output structure as in [Caliendo and Parro \(2015\)](#) but extended to allow for multiple periods, an upward sloping labor supply, and downward nominal wage rigidity. In this section, we present an abridged description of the model, focusing on its non-standard elements and relegating some of the details to [Appendix B](#).

2.1 Basic Assumptions

We assume that the United States is composed of multiple economies or “regions.” There are M regions in the U.S., plus $I - M$ regions (countries) outside of the U.S. (for a total of I regions). To focus on the role of nominal rigidities on employment, we assume that there is no labor mobility across these regions. This is a reasonable assumption given our focus on the short to medium term.

There are S sectors in the economy (indexed by s or k). In each region (indexed by i or j) and each period t , a representative consumer devotes all income to expenditure $P_{j,t}C_{j,t}$, where $C_{j,t}$ and $P_{j,t}$ are aggregate consumption and the price index in region j in period t , respectively. Aggregate consumption is a Cobb-Douglas aggregate of consumption across the S different sectors with expenditure shares $\alpha_{j,s}$. As in a multi-sector Armington trade model, consumption in each sector is a CES aggregate of consumption of the good of each of the I regions, with an elasticity of substitution $\sigma_s > 1$ in sector s .

We allow for the possibility of “broad sectors,” indexed with $b \in 1, \dots, B$. We assume that all individual sectors within a given broad sector have the same wage, and there is free mobility of

workers across the sectors belonging to a broad sector. In contrast, there are mobility frictions and hence potentially different wages across different broad sectors. We use the function $b(k)$ to denote the broad sector b that a given sector k belongs to. The concept of broad sectors allows us to write a single parsimonious model that can still capture different levels at which mobility frictions might occur. Our baseline specification assumes free mobility between all market sectors (i.e., $b(k) = 1 \forall k$, a single broad sector), but in the extensions we explore the consequences of having two broad sectors (one comprising manufacturing and the other one comprising services and agriculture).

Each region produces good k with a Cobb-Douglas production function, using labor with share $\phi_{j,k}$ and intermediate inputs with the shares $\phi_{j,sk}$, where $\phi_{j,k} + \sum_s \phi_{j,sk} = 1$. Additionally, region j has a total factor productivity in sector k and time t equal to $A_{j,k,t}$. We assume perfect competition, and that regions trade with iceberg trade costs $\tau_{ij,k,t} \geq 1$ for exports from i to j in sector k . We also assume that intermediates from different origins are aggregated in the same way as consumption goods (i.e., CES with elasticity σ_s). Let $W_{i,b(k),t}$ denote the wage in region i , in the broad sector $b(k)$ that sector k belongs to, at time t . The previous assumptions imply that the price in region j of good k produced by region i at time t is $\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,b(k),t}^{\phi_{i,k}} \prod_s P_{i,s,t}^{\phi_{i,sk}}$, where $P_{i,s,t}$ is the price index of sector s in region i at time t , and that

$$P_{j,k,t}^{1-\sigma_k} = \sum_{i=1}^I \left(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,b(k),t}^{\phi_{i,k}} \prod_{s=1}^S P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}. \quad (1)$$

For future purposes, note also that

$$P_{i,t} = \prod_{s=1}^S P_{i,s,t}^{\alpha_{i,s}}. \quad (2)$$

2.2 Labor Supply

We denote the total population of region i with \bar{L}_i , which we assume to be time-invariant given the brief period we consider in our analysis. Agents can either engage in home production or look for work in the labor market. If they engage in home production, they receive a utility flow of μ_i . If they participate in the labor market, they can be employed in any of the

B broad sectors. The expected real income from participating in broad sector b is denoted by $\omega_{i,b,t}$. We denote the number of agents that look for work in broad sector b by $\ell_{i,b,t}$ and we let $\pi_{i,t} \equiv \sum_b \ell_{i,b,t} / \bar{L}_i$ and $\pi_{i,b,t} \equiv \ell_{i,b,t} / \sum_{b'} \ell_{i,b',t}$ denote the labor force participation rate and the participation rate in broad sector b , respectively. This implies that

$$\ell_{i,b,t} = \pi_{i,t} \pi_{i,b,t} \bar{L}_i. \quad (3)$$

Each agent has utility parameters z_b for $b \in \{0, 1, \dots, B\}$ so that agent's utility is $\ln \mu_i + z_0$ if choosing home production and $\ln \omega_{i,b,t} + z_b$ if choosing broad sector $b \in \{1, \dots, B\}$. We assume that these utility parameters are drawn from a nested Type-I Extreme Value distribution involving parameters κ and η (satisfying $\eta \geq \kappa$), so that the cumulative distribution of $Z = (Z_0, Z_1, \dots, Z_B)$ is

$$H(z) = \exp \left(- \exp(-\kappa z_0) - \left(\sum_{b=1}^B \exp(-\eta z_b) \right)^{\kappa/\eta} \right).$$

In Appendix B.2 we show that with this setup the following conditions hold

$$\pi_{i,t} = \frac{\omega_{i,t}^\kappa}{\mu_i^\kappa + \omega_{i,t}^\kappa}, \quad (4)$$

and

$$\pi_{i,b,t} = \frac{\omega_{i,b,t}^\eta}{\omega_{i,t}^\eta}, \quad (5)$$

where

$$\omega_{i,t} \equiv \left(\sum_{b=1}^B \omega_{i,b,t}^\eta \right)^{1/\eta}. \quad (6)$$

Our welfare function will be the exponential of the ex-ante instantaneous utility (before the utility shocks are realized), which satisfies

$$u_{i,t} \propto (\mu_i^\kappa + \omega_{i,t}^\kappa)^{1/\kappa}.$$

We consider the exponential of the ex-ante instantaneous utility so that welfare is homogeneous of degree one with respect to wages (and μ_i).

2.3 Downward Nominal Wage Rigidity

We denote the number of agents that are actually employed in region i and broad sector b at time t with $L_{i,b,t}$. In the standard trade model, labor market clearing requires that the labor used in a broad sector in a region be equal to labor supplied to that broad sector, i.e. $L_{i,b,t} \equiv \sum_{s \in b} L_{i,s,t} = \ell_{i,b,t}$. We depart from this assumption and instead follow [Schmitt-Grohe and Uribe \(2016\)](#) by allowing for a downward nominal wage rigidity, which might lead to an employment level that is strictly below labor supply,

$$L_{i,b,t} \leq \ell_{i,b,t}. \quad (7)$$

All prices and wages up to now have been expressed in U.S. dollars. In contrast, a given region faces DNWR in terms of its local currency unit. Letting $W_{i,b,t}^{LCU}$ denote wages in local currency units, DNWR takes the following form:

$$W_{i,b,t}^{LCU} \geq \delta_b W_{i,b,t-1}^{LCU}, \quad \delta_b \geq 0.$$

Denote the exchange rate between the local currency unit of region i and the local currency unit of region 1 (which is the U.S. dollar) in period t with $E_{i,t}$ (in units of dollars per local currency of region i). This implies that $W_{i,b,t} = W_{i,b,t}^{LCU} E_{i,t}$, and hence the DNWR in dollars entails

$$W_{i,b,t} \geq \frac{E_{i,t}}{E_{i,t-1}} \delta_b W_{i,b,t-1}.$$

Since all regions within the U.S. share the dollar as their local currency unit, then $E_{i,t} = 1$ and $W_{i,b,t}^{LCU} = W_{i,b,t} \forall i \leq M$. This means that the DNWR in states of the U.S. takes the familiar form $W_{i,b,t} \geq \delta_b W_{i,b,t-1}$. For the $I - M$ regions outside of the U.S., the LCU is not the dollar and so the behavior of the exchange rate impacts how the DNWR affects the real economy. The DNWR in

dollars can then be captured using a country-specific parameter $\delta_{i,b}$ for each broad sector, i.e.:

$$W_{i,b,t} \geq \delta_{i,b} W_{i,b,t-1}, \quad \delta_{i,b} \geq 0. \quad (8)$$

In our baseline specification we assume that all regions outside of the U.S. have a flexible exchange rate and so the DNWR never binds. We capture these assumptions by setting $\delta_{i,b} = \delta_b \forall i \leq M$ and $\delta_{i,b} = 0 \forall i > M$. In an extension, we consider an alternative scenario in which other countries have fixed exchange rates to the U.S. so that $\delta_{i,b} = \delta_b \forall i$.

Besides equations (7) and (8), we additionally have the complementary slackness condition:

$$(\ell_{i,b,t} - L_{i,b,t})(W_{i,b,t} - \delta_{i,b} W_{i,b,t-1}) = 0. \quad (9)$$

Since we know that people in broad sector b get the real wage of $W_{i,b,t}/P_{i,t}$ with probability $L_{i,b,t}/\ell_{i,b,t}$ we can express the real income from working in broad sector b as

$$\omega_{i,b,t} = \frac{W_{i,b,t} L_{i,b,t}}{P_{i,t} \ell_{i,b,t}}. \quad (10)$$

This assumes that the income generated in a broad sector is equally shared between all people participating in that broad sector, instead of giving zero to unemployed workers and a full wage to employed workers. This simplifies the calculations and represents a form of risk sharing between households in a given broad sector. It is also worth noting that our setup does not allow unemployed workers to engage in home production. As we discuss below, this implies that the threat of unemployment discourages labor force participation, which is a desirable feature that allows the model to match the ADH targets with a reasonable labor supply elasticity.

2.4 Nominal Anchor

So far, we have introduced nominal elements to the model (i.e., the DNWR), but we have not introduced a nominal anchor that prevents nominal wages from rising so much in each period as to make the DNWR always non-binding. Roughly speaking, we now assume that each country has a central bank that is unwilling to allow inflation to be too high because of

associated costs that are left out of the model. In traditional macro models, this is usually implemented via a Taylor rule, where the nominal interest rate reacts to inflation in order to keep price growth in check. Instead, we use a nominal anchor that captures the same idea in a way that naturally lends itself to quantitative implementation in our trade model.

In particular, we assume that world nominal GDP in dollars grows at a constant rate γ across years,

$$\sum_{i=1}^I \sum_{b=1}^B W_{i,b,t} L_{i,b,t} = \gamma \sum_{i=1}^I \sum_{b=1}^B W_{i,b,t-1} L_{i,b,t-1}. \quad (11)$$

Although this nominal anchor might seem simplistic, it nonetheless has some desirable properties. First, it is flexible enough to allow for unemployment even in the context of two countries that have a single region each. Second, it can be seen as capturing a given level of world aggregate demand in the context of a global savings glut (or zero lower bound), in the spirit of papers like [Caballero et al. \(2015\)](#). We further discuss the advantages of this rule as a modeling device in Section 5.3, where we also discuss the consequences of changing the nominal anchor to a constant growth of nominal GDP for the U.S. rather than the whole world.

Consider a shock that requires the relative wage of some region i in broad sector b to fall in order to maintain full employment in that sector-region. The cause could be, for example, a negative productivity shock, an increase in productivity in that sector abroad, or a decline in transfers to the region. If δ_b is low enough, or the exchange rate can depreciate (e.g., $\delta_{i,b}$ is low), then wages can adjust downwards in the required magnitude to avoid unemployment. Alternatively, if γ is high enough then again there would be no unemployment, since no downward adjustment is needed in the wage. However, there are combinations of parameters $\delta_{i,b}$ and γ that can lead to unemployment after the shock, although there would then be a gradual decline in unemployment as the DNWR and the nominal anchor allow for adjustment year after year.

We clarify that having multiple regions is not critical for the shock to lead to unemployment, given our nominal anchor. To see this, imagine that the U.S. were composed of a single region and that there was a positive productivity shock in the rest of the world. If γ was high enough, then the adjustment could take place without unemployment in the U.S., since wages in dollars in the rest of the world could increase enough to generate the necessary relative wage

adjustment. However, if γ is low and δ is high, this full adjustment would not be possible, and there would be (temporary) unemployment in the U.S.

2.5 Equilibrium

Let $R_{i,s,t}$ denote total revenues in sector s of country i . Noting that the demand of industry k of country j of intermediates from sector s is $\phi_{j,sk}R_{j,k,t}$ and allowing for exogenous deficits as in Dekle et al. (2007), the market clearing condition for sector s in country i can be written as

$$R_{i,s,t} = \sum_{j=1}^I \lambda_{ij,s,t} \left(\alpha_{j,s} \left(\sum_{b=1}^B W_{j,b,t} L_{j,b,t} + D_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R_{j,k,t} \right), \quad (12)$$

where $D_{j,t}$ are transfers received by region j , with $\sum_j D_{j,t} = 0$, and where sector- k trade shares in period t are given by

$$\lambda_{ij,k,t} \equiv \frac{(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,b(k),t}^{\phi_{i,k}} \prod_{s=1}^S P_{i,s,t}^{\phi_{i,sk}})^{1-\sigma_k}}{\sum_{r=1}^I (\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,b(k),t}^{\phi_{r,k}} \prod_{s=1}^S P_{r,s,t}^{\phi_{r,sk}})^{1-\sigma_k}}. \quad (13)$$

In turn, labor market clearing in each region and broad sector b requires that

$$W_{i,b,t} L_{i,b,t} = \sum_{s \in b} \phi_{i,s} R_{i,s,t}. \quad (14)$$

Given last-period wages $\{W_{i,b,t-1}\}$ and last period employment $\{L_{i,b,t-1}\}$, the period t equilibrium is a set of wages $\{W_{i,b,t}\}$, employment $\{L_{i,b,t}\}$, trade shares $\{\lambda_{ij,s,t}\}$, country and sector-country prices indices $\{P_{i,t}\}$ and $\{P_{i,s,t}\}$, revenues $\{R_{i,s,t}\}$, and additional labor market variables (i.e., $\ell_{i,b,t}$, $\omega_{i,b,t}$, $\omega_{i,t}$, $\pi_{i,b,t}$, and $\pi_{i,t}$) such that equations (1) - (14) hold.

2.6 Hat Algebra

Our goal is to use a calibrated version of the model above to compute the employment and welfare effects of a trade shock. We do this using data for U.S. states as well as other countries, but without calibrating technology levels and iceberg trade costs along the transition and without requiring data on nominal wages or available labor (since this would require taking a

stance on what efficiency units we measure labor in). To do so, we follow the exact hat algebra methodology of Dekle et al. (2007) and the extension of that methodology to dynamic settings proposed in Caliendo et al. (2019). Consequently, our counterfactual exercises only require data on revenues $R_{i,s,t}$, value added $Y_{i,b,t} \equiv W_{i,b,t}L_{i,b,t}$, trade deficits $D_{i,t}$, the fraction of workers in each broad sector $\pi_{i,b,t}$, and trade shares $\lambda_{ij,s,t}$ in period zero ($t = t_0$), whatever shocks we are interested in, and the model's parameters, namely $\delta_{i,b}$, γ , κ , η , $\{\sigma_s\}$, $\{\alpha_{j,s}\}$, $\{\phi_{i,s}\}$, and $\{\phi_{i,sk}\}$.

We use \hat{x}_t to denote x_t/x_{t-1} for any variable x . To express the equilibrium system in hats and only leave it in terms of observable data in period zero, we assume that the economy starts from a point where every region had full employment.⁵ The equilibrium system is given by:

$$\begin{aligned}
\hat{R}_{i,s,t}R_{i,s,t-1} &= \sum_{j=1}^I \hat{\lambda}_{ij,s,t}\lambda_{ij,s,t-1} \left(\alpha_{j,s} \left(\sum_b \hat{W}_{j,b,t}\hat{L}_{j,b,t}Y_{j,b,t-1} + \hat{D}_{j,t}D_{j,t-1} \right) + \sum_{k=1}^S \phi_{j,sk}\hat{R}_{j,k,t}R_{j,k,t-1} \right) & \forall i, \forall s \\
\hat{\lambda}_{ij,s,t} &= \frac{(\hat{\tau}_{ij,s,t}\hat{A}_{i,s,t}^{-1}\hat{W}_{i,b(s),t}^{\phi_{i,s}} \prod_{k=1}^S \hat{p}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^I \lambda_{rj,s,t-1}(\hat{\tau}_{rj,s,t}\hat{A}_{r,s,t}^{-1}\hat{W}_{r,b(s),t}^{\phi_{r,s}} \prod_{k=1}^S \hat{p}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} & \forall i, \forall s \\
\hat{p}_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \lambda_{ji,s,t-1} \left(\hat{\tau}_{ji,s,t}\hat{A}_{j,s,t}^{-1}\hat{W}_{j,b(s),t}^{\phi_{j,s}} \prod_{k=1}^S \hat{p}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} & \forall i, \forall s \\
\hat{W}_{i,b,t}\hat{L}_{i,b,t}Y_{i,b,t-1} &= \sum_{s \in b} \phi_{i,s}\hat{R}_{i,s,t}R_{i,s,t-1} & \forall i, \forall b \\
\prod_{q=1}^t \hat{L}_{i,b,q} &\leq \prod_{q=1}^t \hat{\ell}_{i,b,q}, \hat{W}_{i,b,t} \geq \delta_{i,b}, \text{ Complementary Slackness} & \forall i, \forall b \\
\hat{\ell}_{i,b,t} &= \frac{\hat{\omega}_{i,t}^\kappa}{1 - \pi_{i,t-1} + \pi_{i,t-1}\hat{\omega}_{i,t}^\kappa} \frac{\hat{\omega}_{i,b,t}^\eta}{\hat{\omega}_{i,t}^\eta} & \forall i, \forall b \\
\hat{\omega}_{i,b,t} &= \frac{\hat{W}_{i,b,t}\hat{L}_{i,b,t}}{\hat{P}_{i,t}\hat{\ell}_{i,b,t}} & \forall i, \forall b \\
\hat{\omega}_{i,t}^\eta &= \sum_b \pi_{i,b,t-1}\hat{\omega}_{i,b,t}^\eta & \forall i \\
\hat{P}_{i,t} &= \prod_{s=1}^S \hat{P}_{i,s,t}^{\alpha_{i,s}} & \forall i \\
\sum_{i=1}^I \sum_b \hat{W}_{i,b,t}\hat{L}_{i,b,t}Y_{i,b,t-1} &= \gamma \sum_{i=1}^I \sum_b Y_{i,b,t-1}. & \text{single}
\end{aligned}$$

For each period t , we use this system of equations to solve for the quantities of interest ($\hat{R}_{i,s,t}$, $\hat{\lambda}_{ij,s,t}$, and $\hat{P}_{i,s,t}$ for all i and s ; $\hat{W}_{i,b,t}$, $\hat{L}_{i,b,t}$, $\hat{\omega}_{i,b,t}$, and $\hat{\ell}_{i,b,t}$ for all i and b ; and $\hat{P}_{i,t}$ and $\hat{\omega}_{i,t}$ for

⁵Assuming that the U.S. had full employment in the year 2000 is not problematic, since that year was the peak of a business cycle, with an unemployment rate of just 4%. This is the lowest unemployment rate observed in the U.S. in the last 40 years (except for the period from 2018 onward). The existence of 4% unemployment is consistent with our assumption of "full employment" because the concept of unemployment in our model is that of "cyclical" unemployment, i.e., the unemployment in excess of the natural rate of unemployment.

all i) given the objects that we already know from the previous period ($Y_{i,b,t-1}$, $\lambda_{ij,s,t-1}$, $D_{i,t-1}$, $R_{i,s,t-1}$, $\pi_{i,t-1}$, $\pi_{i,b,t-1}$, $\{\hat{\ell}_{i,b,q}\}_{q=1}^{t-1}$ and $\{\hat{L}_{i,b,q}\}_{q=1}^{t-1}$ for all i, j, s) and the time t shocks ($\hat{A}_{i,s,t}$, $\hat{D}_{i,t}$ and $\hat{\tau}_{ij,s,t}$ for all i, j, s). Thus, starting at $t = 1$ we can solve this system with information on $Y_{i,0}$, $\lambda_{ij,s,0}$, $D_{i,0}$, $R_{i,s,0}$, $\pi_{i,0}$ and $\pi_{i,b,0}$ for all i, j, s, b (assuming that we depart from a steady state where $L_{i,0} = \ell_{i,0}$) and the shocks ($\hat{A}_{i,s,1}$, $\hat{D}_{i,1}$ and $\hat{\tau}_{ij,s,1}$ for all i, j, s). We obtain $\hat{W}_{i,b,1}$, $\hat{L}_{i,b,1}$ and $\ell_{i,b,1}$ for all i and b . From the previous elements we can also obtain $Y_{i,b,1}$, $\lambda_{ij,s,1}$, $D_{i,1}$, $R_{i,s,1}$, $\pi_{i,1}$, and $\pi_{i,b,1}$ for all i, j, s, b . We can proceed like this for all other periods to solve the system forward while requiring only period zero information and the shocks hitting the economy.

Our general equilibrium model also allows us to compute the welfare effects of the shock. From $u_{i,t} \propto \left(\mu_i^\kappa + \omega_{i,t}^\kappa\right)^{1/\kappa}$ and $\pi_{i,t} = \omega_{i,t}^\kappa / \left(\mu_i^\kappa + \omega_{i,t}^\kappa\right)$ we can express the change in instantaneous utility as $\hat{u}_{i,t} = \hat{\pi}_{i,t}^{1/\kappa} \hat{\omega}_{i,t}$. To provide our welfare calculations, we combine this expression for instantaneous utility with a standard lifetime utility function which is time-separable with discount factor β .

3 Data, Calibration and Exposure to the China Shock

3.1 Data Description

We use trade and production data for 50 U.S. states, 36 additional countries, and an aggregate rest of the world region, for a total of 87 regions from 2000 to 2007. We consider 14 sectors: 12 manufacturing sectors, one service sector, and one agricultural sector. All sectors are classified according to the North American Industry Classification System (NAICS). We provide a brief description of the data here and relegate additional details to Appendix A.

For each region j and each sector k , our model requires data to compute the share of labor in production $\phi_{j,k}$, the share of intermediates from all other sectors $\phi_{j,sk} \forall s$, and the aggregate consumption shares $\alpha_{j,k}$. We use data from the BEA (for U.S. states) and from WIOD to calculate the share of value-added in gross output of region j , which in our model is equivalent to $\phi_{j,k}$. We also scale the relative importance of each U.S. state in the total value added of the U.S. so that the sum of value added across states matches the aggregate value-added of the U.S. according to WIOD. We compute $\phi_{j,sk}$ as the share of purchases of sector k coming from sector

s (the input-output coefficient) using WIOD data.⁶

Our model also requires data on bilateral trade flows for all sectors and all regions in our sample in order to compute deficits, revenue, and trade shares for the year 2000. We also require the bilateral trade flows (combined with the input output coefficients) to infer the $\alpha_{j,k}$'s. We construct the bilateral trade flow dataset in four steps. In the first step, we take sector-level bilateral trade between countries directly from WIOD.

In the second step, we use the Import and Export Merchandise Trade Statistics, a dataset compiled by the U.S. Census Bureau, to compute – for manufacturing and agriculture – the sector-level bilateral trade flows between each U.S. state and each of the other countries in our sample. The U.S. Census data on sector-by-state-by-country exports starts in 2002, and the data on imports starts in 2008. We use these starting years to project our bilateral trade matrix for previous years until 2000 by assuming that the importance of each state in the total exports (imports) to (from) other countries in each sector remains constant at the 2002 (2008) levels. We use a proportionality rule for the bilateral trade flows between the U.S. and other countries to match the values from WIOD in each sector. We provide more details on this in Appendix A.3.

In the third step, we follow [Caliendo et al. \(2019\)](#) to calculate the bilateral trade flows in manufacturing among U.S. states by combining WIOD and the Commodity Flow Survey (CFS). We first compute the bilateral expenditure shares across regions and sectors from the CFS, and then we use a proportionality rule to assign the total U.S. domestic sales from WIOD according to those bilateral shares. The bilateral trade flows matrix for the 50 U.S. states then match the total U.S. domestic sales from WIOD in each sector.

In the fourth and last step, we combine data for region-level production and expenditure in services from the Regional Economic Accounts of BEA, WIOD data, and data on bilateral distances to construct the trade flows in services among all regions consistent with a gravity structure. We follow a similar gravity approach for the case of trade flows in agriculture using data from the Agriculture Census, the National Marine Fisheries Service Census, and WIOD. By construction, the bilateral trade flows in services and agriculture match the aggregates of trade in services and agriculture between all countries (including the U.S.) and the total production of U.S. services and agriculture consumed by the U.S. The details of this procedure are

⁶We assume a common input-output matrix for all U.S. states, which is equal to the one of the U.S. as a whole.

explained in Appendix A.2.4.

3.2 Calibration of the China Shock and Exposure Measure

We focus on the effect of the China shock as captured by a set of productivity shocks in China given by $\{\hat{A}_{China,s,t}\}$ for the manufacturing sectors (i.e. $s \leq 12$).⁷ Inspired by ADH, and following Caliendo et al. (2019) and Galle et al. (2020), we calibrate these productivity shocks to match the changes in U.S. imports from China predicted from the changes in imports from China to other high-income countries.⁸

We decompose the total productivity shock in sector s and time t into a component coming from a sector-level productivity increase that is constant from 2000 to 2007 and a component coming from a productivity increase over time that is constant across sectors, i.e. $\hat{A}_{China,s,t} = \hat{A}_{China,t}^1 \hat{A}_{China,s}^2$. This means we have to estimate 19 parameters. We choose $\{\hat{A}_{China,t}^1\}$ and $\{\hat{A}_{China,s}^2\}$ to match two targets. The first target is the vector of annual predicted changes in U.S. imports from China in all manufacturing sectors combined. We obtain the predicted changes from the following regression:

$$\Delta X_{C,US,t} = a + b_1 \Delta X_{C,OC,t} + \varepsilon_t,$$

where $\Delta X_{C,US,t}$ is the change in U.S. imports from China between year $t - 1$ and year t in all manufacturing sectors, $\Delta X_{C,OC,t}$ is the change in imports from China by the other high-income countries between year $t - 1$ and year t in all manufacturing sectors, and b_1 is the coefficient of interest. We denote the predicted values from this regression by $\{\widehat{\Delta X_{C,US,t}}\}$.⁹

The second target is the vector of predicted changes in U.S. imports from China between 2000 and 2007 across sectors. We obtain this vector from the following regression

$$\Delta X_{C,US,s}^{2007-2000} = b_2 \Delta X_{C,OC,s}^{2007-2000} + \varepsilon_s,$$

⁷For the service and agriculture sectors, we assume that there was no change in Chinese productivity.

⁸We use the subset of ADH countries that are also present in the 2013 version of the WIOD, namely Australia, Germany, Denmark, Spain, Finland, and Japan. New Zealand and Switzerland are included in the “other high-income countries” category of ADH but are not included in WIOD.

⁹This regression only has seven data points, but it still has a high R^2 of 0.87.

where $\Delta X_{C,US,s}^{2007-2000}$ is the change in U.S. imports from China between 2000 and 2007 in sector s , $\Delta X_{C,OC,s}^{2007-2000}$ is the change in imports from China by the other high-income countries between 2000 and 2007 in sector s , and b_2 is the coefficient of interest. The predicted values from this regression are denoted $\{\widehat{\Delta X_{C,US,s}^{2007-2000}}\}$.¹⁰

We choose $\{\hat{A}_{China,t}^1\}$ and $\{\hat{A}_{China,s}^2\}$ such that the total productivity changes in China $\{\hat{A}_{China,s,t}\}$ deliver changes in imports in our model that simultaneously match the 7 values of $\{\widehat{\Delta X_{C,US,t}}\}$ and the 12 values of $\{\widehat{\Delta X_{C,US,s}^{2007-2000}}\}$.¹¹

Besides the China shock, there are some other parameters that we still need to set. We assume that the trade elasticity σ_s is constant across sectors and takes the value of 6, consistent with the trade literature (e.g. Costinot and Rodriguez-Clare, 2014). For inter-temporal comparisons when computing welfare, we use a discount factor β of 0.95 (at the annual level).

The calibration of the key model parameters below is based on matching moments that capture the relative effect of the China shock on labor force participation and unemployment. These moments come from regressions of changes in these variables across regions differentially exposed to the China shock, as captured by an exposure measure that is analogous to the one proposed by ADH. Specifically,

$$\text{Exposure}_i \equiv \sum_{s=1}^S \frac{VA_{i,s,2000}}{VA_{i,2000}} \frac{\widehat{\Delta X_{C,US,s}^{2007-2000}}}{R_{US,s,2000}}, \quad (15)$$

where $R_{US,s,2000}$ is total U.S. production in sector s in the year 2000, $VA_{i,s,2000}$ is value-added of region i in sector s in year 2000 (corresponding to $\phi_{i,s}R_{i,s,2000}$ in the model), $VA_{i,2000} \equiv \sum_s VA_{i,s,2000}$, and $\widehat{\Delta X_{C,US,s}^{2007-2000}}$ is the predicted 2000-2007 change in U.S. imports in sector s from

¹⁰We exclude the constant in this regression because it can lead to negative predicted imports from China, which is impossible. While the regression only has 12 observations, it has an R^2 of 0.99.

¹¹The multiplicative nature of our decomposition, $\hat{A}_{China,s,t} = \hat{A}_{China,t}^1 \hat{A}_{China,s}^2$, implies that their level is not identified. For example, if we multiply all the $\hat{A}_{China,s}^2$ by a constant c and we divide all the $\hat{A}_{China,t}^1$ by c , then we would have the same $\hat{A}_{China,s,t}$. Thus, to proceed, we use the normalization $\sum_{s=1}^S \hat{A}_{China,s}^2 = 1$. Correspondingly, the model is only able to produce changes in imports that satisfy $\sum_{t=2001}^{2007} \Delta X_{C,US,t}^{model} = \sum_{s=1}^S \Delta X_{C,US,s}^{2007-2000,model}$. This condition is automatically satisfied by the actual changes, i.e. $\sum_{t=2001}^{2007} \Delta X_{C,US,t} = \sum_{s=1}^S \Delta X_{C,US,s}^{2007-2000}$, but not necessarily by the predicted changes, due to the lack of a constant in the second regression. We adjust the predicted changes in manufacturing so that they satisfy: $\sum_{t=2001}^{2007} \widehat{\Delta X_{C,US,t}} = \sum_{s=1}^S \widehat{\Delta X_{C,US,s}^{2007-2000}}$, this adjustment is very small. In all of our applications we are able to match our targets with an accuracy greater than 99.9%.

China as in ADH and explained above.¹² Besides the calibration, we will also use this exposure measure to present the results of the model for non-targeted variables such as manufacturing and non-manufacturing employment, as well as for welfare, so that we can see how these predictions vary across states differentially exposed to the China shock.

4 Effects of the China Shock under the Baseline Specification

4.1 Calibration of DNWR and Labor Supply Elasticity

For our baseline specification, we assume that there is a single broad sector in the labor market. This implies that we do not need to calibrate the parameter η , which governs the degree of mobility between different broad sectors, although we do need to take a stand on κ , which governs labor supply choice. We also assume that that all countries outside the U.S. have a flexible exchange rate that adjusts in such a way that they retain full employment, implying that $\delta_i = 0$ for all $i > M$. We do not calibrate γ and δ separately – since only their relative value matters – and instead assume that γ is 1, so that the burden of adjustment falls entirely on δ , as in [Schmitt-Grohe and Uribe \(2016\)](#).

We choose δ and κ simultaneously to match two critical empirical estimates obtained by ADH. The first one is that a \$1,000 per worker increase in import exposure to China increases the unemployment to population rate by 0.22 percentage points. The second one is that the same rise in import exposure increases the labor force non-participation to population rate by 0.55 percentage points.¹³ In broad terms, δ governs the amount of unemployment generated by exposure to China for a given κ , while κ governs the fall in the labor force generated by exposure to China for a given δ .

The calibration results in values of $\delta = 0.984$ and $\kappa = 5.9$. [Figure 1](#) provides an illustration

¹²One difference between our exposure measure and the one in ADH is the use of value-added instead of employment shares as weights. However, note that in our model, labor is the factor of value added, and hence $VA_{i,s,2000} = W_{i,b(s),2000}L_{i,s,2000}$. If there is a single broad sector then $W_{i,b(s),2000} = W_{i,2000} \forall s$ and hence $\frac{VA_{i,s,2000}}{VA_{i,s,2000}} = \frac{L_{i,s,2000}}{L_{i,s,2000}}$. We re-normalize our exposure measure to have the same mean as the ADH measure for comparability purposes.

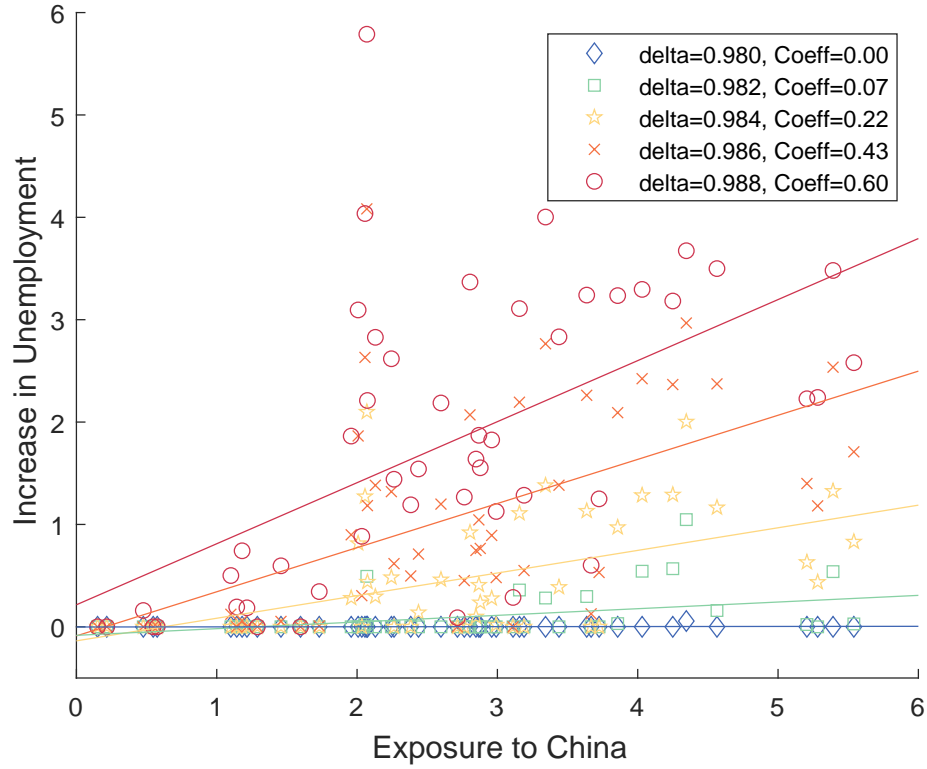
¹³These results correspond to the ones in Panel B of Tables 5 in ADH. Following ADH, we also take the 2006-2008 averages of unemployment and labor force participation in our estimation. We note that although ADH run their regressions at the level of commuting zones rather than states, when we run the same regressions at the state level we get very similar results – see discussion in [Section 6.4](#).

of how the identification of δ and κ works. Panel (a) of that figure shows a scatter plot of the increase in unemployment against the exposure to China for the calibrated level of $\kappa = 5.9$ and different levels of δ . We see that a higher δ leads to a steeper slope in the regression of unemployment on exposure to China (the coefficient is reported in the legend for convenience). For the calibrated parameter value of $\delta = 0.984$, the coefficient obtained in the regression is 0.22, which is the target that we obtained from ADH. Similarly, panel (b) of Figure 1 shows a scatter plot of the decrease in labor force participation against the exposure to China for the calibrated level of $\delta = 0.984$ and different levels of κ . We see that a higher κ leads to a steeper slope in the regression of labor force participation on exposure to China (the coefficients are also reported in the legend). For the calibrated parameter value of $\kappa = 5.9$, the coefficient obtained in the regression is 0.55, which is the target that we obtained from ADH.

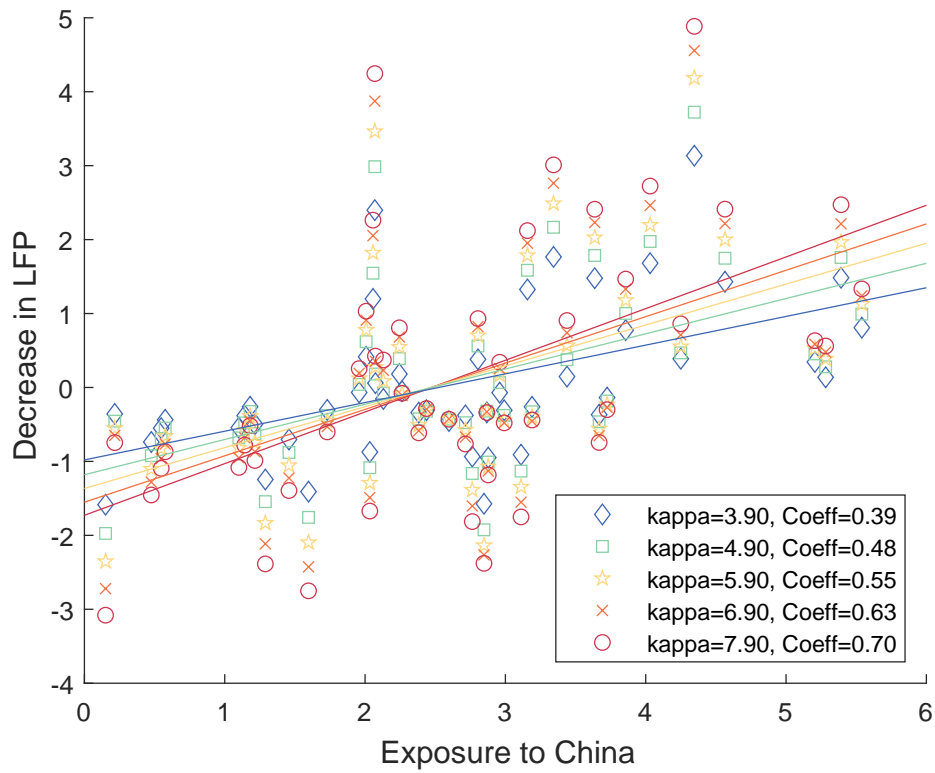
Our estimate for δ falls squarely in the range advocated by [Schmitt-Grohe and Uribe \(2016\)](#) who coincidentally also obtain an annual δ of 0.984 (after “normalizing” γ to one as we do).¹⁴ This estimate implies that wages can fall up to 1.6% annually. On the other hand, since the labor force participation is around 2/3, our estimate for κ implies a labor supply elasticity of approximately 2.¹⁵ This elasticity is relatively high compared to the common intertemporal micro estimates in the literature ([Chetty et al., 2012](#); [Martinez et al., 2019](#); [Tortarolo et al., 2019](#)). More recently and for the case of the U.S., [Mui and Schoefer \(2020\)](#) nonparametrically measure the global extensive-margin aggregate labor supply curve. They find non-constant elasticities along the curve, with smaller values of around 0.6 for large perturbations (consistent with the micro evidence), but larger values of around 3 for minor deviations. Overall, they find that for the case of local fluctuations, the curve appears well approximated by elasticities that are well above the micro-estimates and much closer to our calibrated value.

¹⁴Using a set of countries that excludes the U.S., [Schmitt-Grohe and Uribe \(2016\)](#) obtain a quarterly value of $\delta = 0.996$. This value corresponds to an annual δ of 0.984. However, they end up using a δ of 0.96 in their paper to be conservative.

¹⁵Here we have used the fact that, in our model, labor force participation is $\frac{l}{L} = \frac{\omega^\kappa}{\omega^\kappa + \mu^\kappa}$, implying that $\frac{\partial \ln(l/L)}{\partial \ln \omega} = \kappa(1 - l/L)$.



(a) Unemployment increase vs. exposure for different deltas



(b) LFP decrease vs exposure for different kappas

Figure 1: Illustration of the Identification of κ and δ

4.2 Comparison of Cross-Sectional results with ADH

We now use the calibrated model to study the effects of the China shock across the different U.S. states. We first obtain the changes in wages, employment, unemployment, labor force participation, and real wages for all the 87 regions included in our model. Then we run OLS regressions across U.S. states of the changes in the variables of interest on the exposure measure in Equation (15). We present the resulting coefficients in Table 1, along with the analogous coefficients from ADH.

Column (1) of Table 1 reports the results of ADH.¹⁶ The first two rows correspond to our targeted regression coefficients. As mentioned above, these coefficients indicate that an additional \$1,000 of exposure to the China shock leads to an increase in the unemployment to population share by 0.22 percentage points and an increase the labor force non-participation to population ratio by 0.55 percentage points. The coefficients in the third and fourth rows imply that additional exposure to China decreases the manufacturing employment to population share by 0.59 percentage points and the non-manufacturing employment to population share by 0.18 percentage points (this result is not significantly different from zero). Finally, the coefficients in the fifth and sixth rows show that additional exposure to China lowers non-manufacturing wages by 0.76 percent and increases manufacturing wages by 0.15 percent (this result is not significantly different from zero).

Column (2) of Table 1 presents the results of our baseline model. We focus on the results related to employment and wages in this section, and discuss the welfare effects in Section 4.4. Columns (3) and (4) present results for versions of the model that incorporate mobility frictions between manufacturing and non-manufacturing, which we discuss in Section 5.

Our results in column (2) show that exposure to China measured as in ADH leads to a fall in manufacturing and non-manufacturing employment of 0.32 percentage points and 0.45 percentage points, respectively. These are moments that we did not target in our calibration.¹⁷ Our results from the baseline model understate the fall in manufacturing employment and overstate the fall in non-manufacturing employment. As we discuss in Section 5, the model can

¹⁶Specifically, we use the ADH estimates presented in their panel B of Table 5, and panel B of Table 7.

¹⁷The only restriction is that the coefficients have to add up to 0.77 since this is the sum of the targeted unemployment and NILF coefficients.

Table 1: Employment, wage, and welfare effects of exposure to China across U.S. regions and associated parameters generating them

	ADH (1)	Baseline (2)	MF (3)	DNWRM (4)
<i>Change in Population Shares</i>				
Unemployment (targeted)	0.221***	0.221	0.221	0.221
NILF (targeted)	0.553***	0.553	0.553	0.553
Mfg Employment	-0.596***	-0.320	-0.269	-0.555
Non-mfg Employment	-0.178	-0.453	-0.505	-0.219
<i>Percentage Changes</i>				
Mfg Wage	0.150	-0.577	-0.397	-0.027
Non-mfg Wage	-0.761***	-0.577	-0.652	-1.201
<i>Welfare</i>				
Welfare vs exposure		-0.091	-0.096	-0.109
Mean welfare change		0.218	0.267	0.120
Mean welfare change no DNWR		0.315	0.367	0.355
<i>Parameters</i>				
κ		5.898	5.642	5.623
η		Inf	5.642	5.623
δ_{Mfg}		0.984	0.982	0.992
$\delta_{\text{Non-Mfg}}$		0.984	0.982	0.000

Notes: The changes for the first four coefficients (the employment results) are measured from 2000 to an average of 2006-2008, multiplied by 10/7 to turn into decadal changes, and the shares of employment are measured as percentage of the population. Wages are simply measured in percentage change (between 2000 and 2006-2008), still turned into decadal changes. Welfare is the present value of per period utility change. κ is the parameter that governs substitution between home and market production, η is the one that governs substitution between manufacturing and non-manufacturing and the δ 's govern the DNWR. Column 1 reproduces the ADH results from their Tables 5 (panel B, first row) and 7 (Panel B, columns 1 and 4), stars denote significance, one star for 10%, two for 5%, and three for 1%. Column 2 gives the results in our baseline model with no mobility frictions. Column 3 incorporates mobility frictions and has the same delta in both broad sectors. Column 4 keeps mobility frictions but imposes the DNWR only in manufacturing. In column (2) η is reported as "Inf" because the baseline model does not incorporate mobility frictions, which can be interpreted as implying that the elasticity between broad sectors is infinite.

match these two ADH moments better if we allow for mobility frictions across manufacturing and non-manufacturing, and assume that the DNWR only applies to the manufacturing sector.

Regarding the effect of exposure to China on wages, our baseline model without mobility frictions displays the same impact on manufacturing and non-manufacturing wages by design. However, given this restriction, the model still performs relatively well, since the fall in wages

of 0.57 percent that we find in our model is very close to the weighted average of the ADH results using employment shares as weights.

4.3 Aggregate Employment Effects

One of the advantages of our general equilibrium model is that we can go beyond cross-sectional estimates to obtain the aggregate effects of the China shock on employment and other variables. In Figure 2, we plot three variables related to the labor market as an average across all U.S. states. The top panel plots the cumulative change in employment over population. This variable falls by 1.8% from 2000 to 2004 and subsequently recovers to end roughly 1% higher in 2010. In the middle panel, we plot the cumulative change in labor force participation over population. This variable falls by 1% from 2000 to 2004 before recovering to end the period roughly 1% higher.¹⁸ Finally, in the bottom panel, we plot the cumulative change in unemployment. This variable increased by 0.8 percentage points from 2000 to 2004, falling back to zero in 2010, a result of the fact that in our calibration there are no shocks after 2007.¹⁹

It is interesting to note that the effect of the China shock on labor force participation reverses sign throughout the transition. On impact, the shock leads to a temporary decline in participation, stemming from the fact that unemployment discourages participation due to the risk of participating in the labor market but not being able to obtain a job. However, when the China shock stops hitting the economy, and the nominal wage has room to fully adjust, labor force participation ends up increasing. This increase happens because, in the absence of nominal rigidities, the China shock is a positive terms-of-trade shock for the U.S., which translates to a higher real wage and an increase in labor supply.

The results imply that most states experience both a long-run increase in the real wage and a temporary increase in unemployment. This may seem paradoxical, but it is a natural consequence of a shock that implies both an improvement in the terms of trade and a decline in the export price index. To see this more clearly, consider a small open economy and imagine that the price index of its exports falls while the price index of its imports falls even more.

¹⁸The final cumulative change in employment and labor force participation must be the same because all unemployment disappears in the long run.

¹⁹We view this feature of the model as desirable, since it is hard to square a permanent unemployment effect of the China shock with the historically low levels of unemployment observed in the U.S. between 2016 and 2019.

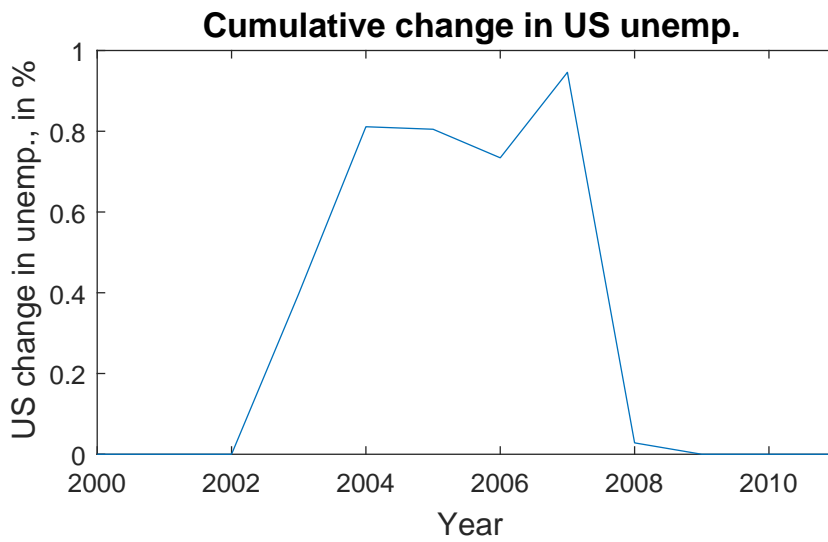
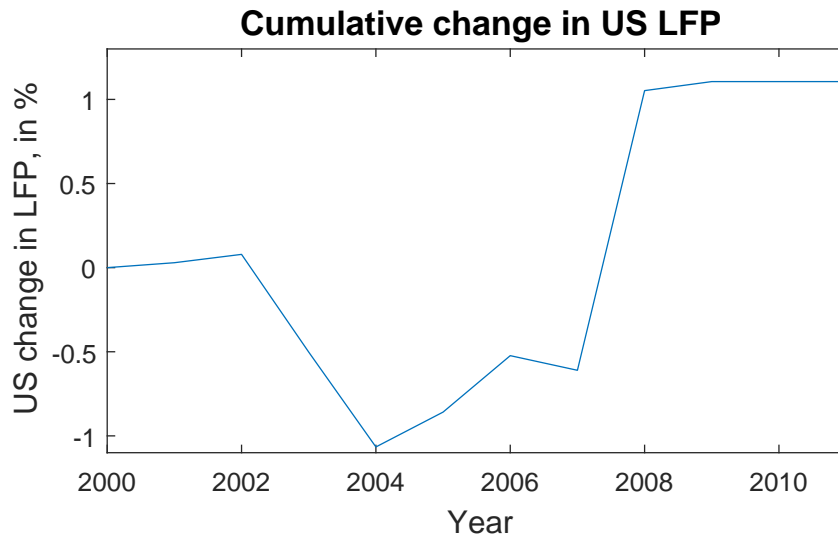
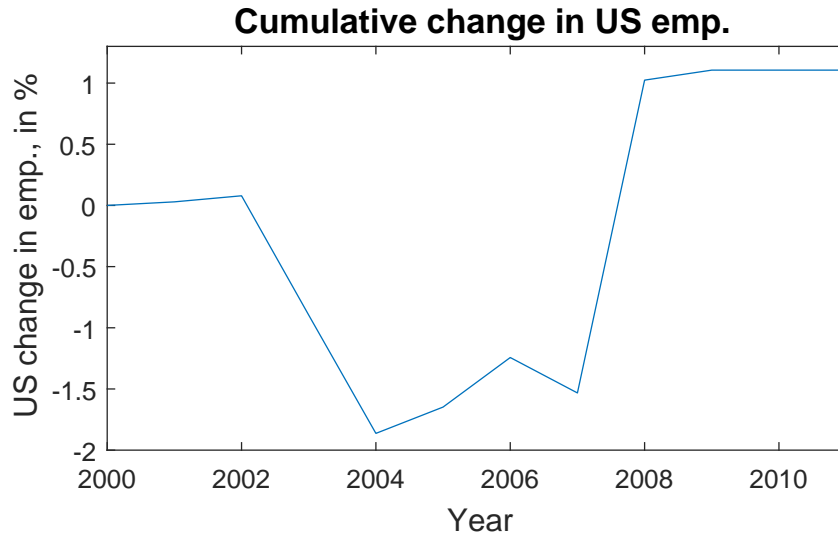


Figure 2: Paths of changes in different employment-related variables for the U.S. average

Since the terms of trade have improved, the real wage and employment would increase in the absence of nominal frictions. However, the fact that the price index of its exports has fallen requires the nominal wage to decline, and if this decline is higher than $1 - \delta$, there would be temporary unemployment.

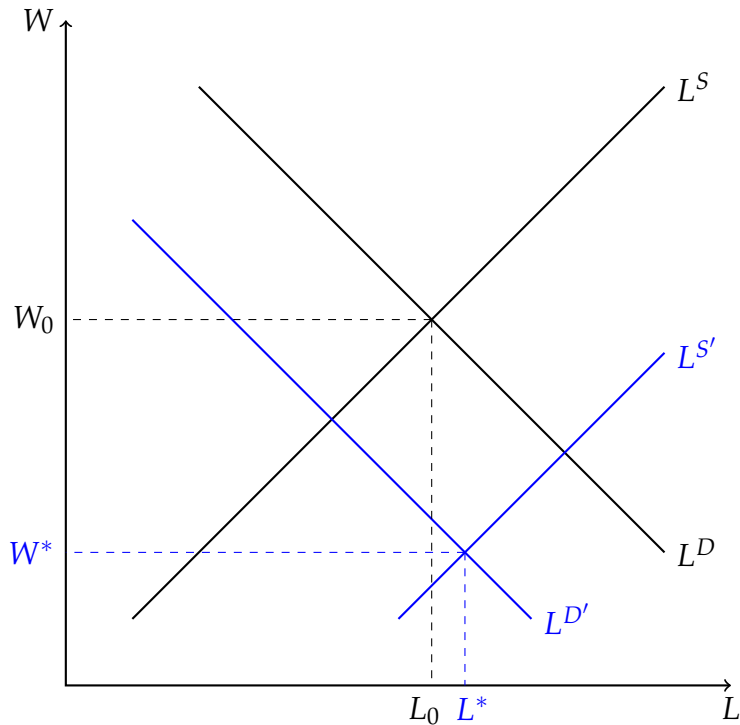
We illustrate this mechanism via Figure 3. Both panels in the figure have the nominal wage in the vertical axis and employment in the horizontal axis.²⁰ Panel (a) illustrates the situation when there is no DNWR. The China shock leads to a fall in labor demand illustrated by a movement from L^D to $L^{D'}$. However, the China shock also leads to a fall in prices, which increases the real wage and leads to an increase in labor supply from L^S to $L^{S'}$. The final result is a fall in the nominal wage from W_0 to W^* , a fall in prices from P_0 to P^* (not illustrated), an increase in the real wage from W_0/P_0 to W^*/P^* (prices fall more than wages), and an increase in the amount of labor supplied from L_0 to L^* .

Panel (b) of Figure 3 shows the adjustment in the presence of DNWR assuming that $\delta^3 W_0 < W^* < \delta^2 W_0$. In the first year, the wage only falls from W_0 to $W_1 \equiv \delta W_0$ and employment falls from L_0 to L_1 , as determined by the demand curve. Since the wage does not fully adjust in the first year, the fall in prices is also smaller than in the frictionless case, and hence the labor supply curve only moves from L^S to L_1^S . The gap between the labor supplied at point A and labor demanded L_1 is the level of unemployment. In the second year wages adjust further down (to $W_2 \equiv \delta W_1 = \delta^2 W_0$), the labor supply curve moves to L_2^S , employment increases from L_1 to L_2 , labor supplied moves from point A to point B, and unemployment decreases. In the third year, wages finally adjust fully and there is no longer unemployment. Notice that the final equilibrium of the economy is the same with and without DNWR, and it involves higher labor supply, a higher real wage, and no unemployment.

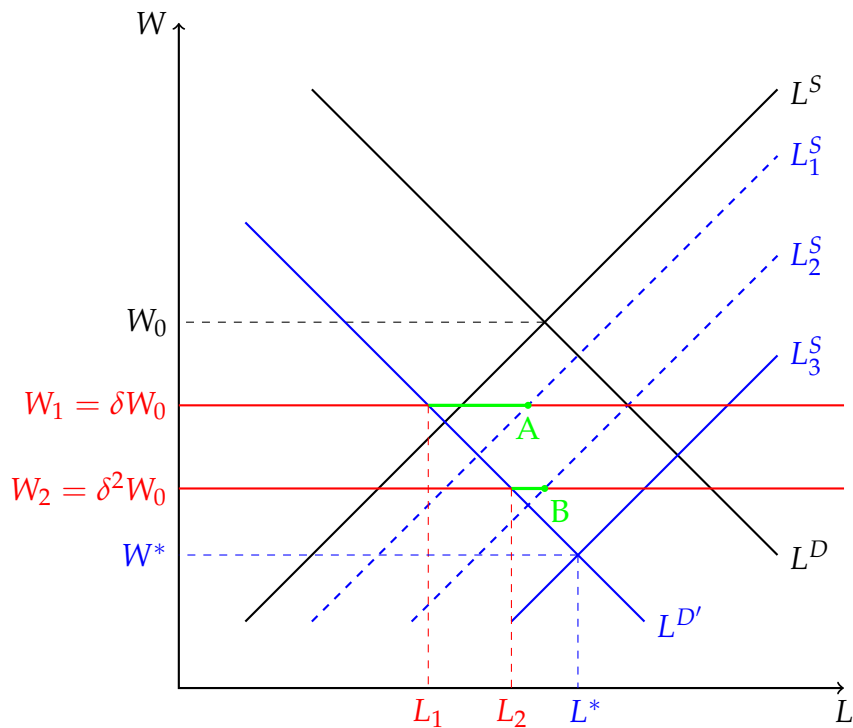
4.4 Welfare Effects

We find that U.S. states more exposed to the China shock experience lower model-implied welfare gains: a \$1,000 per worker increase in exposure to China decreases welfare by around 9 basis points (this is the coefficient displayed in Table 1, column (2), row 7). Figure 4 presents a scatter plot of the percentage change in welfare across states against exposure to China, while

²⁰Since the nominal wage is in the vertical axis, movements in prices lead to a shift in the labor supply curve.



(a) Case without DNWR



(b) Case with DNWR

Figure 3: Illustration of wage and employment effects, with and without DNWR. The nominal wage is in the vertical axis, hence price movements results in shifts in the labor supply curve. Employment is in the horizontal axis.

Figure 5 displays a welfare map across the 50 U.S. states. There are 37 states that gain from the China shock while 13 states experience losses. Of these 13 states, only 3 experience a worsening of their terms of trade, which imply a lower steady state real wage. The other 10 states that suffer losses actually experience improvements of their terms of trade, but these are dominated by large temporary increases in unemployment due to the DNWR.

The unemployment and labor-force participation margins play very different roles in determining these welfare changes: increases in unemployment due to a binding DNWR lead to worse welfare outcomes, while changes in labor-force participation allow regions to mitigate negative shocks and benefit more from positive shocks. To understand the role of DNWR, we use a model that has no DNWR but where we still match the overall effect of the China shock on employment (i.e., the sum of the first two rows of column 1 of Table 1, amounting to 0.77). This implies a halving of the effect of exposure to China on welfare (from 9 basis points to 4.5 basis points). To understand the role of the labor-force participation margin, we can set $\kappa = 0$ (which implies that households cannot reallocate from home production to employment), and recalibrate δ so as to match the effect of exposure to China on unemployment (i.e., the 0.22 in the first row of column 1 of Table 1). This leads to an increase in the effect of exposure to China on welfare from 9 basis points to 11 basis points.

When we consider the U.S. as a whole, and measure overall welfare by the population-weighted average across U.S. states, we see that the China shock leads to an increase in welfare of 22 basis points. This is true even though we match the unemployment effects captured by ADH, which have sometimes been interpreted as implying that the China shock had adverse overall welfare effects.

It is interesting to compare the results of our baseline model against those from a model without nominal rigidity (i.e., with $\delta = 0$) and with κ recalibrated in order to match a 0.77 effect of exposure on the share of the population that is not employed. In this alternative version of the model, all but 3 states experience welfare gains from the China shock, and the U.S. as a whole experiences gains of 32 basis points. This is comparable to the gains obtained in recent papers studying the same setting (Caliendo, Dvorkin, and Parro, 2019; Galle, Rodriguez-Clare, and Yi, 2020). However, to match our 0.77 target, the model without DNWR requires a value of κ of approximately 46, implying an unrealistically high labor supply elasticity of around 15.

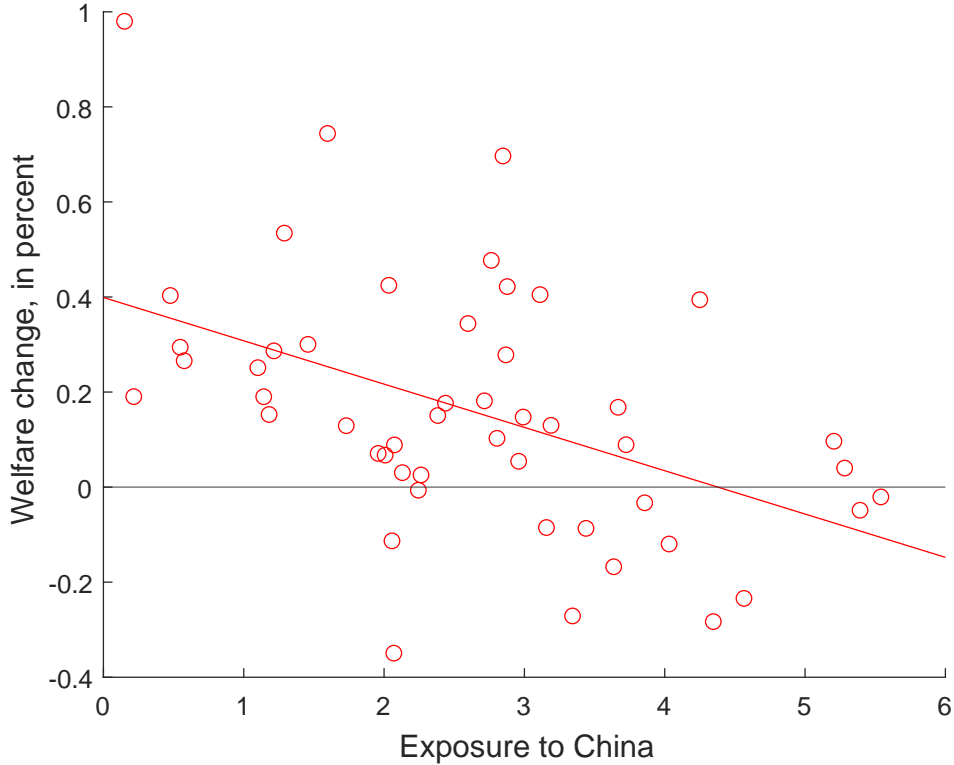


Figure 4: Welfare change vs exposure to China across U.S. states

Finally, Table 2 shows the different welfare changes in the model with and without DNWR, for several values of the discount factor β . For $\beta = 0.95$, the model without DNWR overestimates the welfare gains by approximately 33% relative to our baseline model. For $\beta = 0.99$ this overestimation is just 8%, while for $\beta = 0.91$ it is 56%.

5 Extensions

In this section we discuss several extensions of the baseline model. First, we introduce mobility frictions between manufacturing and non-manufacturing. Second, we discuss how to introduce some of the increases in trade surpluses that happened in China between 2000 and 2007 as an integral part of the China shock. Finally, we analyze how our baseline results change under a different exchange rate regime for other countries, or a different nominal anchor. For each of these alternative models we recalibrate the China shock, $\{\hat{A}_{China,s,t}\}$, following the procedure described in Section 3.

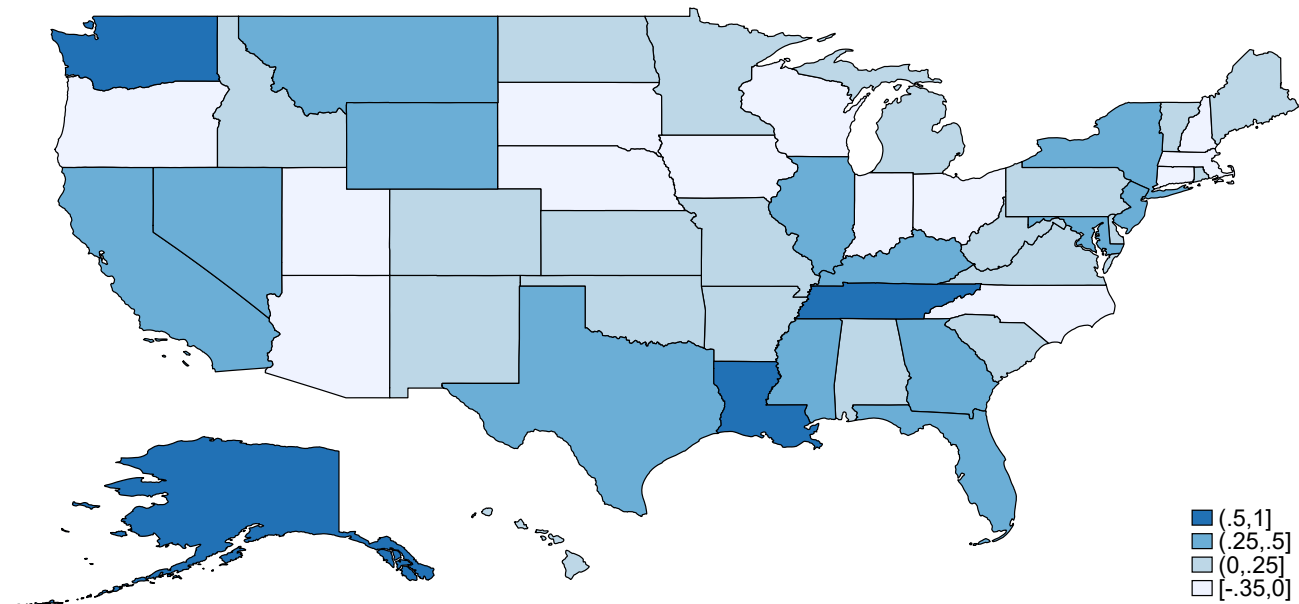


Figure 5: Welfare across U.S. states for calibrated parameter values in the baseline model

5.1 Mobility Frictions between Manufacturing and Non-Manufacturing

Our baseline specification assumes a single broad sector, which implies free mobility between all sectors and a single wage for all workers, as shown in column (2) of Table 1. We now explore the implications of mobility frictions in our model, by allowing for two broad sectors with costly mobility between them. The first broad sector is composed of all manufacturing sectors, and the second one is composed of the remaining non-manufacturing sectors.²¹

Besides the parameters κ and δ , we now also need to calibrate the parameter η governing the elasticity of switching between the two broad sectors. We could also calibrate two different δ parameters, one for each broad sector. We perform two separate exercises in the model with mobility frictions, where we make different assumptions regarding the additional parameters to be calibrated.

In the first exercise, the manufacturing (Mfg) and non-manufacturing (Non-Mfg) broad sectors have the same DNWR parameter, i.e., $\delta_{Mfg} = \delta_{Non-Mfg}$, while in the second one only the broad manufacturing sector faces DNWR, (i.e., $\delta_{Mfg} > 0$ and $\delta_{Non-Mfg} = 0$). For parsimony, we impose that $\eta = \kappa$ in both exercises, which represents a natural benchmark where

²¹This means that $b(s) = 1 \forall s \leq 12$, and $b(s) = 2 \forall s > 12$. Manufacturing sectors (1-12) are part of the broad manufacturing sector, while services and agriculture (13 and 14) make up the non-manufacturing broad sector.

Table 2: Welfare gains from the China shock across different discount factors

β	$\delta = 0$ (1)	calibrated δ (2)	% decrease (3)
0.99	0.381	0.349	8.48
0.97	0.351	0.279	20.59
0.95	0.324	0.218	32.61
0.93	0.298	0.166	44.46
0.91	0.274	0.120	56.12

Notes: This table displays the average welfare gains from the China shock, for the U.S. as a whole, across different values of the discount factor β . Column (1) displays the gains in percent when the DNWR is inactive ($\delta = 0$). Column (2) displays the gains in percent for our calibrated δ value of 0.984. Finally, column (3) displays the percentage decrease in the welfare gain when going from $\delta = 0$ to the calibrated δ .

mobility frictions across sectors are the same as those between home and market production. We thus calibrate two parameters in each exercise: the labor elasticity parameter ($\eta = \kappa$) and the DNWR parameter ($\delta_{Mfg} = \delta_{Non-Mfg}$ in the first exercise and δ_{Mfg} in the second one). We use those parameters to match the same two ADH targets as in the baseline calibration.

Column (3) of Table 1 presents the results of the first exercise. Relative to our baseline model, we find that δ falls to 0.982 while κ remains relatively similar, at 5.64. As expected, the presence of mobility frictions results in different wage responses in the different broad sectors. In particular, we find that the impact of exposure to China on the wage in non-manufacturing is almost twice the one in manufacturing (approaching the ADH empirical pattern). However, the sectoral employment changes move in the wrong direction: the coefficient of exposure to China on non-manufacturing employment becomes more negative and the one for manufacturing gets closer to zero (moving away from the ADH pattern). The welfare results in this exercise are overall more favorable than in the baseline model: both with and without DNWR the economy experiences more welfare gains from the China shock.

Intuitively, mobility frictions lead to a stronger required decline in the manufacturing wage, since manufacturing is the sector most negatively affected by the China shock. This leads the manufacturing wage to hit the lower bound imposed by the DNWR in most U.S. states (in 44 states in 2007), limiting the variation in the manufacturing wage across states more and less exposed to the China shock. In contrast, the non-manufacturing wage hits the DNWR

less often (in 30 states in 2007) and this allows for a higher slope in the relationship between exposure and the wage change in non-manufacturing.

Column (4) of Table 1 presents the results of the second exercise. The calibrated κ is almost the same as in column (3), but the calibrated δ of 0.992 is higher than before. A higher value of δ is intuitive, because the DNWR only applies to the manufacturing sector. Consequently, it needs to bind more strongly in order to match the required response of unemployment to exposure. In this exercise the responses of the employment shares to exposure in each broad sector are very close to the ones in ADH, even though these moments were not targeted in the calibration. The wage responses are also similar to ADH, with the manufacturing wage exhibiting a coefficient that is very close to zero and the non-manufacturing wage responding strongly to exposure (actually this responds too much relative to ADH). Overall, the results in column (4) of Table 1 are very close to the results obtained by ADH and hence this model provides a good benchmark to understand the effects of trade shocks on unemployment, labor force participation, wages, and welfare. Using this version of the model, we find that the U.S. experiences an average welfare gain of 12 basis points (weighing each state by its population), which is only around one-third of what the model without DNWR would predict.

Given the improved performance of the version of the model in which there is DNWR only in the manufacturing sector, it is worth discussing whether this is a realistic assumption. There are a few papers documenting a substantial degree of heterogeneity in wage rigidity across sectors and occupations for different contexts (Radowski and Bonin, 2010; Du Caju et al., 2012). More recently and for the U.S., Hazell and Taska (2019) explore this heterogeneity using a dataset containing wages for new vacancies with specific job descriptions for each establishment. Their paper finds that production workers face a higher degree of DNWR than workers in non-production occupations. Several elements could explain this fact, for example stronger unionization in manufacturing relative to services. However, we prefer to interpret the model with DNWR only in manufacturing as a rough way to capture forces pushing for reallocation of labor from manufacturing to services. Even with the same DNWR in the two sectors, these forces would make the DNWR in manufacturing more likely to be binding.

Table 3: Employment, wage, and welfare effects of exposure to China across U.S. regions and associated parameters generating them (continued)

	Baseline (2)	Def. Low (5)	Def. High (6)	Fixed ER (7)	U.S. NA (8)
<i>Change in Population Shares</i>					
Unemployment (targeted)	0.221	0.221	0.221	0.221	0.142
NILF (targeted)	0.553	0.553	0.553	0.553	0.553
Mfg Employment	-0.320	-0.331	-0.376	-0.299	-0.260
Non-mfg Employment	-0.453	-0.443	-0.398	-0.474	-0.435
<i>Percentage Changes</i>					
Mfg Wage	-0.577	-0.553	-0.442	-0.582	-0.634
Non-mfg Wage	-0.577	-0.553	-0.442	-0.582	-0.634
<i>Welfare</i>					
Welfare vs exposure	-0.091	-0.088	-0.075	-0.086	-0.129
Mean welfare change	0.218	0.232	0.299	0.216	0.011
Mean welfare change no DNWR	0.315	0.333	0.419	0.299	0.236
<i>Parameters</i>					
κ	5.898	6.034	6.761	5.992	8.639
η	Inf	Inf	Inf	Inf	Inf
δ_{Mfg}	0.984	0.985	0.988	0.989	1.000
$\delta_{\text{Non-Mfg}}$	0.984	0.985	0.988	0.989	1.000

Notes: All definitions are the same as the ones in Table 1. Column 2, which contains the results from the baseline specification, repeats Column 2 from Table 1 to facilitate comparison. Column 5 gives the results from our baseline model where we introduce a modest increase in Chinese surplus as part of the China shock, while in column 6 this increase is larger. Column 7 gives the results when other countries have fixed exchange rates relative to the U.S. and column 8 gives the results when the baseline nominal anchor is replaced by a nominal anchor which indicates that U.S. GDP grows at a constant rate.

5.2 Changing Deficits as Part of the China Shock

In the previous versions of our model, we kept the deficits for all regions constant in terms of world GDP. However, it is plausible that part of the increase in Chinese surpluses that occurred between 2000 and 2007 could be part of the China shock. In this section, we explore what happens if, besides a Chinese productivity increase, the China shock also entails an increase in the Chinese surplus, which has to be offset by a rise in the deficits of other regions. In particular, we perform two separate exercises where we explore different assumptions regarding how we treat the increase in the Chinese surplus.

In the first exercise, we only incorporate the increase in Chinese surpluses that occurred

because China's GDP increased relative to world GDP (i.e., we keep the surplus to GDP ratio constant in China), while in the second exercise we incorporate the changes in the Chinese surplus observed in the data. An increase in the surplus of China implies an equal change in the combined deficits of the other countries, since total deficits must always sum to zero. We keep the deficits of all other countries besides the U.S. unchanged in terms of world GDP in both exercises, thus having the U.S. deficit offset the whole increase in China's surplus. We distribute this increased deficit across U.S. states according to their shares in U.S. GDP.

The results for these two exercises are shown in Table 3, which has the same structure as Table 1 (the first column repeats Column (2) in Table 1, to facilitate comparisons of the new results with those in the baseline). Column (5) presents the results of the first exercise, while column (6) shows the results from the second exercise. Column (5) is labeled "Def. Low", because the U.S. deficit increases moderately (i.e., around 10% in total over the 2000-2007 period), while column (6) is labeled "Def. High", because the U.S. deficit rises substantially (i.e., around 57% in total over the 2000-2007 period).

The results in column (5) are very similar to those of our baseline calibration, although manufacturing employment falls slightly more, wages fall marginally less, and κ and δ both increase slightly. We also find that welfare reacts less to exposure and that the welfare increase is higher compared to our baseline model.²² The higher gain happens because, in the presence of trade costs, a transfer from abroad leads to a worldwide shift in demand towards domestic goods and this in turn improves the recipient's terms of trade. The changes from the baseline model to column (5) are amplified in column (6), because the U.S. deficit grows more in response to the China shock, but the overall pattern is the same.

More importantly, the changes in trade imbalances in both exercises require an *increase* in the wage in U.S. states relative to China, thereby relieving the pressures caused by the productivity increase in China and making the DNWR less binding. This is reflected in a smaller increase in U.S. unemployment: whereas the U.S. 2007 unemployment rate corresponding to column 2 is 0.95%, the one corresponding to column 5 is 0.9% and the one corresponding to column 6 is 0.7%.

²²As in Costinot and Rodriguez-Clare (2014), we measure welfare as real income rather than real expenditure. This avoids attributing a positive direct gain to the foreign transfer. Taking into consideration the direct gain would risk treating deficits as a gift and assuming away their future costs.

5.3 Other Exchange Rate Regimes and Nominal Anchors

So far, we have assumed that all countries outside of the U.S. have a fully flexible exchange rate with respect to the U.S. dollar. Adjustments in their exchange rate then ensure that the DNWR never binds, and hence countries outside of the U.S. never experience any unemployment. Additionally, our nominal anchor assumed that the world nominal GDP grows at a constant rate. In this section, we consider the consequences of making different assumptions for these features, presenting two exercises. The first exercise imposes that all other countries have a fixed exchange rate with respect to the U.S. dollar. Thus, compared to the baseline, we will no longer have countries devaluing their currencies relative to the U.S. dollar, implying less of a need for the wage to fall in the U.S., and hence lower unemployment in U.S. labor markets. The second exercise modifies the nominal anchor, assuming that U.S. nominal GDP (instead of world nominal GDP) grows at a constant rate. Here the DNWR will only prevent the necessary adjustments in the relative wages across U.S. states, but not between U.S. states and China, implying weaker effects.

In the first exercise, a fixed exchange rate implies that now all countries face a potentially binding DNWR, $W_{i,b,t} \geq \delta_b W_{i,b,t-1} \quad \forall i$. The nominal anchor is still the one in our baseline model, implying that world GDP grows at a rate $\gamma = 1$. Column (7) of Table 3 presents the results of this exercise. Overall, our results are remarkably robust to the new assumption regarding the exchange rate regime. The most notable change is that δ increases from 0.984 to 0.989. This increase in δ occurs because the other countries “absorb” part of the China shock, so a higher δ is needed in the U.S. to match the target response of unemployment to exposure. All other results are very close to the ones in our baseline model.

In the second exercise, the new nominal rule, in changes, is given by

$$\sum_{i=1}^M \hat{W}_{i,t} \hat{L}_{i,t} Y_{i,t-1} = \gamma \sum_{i=1}^M Y_{i,t-1}.$$

Column (8) of Table 3 presents the results of the model with this nominal rule. In this case, it is not possible to match both ADH targets simultaneously, even when setting $\delta = 1$. The reason is that now the DNWR only imposes limits to the adjustments in relative wage changes across

U.S. states, and the China shock only requires mild changes in such relative wages.²³

At first pass, a U.S. nominal anchor might seem like a more natural approach than the world nominal anchor used in our baseline specification. However, the fact that the baseline specification works better to explain the effects of the China shock, indicates that perhaps some nominal features cannot be captured by the U.S. nominal anchor. For example, suppose that China wanted to avoid large nominal wage increases to control inflation and also wanted to avoid appreciations to preserve competitiveness (a practice they are widely regarded as having pursued during the 2000-2007 period). Such a scenario would require a fall in the U.S. wage in dollars that would then hit the DNWR. This is something that can be parsimoniously captured (albeit in a reduced form) with the world nominal anchor, but not with the U.S. nominal anchor. Another type of nominal features that are not present with the U.S. nominal anchor, but are captured in a reduced-form but parsimonious manner by the world nominal anchor, are those related to the “Dominant Currency Paradigm” described by [Gopinath et al. \(2020\)](#).

6 Discussion

6.1 Different Exposure Measures

The measure of exposure to China that we have been using throughout the paper (defined in Equation 15) follows the one in ADH. This measure is a Bartik instrument where the “shift component” is given by the predicted change in imports from China to the U.S. in a sector and the “share component” is given by the share of employment (or value-added) in that sector in that region. This exposure measure cannot fully capture the welfare effects of the China shock, because it misses the impact through consumer prices.²⁴

As we show in Appendix C, in a simple neoclassical environment with an upward sloping labor supply curve but without nominal rigidities, a sufficient statistic for the first-order

²³It is also worth noting that, with $\delta = 1$, the unemployment generated by the China shock stays in the respective states forever, so in this case, the model displays “permanent” unemployment effects. Consequently, the welfare gains from the China shock are much lower, as the DNWR leads to the loss of over 90% of the gains that would accrue without rigidities.

²⁴Consider for example a region that did not produce a good at all (and hence would have a zero value-added share) but consumed it in a positive amount. This region would benefit from an increase in Chinese productivity in that sector, even though the ADH measure would imply a zero exposure of that sector to the shock.

changes in employment resulting from the China shock would use net exports as the “share” component, as in

$$\text{Exposure}_i^{NX} \equiv \sum_{s=1}^S \frac{TX_{i,s,2000} - TM_{i,s,2000}}{R_{i,2000}} \frac{\widehat{\Delta X_{C,US,s}^{2007-2000}}}{R_{US,s,2000}}, \quad (16)$$

where $TX_{i,s,2000}$ are the total sales of region i in sector s in year 2000, and $TM_{i,s,2000}$ is total expenditure of region i on sector s in year 2000. This captures the effect of the shock on the economy’s terms of trade, which in turn affects the equilibrium real wage and employment according to labor demand and supply elasticities. In contrast, when the wage does not adjust because of the DNWR, the labor or value-added shares become relevant, since the change in employment is determined entirely by the shift in the demand curve. Of course, in a more realistic situation where wages are sometimes fixed due to the DNWR but can eventually adjust to their frictionless level, then both measures of exposure are expected to be relevant.

To illustrate this point, we regress the state-level changes in welfare and employment generated by the model on both exposure measures (and a constant), with and without the DNWR. When there is no DNWR, we expect that only the net export exposure measure would be significant, while when there is DNWR we expect both exposure measures to be significant.

The results are reported in Table 4. Columns (1) and (3) reveal that, without DNWR, only the net export exposure measure is significant (at the 1% level) for employment and welfare, while ADH exposure is not significant. In contrast, columns (2) and (4) show that in the model with DNWR both the ADH exposure measure and the net export exposure measure are significant. These results indicate that a mechanism similar to DNWR is likely to be active in the U.S. economy, and this is what leads to the ADH exposure measure being relevant.²⁵

6.2 Quantifying Job Losses

In this section we attempt to put some numbers to the employment changes that we have documented in previous sections of the paper. Recall that ADH find a cross-sectional estimate

²⁵In our framework, nominal rigidities lead to a separate effect of labor demand on employment over and above those that would come through terms-of-trade effects. [Adao et al. \(2019\)](#) obtain similar effects through a more general reduced-form specification of the labor market where labor supply is a function of the wage and the consumption price entering separately rather than through the real wage.

Table 4: "Horse race" between different exposure measures in the baseline model with and without DNWR

	(1)	(2)	(3)	(4)
	Welf. No DNWR	Welf. DNWR	Empl. No DNWR	Empl. DNWR
Constant	0.539*** (0.059)	0.564*** (0.069)	1.799*** (0.200)	3.305*** (0.808)
ADH Exposure	-0.022 (0.019)	-0.046** (0.022)	-0.048 (0.064)	-0.855*** (0.261)
NX Exposure	-0.088*** (0.019)	-0.107*** (0.022)	-0.312*** (0.064)	-0.879*** (0.261)
N	50	50	50	50
R squared	0.408	0.473	0.409	0.444
Mean dep. var.	0.247	0.159	0.849	-1.255

Notes: This table shows the results of regressing several variables of interest on a constant, ADH exposure, and net export exposure. The exposure variables are described in the text. The dependent variables are: welfare change from the China shock in the baseline model without DNWR (column 1), welfare change from the China shock in the baseline model with DNWR (column 2), percentage change in total employment between 2000 and 2007 in the baseline model without DNWR (column 3), and percentage change in total employment between 2000 and 2007 in the baseline model with DNWR (column 4).

indicating that a \$1,000 per worker increase in import exposure to China leads to a decrease in the employment to population ratio of 77 basis points (22 basis points from increased unemployment and 55 basis points from reduced labor force participation). We start by using the ADH estimate in a naive calculation that assumes that a U.S. region with zero ADH exposure would have no employment changes (meaning that the cross-sectional regression has an absolute intercept of zero). This calculation implies that the China shock generated employment losses of $0.77 \cdot 2.63 \cdot 220$ million = 4.4 million jobs (where 2.63 is the mean exposure, and 220 million is approximately the U.S. population over 16 years between 2000 and 2007). However, according to our model, U.S. states with zero ADH exposure to the China shock increase their employment because they experience a positive terms-of-trade shock. This means that the cross-sectional regression of employment on exposure to China has a negative intercept. In particular, the intercept in the regression is approximately -1.5. A back-of-the-envelope calculation would add $1.5 \cdot 220 = 3.3$ million jobs because of the intercept. Combining the intercept number of 3.3 million jobs gained with the cross-sectional estimate of 4.4 million jobs lost,

would indicate that the net effect is a loss of 1.1 millions jobs.

The previous discussion is based on a simple regression in order to compare with ADH and discuss the importance of the intercept in the regression. However, with our full general equilibrium model we do not need to perform such simplified calculations, and we can compute the actual G.E. effects of the China shock in 2007. Our model implies that 1.28 millions jobs were lost by 2007 due to the China shock. This number is similar to the 1.1 million estimate obtained from our back-of-the-envelope calculations incorporating a non-zero intercept.

It is important to point out that in all of these estimates we stop the accounting of job losses in 2007. If we continue the analysis into further years, we would obtain that the China shock actually led to a net job gain in the U.S., since labor force participation is roughly 1% higher once the China shock completely dissipates.

6.3 Search and Matching vs. DNWR

While we have focused on DNWR as a way to generate unemployment in an otherwise standard quantitative trade model, we acknowledge that another approach would be to add search and matching frictions in the labor market. This has been done by papers like [Kim and Vogel \(2020a\)](#), [Kim and Vogel \(2020b\)](#), [Galle et al. \(2020\)](#) and [Dix-Carneiro et al. \(2020\)](#). As shown in [Kim and Vogel \(2020b\)](#), if the cost of posting vacancies is in terms of the final good, then a deterioration of the terms of trade leads to a decline in the real wage and an increase in unemployment. Broadly speaking, this is as if the economy moved downwards along an upward sloping "pseudo labor-supply curve." The problem with this approach, however, is that – as shown above – only three U.S. states experience a deterioration of their terms of trade, implying that unemployment would actually fall in most states (see [Galle et al., 2020](#)). In contrast, DNWR can lead to temporary increases in unemployment even in states that experience improving terms of trade

6.4 States vs. Commuting Zones

As discussed in previous sections of the paper, we have used the 0.22 and 0.55 response-to-exposure coefficients from ADH when calibrating our model. One concern is that the cross-

sectional regressions in ADH are done at the commuting-zone level, while our analysis is at the state level. The different sources of variation might lead to different response-to-exposure coefficients. To alleviate this concern, we ran the same ADH regressions but at the state rather than the commuting-zone level. Using actual (not model-based) data on state-level unemployment and labor force participation, we regressed the change in unemployment and the change in the fraction of the population not-in-the-labor-force between 2000 and 2007 (where 2007 is measured as an average between 2006 and 2008 like in ADH and turned into decadal changes) on exposure to China. We obtain a response-to-exposure coefficient on unemployment of 0.25 at the state level, compared to the 0.22 obtained in ADH at the commuting-zone level. The 95% confidence interval for the 0.25 estimate is between 0.15 and 0.35, which comfortably includes the 0.22 point estimate obtained in ADH. These two estimates are very close, indicating that the usage of the ADH targets for the state-level analysis does not seem to be problematic.

For the fraction of the population not-in-the-labor-force, we obtain a response-to-exposure coefficient of 0.37 at the state level, compared to the 0.55 obtained in ADH at the commuting-zone level. The two estimates are not as close as in the unemployment case, but the 95% confidence interval for the 0.37 estimate is between 0.16 and 0.58, which includes the 0.55 point estimate obtained in ADH. Given these results, we decided to use the ADH results as targets for our calibration rather than our own (necessarily less precise) state-level estimates. Our analysis would not change substantially if we used the state-level estimates instead.

6.5 Costs of Mobility Across Sectors

Workers do not incur in any costs of reallocation between home production and work in our baseline model, or between sectors in our extension with mobility frictions. Allowing for such costs would imply that agents' reallocation decisions would now depend on their expectations about future shocks. If agents were myopic and never expected the China shock to hit (even after it has already occurred in past years), our conclusions would not change. On the other extreme, if (as in [Caliendo et al., 2019](#)) agents had perfect foresight, then some workers would preemptively leave the manufacturing sector at the beginning of the 2000-2007 period. For a given value of δ , this would decrease unemployment in manufacturing, and so a higher δ would be required to match the same ADH coefficients.

7 Conclusion

In this paper we have added downward nominal wage rigidity to an otherwise standard quantitative trade model to study the path of adjustment in employment after a trade shock. Qualitatively, even a trade shock that improves an economy's terms of trade can lead to unemployment if it leads to a contraction in the economy's labor demand relative to overall nominal demand.

We calibrate the model to match the reduced-form evidence in [Autor et al. \(2013\)](#), and find that the China shock is responsible of up to 0.9 percentage points of the increase in unemployment in the U.S. over the period 2000-2007. This unemployment increase can go as high as 3 percentage points for the states affected the most. In spite of this increase in unemployment, the U.S. as a whole still gains from the China shock. However, such gains amount to between one third and two thirds of the gains without nominal rigidities, and there are 10 states that experience welfare losses despite the improvement in their terms of trade.

We acknowledge that the way we have captured nominal forces and trade imbalances in our model is simplistic relative to the state of the art in macroeconomics. A more satisfactory approach from a macroeconomic perspective would model monetary policy by adding a Taylor Rule with a zero lower bound, forward-looking agents making savings and investment decisions, and international financial flows affecting exchange rates, among other features. We have instead chosen to capture these forces via simple mechanical rules so that we can have a rich trade structure with many countries and sectors while still being able to conduct the quantitative analysis in a transparent way. Our hope is that this serves to identify the key elements that future models need to incorporate.

It is also important to emphasize that our welfare analysis is conducted at a fairly aggregate level. In particular, we do not capture the possibly large losses accruing to households experiencing unemployment spells in the absence of an appropriate safety net. Such large losses have been documented by many authors recently, for example [Autor et al. \(2014\)](#) and [Pierce and Schott \(2020\)](#). Thus, our results should be interpreted with caution; the decline in the gains from the China shock could be larger if our analysis were conducted at a more granular level.

References

- ADAO, R., C. ARKOLAKIS, AND F. ESPOSITO (2019): "Spatial linkages, global shocks, and local labor markets: Theory and evidence," Tech. rep., National Bureau of Economic Research.
- AUTOR, D. H., D. DORN, AND G. H. HANSON (2013): "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, 103, 2121–68.
- AUTOR, D. H., D. DORN, G. H. HANSON, AND J. SONG (2014): "Trade Adjustment: Worker-Level Evidence," *The Quarterly Journal of Economics*, 129, 1799–1860.
- BERAJA, M., E. HURST, AND J. OSPINA (2016): "The Aggregate Implications of Regional Business Cycles," Working Paper 21956, National Bureau of Economic Research.
- CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2015): "Global Imbalances and Currency Wars at the ZLB," Working Paper 21670, National Bureau of Economic Research.
- CALIENDO, L., M. DVORKIN, AND F. PARRO (2019): "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 87, 741–835.
- CALIENDO, L. AND F. PARRO (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 82, 1–44.
- CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2012): "Does Indivisible Labor Explain the Difference Between Micro and Macro Elasticities? A Meta-analysis of Extensive Margin Elasticities," *NBER macroeconomics Annual*, 27, 1–56.
- CHODOROW-REICH, G. AND J. WIELAND (2017): "Secular Business Reallocation and Business Cycles," Unpublished Manuscript.
- CHOUDHRI, E. U., H. FARUQEE, AND S. TOKARICK (2011): "Trade Liberalization, Macroeconomic Adjustment, and Welfare: How Costly Are Nominal Rigidities," *IMF Economic Review*, 59, 455–487.

- CLARIDA, R., J. GALI, AND M. GERTLER (2002): "A simple framework for international monetary policy analysis," *Journal of Monetary Economics*, 49, 879–904.
- COSTINOT, A. AND A. RODRIGUEZ-CLARE (2014): *Trade Theory with Numbers: Quantifying the Consequences of Globalization*, Elsevier, vol. 4 of *Handbook of International Economics*, 197–261.
- DALY, M. C. AND B. HOBIJN (2014): "Downward Nominal Wage Rigidities Bend the Phillips Curve," *Journal of Money, Credit and Banking*, 46, 51–93.
- DAVIDSON, C. AND S. J. MATUSZ (2004): *International trade and labor markets: Theory, evidence, and policy implications*, WE Upjohn Institute.
- DEKLE, R., J. EATON, AND S. KORTUM (2007): "Unbalanced Trade," *American Economic Review*, 97, 351–355.
- DICKENS, W. T., L. GOETTE, E. L. GROSHEN, S. HOLDEN, J. MESSINA, M. E. SCHWEITZER, J. TURUNEN, AND M. E. WARD (2007): "How Wages Change: Micro Evidence from the International Wage Flexibility Project," *Journal of Economic Perspectives*, 21, 195–214.
- DIX-CARNEIRO, R., J. PESSOA, R. REYES-HEROLES, AND S. TRAIBERMAN (2020): "Globalization, Trade Imbalances, and Labor Market Adjustment," *Working Paper*.
- DU CAJU, P., C. FUSS, AND L. WINTR (2012): "Sectoral Differences in Downward Real Wage Rigidity: Workforce Composition, Institutions, Technology and Competition," *Journal for Labour Market Research*, 45, 7–22.
- EATON, J., S. KORTUM, AND B. NEIMAN (2013): "On Deficits and Unemployment," *Revue Economique*, 64, 405–420.
- FALLY, T. (2015): "Structural gravity and fixed effects," *Journal of International Economics*, 97, 76–85.
- GALI, J. AND T. MONACELLI (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707–734.
- (2008): "Optimal Monetary and Fiscal Policy in a Currency Union," *Journal of International Economics*, 76, 116–132.

- (2016): “Understanding the Gains from Wage Flexibility: The Exchange Rate Connection,” *American Economic Review*, 106, 3829–68.
- GALLE, S., A. RODRIGUEZ-CLARE, AND M. YI (2020): “Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade,” Working Paper 23737, National Bureau of Economic Research.
- GOPINATH, G., E. BOZ, C. CASAS, F. J. DEZ, P.-O. GOURINCHAS, AND M. PLAGBORG-MLLER (2020): “Dominant Currency Paradigm,” *American Economic Review*, 110, 677–719.
- GRIGSBY, J., E. HURST, AND A. YILDIRMAZ (2019): “Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data,” NBER Working Papers 25628, National Bureau of Economic Research, Inc.
- HASAN, R., D. MITRA, P. RANJAN, AND R. N. AHSAN (2012): “Trade liberalization and unemployment: Theory and evidence from India,” *Journal of Development Economics*, 97, 269–280.
- HAZELL, J. AND B. TASKA (2019): “Downward Rigidity in the Wage for New Hires,” *Working Paper*.
- HEAD, K. AND T. MAYER (2002): *Illusory border effects: Distance mismeasurement inflates estimates of home bias in trade*, vol. 1, Citeseer.
- HEID, B. AND M. LARCH (2016): “Gravity with unemployment,” *Journal of International Economics*, 101, 70–85.
- HELPMAN, E., O. ITSKHOKI, AND S. REDDING (2010): “Inequality and unemployment in a global economy,” *Econometrica*, 78, 1239–1283.
- KIM, R. AND J. VOGEL (2020a): “Trade and welfare (across local labor markets),” *Working Paper*.
- (2020b): “Trade shocks and labor market adjustment,” *Working Paper*.
- MARTINEZ, I. Z., E. SAEZ, AND M. SIEGENTHALER (2019): “Intertemporal Labor Supply Substitution? Evidence from the Swiss Income Tax Holidays,” *Working Paper*.

- MUI, P. AND B. SCHOEFER (2020): "Reservation Raises: The Aggregate Labor Supply Curve at the Extensive Margin," *Working Paper*.
- NAKAMURA, E. AND J. STEINSSON (2014): "Fiscal Stimulus in a Monetary Union: Evidence from US Regions," *American Economic Review*, 104, 753–92.
- PIERCE, J. R. AND P. K. SCHOTT (2020): "Trade liberalization and mortality: evidence from US counties," *American Economic Review: Insights*, 2, 47–64.
- RADOWSKI, D. AND H. BONIN (2010): "Downward Nominal Wage Rigidity in Services: Direct Evidence from a Firm Survey," *Economics Letters*, 106, 227–229.
- SCHMITT-GROHE, S. AND M. URIBE (2016): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124, 1466–1514.
- TORTAROLO, D., G. CRUCES, AND V. CASTILLO (2019): "It Takes Two to Tango: Labor Responses to an Income Tax Holiday in Argentina," *Working Paper*.

Appendix

A Data Construction

In this appendix section, we provide details on the construction of the data we briefly described in Section 3.1. We divide this appendix into three parts. Appendix A.1 describes all data sources. Appendix A.2 discusses how we combine the different data sources to compute an internally consistent bilateral trade-flow matrix for all sectors for the years when all the data is available. There is limited availability for the state \times sector-level trade data coming from the CENSUS. Data for exports at the state \times sector-level starts in 2002 and data for imports starts in 2008. Finally, Appendix A.3 discusses how we use the previous step to construct a bilateral trade-flows for the years before full data availability.

A.1 Data Description and Sources

List of sectors: We use a total of 14 sectors. The list includes 12 manufacturing sectors, one catch-all services sector, and one agriculture sector. We follow [Caliendo et al. \(2019\)](#) in the selection of the 12 manufacturing sectors. These are: **1)** Food, beverage, and tobacco products (NAICS 311-312, WIOD sector 3); **2)** Textile, textile product mills, apparel, leather, and allied products (NAICS 313-316, WIOD sectors 4-5); **3)** Wood products, paper, printing, and related support activities (NAICS 321-323, WIOD sectors 6-7); **4)** Mining, petroleum and coal products (NAICS 211-213, 324, WIOD sectors 2, 8); **5)** Chemical (NAICS 325, WIOD sector 9); **6)** Plastics and rubber products (NAICS 326, WIOD sector 10); **7)** Nonmetallic mineral products (NAICS 327, WIOD sector 11); **8)** Primary metal and fabricated metal products (NAICS 331-332, WIOD sector 12); **9)** Machinery (NAICS 333, WIOD sector 13); **10)** Computer and electronic products, and electrical equipment and appliance (NAICS 334-335, WIOD sector 14); **11)** Transportation equipment (NAICS 336, WIOD sector 15); **12)** Furniture and related products, and miscella-

neous manufacturing (NAICS 337- 339, WIOD sector 16). There is a **13**) Services sector which includes Construction (NAICS 23, WIOD sector 18); Wholesale and retail trade sectors (NAICS 42-45, WIOD sectors 19-21); Accommodation and Food Services (NAICS 721-722, WIOD sector 22); transport services (NAICS 481-488, WIOD sectors 23-26); Information Services (NAICS 511-518, WIOD sector 27); Finance and Insurance (NAICS 521-525, WIOD sector 28); Real Estate (NAICS 531-533, WIOD sectors 29-30); Education (NAICS 61, WIOD sector 32); Health Care (NAICS 621-624, WIOD sector 33); and Other Services (NAICS 493, 541, 55, 561, 562, 711-713, 811-814, WIOD sector 34).²⁶

List of countries: We use data for 50 U.S. states, 37 other countries including a constructed rest of the world. The list of countries is: Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, the Slovak Republic, Slovenia, South Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.

Data on bilateral trade between countries: World Input-Output Database (WIOD). Release of 2013. We use data for 2000-2007. We map the sectors in the WIOD database to our 14 sectors in the following way: **1**) Food Products, Beverage, and Tobacco Products (c3); **2**) Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4-c5); **3**) Wood Products, Paper, Printing, and Related Support Activities (c6-c7); **4**) Petroleum and Coal Products (c8); **5**) Chemical (c9); **6**) Plastics and Rubber Products (c10); **7**) Nonmetallic Mineral Products (c11); **8**) Primary Metal and Fabricated Metal Products (c12); **9**) Machinery (c13); **10**) Computer and Electronic Products, and Electrical Equipment and Appliances (c14); **11**) Transportation Equipment (c15); **12**) Furniture and Related Products, and Miscellaneous Manufacturing (c16); **13**) Construction (c18), Wholesale and Retail Trade (c19-c21), Transport Services (c23-c26), Information Services (c27), Finance and Insurance (c28), Real Estate (c29- c30); Education (c32); Health Care (c33), Accommodation and Food Services (c22), and Other Services (c34); **14**) Agriculture and Mining (c1-c2). We follow [Costinot and Rodriguez-Clare \(2014\)](#) to remove the negative values in the trade data from WIOD.

²⁶The only difference with respect to [Caliendo et al. \(2019\)](#) in the definition of manufacturing sectors is that we include Mining (NAICS 211-213) together with Petroleum and Coal Products (NAICS 324) in our sector 4.

Data on bilateral trade in manufacturing between U.S states: We combine the 2002 and 2007 Commodity Flow Survey (CFS) with the WIOD database. The CFS records shipments between U.S states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). We follow [Caliendo et al. \(2019\)](#) and use CFS 2007 tables that cross-tabulate establishments by their assigned NAICS codes against commodities (SCTG) shipped by establishments within each of the NAICS codes. These tables allow for mapping of SCTG to NAICS.

Data on bilateral trade in manufacturing and agriculture between U.S states and the rest of the countries: We obtain sector-level imports and exports between the 50 U.S. states and the list of other countries from the Import and Export Merchandise Trade Statistics, which is compiled by the U.S. Census Bureau. This dataset reports imports and exports in each NAICS sector between each U.S. state and each other country in the world. Data for exports at the state \times sector level starts in 2002. Data for imports at the state \times sector level starts in 2008.

Data on sectoral and regional value added share in gross output: Value added for each of the 50 U.S. states and 14 sectors can be obtained from the Bureau of Economic Analysis (BEA) by subtracting taxes and subsidies from GDP data. In the cases when gross output was smaller than value added we constrain value added to be equal to gross output. For the list of other countries we obtain the share of value added in gross output using data on value added and gross output data from WIOD.

Data on services expenditure and production: We compute bilateral trade in services using a gravity approach explained in [Appendix A.2.4](#). As part of this calculations we require data on production and expenditure in services by region. We obtain U.S. state-level services GDP from the Regional Economic Accounts of the Bureau of Economic Analysis (BEA). We obtain U.S. state-level services expenditure from the Personal Consumption Expenditures (PCE) database of BEA. Finally, for the list of other countries we compute total production and expenditure in services from WIOD.

Data on agriculture expenditure and production: We also compute bilateral trade in agriculture using a gravity approach explained in [Appendix A.2.4](#). To get production in agriculture for the U.S. states we combine the 2002 and 2007 Agriculture Census with the National Marine Fisheries Service Census to get state-level production data on crops and livestock and

seafood. We infer state-level expenditure in agriculture from our gravity approach explained in Appendix A.2.4. Finally, for the list of other countries we compute total production and expenditure in agriculture from WIOD.

Data on population and geographic coordinates: As part of the gravity approach to compute bilateral trade in services, we also need to compute bilateral distances between regions. We follow the procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. We thus require data on the most populated cities in each country, the cities' coordinates and population, and each country's population. We obtain this information from the United Nations' Population Division website. In particular, we use the population of urban agglomerations with 300,000 inhabitants or more in 2018, by country, for 2000-2007. For Austria, Cyprus, Denmark, Estonia, Hungary, Ireland, Lithuania, Slovakia and Slovenia we use the two most populated cities.²⁷ For the case of U.S. states, we use population and coordinates data for each U.S county within each U.S state. The data for the U.S. counties comes from the U.S. CENSUS.

A.2 Construction of Bilateral Trade Flows Between Regions

We follow the notation from Costinot and Rodriguez-Clare (2014) and omit the time subscripts t that are relevant in our quantitative model. Define $X_{ij,ks}$ as sales of intermediate goods from sector k in region i to sector s in region j , and $X_{ij,kF}$ as the sales of sector k in region i to the final consumer of region j . Our final objective is to construct a bilateral trade flows matrix between all regions in our sample with elements equal to $X_{ij,k} = \sum_s X_{ij,ks} + X_{ij,kF}$. This matrix allows us to compute the trade shares $\lambda_{ij,k}$, and the sector-level revenues $R_{j,k} = \sum_l X_{jl,k}$ for each region, which are crucial elements in our hat algebra described in Section 2.6.

As additional definitions, take $E_{j,k} = \sum_i X_{ij,k}$ as the total expenditure of region j in sector k , $F_{j,k} = \sum_i X_{ij,kF}$ as the final consumption in region j of sector k , $F_j = \sum_k F_{j,k}$ as the total final consumption of region j , and $X_{j,ks} = \sum_i X_{ij,ks}$ as the total purchases that sector s in region j makes from sector k . We construct the matrix of $X_{ij,k}$ in four parts explained below. With some abuse of notation, we refer to a region i as a U.S. state (country) by using the notation $i \in US$ ($i \notin US$).

²⁷For the specific case of Cyprus, the cities' information comes from the country's Statistical Service.

A.2.1 Part 1: Bilateral Trade between Countries

In the first part we focus on the case where both i and j are countries. Thus, we simply take $X_{ij,k} = X_{ij,k}^{WIOD}$, where $X_{ij,k}^{WIOD}$ are the bilateral trade flows that come directly from the WIOD database without any further calculations.

A.2.2 Part 2: Manufacturing Trade between U.S. States and Countries

For the second part, we combine Census and WIOD data to calculate the trade flows between each of the 50 U.S. states and the other 37 country regions. We scale state-level imports and exports data from the Import and Export Merchandise Trade Statistics to match the U.S. totals in WIOD. More precisely, the exports (imports) of state i to (from) country j in manufacturing sector k are computed as a proportion of WIOD's U.S. export (imports) to (from) country j in sector k . This proportion is equal to the exports (imports) of state i to (from) country j in sector k relative to the total U.S. exports (imports) to (from) country j in sector k .

Mathematically, let $X_{ij,k}^{census}$ be the bilateral trade flows between regions i and j , in sector k , according to the Import and Export Merchandise Trade Statistics database. Define the share of sector k exports of state i to country j relative to the total U.S. exports of sector k as:

$$y_{ij,k}^{census} \equiv \frac{X_{ij,k}^{census}}{\sum_{h \in US} X_{hj,k}^{census}}.$$

Analogously, define the share of sector k imports of state j to country i as:

$$e_{ij,k}^{census} \equiv \frac{X_{ij,k}^{census}}{\sum_{l \in US} X_{il,k}^{census}},$$

then we define our object of interest:

$$X_{ij,k} = \begin{cases} y_{ij,k}^{census} X_{US,j,k}^{WIOD} & \forall i \in US, \forall j \notin US \\ e_{ij,k}^{census} X_{i,US,k}^{WIOD} & \forall i \notin US, \forall j \in US \end{cases}.$$

A.2.3 Part 3: Manufacturing Trade among U.S. States

In the third part we focus on manufacturing bilateral trade between U.S. States. For this, we combine WIOD Data for the total trade of the U.S. with itself, and the closest Commodity Flow Survey (CFS) for each year. We first compute the shares that each state i exports to state j in sector k represent in the total trade of sector k according to CFS. Then, we calculate the total exports of state i to state j in sector k as WIOD's U.S. trade with itself in sector k multiplied by the share computed in the previous step.

Mathematically, define $X_{ij,k}^{CFS}$ as the bilateral trade flows between state i and state j , in manufacturing sector k , according to the CFS. We first construct:

$$x_{ij,k}^{CFS} \equiv \frac{X_{ij,k}^{CFS}}{\sum_h \sum_l X_{hl,k}^{CFS}} \quad \forall (i \in US, \& j \in US),$$

then we define our object of interest: $X_{ij,k} = x_{ij,k}^{CFS} X_{US,US,k}^{WIOD} \quad \forall (i \in US, \& j \in US)$.

A.2.4 Part 4: Trade in Services and Trade in Agriculture

We compute bilateral trade flows for services and agriculture separately using a gravity structure that matches WIOD totals for trade between countries (including the U.S.).

Theory. Start with the standard gravity equation (for simplicity, we remove the subscript of the sector):

$$X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j} \right)^{-\varepsilon} E_j,$$

where $P_j^{-\varepsilon} = \sum_i (w_i \tau_{ij})^{-\varepsilon}$. We know that $\sum_j X_{ij} = R_i$ and hence $\sum_j \left(\frac{w_i \tau_{ij}}{P_j} \right)^{-\varepsilon} E_j = R_i$. This implies $w_i^{-\varepsilon} \Pi_i^{-\varepsilon} = R_i$, where $\Pi_i^{-\varepsilon} = \sum_j \tau_{ij}^{-\varepsilon} P_j^\varepsilon E_j$. Let $\tilde{P}_j \equiv P_j^{-\varepsilon}$ and $\tilde{\Pi}_i \equiv \Pi_i^{-\varepsilon}$, and $\tilde{\tau}_{ij} \equiv \tau_{ij}^{-\varepsilon}$. Given $\{E_j\}$, $\{R_i\}$, and $\{\tilde{\tau}_{ij}\}$, one we can get $\{\tilde{P}_j\}$ and $\{\tilde{\Pi}_i\}$ for all regions from the following system:

$$\begin{aligned} \tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \\ \tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \end{aligned} \quad (17)$$

The solution for $\{\tilde{P}_j, \tilde{\Pi}_i\}$ is unique up to a constant (Fally, 2015). This indeterminacy requires a normalization. We thus impose $\tilde{P}_1 = 100$ in each exercise. Then one can compute our outcome of interest $\{X_{ij}\}$ from

$$X_{ij} = \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j. \quad (18)$$

Computation of the bilateral resistance $\tilde{\tau}_{ij}$. To solve the gravity system, we must first compute $\tilde{\tau}_{ij} \forall i, j$. We proceed by assuming the following functional form:

$$\tilde{\tau}_{ij} = \beta_0^{l_{ij}} dist_{ij}^{\beta_1} \exp(\xi_{ij}),$$

where l_{ij} is an indicator variable equal to 1 if $i = j$, and ξ_{ij} is an idiosyncratic error term. β_1 captures the standard distance elasticity and β_0 captures the additional *inverse* resistance of trading with others versus with oneself.

To calculate $dist_{ij}$, we follow the same procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. The idea is to calculate the distance between two countries based on bilateral distances between the largest cities of those two countries, those inter-city distances being weighted by the share of the city in the overall country's population (Head and Mayer, 2002).

We use population for 2010 and coordinates data for all U.S. counties, and all cities around the world with more than 300,000 inhabitants. For those countries with less than two cities of this size, we take the largest cities. Coordinates are important to calculate the physical bilateral distances in kms between each county r in state i and county s in state j ($d_{rs} \forall r \in i, s \in j$ and $\forall i, j = 1, \dots, 50$), and define $dist(ij)$ as:

$$dist(ij) = \left(\sum_{r \in i} \sum_{s \in j} \left(\frac{pop_r}{pop_i} \right) \left(\frac{pop_s}{pop_j} \right) d_{rs}^\theta \right)^{1/\theta}, \quad (19)$$

where pop_h is the population of country/state h . We set $\theta = -1$.

Given our definition of $\tilde{\tau}_{ij}$ we can write the gravity equation between countries in the

following way.

$$X_{ij} = \beta_0^{l_{ij}} dist_{ij}^{\beta_1} \exp(\zeta_{ij}) \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j.$$

Taking logs we can write the previous equation as:

$$\ln X_{ij} = \delta_i^o + \delta_j^d + \tilde{\beta}_0 l_{ij} + \beta_1 \ln dist_{ij} + \zeta_{ij}, \quad (20)$$

where $\tilde{\beta}_0 = \ln \beta_0$ and the δ s are fixed effects. We first estimate the equation above separately for services and agriculture using a 2000-2011 panel of bilateral trade flows between countries from WIOD. We present our OLS estimation results in Table 5. Columns (1) and (2) refer to the estimated coefficients for the case of services and agriculture, respectively. Both regressions include year-by-origin and year-by-destination fixed effects. We take these estimates and compute the bilateral resistance term in each sector as $\hat{\tau}_{ij} = \exp(\hat{\beta}_0 l_{ij} + \hat{\beta}_1 \ln dist_{ij})$.

Table 5: Estimation of Own-Country Dummy and Distance Elasticity

Dep. Var.: $\ln X_{ij,t}$	(1) Services	(2) Agriculture
l_{ij}	7.357 *** (0.126)	4.143 *** (0.145)
$\ln dist_{ij}$	-0.376 *** (0.037)	-1.745 *** (0.020)
Year \times Orig.	Yes	Yes
Year \times Dest.	Yes	Yes
Observations	17,328	17,328
Adjusted R^2	0.66	0.76

Notes: This table displays the OLS estimates of specifications analogous to the one in equation (20). The outcome variable $\ln X_{ij,t}$ is the log exports of country i sent to country j . The own-country dummy l_{ij} is defined as an indicator function equal to one whenever country i is the same as country j . Finally, $\ln dist_{ij}$ is the log distance between country i and country j . This variable is computed according to equation (19). Robust standard errors are presented in parenthesis. *** denotes statistical significance at the 1%.

Trade in Services: As inputs, we need total expenditures in services for each region (E_i), as well as total production in services (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we take these variables from the Regional Economic Accounts of the Bureau of Economic Analysis. We scale the state-level services production and expenditures so that they aggregate to the U.S. totals in WIOD.

We incorporate the information on bilateral trade in services between countries (including the U.S.) that comes from WIOD to the gravity system of equation (17) by first writing the system as:

$$\begin{aligned}\tilde{P}_j &= \sum_{i \notin US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i + \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \\ \tilde{\Pi}_i &= \sum_{j \notin US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j + \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j.\end{aligned}$$

Then, we define $\tilde{\lambda}_j \equiv 1 - \frac{\sum_{i \notin US} X_{ij}}{E_j}$ for $j \notin US$ (the share of imports of region $j \notin US$ coming from the U.S.) and $\tilde{\lambda}_i^* \equiv 1 - \frac{\sum_{j \notin US} X_{ij}}{R_i}$ for $i \notin US$ (total exports of region $i \notin US$ to other regions not in the U.S.). Using these two definitions and substituting $\tilde{\tau}_{ij} = X_{ij} \tilde{\Pi}_i \tilde{P}_j^{-1} Y_i^{-1} X_j^{-1}$ whenever $i, j \notin US$ in the previous system of equations we have the final system we solve for services:

$$\begin{aligned}\tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \in US \\ \tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \in US \\ \tilde{\lambda}_j \tilde{P}_j &= \sum_{i \notin US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \notin US \\ \tilde{\lambda}_i^* \tilde{\Pi}_i &= \sum_{j \notin US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \notin US\end{aligned}$$

Once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the final bilateral trade matrix according to equation (18).

Trade in Agriculture: As inputs, we need total expenditures in services for each region (E_i), as well as total production in agriculture (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we compute total production (R_i) by combining data from the Agriculture Census and the National Marine Fisheries Service Census. We scale the

state-level agriculture production so that it aggregates to the U.S. total in WIOD. However, it is not possible to find state-level agriculture expenditure for U.S. states. To overcome this data unavailability, we combine the U.S. input-output matrix ($\phi_{j,ks}$) together with the shares of value-added in gross production (i.e., the labor share) ($\phi_{j,k}$) in order to compute a value of (E_i) that is consistent with the full bilateral trade matrix for all regions and all sectors.

In order to describe our procedure, note that the total expenditure of region j in sector k ($E_{j,k}$) could be written as:

$$E_{j,k} = \sum_s \tilde{\phi}_{j,ks} R_{j,s} + F_{j,k},$$

where $\tilde{\phi}_{j,ks} = \phi_{j,ks}(1 - \phi_{j,s})$. We make two assumptions. First, we assume that $\tilde{\phi}_{j,ks} = \tilde{\phi}_{US,ks} \forall j \in US$, which means that we assume common input-output matrix and value-added shares across U.S. states and equal to the ones of the U.S. as a whole. Second, when $j \in US$ we assume that $F_{j,k} = \frac{F_j}{F_{US}} F_{US,k} = F_j \gamma_k$, where $\gamma_k \equiv \frac{F_{US,k}}{F_{US}}$. This second assumption relies on the identical Cobb-Douglas preferences across U.S. states. Using these two assumptions we get that:

$$F_j = E_{j,k} - \sum_s \tilde{\phi}_{j,ks} R_{j,s} + \sum_{r \neq k} \left(E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s} \right).$$

Substituting the previous equation in the definition of $E_{j,k}$ for the agriculture sector ($k = AG$), and $j \in US$ we find:

$$E_{j,AG} = \sum_s \tilde{\phi}_{j,AGs} R_{j,s} + \frac{\gamma_{AG}}{1 - \gamma_{AG}} \sum_{r \neq AG} \left(E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s} \right),$$

which can be computed using state-level production of all sectors and state-level expenditure data of all other sectors (excluding agriculture), combined with the U.S.-level input-output matrix, value-added shares, and sector-level consumption shares.

Once we obtain the state-level expenditure values in agriculture, we can proceed with the gravity system in equation (17). As in the case of services, we incorporate the information on bilateral trade in agriculture between countries (including the U.S.) that comes from WIOD. We also incorporate the bilateral trade in agriculture between U.S. states and other countries coming from the Import and Export Merchandise Trade Statistics. Thus, we only need to focus

on $\{\tilde{P}_j\}_{j \in US}$ and $\{\tilde{\Pi}_i\}_{i \in US}$.

Define $\chi_i^* = 1 - \sum_{j \notin US} \frac{X_{ij}}{R_i}$ for $i \in US$ (the share of sales of state i that stay in the U.S.) and $\chi_j = 1 - \sum_{i \notin US} \frac{X_{ij}}{E_{j,k}}$ for $j \in US$ (the share of purchases of state i that come from the U.S.). The final system we solve for agriculture becomes:

$$\begin{aligned}\chi_j \tilde{P}_j &= \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i, \forall j \in US \\ \chi_i^* \tilde{\Pi}_i &= \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j, \forall i \in US\end{aligned}$$

As before, once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the bilateral trade in agriculture between U.S. states according to equation (18).

A.3 Projection of Bilateral Trade Flows between Regions

Since the Import and Export Merchandise Trade Statistics data for exports starts in 2002 and for imports starts in 2008, the bilateral trade flows between regions for the years before the data starts cannot be computed directly from the data. In this section, we adapt our computation method to take into account this issue. All previous procedures with the exception of the manufacturing, agriculture, and mining trade between U.S. states and countries remains the same. For the exception case we proceed as follows. Denote $X_{ij,k}^{base}$ as the matrix $X_{ij,k}$ for the first year where the exports or imports data is available (the base year). Define the share of exports of U.S. State i in sector k , going to country j in the base year as:

$$y_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{h \in US} X_{hj,k}^{base}} \quad \forall i \in US, j \notin US.$$

Similarly, define the share of imports of U.S. state j in sector k , coming from country i in the base year as:

$$e_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{l \in US} X_{il,k}^{base}} \quad \forall i \notin US, j \in US.$$

Finally for each sector k in manufacturing or agriculture; and any year before the base year define:

$$X_{ij,k} = \begin{cases} e_{ij,k}^{base} X_{iUS,k}^{WIOD} & \forall i \notin US, \forall j \in US \\ y_{ij,k}^{base} X_{USj,k}^{WIOD} & \forall i \in US, \forall j \notin US \end{cases} .$$

B Model Details

B.1 Production

Here we elaborate on the way the Input-Output loop works. There are I regions and S sectors, and to produce output in each region and sector firms need to combine labor with all the sectoral aggregates (the version of them available in that region). Specifically, the technology to produce the differentiated good of industry s in region i at time t is

$$Y_{i,s,t} = \left(\phi_{i,s}^{-\phi_{i,s}} \prod_{k=1}^S \phi_{i,ks}^{-\phi_{i,ks}} \right) A_{i,s,t} L_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S M_{i,ks,t}^{\phi_{i,ks}}$$

where $M_{i,ks,t}$ is the quantity of the composite good of industry k used in region i to produce in sector s at time t , $\phi_{i,s}$ is the labor share in region i , sector s , $\phi_{i,ks}$ is the share of inputs that sector s uses from sector k in region i , and $1 - \phi_{i,s} = \sum_{k=1}^S \phi_{i,ks}$. The resource constraint for the composite good produced in region j , sector k , at time t is

$$M_{j,k,t} = C_{j,k,t} + \sum_{s=1}^S M_{j,ks,t}.$$

In turn, the resource constraint for good s produced by region i at time t is

$$Y_{i,s,t} = \sum_{j=1}^I \tau_{ij,s,t} Y_{ij,s,t}.$$

The composite in sector k is produced according to

$$M_{j,k,t} = \left(\sum_{i=1}^I Y_{ij,k,t}^{\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}} .$$

Now let's move to the equations in terms of the prices and values. Let's start with prices. Let $P_{i,s,t}$ be the price of $M_{i,s,t}$ and $p_{ij,s,t}$ be the price of $Y_{i,s,t}$ in j at time t . Recall that the wage can vary between different sectors in the same region because of mobility frictions, so let $W_{i,s,t}$ be the nominal wage in region i , sector s , at time t . We know that

$$\begin{aligned} p_{ii,s,t} &= A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S P_{i,k,t}^{\phi_{i,ks}}, \\ p_{ij,s,t} &= \tau_{ij,s,t} p_{ii,s,t}, \\ P_{j,s,t} &= \left(\sum_{i=1}^I p_{ij,s,t}^{1-\sigma_s} \right)^{1/(1-\sigma_s)}, \end{aligned}$$

Combining the last three equations we obtain:

$$P_{j,s,t}^{1-\sigma_s} = \sum_{i=1}^I \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S P_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s},$$

which, for each time period t , is a system of $I \times S$ equations in $I \times S$ unknowns that can be used to solve for the $P_{j,s,t}$'s given the trade costs ($\tau_{ij,s,t}$'s), technologies ($A_{i,s,t}$'s), wages ($W_{i,s,t}$'s), labor shares ($\phi_{i,s}$'s) and input output coefficients ($\phi_{i,ks}$), note that we do not allow the labor shares and input output coefficients to vary with time. This system of $I \times S$ equations in $I \times S$ unknowns is well behaved and can be solved using contraction mapping techniques, where you start with a guess for the $I \times S$ prices (denoted $PI_{j,s,t}$), and obtain a new guess (denoted $PE_{j,s,t}$) as follows:

$$PE_{j,s,t} = \left(\sum_{i=1}^I \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S PI_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$$

We iterate until the difference between PE and PI is very small and this provides a solution to the system. This is a similar method to the one followed in [Caliendo and Parro \(2015\)](#). Getting back to the description of the setup of the model, the price of final output in region j at time t

is given by

$$P_{j,t} = \prod_{s=1}^S P_{j,s,t}^{\alpha_{j,s}}.$$

Now let's move on to resource constraints in value. Multiplying the resource constraint for $M_{j,k,t}$ by $P_{j,k,t}$ we get

$$Z_{j,k,t} = P_{j,k,t} C_{j,k,t} + \sum_{s=1}^S P_{j,k,t} M_{j,ks,t},$$

where $Z_{j,k,t} \equiv P_{j,k,t} M_{j,k,t}$ denotes the total expenditure of region j in industry k at time t . Let $\lambda_{ij,k,t}$ be the share of that expenditure spent on imports from i ,

$$\lambda_{ij,k,t} \equiv \frac{p_{ij,k,t} Y_{ij,k,t}}{Z_{j,k,t}}.$$

We know that

$$\lambda_{ij,k,t} = \frac{p_{ij,k,t}^{1-\sigma_k}}{\sum_l p_{lj,k,t}^{1-\sigma_k}} = \frac{p_{ij,k,t}^{1-\sigma_k}}{P_{j,k,t}^{1-\sigma_k}} = \frac{\left(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,k,t}^{\phi_{i,k}} \prod_{s=1}^S P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}}{\sum_{r=1}^I \left(\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,k,t}^{\phi_{r,k}} \prod_{s=1}^S P_{r,s,t}^{\phi_{r,sk}} \right)^{1-\sigma_k}}.$$

Let $R_{i,k,t} = p_{ii,k,t} Y_{i,k,t}$ represent the sales of good k by region i at time t . Multiplying the resource constraint for $Y_{i,k,t}$ above by $p_{ii,k,t}$ we get

$$p_{ii,k,t} Y_{i,k,t} = \sum_{j=1}^I \tau_{ij,k,t} p_{ii,k,t} Y_{ij,k,t},$$

and hence

$$R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} Z_{j,k,t}.$$

Plugging in from the resource constraint above for $Z_{j,k,t}$ we then have

$$R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} \left(P_{j,k,t} C_{j,k,t} + \sum_s P_{j,k,t} M_{j,ks,t} \right).$$

Note that

$$P_{j,k,t}M_{j,ks,t} = \phi_{j,ks}R_{j,s,t}.$$

Additionally, the total amount available for consumption in region j at time t is the sum of three things, total labor income which includes rebates from the government (denote it $I_{j,t}$) and the deficit (denoted $D_{j,t}$). So we get

$$P_{j,k,t}C_{j,k,t} = \alpha_{j,k} (I_{j,t} + D_{j,t}),$$

hence

$$R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} \left(\alpha_{j,k} (I_{j,t} + D_{j,t}) + \sum_s \phi_{j,ks} R_{j,s,t} \right).$$

For each time period t , this is a linear system of $I \times S$ equations in $I \times S$ unknowns that can be used to solve for the $R_{i,k,t}$'s given the trade shares ($\lambda_{ij,k,t}$'s), Cobb-Douglas shares ($\alpha_{j,k}$'s), labor incomes ($I_{j,t}$'s), deficits ($D_{j,t}$'s), government revenues ($G_{j,t}$'s), and input output coefficients ($\phi_{j,ks}$). Since this is a linear system in the R 's, it is relatively easy to solve. Of this total production ($R_{i,k,t}$), we know that a fraction $\phi_{i,k}$ is paid to labor, so we can write:

$$W_{i,b,t}L_{i,b,t} = W_{i,b,t} \sum_{s \in b} L_{i,s,t} = \sum_{s \in b} \phi_{i,s} R_{i,s,t}$$

B.2 Labor Supply

We ignore region and time subscripts here, so that we are effectively focusing on a single region and period. Each agent has a discrete choice between home production with utility flow μ and work in broad sector $b \in \{1, \dots, B\}$ with real wage ω_b . For ease of notation, we think of home production as broad sector 0, so that the set of broad sectors is now $\{0, 1, \dots, B\}$, and we use $\omega_0 = \mu$. Each agent has utility parameters z_b for $b \in \{0, 1, \dots, B\}$ so that agent's utility is $\ln \omega_b + z_b$ if she chooses broad sector b . We assume that these utility parameters are drawn from a nested Gumbel distribution so that the cumulative distribution of $Z = (Z_0, Z_1, \dots, Z_B)$ is

$$H(z) = \exp \left(- \exp(-\kappa z_0) - \left(\sum_{b=1}^B \exp(-\eta z_b) \right)^{\kappa/\eta} \right), \quad (21)$$

with $0 \leq \kappa \leq \eta$.²⁸

Imagine that an agent is maximizing $\ln \omega_b + z_b$ (over choice of b). Below we show that the probability that the agent chooses $b \in \{1, \dots, B\}$ is given by

$$\tilde{\pi}_b = \frac{\omega_b^\eta}{\sum_{b'=1}^B \omega_{b'}^\eta} \frac{\left(\sum_{b'=1}^B \omega_{b'}^\eta\right)^{\kappa/\eta}}{\omega_0^\kappa + \left(\sum_{b'=1}^B \omega_{b'}^\eta\right)^{\kappa/\eta}}. \quad (22)$$

Moreover, the exponential of the expectation of the maximized value is

$$\exp(E(\max \ln \omega_b + Z_b)) = e^\gamma \left(\omega_0^\kappa + \left(\sum_{b=1}^B \omega_b^\eta \right)^{\kappa/\eta} \right)^{1/\kappa}, \quad (23)$$

where γ is the Euler-Mascheroni constant.

The result in Equation (22) further implies that the probability of choosing $b \in \{1, \dots, B\}$ is

$$\pi \equiv \sum_{b=1}^B \tilde{\pi}_b = \frac{\left(\sum_{b=1}^B \omega_b^\eta\right)^{\kappa/\eta}}{\omega_0^\kappa + \left(\sum_{b=1}^B \omega_b^\eta\right)^{\kappa/\eta}}$$

while the probability of choosing $b \in \{1, \dots, B\}$ conditional on not choosing $b = 0$ is

$$\pi_b \equiv \frac{\tilde{\pi}_b}{\pi} = \frac{\omega_b^\eta}{\sum_{b'=1}^B \omega_{b'}^\eta}.$$

This implies that $\tilde{\pi}_b = \pi \cdot \pi_b$. Of course, we also have that the probability that the agent chooses $b = 0$ is

$$\tilde{\pi}_0 = 1 - \sum_{b=1}^B \tilde{\pi}_b = \frac{\omega_0^\kappa}{\omega_0^\kappa + \left(\sum_{b'=1}^B \omega_{b'}^\eta\right)^{\kappa/\eta}}.$$

We now prove the result in Equation (22). Without loss of generality, we derive the probability that $1 = \arg \max_b \ln \omega_b + Z_b$. This is the same as the probability that $z_b \leq a_b + Z_1$ for all

²⁸This conditions is sufficient to ensure that the density function is positive everywhere in $z \in \mathbb{R}_+^{B+1}$.

b , where $a_b \equiv \ln(\omega_1/\omega_b)$. Thus,

$$\tilde{\pi}_1 = \int_{-\infty}^{\infty} H_1(a_0 + z, z, a_2 + z, \dots, a_B + z) dz. \quad (24)$$

From (21) we get

$$\begin{aligned} H_1(z_0, z_1, \dots, z_B) &= \kappa \left(\sum_{b=1}^B \exp(-\eta z_b) \right)^{\kappa/\eta-1} \exp(-\eta z_1) \\ &\cdot \exp \left(-\exp(-\kappa z_0) - \left(\sum_{b=1}^B \exp(-\eta z_b) \right)^{\kappa/\eta} \right), \end{aligned}$$

and hence

$$\begin{aligned} H_1(a_0 + z, z, a_2 + z, \dots, a_B + z) &= \kappa A^{\kappa-\eta} \exp(-\kappa z) \exp(-\eta a_1) \\ &\cdot \exp(-[\exp(-\kappa a_0) - A^\kappa] \exp(-\kappa z)), \end{aligned} \quad (25)$$

where

$$A \equiv \left(\sum_{b=1}^B \exp(-\eta a_b) \right)^{1/\eta}.$$

Plugging into (24) and integrating we get

$$\tilde{\pi}_1 = \frac{A^{\kappa-\eta}}{\exp(-\kappa a_0) + A^\kappa} = \frac{\left(\sum_{b=1}^B \exp(-\eta a_b) \right)^{\kappa/\eta-1}}{\exp(-\kappa a_0) + \left(\sum_{b=1}^B \exp(-\eta a_b) \right)^{\kappa/\eta}}.$$

Plugging in for $a_b \equiv \ln \omega_1/\omega_b$ and simplifying we get

$$\tilde{\pi}_1 = \frac{\omega_1^\eta}{\sum_{b=1}^B \omega_b^\eta} \frac{\left(\sum_{b=1}^B \omega_b^\eta \right)^{\kappa/\eta}}{\omega_0^\kappa + \left(\sum_{b=1}^B \omega_b^\eta \right)^{\kappa/\eta}}.$$

Generalizing to $b \geq 1$ leads to the result in Equation (22).

To establish the result in Equation (23), we again assume that the choice is $b = 1$. The probability that $\max_b \ln(Z_b \omega_b) \leq x$ and $1 = \arg \max_b Z_b \omega_b$ is $\int_0^{(1/\omega_1) \exp x} H_1(a_0 z, z, a_2 z, \dots, a_B z) dz$.

Using (25) and integrating we then have

$$\int_0^{(1/\omega_1) \exp x} H_1(a_0 z, z, a_2 z, \dots, a_B z) dz = \frac{A^{\kappa-\eta}}{a_0^{-\kappa} + A^\kappa} \exp\left(-(\exp x)^{-\kappa} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa]\right).$$

This implies that

$$\begin{aligned} E(\max \ln(Z_b \omega_b) \mid \arg \max Z_b \omega_b = 1) &= \frac{1}{\tilde{\pi}_1} \frac{A^{\kappa-\eta}}{a_0^{-\kappa} + A^\kappa} \int_{-\infty}^{\infty} x \kappa (\exp x)^{-\kappa-1} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa] \\ &\cdot \exp\left(-(\exp x)^{-\kappa} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa]\right) dx, \end{aligned}$$

and hence, using $\tilde{\pi}_1 = \frac{A^{\kappa-\eta}}{a_0^{-\kappa} + A^\kappa}$, we have

$$\begin{aligned} E(\max \ln(Z_b \omega_b) \mid \arg \max Z_b \omega_b = 1) &= \int_{-\infty}^{\infty} x \kappa (\exp x)^{-\kappa-1} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa] \\ &\cdot \exp\left(-(\exp x)^{-\kappa} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa]\right) dx. \end{aligned}$$

So we need to know

$$\int_{-\infty}^{\infty} x \kappa T (\exp x)^{-\kappa-1} \exp\left(-T (\exp x)^{-\kappa}\right) dx = \int_{-\infty}^{\infty} x d \exp\left(-T (\exp x)^{-\kappa}\right).$$

But note that

$$T (\exp x)^{-\kappa} = \left(T^{-1/\kappa} \exp x\right)^{-\kappa} = \left(\exp\left(-\frac{1}{\kappa} \ln T\right) \exp x\right)^{-\kappa} = \left(\exp\left(x - \ln T^{1/\kappa}\right)\right)^{-\kappa}.$$

This is the Gumbel distribution with location parameter $\mu = \ln T^{1/\kappa}$ and scale parameter $\beta = 0$. But we know that the expectation of a variable distributed Gumbel with μ and β is $\mu + \beta\gamma$, where γ is the Euler-Mascheroni constant, hence we have

$$\int_{-\infty}^{\infty} x d \exp\left(-T (\exp x)^{-\kappa}\right) = \ln T^{1/\kappa} + \gamma.$$

This implies that

$$\int_{-\infty}^{\infty} x \kappa (\exp x)^{-\kappa-1} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa] \exp\left(-(\exp x)^{-\kappa} \omega_1^\kappa [a_0^{-\kappa} + A^\kappa]\right) dx$$

$$= \ln \left[\omega_1 (a_0^{-\kappa} + A^\kappa)^{1/\kappa} \right] + \gamma$$

and hence

$$E(\max Z_b \omega_b | \arg \max Z_b \omega_b = 1) = \ln \left[\omega_1 (a_0^{-\kappa} + A^\kappa)^{1/\kappa} \right] + \gamma.$$

Substituting for A and a_b , we then have

$$E(\max Z_b \omega_b | \arg \max Z_b \omega_b = 1) = \ln \left[\omega_1 \left((\omega_1 / \omega_0)^{-\kappa} + \left(\sum_{b=1}^B (\omega_1 / \omega_b)^{-\eta} \right)^{\kappa/\eta} \right)^{1/\kappa} \right] + \gamma$$

and hence

$$E(\max Z_b \omega_b | \arg \max Z_b \omega_b = 1) = \ln \left[\left(\omega_0^\kappa + \left(\sum_{b=1}^B \omega_b^\eta \right)^{\kappa/\eta} \right)^{1/\kappa} \right] + \gamma$$

Of course, this implies that

$$\exp(E(\max Z_b \omega_b)) = e^\gamma \left(\omega_0^\kappa + \left(\sum_{b=1}^B \omega_b^\eta \right)^{\kappa/\eta} \right)^{1/\kappa}.$$

B.3 Equilibrium and Utility

In this model, the equilibrium system in each period t is described by equations (1) - (14). After introducing equations (4) and (5) into equation (3), we can express the equilibrium as follows:

$$\begin{aligned} R_{i,s,t} &= \sum_{j=1}^I \lambda_{ij,s,t} \left(\alpha_{j,s} \left(\sum_b W_{j,b,t} L_{j,b,t} + D_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R_{j,k,t} \right) & \forall i, \forall s \\ \lambda_{ij,s,t} &= \frac{(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,b(s),t}^{\phi_{i,s}} \prod_k P_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^I (\tau_{rj,s,t} A_{r,s,t}^{-1} W_{r,b(s),t}^{\phi_{r,s}} \prod_k P_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} & \forall i, \forall s \\ P_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \left(\tau_{ji,s,t} A_{j,s,t}^{-1} W_{j,b(s),t}^{\phi_{j,s}} \prod_{k=1}^S P_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} & \forall i, \forall s \end{aligned}$$

$$\begin{aligned}
W_{i,b,t}L_{i,b,t} &= \sum_{s \in b} \phi_{i,s} R_{i,s,t} && \forall i, \forall b \\
L_{i,b,t} &\leq \ell_{i,b,t}, W_{i,b,t} \geq \delta_{i,b} W_{i,b,t-1}, CS && \forall i, \forall b \\
\ell_{i,b,t} &= \frac{\omega_{i,t}^\kappa}{\mu_i^\kappa + \omega_{i,t}^\kappa} \frac{\omega_{i,b,t}^\eta}{\omega_{i,t}^\eta} \bar{L}_i && \forall i, \forall b \\
\omega_{i,b,t} &= \frac{W_{i,b,t}L_{i,b,t}}{P_{i,t}\ell_{i,b,t}} && \forall i, \forall b \\
\omega_{i,t}^\eta &= \sum_b \omega_{i,b,t}^\eta && \forall i \\
P_{i,t} &= \prod_{s=1}^S P_{i,s,t}^{\alpha_{i,s}} && \forall i \\
\sum_{i=1}^I \sum_b W_{i,b,t}L_{i,b,t} &= \gamma \sum_{i=1}^I \sum_b Y_{i,b,t-1} && \text{single}
\end{aligned}$$

The equilibrium in changes is then the one described in the main text. Recall that in our setup we can express instantaneous utility as:

$$u_{i,t} \propto (\mu_i^\kappa + \omega_{i,t}^\kappa)^{1/\kappa} = (\omega_{i,t}^\kappa / \pi_{i,t})^{1/\kappa} = \omega_{i,t} \pi_{i,t}^{-1/\kappa}.$$

The change in utility can then be expressed as:

$$\hat{u}_{i,t} = \hat{\omega}_{i,t} \hat{\pi}_{i,t}^{-1/\kappa} = \hat{\omega}_{i,t} \left(\frac{\hat{\omega}_{i,t}^\kappa}{1 - \pi_{i,t-1} + \pi_{i,t-1} \hat{\omega}_{i,t}^\kappa} \right)^{-1/\kappa} = (1 - \pi_{i,t-1} + \pi_{i,t-1} \hat{\omega}_{i,t}^\kappa)^{1/\kappa}.$$

C Exposure measures

Consider an economy producing a set of homogeneous goods across sectors $s = 1, \dots, S$ with prices p_s . Labor is the only factor of production that is mobile across sectors, and there are decreasing returns to labor in each sector so that $q_s = F_s(l_s)$ with $F'_s(\cdot) > 0$ and $F''_s(\cdot) < 0$. Preferences are given by $U(c) - V(l)$, where $l \equiv \sum_s l_s$, $U(c)$ is homogeneous of degree one, and $V'(\cdot) > 0$ and $V''(\cdot) < 0$. We are interested in the effect of a foreign shock on employment in two different cases. In the first case the wage w is fixed and labor is fully determined by labor demand (we assume that labor supply is higher than labor demand at the fixed wage w), while

in the second case the wage is fully flexible and clears the labor market. Below we show that further assuming that $\varepsilon(l_s) \equiv -\frac{F_s''(l_s)l_s}{F_s'(l_s)} = \varepsilon$ for all s and $\mu(l) \equiv \frac{V''(l)l}{V'(l)} = \mu$, then in the case of a fixed wage we have

$$d \ln l = \frac{1}{\varepsilon} \sum_s \left(\frac{p_s q_s}{I} \right) d \ln p_s \quad (26)$$

while in the case of flexible wages we have

$$d \ln l = \frac{1}{\varepsilon + \mu} \sum_s \left(\frac{p_s q_s - p_s c_s}{I} \right) d \ln p_s, \quad (27)$$

where $I \equiv \sum_s p_s q_s$. Thus, if the wage is fixed and if we know the log changes in prices resulting from the foreign shock then we can interact them with revenue shares, $\frac{p_s q_s}{I}$, to construct a Bartik-style sufficient statistic for the first order effect on employment. In contrast, if the wage fully adjusts to equalize labor supply and demand, then the appropriate weights (share components in the Bartik measure) for the price changes are instead given by net exports as a share of GDP, to capture the implied terms-of-trade effects. If the economy is small, then prices are exogenous and one could further replace $d \ln p_s$ by the underlying Chinese productivity shocks.

Let's start with the case where w is fixed. Fully differentiating the equilibrium condition $p_s F_s'(l_s) = w$ implies

$$d \ln l_s = \frac{d \ln p_s}{\varepsilon_s(l_s)},$$

where $\varepsilon(l_s) \equiv -\frac{F_s''(l_s)l_s}{F_s'(l_s)}$. We then have

$$d \ln l = \sum_s m_s \frac{d \ln p_s}{\varepsilon_s(l_s)}$$

where $m_s \equiv \frac{l_s}{\sum_s l_s}$. Assuming that $\varepsilon_s(l_s) = \varepsilon$ we know that $p_s q_s / I = m_s$ and hence we get (26).

Now let's consider the case with a flexible wage. The equilibrium is given by w, l, λ and $\{l_s, c_s\}_s$ such that the following equations hold

$$p_s F_s'(l_s) = w \quad (28)$$

$$\frac{\partial U_s}{\partial c_s} = \lambda p_s \quad (29)$$

$$V'(l) = \lambda w \quad (30)$$

$$\sum_s l_s = l \quad (31)$$

$$\sum_s p_s c_s = \sum_s p_s f_s(l_s). \quad (32)$$

Differentiating equation (30) yields

$$\mu(l)d \ln l = d \ln \lambda + d \ln w,$$

where $\mu(l) \equiv \frac{V''(l)l}{V'(l)}$. This implies that

$$d \ln l = \frac{d \ln (w/P)}{\mu(l)}, \quad (33)$$

with $P \equiv 1/\lambda$. Next, totally differentiating equations (28) and (31) yields

$$\begin{aligned} d \ln p_s - \varepsilon d \ln l_s &= d \ln w \\ \sum_s m_s d \ln l_s &= d \ln l. \end{aligned}$$

Combined, the previous two equations imply

$$\sum m_s d \ln p_s - \varepsilon d \ln l = d \ln w,$$

which combined with (33) implies (after some rearranging)

$$d \ln (w/P) = \frac{\mu}{\mu + \varepsilon} (\sum m_s d \ln p_s - d \ln P). \quad (34)$$

But equation (29) implies that

$$\sum_s \frac{\partial U_s}{\partial c_s} c_s = \lambda \sum_s p_s c_s.$$

Since $U(c)$ is h.d.g. 1 this implies $U(c) = \lambda \sum_s p_s c_s$. Totally differentiating this equation yields

$$\sum_s \frac{\partial U_s}{\partial c_s} dc_s = \left(\sum_s p_s c_s \right) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s.$$

Using equation (29) this implies

$$\sum_s \lambda p_s dc_s = \left(\sum_s p_s c_s \right) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s,$$

which, after simplifying, implies

$$d \ln P = d \ln (1/\lambda) = \sum_s \theta_s d \ln p_s, \quad (35)$$

where $\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s}$. Plugging into (34) and combining with (33) we get

$$d \ln l = \frac{1}{\mu + \varepsilon} \sum (m_s - \theta_s) d \ln p_s.$$

Finally, note that $m_s \equiv \frac{l_s}{\sum_s l_s} = \frac{w l_s}{\sum_s w l_s} = \frac{p_s F'_s(l_s) l_s}{\sum_s p_s F'_s(l_s) l_s}$. Using $\varepsilon(l_s) \equiv -\frac{F''_s(l_s) l_s}{F'_s(l_s)} = \varepsilon$, we know that $F_s(l_s) \propto l_s^{1-\varepsilon}$ and $F'_s(l_s) \propto (1-\varepsilon) l_s^{-\varepsilon}$, hence

$$m_s = \frac{p_s F_s(l_s)}{\sum_s p_s F_s(l_s)} = \frac{p_s q_s}{\sum_s p_s q_s} = \frac{p_s q_s}{I}.$$

On the other hand, using (32) we have

$$\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s} = \frac{p_s c_s}{I}.$$

Combined, the last three displayed equations imply (27).

D Modifying the Data to Fit a New Set of Alphas

From our data on bilateral trade flows, labor shares, and input-output coefficients we can back out a set of Cobb-Douglas parameters which are the $\alpha_{i,s}$'s. There are certain situations

where we might want to change these α 's. One reason that we might want to do this is because the original α 's implied by our data might be slightly negative, which is not ideal. Another reason might be that we want to equalize the α 's between all regions of the United States or all regions of the World (as done in [Caliendo, Dvorkin, and Parro \(2019\)](#)).

Imagine that we know the new set of alphas that we want to obtain, and we want to recover the data (bilateral trade flows) that is compatible with these new alphas. The equilibrium system to obtain the data for the new alphas is the following (notice that we are basically applying the system for a given year, likely 2000, without the DNWR, without mobility, and with $\gamma = 1$ in the nominal anchor):

$$\begin{aligned}
\dot{p}_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \lambda_{ji,s,t-1} \left(\dot{W}_{j,s,t}^{\phi_{j,s}} \prod_{k=1}^S \dot{p}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} \\
\lambda_{ij,s,t} &= \frac{\lambda_{ij,s,t-1} (\dot{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S \dot{p}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^I \lambda_{rj,s,t-1} (\dot{W}_{r,s,t}^{\phi_{r,s}} \prod_{k=1}^S \dot{p}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} \\
R_{i,s,t} &= \sum_{j=1}^I \lambda_{ij,s,t} \left(\alpha'_{j,s} \left(\sum_s \dot{W}_{j,s,t} Y_{j,s,t-1} + D_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R_{j,k,t} \right) \\
\dot{W}_{i,s,t} Y_{i,s,t-1} &= \phi_{i,s} R_{i,s,t} \\
\sum_{i=1}^I \sum_{s=1}^S \dot{W}_{i,s,t} Y_{i,s,t-1} &= \sum_{i=1}^I \sum_{s=1}^S Y_{i,s,t-1}
\end{aligned}$$

In this system the λ_{t-1} , α' , Y_{t-1} , D_t 's are all data, and the \dot{W} , \dot{P} , R_t and λ_t 's are the outcomes. From these outcomes we can construct the new bilateral trade flow matrix.