

Debt as Safe Asset: Mining the Bubble*

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December 31, 2020
Preliminary and Incomplete

Abstract

How much government debt can the market absorb? At what interest rate? Is there a limit, a “Debt Laffer Curve”? What is the impact on inflation? When can governments run a permanent (primary) deficit without ever paying back its debt, like a Ponzi scheme, and nevertheless individual citizens’ transversality conditions hold? What is a safe asset? What are its features? Why is government debt a safe asset? When does one lose the safe asset status? How do we have to modify representative agent asset pricing and the FTPL?

Keywords: Government Debt, Debt Laffer Curve, FTPL, Fiscal Capacity, I Theory of Money

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1 Introduction

How much government debt can the market absorb? At what interest rate? Is there a limit, a “Debt Laffer Curve”? What is the impact on inflation? When can governments run a permanent (primary) deficit without ever paying back its debt, like a Ponzi scheme, and nevertheless individual citizens’ transversality conditions hold? What is a safe asset? What are its features? Why is government debt a safe asset? When does one lose the safe asset status? Why is there debt valuation puzzle for governments for advanced countries like the US and Japan? How do we have to modify representative agent asset pricing and the Fiscal Theory of the Price Level (FTPL) equation?

These are the economic questions of our times in light of record level debt levels after the COVID crisis around the globe. This paper attempts to address them within a setting in which citizens face uninsurable idiosyncratic risks and hence save for precautionary reasons. Each citizen lives forever and adjusts his portfolio consisting of physical capital and the government bond. Idiosyncratic (and aggregate) shocks make capital risky, which cannot be diversified away. This makes government bonds attractive since they can be sold after an adverse shock. From an individual citizen’s perspective it is this ability to retrade, which makes the government bond a desirable hedging instrument. His planned dynamic trading strategy generates a payoff stream that is a good hedge. This is the first of the two key characteristics of a safe asset, the *Good Friend Analogy*.¹ A safe asset is like a good friend, it is around. That is, it is (i) valuable and (ii) liquid when one needs it.² The government bond is a safe asset and is desirable even if it does not yield any interest or dividend.

In other words, the classic asset pricing equation consists not only of the appropriately discounted cash flow stream but has to be complemented with a discounted stream of service flows. The retrading allows citizens in the economy to partially insure each other and overcome the incomplete markets friction. Hence, the real value of government debt, i.e. the nominal value \mathcal{B} divided by the price level \mathcal{P} is

$$\mathcal{B}/\mathcal{P} = \mathbb{E}[PV[\textit{primary surpluses}]] + \mathbb{E}[PV[\textit{service flows}]].$$

¹The two key characteristics of a safe asset were first proposed in [Brunnermeier and Haddad \(2012\)](#).

²Hence, it makes sense for central banks to act as market maker of last resort to ensure that bid-ask spreads remain low. Viewed this way John Law’s big achievement was to create a safe asset status for English and French government debt early in the 18th century.

In the same spirit, we propose to include the additional second term also in the FTPL equation.

Importantly, from an individual perspective the transversality condition holds since his discount factor also reflects the idiosyncratic risk she cannot hedge away. On the other hand, from the aggregate perspective, the issuer's perspective, the safe asset component of the asset pricing equation is a bubble.

When adding aggregate shocks the full feature of safe assets emerges. We consider economies when entering a recession, aggregate output declines and at the same time idiosyncratic risk rises. Let us consider both components of the asset pricing equation, which also underlines the FTPL. The first term reflects the mainstream view, prevalent in the representative agent asset pricing. A drop in output reduces payoffs and increases the marginal utility, leading to the traditional positive β in the asset pricing equation. The second term, the safe asset term, captures the discounted stream of service flows, which in our setting yields partial insurance benefit. This term behaves very differently. As idiosyncratic risk rises in recessions, citizens prefer to shift their portfolio away from capital towards the government bond, resulting in a force that pushes up the real value of government debts. That is, the second term due to the discounted stream of service flows has a negative β .³ In a sense, the Jiang et al. (2019)'s "debt valuation puzzle" for the US can be seen as an empirical vindication of the importance of the second term in our analysis. Even more pronounced the primary surplus in Japan was negative for 50 out of the last 60 years, also suggesting a large second term overpowering the first term.

The second characteristic feature of safe assets is the *Safe Asset Tautology*. A safe asset is safe when it is perceived to be safe so that in times of crisis investors flock to it. In other words, the safe asset status is highly endogenous and part of a multiple equilibrium structure. From an aggregate perspective a safe asset is a "bubble" and bubbles can pop. As a consequence, an asset can lose its safe asset status. Government debt is special as long as the government has sufficient fiscal space to fend off a possible jump to a non-safe asset equilibrium. Note that the ability alone to permanently raise taxes to back the debt is sufficient to prevent such a jump. This ability should be an impor-

³The second term can also be due to binding collateral constraints, (narrow) money as medium of exchange benefits or any other form of convenience yield as in Krishnamurthy and Vissing-Jorgensen (2012). However, it is not obvious whether the last two forms of convenience yield would generate a negative β .

tant element in any debt sustainability analysis. Private companies do not have taxing power and hence can not fully replicate the off-equilibrium backing. An explicit and formal characterization of the fragility of the safe asset status goes beyond the scope of this paper. In [Brunnermeier et al. \(2021\)](#) we discuss this in the context of an international framework for emerging market economies. Emerging market government bonds' safe asset status competes with advanced economies safe assets and hence are deeply affected by spillovers from US monetary policy.

As long as the safe asset status can be maintained, the government can issue debt at favorable interest rates. Citizens are willing to receive a low interest rate since they enjoy the service flow. Indeed, the government can even run a Ponzi scheme: Pay off the maturing bonds with newly issued debt and issuing more for additional expenditures. In other words, the government can “mine the bubble”. As the growth rate of the supply of bonds increases, the citizens' cash-flow return of holding the government bonds declines. “Printing” bonds at a faster rate acts like a tax on bond holdings and consequently lowers the “tax base”, the value of the bonds. A “*Debt Laffer Curve*” arises. When tax exceeds a certain level overall tax revenue from bubble mining declines. As the government issues bonds at a rate so high that the price of bonds collapses, the tax revenue vanishes as well.

Overall, a safe asset perspective sheds a different light on the valuation of government debt and stresses that any debt sustainability analysis (DSA) should include the fragility of the safe asset status.

Literature. – To be written –

2 Model

2.1 Model Setup

There is a continuum of households indexed by $i \in [0, 1]$. All households have identical logarithmic preferences

$$\mathbb{E} \left[\int_0^\infty e^{-\rho t} \log c_t^i dt \right]$$

with discount rate ρ .

Each agent operates one firm that produces an output flow $a_t k_t^i dt$, where k_t^i is the capital input chosen by the firm and a_t is an exogenous productivity process that is common for all agents. Capital of firm i evolves according to

$$\frac{dk_t^i}{k_t^i} = \left(\Phi \left(i_t^i \right) - \delta \right) dt + \tilde{\sigma}_t d\tilde{Z}_t^i + d\Delta_t^{k,i},$$

where $d\Delta_t^{k,i}$ represents firm i 's market transactions in physical capital, $i_t^i k_t^i dt$ are the firm's physical investment expenditures (in output goods), Φ is a concave function that captures adjustment costs in capital accumulation, δ is the depreciation rate, and \tilde{Z}^i is an agent-specific Brownian motion that is i.i.d. across agents i . \tilde{Z}^i introduces firm-specific idiosyncratic risk. $\tilde{\sigma}_t$ is an exogenous process that governs the magnitude of idiosyncratic risk faced by agents. To obtain simple closed-form expressions, we choose the functional form $\Phi(\iota) = \frac{1}{\phi} \log(1 + \phi\iota)$ with adjustment cost parameter $\phi \geq 0$ for the investment technology.

The key friction in the model is that agents are not able to share idiosyncratic risk. While they are allowed to trade physical capital and any type of claim contingent on aggregate risk, they cannot write financial contracts contingent on individual \tilde{Z}^i histories. As a consequence, all agents have to bear the idiosyncratic risk inherent in their physical capital holdings.

Besides households, there is a government that funds government spending, imposes taxes on firms, and issues nominal government bonds. The government has an exogenous need for real spending $g_t K_t dt$, where K_t is the aggregate capital stock and g_t is an exogenous process. The government imposes a proportional output tax (subsidy, if negative) τ_t on firms. Outstanding nominal government debt has a face value of \mathcal{B}_t and pays nominal interest i_t . \mathcal{B}_t follows a continuous process $d\mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t dt$, where the growth rate $\mu_t^{\mathcal{B}}$ is a policy choice of the government. In short, the government chooses the policy instruments $\tau_t, i_t, \mu_t^{\mathcal{B}}$ contingent on histories of prices taking g_t as given and subject to the nominal budget constraint

$$i_t \mathcal{B}_t + \mathcal{P}_t g_t K_t = \mu_t^{\mathcal{B}} \mathcal{B}_t + \mathcal{P}_t \tau_t a_t K_t, \quad (1)$$

where \mathcal{P}_t denotes the price level.

We assume that the exogenous processes $a_t, \tilde{\sigma}_t, \mathfrak{g}_t$ follow a joint Markov diffusion process that is driven by some Brownian motion Z_t , which captures aggregate risk and is independent of all the idiosyncratic Brownian motions \tilde{Z}_t^i .

The model is closed by the aggregate resource constraint

$$C_t + \mathfrak{g}_t K_t + \iota_t K_t = a_t K_t, \quad (2)$$

where $C_t := \int c_t^i di$ is aggregate consumption and $\iota_t = \int \iota_t^i k_t^i / K_t di$ is the average investment rate.

2.2 Model Solution

Price Processes and Returns. Let q_t^K be the market price of a single unit of physical capital. Then, $q_t^K K_t$ is private capital wealth. Let further $q_t^B := \frac{B_t / \mathcal{P}_t}{K_t}$ be the ratio of the real value of government debt to total capital in the economy.⁴ Then, the real value of the total stock of government bonds is $q_t^B K_t$ and the real value of a single government bond is $\frac{q_t^B K_t}{B_t}$. It is convenient to define the share of total wealth in the economy that is due to bond wealth,

$$\vartheta_t := \frac{q_t^B K_t}{(q_t^B + q_t^K) K_t}.$$

We postulate that q_t^B and q_t^K have a generic Ito evolution

$$dq_t^B = \mu_t^{q,B} q_t^B dt + \sigma_t^{q,B} q_t^B dZ_t, \quad dq_t^K = \mu_t^{q,K} q_t^K dt + \sigma_t^{q,K} q_t^K dZ_t.$$

Whenever $q_t^B, q_t^K \neq 0$, the unknown (geometric) drifts $\mu_t^{q,B}, \mu_t^{q,K}$ and volatilities $\sigma_t^{q,B}, \sigma_t^{q,K}$ are uniquely determined by the local behavior of q_t^B and q_t^K , respectively. In the following, we also use the notation μ_t^ϑ and σ_t^ϑ for the (geometric) drift and volatility of ϑ_t .⁵

Households can trade two assets in positive net supply (if $q_t^B \neq 0$), bonds and capital. Assume that in equilibrium $\iota_t = \iota_t^i$ for all i (to be verified below) such that aggregate

⁴It is more convenient to work with this normalized version of the inverse price level $1/\mathcal{P}_t$, because the latter depends on the scale of the economy and the nominal quantity of outstanding bonds in equilibrium, whereas q_t^B does not.

⁵This means, $d\vartheta_t = \mu_t^\vartheta \vartheta_t dt + \sigma_t^\vartheta \vartheta_t dZ_t$.

capital grows locally deterministically at rate $\Phi(\iota_t) - \delta$. Then, the return on bonds is

$$\begin{aligned} dr_t^B &= i_t dt + \frac{d\left(q_t^B K_t / \mathcal{B}_t\right)}{q_t^B K_t / \mathcal{B}_t} = \frac{d\left(q_t^B K_t\right)}{q_t^B K_t} - \overbrace{\left(\mu_t^B - i_t\right)}^{=: \check{\mu}_t^B} dt \\ &= \left(\Phi(\iota_t) - \delta + \mu_t^{q,B} - \check{\mu}_t^B\right) dt + \sigma_t^{q,B} dZ_t. \end{aligned} \quad (3)$$

The return on agent i 's capital is

$$\begin{aligned} dr_t^{K,i}(\iota_t^i) &= \frac{(1 - \tau_t) a_t - \iota_t^i}{q_t^K} + \frac{d(q_t^K k_t^i)}{q_t^K k_t^i} \\ &= \left(\frac{(1 - \tau_t) a_t - \iota_t^i}{q_t^K} + \Phi(\iota_t^i) - \delta + \mu_t^{q,K}\right) dt + \sigma_t^{q,K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned}$$

Using the government budget constraint (1) to substitute out $\tau_t a$ yields

$$dr_t^{K,i}(\iota_t^i) = \left(\frac{a_t - g_t - \iota_t^i}{q_t^K} + \frac{q_t^B}{q_t^K} \check{\mu}_t^B + \Phi(\iota_t^i) - \delta + \mu_t^{q,K}\right) dt + \sigma_t^{q,K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i.$$

Household Problem and Equilibrium. We formulate the household problem as a standard consumption-portfolio-choice problem that does not make explicit reference to the capital trading process $d\Delta_t^{K,i}$ as a choice variable. For this purpose, denote by n_t^i the net worth of household i and let θ_t^i be the fraction of net worth invested into bonds. Then net worth evolves according to

$$\frac{dn_t^i}{n_t^i} = -\frac{c_t^i}{n_t^i} dt + dr_t^B + (1 - \theta_t^i) \left(dr_t^{K,i}(\iota_t^i) - dr_t^B\right). \quad (4)$$

The household chooses consumption c_t^i , real investment ι_t^i , and the portfolio share θ_t^i to maximize utility V_0^i subject to (4). The HJB equation for this problem is (using the returns expressions from the previous paragraph)

$$\begin{aligned} &\rho V_t(n^i) - \partial_t V_t(n^i) \\ &= \max_{c^i, \theta^i, \iota^i} \left\{ \log c^i + V_t'(n^i) \left[-c^i + n^i \left(\frac{\mathbb{E}_t[dr_t^B]}{dt} + (1 - \theta^i) \overbrace{\left(\frac{a_t - g_t - \iota^i}{q_t^K} + \Phi(\iota^i) - \Phi(\iota_t) - \frac{\mu_t^\theta - \check{\mu}_t^B - (\sigma_t^{q,B} - \sigma_t^\theta) \sigma_t^\theta}{1 - \theta_t} \right)}^{\frac{\mathbb{E}_t[dr_t^{K,i}(\iota_t^i)]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt}} \right) \right] \right\} \end{aligned}$$

$$+ \frac{1}{2} V_t''(n^i) (n^i)^2 \left(\left(\sigma_t^{q,B} - (1-\theta^i) \frac{\sigma_t^\theta}{1-\vartheta_t} \right)^2 + (1-\theta^i)^2 \tilde{\sigma}_t^2 \right) \Big\},$$

where we have used $\sigma_t^{q,K} - \sigma_t^{q,B} = \frac{\sigma_t^\theta}{1-\vartheta_t}$. As this is a standard portfolio choice problem, we conjecture a functional form $V_t(n^i) = \alpha_t + \frac{1}{\rho} \log n^i$ for the value function,⁶ where α_t depends on (aggregate) investment opportunities, but not on individual net worth n^i . Substituting this into the HJB and taking first-order conditions yields the Tobin's q investment equation, the permanent income consumption equation, and the Merton portfolio choice equation,⁷

$$\begin{aligned} q_t^K &= \frac{1}{\Phi'(l_t^i)}, \\ c_t^i &= \rho n_t^i, \\ \frac{a_t - g_t - l_t}{q_t^K} - \frac{\mu_t^\theta - \check{\mu}_t^B}{1-\vartheta_t} + \frac{(\sigma_t^{q,B} - \sigma_t^\theta) \sigma_t^\theta}{1-\vartheta_t} &= (1-\theta^i) \tilde{\sigma}_t^2 + \left(\sigma_t^{q,B} - \frac{1-\theta^i}{1-\vartheta_t} \sigma_t^\theta \right) \frac{\sigma_t^\theta}{1-\vartheta_t}. \end{aligned}$$

Using the functional form $\Phi(l) = \frac{1}{\phi} \log(1 + \phi l)$ and goods market clearing (2), the first two equations aggregated across agents imply

$$\begin{aligned} l_t &= \frac{(1-\vartheta_t)(a_t - g_t) - \rho}{1-\vartheta_t + \phi\rho}, \\ q_t^B &= \vartheta_t \frac{1 + \phi(a_t - g_t)}{1-\vartheta_t + \phi\rho}, \\ q_t^K &= (1-\vartheta_t) \frac{1 + \phi(a_t - g_t)}{1-\vartheta_t + \phi\rho}, \end{aligned}$$

which determines the equilibrium uniquely up to the nominal wealth share ϑ_t . Bond market clearing and the fact that all households choose the same θ^i imply $\theta_t^i = \vartheta_t$ and substituting this and goods market clearing into the first-order condition for θ^i gives the additional condition (after solving for μ^θ)

$$\mu_t^\theta = \rho + \check{\mu}_t^B - (1-\vartheta_t)^2 \tilde{\sigma}_t^2.$$

⁶We relegate the technical but standard verification argument to Online Appendix ??.

⁷In particular, the first condition verifies $l_t^i = l_t$, and the last condition already uses this fact to eliminate $\Phi(l^i) - \Phi(l_t)$.

This is a backward equation for ϑ_t that has been derived under the assumption that bonds have a positive value ($\vartheta_t > 0$). In particular, in these cases multiplying the equation by ϑ_t represents an equivalence transformation. Furthermore, if $\vartheta_t = 0$, then by no arbitrage, agents must expect also $d\vartheta_t = 0$; otherwise, they could earn an infinite risk-free return from investing into bonds. Consequently, the backward stochastic differential equation (BSDE)

$$\mathbb{E}_t [d\vartheta_t] = \left(\rho + \check{\mu}_t^B - (1 - \vartheta_t)^2 \check{\sigma}_t^2 \right) \vartheta_t dt \quad (5)$$

must hold along any equilibrium path, regardless of whether bonds have positive value or not.

Together with a specification for the evolution of the exogenous states $\tilde{\sigma}_t$, a_t , and g_t and for policy $\check{\mu}_t^B$, equation (5) determines the equilibrium process for ϑ_t .

2.3 Safe Asset Debt Valuation Equation: Two Perspectives

Pricing government debt can be done from two different perspectives. First, the individual perspective recognizes that individual citizens do not intend to buy and hold the government bond, but plan to retrade it whenever they face a shock. After a negative shock, they raise cash flow by selling the bond, while after a positive shock they buy additional bonds. The cash flow stream associated with this optimal trading strategy is stochastic. Instead of pricing the government bond directly, it is insightful to “price” the cash flows from the optimal stochastic trading strategy and then aggregate over all individuals. Second, the aggregate perspective prices government debt from the government perspective. Hence, in a setting without aggregate risk the bond is risk-free and future payoffs are discounted at the risk-free rate. In a setting with aggregate risk, only the aggregate component of the stochastic discount factor enters the debt valuation equation. Note also dynamic programming implies that the transversality condition has to hold only from the individual perspective, for each citizen. Optimality does not imply a transversality condition from the aggregate perspective (where discounting happens at a lower effective rate).

Individual Perspective. We denote the individual SDF process of citizen i with $\tilde{\zeta}_t^i$. This process satisfies $d\tilde{\zeta}_t^i / \tilde{\zeta}_t^i = -r_t^f dt - \zeta_t dZ_t - \tilde{\zeta}_t^i d\tilde{Z}_t^i$, with a negative drift term equal to the risk-free rate and aggregate and idiosyncratic price of risk terms, ζ_t , $\tilde{\zeta}_t^i$ respec-

tively.⁸ Also, let $\eta_t^i := n_t^i/N_t$ be citizens i 's net worth share. The individual perspective asset pricing yields our main valuation equation,

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E} \left[\int_0^\infty \left(\int \zeta_t^i \eta_t^i di \right) s_t K_t dt \right] + \mathbb{E} \left[\int_0^\infty \left(\int \zeta_t^i \eta_t^i di \right) (1 - \vartheta_t)^2 \tilde{\sigma}_t^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt \right]. \quad (6)$$

The real value of all outstanding public debt $\frac{\mathcal{B}_0}{\mathcal{P}_0}$ consists of two terms, the discounted value of future primary surpluses, $s_t K_t := (\tau_t a - g_t) K_t$, plus the discounted value of future service flows, $(1 - \vartheta_t)^2 \tilde{\sigma}_t^2 \frac{\mathcal{B}_t}{\mathcal{P}_t}$. The safe asset service flow is due to partial insurance, which increases in the value of public debt, and the amount of idiosyncratic risk the citizen is exposed to, which in turn depends on his portfolio share on physical capital $(1 - \vartheta_t)$ and $\tilde{\sigma}_t$. Importantly, the stochastic discount factor in this equation is a net-worth-weighted average of individual stochastic discount factors. Since a single citizen's individual net worth weight η_t^i co-moves negatively with his SDF ζ_t^i , the discount factor is lower (discount rate is higher) than the usual unweighted average discount factor (used in aggregate perspective below).

To obtain valuation equation (6) we start valuing citizen i 's bond portfolio at time $t = 0$:⁹

$$n_0^{b,i} = \mathbb{E} \left[\int_0^\infty \zeta_t^i \left(\underbrace{c_t^i}_{\text{consumption}} - \underbrace{k_t^i (a - l_t - \tau_t a_t)}_{\text{production net of investment and taxes}} - \underbrace{q_t^K k_t^i (1 - \vartheta_t) \tilde{\sigma}_t \tilde{\sigma}_t^{\Delta,i}}_{\text{cash flows from trading capital}} \right) dt \right], \quad (7)$$

where $n_0^{b,i} := \theta_0^i n_0^i$ is the initial bond wealth of agent i . This equation says that the initial bond wealth of the household must equal the discounted value of future consumption in excess of the citizen i 's own production net of reinvestment and tax payments and in excess of trading expenses for purchasing new capital (these can be negative if capital is sold). In this model, capital trading only happens in response to idiosyncratic shocks,

⁸In integral form the individual SDF is

$$\zeta_t^i = \underbrace{\exp \left(- \int_0^t r_\tau^f d\tau \right)}_{\text{time discounting}} \cdot \underbrace{\exp \left(- \int_0^t \zeta_\tau dZ_\tau - \frac{1}{2} \int_0^t \zeta_\tau^2 d\tau \right)}_{\text{aggregate risk}} \cdot \underbrace{\exp \left(- \int_0^t \zeta_\tau d\tilde{Z}_\tau^i - \frac{1}{2} \int_0^t \zeta_\tau^2 d\tau \right)}_{\text{idiosyncratic risk}},$$

where the second and third factors are martingales.

⁹This equation is an immediate consequence of the agent's intertemporal budget constraint. In particular, a transversality condition always ensures that there is no additional nonvanishing terminal wealth term.

$$d\Delta_t^{k,i} = \tilde{\sigma}_t^{\Delta,i} d\tilde{Z}_t^i. \text{ }^{10}$$

Next, replacing individual with aggregate variables $c_t^i = \eta_t^i C_t$ and $k_t^i = \eta_t^i K_t$, one obtains

$$n_0^{b,i} = \mathbb{E} \left[\int_0^\infty \tilde{\zeta}_t^i \eta_t^i \left(\tau_t a_t K_t + C_t - (a - \iota_t) K_t + (1 - \vartheta_t) \tilde{\sigma}_t^2 \vartheta_t q_t^K K_t \right) dt \right].$$

Including the aggregate resource constraint (2), $C_t - (a - \iota_t) K_t = \mathfrak{g}_t K_t$, the fact that, $n_0^{b,i} = \vartheta_0 n_0^i = \eta_0^i \frac{\mathcal{B}_0}{\mathcal{P}_0}$, and, $\vartheta_t q_t^K K_t = (1 - \vartheta_t) \frac{\mathcal{B}_t}{\mathcal{P}_t}$ leads to

$$\eta_0^i \frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E} \left[\int_0^\infty \tilde{\zeta}_t^i \eta_t^i s_t K_t dt \right] + \mathbb{E} \left[\int_0^\infty \tilde{\zeta}_t^i \eta_t^i (1 - \vartheta_t)^2 \tilde{\sigma}_t^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt \right]. \quad (8)$$

Finally, integrating over individuals i yields equation (6).

Aggregate Perspective. From the aggregate perspective, individual uninsurable risk does not enter the valuation equation directly. Indeed, absent aggregate shocks (including inflation shocks), the government bond is a risk-free asset. Viewed from this perspective we obtain a different, an aggregate, discount factor process, $d\bar{\zeta}_t / \bar{\zeta}_t = -r_t^f dt - \zeta_t dZ_t$.¹¹ Absent aggregate risk the discount factor is simply $\bar{\zeta}_t = \exp(-\int_0^t r_\tau^f d\tau)$.

The government debt valuation equation at $t = 0$ is

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \lim_{T \rightarrow \infty} \left(\mathbb{E} \left[\int_0^T \bar{\zeta}_t s_t K_t dt \right] + \mathbb{E} \left[\bar{\zeta}_T \frac{\mathcal{B}_T}{\mathcal{P}_T} \right] \right), \quad (9)$$

consisting of two terms: a discounted stream of primary surpluses plus (the limit of) a discounted terminal value. The latter can be positive even in the limit, giving rise to a possible bubble on government debt.¹² The reason is that in our model no private citizen's transversality condition necessary implies $\mathbb{E} \left[\bar{\zeta}_T \frac{\mathcal{B}_T}{\mathcal{P}_T} \right] \rightarrow 0$ because agents do not

¹⁰While agents expect to make as many purchases as sales in the future, so that the expected cash flows from trading are zero, there is nevertheless a trading term in equation (7) that reflects the covariance between cash flows from trading and individual marginal utility.

¹¹The aggregate discount factor is the projection of any individual citizen's SDF onto a common filtration generated by the aggregate Brownian $\{Z_t\}_{t=0}^\infty$. Put differently, $\bar{\zeta}_t := \mathbb{E} \left[\zeta_t^i \mid Z_\tau : \tau \leq t \right]$, takes conditional expectations with respect to the history of aggregate shocks dZ_τ up to time t but without any knowledge of idiosyncratic shocks. Equivalently, $\bar{\zeta}_t = \int \zeta_t^i di$ is the unweighted average of individual SDFs.

¹²The bubble term on government debt is discussed in detail in Brunnermeier et al. (2020).

buy and hold a fixed fraction of the government debt stock but constantly trade bonds. If the discount factor is small enough so that the terminal condition does converge to zero, we obtain the traditional debt valuation equation that says that the value of debt must equal the present value of primary surpluses.

To obtain equation (9), we start by using $d\mathcal{B}_t = \mu_t^B \mathcal{B}_t dt$ to rewrite the government flow budget constraint (1) as

$$-(d\mathcal{B}_t - i_t \mathcal{B}_t dt) = \mathcal{P}_t \underbrace{(\tau a_t - \mathfrak{g}_t)}_{=s_t} K_t dt,$$

where s_t denotes again the government primary surplus normalized by the aggregate capital stock.

We now multiply both sides by the nominal SDF $\zeta_t^i / \mathcal{P}_t$ of agent i and use Ito's product rule to replace $\zeta_t^i / \mathcal{P}_t d\mathcal{B}_t$ with $d\left(\zeta_t^i / \mathcal{P}_t \mathcal{B}_t\right) - \mathcal{B}_t d\left(\zeta_t^i / \mathcal{P}_t\right)$:¹³

$$-d\left(\zeta_t^i \mathcal{B}_t / \mathcal{P}_t\right) + \mathcal{B}_t \left(d\left(\zeta_t^i / \mathcal{P}_t\right) + i_t \zeta_t^i / \mathcal{P}_t dt\right) = \zeta_t^i s_t K_t dt.$$

Integrating this equation from $t = 0$ to $t = T$, taking expectations, and solving for $\zeta_0^i \mathcal{B}_0 / \mathcal{P}_0$ yields

$$\zeta_0^i \frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E} \left[\int_0^T \zeta_t^i s_t K_t dt \right] - \mathbb{E} \left[\int_0^T \mathcal{B}_t \left(d\left(\zeta_t^i / \mathcal{P}_t\right) + i_t \zeta_t^i / \mathcal{P}_t dt \right) \right] + \mathbb{E} \left[\zeta_T^i \frac{\mathcal{B}_T}{\mathcal{P}_T} \right]. \quad (10)$$

Equation (10) is simply an accounting identity, the government flow budget constraint (1) multiplied with the discounting process $\zeta_t^i / \mathcal{P}_t$. We now add economic content by noting that the individual SDF ζ_t^i must price the bond because agent i is marginal in the bond market. This implies that the associated nominal SDF $\zeta_t^i / \mathcal{P}_t$ must decay on average at the nominal market interest rate, so that the second term in equation (10) vanishes. In addition, we can replace the individual SDF ζ_t^i with the average SDF $\bar{\zeta}_t$ because equation (10) holds for all individuals i and $s_t K_t$ and $\mathcal{B}_T / \mathcal{P}_T$ are free of idiosyncratic risk. When taking the limit $T \rightarrow \infty$, we obtain equation ...

COMPARE BOTH SDFs across both perspectives!

– comparison to be written –

¹³There is no quadratic covariation term because $d\mathcal{B}_t$ is absolutely continuous.

2.4 Closed-Form Steady State and Gordon Growth Formulas

In this section, we assume that productivity a , idiosyncratic risk $\tilde{\sigma}$, and government spending per unit of capital g are constant. We also restrict attention to government policies that hold taxes τ constant over time and characterize steady-state equilibria with constant q^B and q^K and a positive value of government bonds, $q^B > 0$. These assumptions immediately imply that also ϑ and $\check{\mu}^B$ must be constant in such a steady state.

Any such equilibrium must thus solve equation (5) with $d\vartheta_t = 0$. The right-hand side is a third-order polynomial, so there are three solutions to this equation, $\vartheta = 0$, $\vartheta = \frac{\tilde{\sigma} + \sqrt{\rho + \check{\mu}^B}}{\tilde{\sigma}}$, and $\vartheta = \frac{\tilde{\sigma} - \sqrt{\rho + \check{\mu}^B}}{\tilde{\sigma}}$. Among these solutions, only the third can be consistent with $q^B, q^K > 0$ and thus a valid steady state equilibrium in which bonds have a positive value.¹⁴ It is consistent with such an equilibrium if in addition the condition

$$\tilde{\sigma} \geq \sqrt{\rho + \check{\mu}^B}$$

is satisfied. Effectively, this inequality imposes a constraint on bond growth in excess of interest payments $\check{\mu}^B$ for the private sector to remain willing to hold government bonds.

In this case, investment is

$$\iota = \frac{\sqrt{\rho + \check{\mu}^B} (a - g) - \rho \tilde{\sigma}}{\sqrt{\rho + \check{\mu}^B} + \phi \rho \tilde{\sigma}}$$

and the (scaled) real asset values are

$$q^B = \frac{(\tilde{\sigma} - \sqrt{\rho + \check{\mu}^B}) (1 + \phi (a - g))}{\sqrt{\rho + \check{\mu}^B} + \phi \rho \tilde{\sigma}}, \quad q^K = \frac{\sqrt{\rho + \check{\mu}^B} (1 + \phi (a - g))}{\sqrt{\rho + \check{\mu}^B} + \phi \rho \tilde{\sigma}}.$$

While these expressions have the advantage of being an explicit model solution in terms of parameters, for interpretation it is helpful to write the last two equations as

¹⁴The second solution never corresponds to a valid equilibrium, while the first is only consistent with equilibrium if government primary surpluses are zero, see [Brunnermeier et al. \(2020\)](#) for details.

Gordon growth formulas

$$q^B = \frac{s + (1 - \vartheta)^2 \tilde{\sigma}^2 q^B}{\mathbb{E}[dr^n]/dt - g}, \quad q^K = \frac{(1 - \tau)a - \iota}{\mathbb{E}[dr^K]/dt - g}.$$

Here, the second equation follows from the fact that the price of a single unit of capital must be the present value of cash flows generated by that unit of capital. The current period cash flow is production net of taxes and reinvestment, $(1 - \tau)a - \iota$, and the expected growth rate of these cash flows is the economy's growth rate $g := \Phi(\iota) - \delta$.¹⁵ Because capital is risky, expected cash flows must be discounted at the expected return on capital $\mathbb{E}[dr^K]/dt$, which includes a risk premium for idiosyncratic risk.

The first equation is a consequence of equation (6), the individual perspective on government debt valuation. Per unit of aggregate capital in the economy, the “cash flow” on bonds consists of the surplus-capital ratio s and the service flow $(1 - \vartheta)^2 \tilde{\sigma}^2 q^B$ from trading bonds to self-insure against idiosyncratic risk. Both types of cash flows grow on average at the economy's growth rate, but are risky from the individual's perspective. The required discount rate is therefore $\mathbb{E}[dr^n]/dt$, where $dr^n = \vartheta dr^B + (1 - \vartheta)dr^K$ denotes the return on the agents (net worth) portfolio, because the idiosyncratic risk of net worth is precisely the residual idiosyncratic risk that the agent has to bear after optimal re-trading of bonds.¹⁶

3 Mining the Bubble: The Debt-Laffer Curve

The potential presence of a bubble in equation (9) in the aggregate perspective and the service flow term in equation (6) in the individual perspective suggest that the safe asset status of government debt represents a fiscal resource that the government can “mine” for revenue instead of taxation. Indeed, when the government chooses a permanently positive bond growth in excess of interest payments $\check{\mu}^B$ (and thus permanently negative primary surpluses), the value of government debt may not collapse despite the negative present value of primary surpluses.

Our model therefore implies that the government may be able to finance govern-

¹⁵ g is both the growth rate of output and of the aggregate capital stock.

¹⁶More formally, $\eta_i^i = n_i^i / N_i$, so that the relative risk in n_i^i is the same as the relative risk in η_i^i and the latter matters for discounting when using the weighted-average discount factor $\int \zeta_i^i \eta_i^i di$.

ment expenditures by “mining the bubble” without ever raising taxes for it. It can do so if idiosyncratic risk is sufficiently severe (high $\tilde{\sigma}$) such that even in the absence of positive surpluses government debt retains a positive value because of a bubble component. In steady state, this is the case under the condition $\tilde{\sigma} > \sqrt{\rho}$, which is equivalent to $r^f \leq g$.

If this condition is satisfied, does the existence of a bubble imply that the government faces no budget constraint and can expand spending without limits? The answer is of course no as real resources are still finite and the real value of government debt reacts to the policy choice. Specifically, primary deficits per unit of capital are given by¹⁷

$$-s_t = \check{\mu}_t^B q_t^B.$$

The first factor, $\check{\mu}_t^B$, measures revenue raised by bond issuance that is not distributed to bond holders in the form of interest payments. If it is positive, the claim of old bond holders is diluted by the issuance of new bonds, i.e., a higher $\check{\mu}_t^B$ represents a tax on existing bond holders. The second factor, q_t^B , is the tax base, the real value of existing debt (per unit of capital). If this tax base reacts negatively to an increase in $\check{\mu}_t^B$, a standard Laffer curve intuition emerges.

This is indeed the case and easiest to see in steady state. Then q^B is explicitly given by

$$q^B = \frac{(\tilde{\sigma} - \sqrt{\rho + \check{\mu}^B}) (1 + \phi (a - g))}{\sqrt{\rho + \check{\mu}^B} + \phi \rho \tilde{\sigma}}.$$

There are two reasons why higher deficits decrease q^B . First, there is a direct effect from increasing $\check{\mu}^B$. This emerges because higher debt growth distorts the portfolio choice between government bonds and capital, making capital more attractive and thereby lowering ϑ . If additional deficits are used to increase capital subsidies $-\tau$, this is the only effect. However, if additional deficits are used to fund government spending by raising g , q^B decreases again due to the presence of the term $a - g$ (at least if $\phi > 0$). This second effect is a consequence of the resource constraint (2): when the government claims a larger share of output, consumption has to decline, which lowers all asset values symmetrically.¹⁸

¹⁷This equation follows immediately from the government budget constraint.

¹⁸This intuition breaks down for $\phi = 0$ as then agents can convert existing capital goods freely into consumption goods and instead the growth rate is reduced.

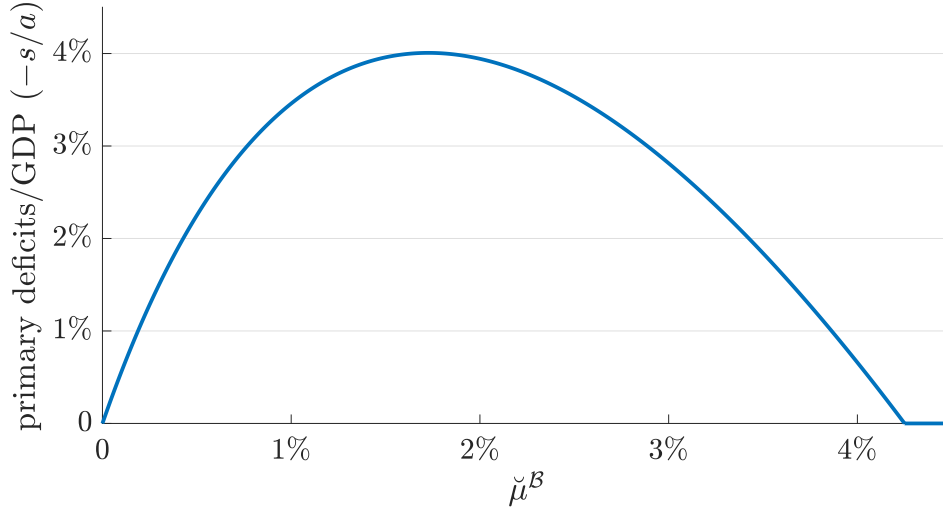


Figure 1: Debt Laffer curve in steady state when there is a bubble on government debt

Figure 1 depicts the typical shape of the debt Laffer curve in our framework when deficits are used to increase capital subsidies. In this example, if the bubble is mined too aggressively so that $\check{\mu}^B > 2\%$, the government fails to raise additional real revenues. In particular, there is a limit to bubble mining and the government still faces a constraint on real spending.

4 Counter-cyclical Safe Asset and 2 Betas

4.1 Model Setup with Stochastic Idiosyncratic Risk

We introduce aggregate risk as shocks to idiosyncratic risk $\tilde{\sigma}_t$. We interpret periods of high idiosyncratic risk as recessions and want them to be associated with lower consumption and higher marginal utility. Rather than microfounding this relationship explicitly, we simply impose exogenous relationships $a_t = a(\tilde{\sigma}_t)$ and $\mathfrak{g} = \mathfrak{g}(\tilde{\sigma}_t)$ that are consistent with the desired correlation structure.¹⁹ Specifically, we assume that $\tilde{\sigma}_t$ follows an Ornstein-Uhlenbeck process in logs,

$$d \log \tilde{\sigma}_t = -\psi \left(\log \tilde{\sigma}_t - \log \tilde{\sigma}^0 \right) dt + \sigma dZ_t.$$

¹⁹For models similar to ours in which output and consumption naturally react negatively to risk shocks, see DiTella and Hall (2020) and Li and Merkel (2020).

To ensure that C_t/K_t is strictly decreasing in $\tilde{\sigma}_t$, we do not directly specify functions $a(\tilde{\sigma})$ and $\mathfrak{g}(\tilde{\sigma})$, but instead impose for the endogenous consumption-capital ratio the equation

$$C/K(\tilde{\sigma}_t) := \alpha_0 - \alpha_1 \tilde{\sigma}_t$$

for some parameters $\alpha_0, \alpha_1 > 0$. Because equation (5) implies that ϑ_t is determined independently of the processes for a_t and \mathfrak{g}_t , we can first solve for the solution function $\vartheta(\tilde{\sigma})$ using just the specification for the $\tilde{\sigma}_t$ process and then invert the formula $C_t/K_t = \rho \frac{1+\phi(a_t-\mathfrak{g}_t)}{1-\vartheta_t+\phi\rho}$ to back out the required function $a - \mathfrak{g}$ to obtain the desired consumption-capital ratio in equilibrium.²⁰

For government policy, we assume that debt growth net of interest payments satisfies a linear relationship

$$\check{\mu}_t^B = \nu_0 + \nu_1 \tilde{\sigma}_t$$

with parameters $\nu_0 < 0$ and $\nu_1 > 0$. Provided ν_1 is sufficiently large, this implies that surpluses $s_t = -\check{\mu}_t^B q_t^B$ are positive for low idiosyncratic risk (in expansions) and negative for high idiosyncratic risk (in recessions). Primary surpluses therefore correlate negatively with marginal utility and any agent in the economy would require a positive risk premium for holding a (hypothetical) claim to primary surpluses.

4.2 Analyzing the Two Asset Pricing Terms Separately

We now consider the two terms in the government debt valuation equation derived from the individual perspective (equation (6)). Figure 2 plots the two present values

$$q^{B,cf}(\tilde{\sigma}) := \mathbb{E} \left[\int_0^\infty \left(\int \tilde{\zeta}_t^i \eta_t^i di \right) s_t K_t dt \mid \tilde{\sigma}_0 = \tilde{\sigma}, K_0 \right] / K_0$$

$$q^{B,sf}(\tilde{\sigma}) := \mathbb{E} \left[\int_0^\infty \left(\int \tilde{\zeta}_t^i \eta_t^i di \right) (1 - \vartheta_t)^2 \tilde{\sigma}_t^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt \mid \tilde{\sigma}_0 = \tilde{\sigma}, K_0 \right] / K_0$$

for a numerical example.²¹ The blue solid line shows the present value of future primary surpluses (cash flows) $q^{B,cf}$ as a function of $\tilde{\sigma}$. This value is strictly decreasing

²⁰The processes a and \mathfrak{g} are not individually relevant for anything of interest here, just their difference $a - \mathfrak{g}$ is.

²¹The parameter values are $\tilde{\sigma}^0 = 0.2$, $\rho = 0.015$, $\alpha_0 = 0.04$, $\alpha_1 = 0.01$, $\nu_0 = -0.0025$, $\nu_1 = 0.01$, $\phi = 3$, $\psi = 0.025$, $\sigma = 0.1$.

in idiosyncratic risk and has a low – and sometimes negative – value. Comparing the present value of surpluses $q^{B,cf}K$ in our model to the market value of government debt q^BK , which is represented by the black dashed line in Figure 2, reveals a large gap $(q^B - q^{B,cf})K$, a “debt valuation puzzle”. In addition, when compared with the present value of surpluses $q^{B,cf}K$, the total value of government debt q^BK has also the opposite correlation with the aggregate state. Yet, there is no puzzle from the perspective of our model: government debt is also a safe asset valued for its service flow from re-trading which is represented by the component $q^{B,sf}(\tilde{\sigma})$. As the red solid line in Figure 2 shows, this value is positive, large and positively correlated with $\tilde{\sigma}_t$. This additional component dominates the overall dynamics of the value of government debt and is the reason that q^B appreciates in bad times despite the simultaneous drop in $q^{B,cf}$. That $q^{B,sf}$ must be positively correlated with $\tilde{\sigma}$ can also be seen from the present value equation: one can show that for our policy specification residual net worth risk $(1 - \theta)\tilde{\sigma}_t$ must be strictly increasing in $\tilde{\sigma}_t$, so that an increase in risk increases the value of insurance service flows from re-trading.²²

The correlation structure apparent in Figure 2 implies that, if the two claims $q^{B,cf}$ and $q^{B,sf}$ could be traded separately, the cash flow claim would be a high- β asset while the service flow claim would be a negative- β asset. The presence of this second, negative- β component makes government debt a safe asset also with respect to aggregate shocks. Figure 3 depicts this explicitly by plotting (weighted) conditional betas for the two hypothetical assets.²³

4.3 The Possibility of Insuring Bond Holders and Tax Payers at the Same Time

In our simple setting citizens are capital owners and bond holders. In this section, we conceptually separate each household into two sub-units, a capital owner and a

²²This is not an entirely rigorous argument as it ignored changes in the discount rate. The effective discount rate in the weighted-average SDF $\int \zeta_t^i \eta_t^i di$ can both increase or decrease with the aggregate state x_t depending on whether the *aggregate* risk premium increases or decreases. Note however, that the level of idiosyncratic risk does not directly matter for the effective discount rate because the risk premium on idiosyncratic risk exactly offsets the lower risk-free rate due to a precautionary motive.

²³We define $\beta_t^i = \sigma_t^i / \zeta_t$, where $i \in \{cf, sf\}$ and dr^i is the return on the respective component and $\tilde{\sigma}_t^i$ is the aggregate risk loading of that return. This definition can be interpreted as $\beta_t = -\frac{\text{cov}_t(d\zeta_t / \zeta_t, dr_t^i)}{\text{var}_t(d\zeta_t / \zeta_t)}$, where $d\zeta_t / \zeta_t$ is the SDF that discounts cash flows from $t + dt$ to time t . In addition, we weight β^i by its share $\omega^i := q^{B,i} / q^B$ of the total government debt claim.

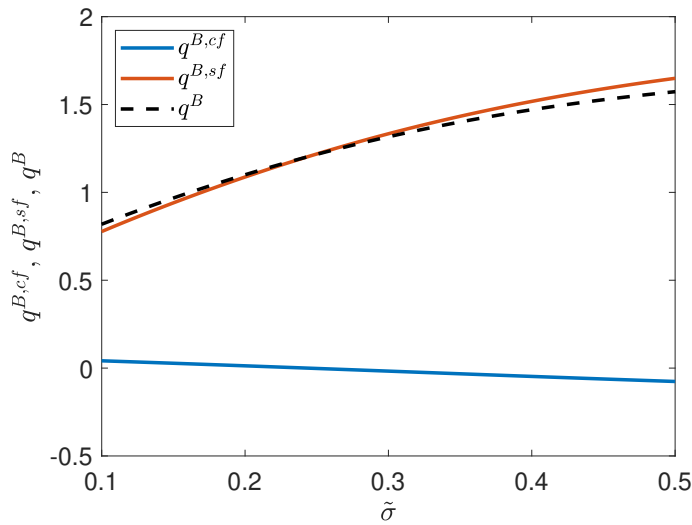


Figure 2: Decomposition of the value of government debt as a function of idiosyncratic risk $\tilde{\sigma}$. The blue solid line shows the present value of primary surpluses ($q^{B,cf}$), the red solid line the present value of service flows ($q^{B,sf}$) and the black dashed line the total value of government debt (q^B), all normalized by the capital stock.

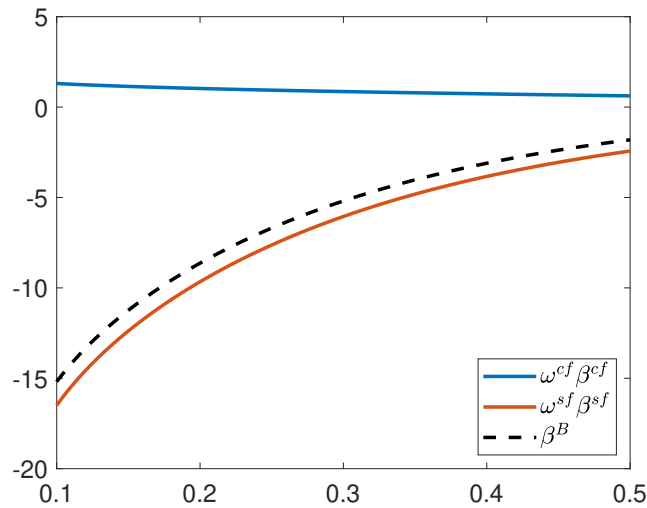


Figure 3: Conditional betas of hypothetical claims to the surplus and risk-sharing components of the government debt value

government debt holder. Surprisingly, it is possible to follow a government policy that provides tax payers insurance against negative aggregate shocks and bond holders at the same time. By cutting taxes (or even granting subsidies) for capital owners in recessions, their tax burden is positively correlated with their income. At the same time, the safe asset premium rises in recessions, which provides insurance to government bond holders. Importantly, this finding in our incomplete market setting with a safe-asset bubble is in sharp contrast to traditional asset pricing in which either tax payers or government bond holders can insured, as pointed out in [Jiang et al. \(2020\)](#).

5 Debt Sustainability Analysis

5.1 Fiscal Capacity (Off Equilibrium)

Government debt as a bubbly safe asset is only one equilibrium next to possible other no-bubble equilibria. What ensures that the bubble does not burst and that we do not end up with the standard bubble-free real debt evaluation, wherein government debt loses its safe-asset status?

First, the government could support the current value of its debt by raising taxes so that it generates a permanently positive surplus stream that backs the current value. This requires that the government has the capacity to raise taxes. Such a policy would of course give up any revenues from bubble mining, but creates a situation where the “conventional” FTPL equation (without second term, the bubble term) applies and determines and supports the price level.

Second, it is sufficient for the government to provide this tax-backing *off-equilibrium*. To see this consider the case in which private investors coordinated on the belief that the bubble on government debt was smaller than in the stationary bubble equilibrium and decided to be no longer willing to hold the debt, then the government could react by permanently reverting to a positive-surplus regime in which debt is fully backed by future surpluses. Such a policy shift would generate capital gains for government debt holders and thus make government debt so attractive *ex ante*, that it would remain optimal for citizens to hold on to their government bonds.

How much *fiscal capacity* is needed to “defend” the bubble on government debt? The off-equilibrium strategy involves permanently positive primary surpluses that grow at

the same rate as the economy. While the (positive) scale of these surpluses can be arbitrarily small, the fiscal authority needs the capacity and commitment to turn equilibrium deficits into surpluses before an inflationary collapse of its currency forces it to do so.²⁴

5.2 Why is the Safe-asset Bubble not on Private Asset?

So far, our argument does not explain why the safe asset bubble is on government debt and not on any other (private) asset. First, note that the bubble cannot be associated with corporate or household/mortgage debt since they are of finite maturity. While most government bonds are also finite-maturity assets, the difference is that they are a legal claim on money, an infinite-maturity government liability. This leaves only infinitely-lived private equity claims as possible bubble-carrying assets. While theoretically possible, the government could eliminate a private bubble by following an (off-equilibrium) tax policy that makes its debt a more attractive safe asset than private equity claims. For example, the government could make its off-equilibrium primary surplus stream a much safer asset. Private corporations do not have such an off-equilibrium threat to eliminate all bubbles and therefore cannot force the bubble onto their stocks.²⁵

5.3 Market Maker of Last Resort to Ensure Safe-asset Status

The bubbly safe-asset status requires that citizens can freely trade the assets as they experience adverse shocks. The government through its central bank can engage as market maker of last resort so that citizens can trade the asset facing only small bid-ask spreads. This ensures that government debt retains the safe asset status. Private assets do not enjoy this privilege. For example, illiquid long-lived real estate assets

²⁴Ultimately, a loss of safe asset status would force the government to give up bubble mining and reduce the deficit by inflating away the real value of government debt. However, to defend the bubble, the government must revert to surpluses and back the debt at its old, pre-inflation, value to generate capital gains for bond holders that rule out this inflationary equilibrium. The mere ability to raise taxes temporarily when inflation dynamics are underway to stop further inflation is insufficient.

²⁵If a private company ever discovered a technology that generated a sufficiently safe cash flow stream growing at the same rate as the economy, the government could still use countercyclical corporate income taxes to make the company's after-tax cash flows more procyclical and thus the company's stock less suitable as a safe asset.

are unlikely to carry a safe-asset bubble since their large trading costs prevent efficient re-trading.

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