Monetary Policy and Wealth Effects: The Role of Risk and Heterogeneity*

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Abstract

We study the role of wealth effects, i.e. the revaluation of real and financial assets, in the monetary policy transmission mechanism. We build an analytical heterogeneous-agents model with two main ingredients: i) rare disasters and ii) risky household debt. The model captures time-varying risk premia and precautionary savings in a linearized setting that nests the textbook New Keynesian model. Quantitatively, the model matches the empirical response of asset prices and the heterogeneous impact on borrowers and savers. We find that wealth effects induced by time-varying risk and household debt account for the bulk of the output response to monetary policy. Defaultable private debt amplifies the effects of monetary policy, but the effect decreases with its duration. Controlling for the fiscal response to a monetary shock is crucial for the quantitative results.

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1 Introduction

A long tradition in monetary economics emphasizes the role of wealth effects, i.e. the revaluation of real and financial assets, in the economy's response to changes in monetary policy. Its importance can be traced back to both classical and Keynesian economists, such as Pigou, Patinkin, Metzler, and Tobin.[1] Keynes himself described the effects of interest rate changes as follows:

> There are not many people who will alter their way of living because the rate of interest has fallen from 5 to 4 per cent, if their aggregate income is the same as before. [...] Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, *depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets*.
> - John Maynard Keynes, The General Theory of Employment, Interest, and Money (emphasis added).

There is a large empirical literature documenting the impact of monetary policy on asset prices. Bernanke and Kuttner (2005) and Kekre and Lenel (2020) study the effects of monetary shocks on stock prices. Gertler and Karadi (2015) and Hanson and Stein (2015) consider the effects on bonds. Moreover, Cieslak and Vissing-Jorgensen (2020) shows that policymakers track the behavior of stock markets because of their impact on households' consumption, while Chodorow-Reich et al. (2021) establish the importance of this channel empirically.

Despite the evidence, relatively little work has been done to theoretically study the role of asset price fluctuations on the monetary transmission mechanism. An important reason for this is that incorporating these channels represents a challenge to standard monetary models. A robust finding of the empirical literature is that changes in asset prices can be explained mainly by fluctuations in future excess returns, related to changes in the risk premia, rather than changes in the risk-free rate. However, the standard approach abstracts from risk premia and generates counterfactual asset-pricing dynamics. Moreover, models that feature richer asset-pricing dynamics require the use of complex global or high-order perturbation methods, which lack the insights of the role of the different channels of transmission provided by analytically tractable models.

[1] The revaluation of government liabilities was central to Pigou (1943) and Patinkin (1965), while Metzler (1951) considered stocks and money. Tobin (1969) focused on how monetary policy interacted with the value of real assets.

In this paper, we propose a new framework that generates rich asset-pricing dynamics and heterogeneous portfolios while preserving the simplicity of the textbook New Keynesian model, and study the role of wealth effects in the economy's response to monetary policy. The model has two main ingredients: i) rare disasters and ii) household debt. Rare disasters allow us to capture both a precautionary savings motive and realistic risk premia. Barro (2009) and Gabaix (2012) argue that the risk of a rare disaster and, in particular, its time-varying component, can successfully explain major asset-pricing facts.[2] Moreover, private debt is a significant component of households' portfolios, representing 75% of GDP, and, as recently shown by Cloyne et al. (2020), borrowers account for the bulk of the response of aggregate consumption to changes in interest rates. By incorporating private debt, we are able to capture the role of revaluations in both gross and net asset positions. We also study the role of default risk and long maturities in household debt, important features of most debt contracts, such as mortgages or student debt.

To capture the time-varying component of risk premia, we assume that the probability of a disaster depends on the level of the nominal interest rate. Using data on a panel of advanced economies since the nineteenth century, Schularick et al. (2021) finds that contractionary monetary shocks significantly increases the probability of a subsequent financial crisis. Based on this evidence, we provide a new micro-foundation for the relationship between monetary policy, rare disasters, and asset prices. In an extension with financial intermediaries, we show that banks' exposure to interest rate risk makes them vulnerable to runs. A contractionary monetary shock then weakens banks' balance sheets and increases the probability of a financial crisis, an endogenous disaster. Our assumption on the probability of disaster enables us to capture this mechanism in our baseline model.

We consider an economy populated by two types of households, borrowers and savers. The model captures key features of heterogeneous-agents New Keynesian (HANK) models, such as precautionary savings and heterogeneous marginal propensities to consume (MPCs), in a setting with positive private debt, a combination that has been elusive in the analytical HANK literature. Despite being stylized, the model captures quantitatively central features of the monetary transmission mechanism, including important asset-pricing moments such as the term

[2] Rare disasters have been widely used to explain a range of asset-pricing "puzzles"; see Tsai and Wachter (2015) for a review.

premium, the equity premium, and corporate spreads, as well as the differential responses of borrowers and savers to monetary shocks observed in the data. Given this success, we use the model to quantitatively assess the importance of the channels through which monetary policy affects households' consumption. We find that time-varying risk and private debt jointly account for more than 80% of the initial response of aggregate consumption to a monetary shock. These results reveal the importance of accounting for risk and portfolio heterogeneity to understand the economy's response to monetary policy.

Our solution method allows us to obtain time-varying risk premia in a linearized setting and provide a complete analytical characterization of the channels involved. The method consists on perturbing the economy around a stationary equilibrium with *positive aggregate risk* instead of adopting the more common approach of approximating around a non-stochastic steady state. By perturbing around the stochastic stationary equilibrium, we are able to obtain time variation in precautionary motives and risk premia using a first-order approximation, while the standard approach would require a third-order approximation (see e.g. Andreasen 2012). Moreover, by linearizing around an economy with zero monetary risk, we are able to solve for the stochastic stationary equilibrium in closed form, avoiding the need to compute the risky steady state numerically, as in Coeurdacier et al. (2011). This hybrid approach allows us to capture the effect of aggregate risk on asset prices in a linearized model.

The first result states that output satisfies an *aggregate Euler equation*, where its sensitivity to interest rates depends on the disaster risk and the level of private debt. With zero private liquidity and disaster probability, our economy features a *discounted* Euler equation, where output is less sensitive to future interest rate changes due to a precautionary motive, as in the incomplete-markets model of McKay et al. (2017). The presence of private debt acts in the opposite direction, as it pushes the economy towards *compounding*. We find that the second effect dominates in our calibration, so the aggregate Euler equation features compounding, even though, at the micro level, the savers' Euler equation always features discounting.

We turn next to the channels through which monetary policy affects the economy. We show that equilibrium output can be characterized as the sum of four terms: an *intertemporal-substitution effect* (ISE); a *time-varying risk effect*; an *inside* *wealth effect*, i.e. the change in valuation of assets in zero net supply; and an *outside wealth effect*, i.e. changes in the valuation of assets in positive net supply.[3]

The ISE corresponds to the output response that operates through changes in the *timing* of output but not its overall (present value) level. While this channel is quantitatively important in the textbook New Keynesian model, we find that it has a marginal impact in the presence of heterogeneous agents and risk, consistent with the findings in Kaplan et al. (2018). Most of the output response can be explained by time-varying risk and wealth effects.

The time-varying risk effect captures the change in the households' precautionary motives due to changes in the probability of a disaster. When the probability of disaster is constant, the model is able to capture important unconditional asset-pricing moments, such as the level of the equity premium and an upward-sloping yield curve, but it fails to generate the observed response of risk premia to monetary shocks. This failure has significant real consequences, as aggregate risk has then only a minor impact on the response of output and inflation. With time-varying disaster risk, the model is able to simultaneously match how long-term bonds, corporate spreads, and equities respond to monetary shocks in the data, and the impact on output increases almost threefold. This highlights the importance of matching the empirical response of asset prices to properly assess the role of risk in determining how monetary policy affects the economy.

The inside wealth effect captures the aggregate implications of the differential response of borrowers and savers to changes in interest payments.[4] An increase in nominal interest rates creates a positive wealth effect on savers, as they receive a higher income from private lending, and a corresponding loss to borrowers. Given the higher MPC for borrowers, this generates a negative aggregate response of output on impact. We also show that allowing for time-varying default risk in household debt substantially amplifies the effect of monetary policy, but this effect gets attenuated when debt has a high duration.

Finally, the outside wealth effect is the sum of the change in wealth for all households. This includes the change in the value of stocks, government bonds,

[3]The notion of inside/outside wealth is reminiscent of inside/outside money as used by Gurley and Shaw (1960), and, more recently, inside/outside liquidity by Holmstrom and Tirole (2011).

[4]Note that previous analytical HANK models focused on either heterogeneous MPC but no private debt, as in Bilbiie (2018), or positive private debt and no heterogeneity in MPCs, as in Acharya and Dogra (2020).

and human wealth, *net* of the impact of discount rates on the present discounted value of consumption. An important result of our analysis is that the outside wealth effect is tightly connected to the response of fiscal policy to monetary shocks.[5] In particular, we show that the outside wealth effect is proportional to the revaluation of public debt and the fiscal backing, that is, the change in taxes and transfers, in response to monetary shocks. Intuitively, in a closed economy, the government is the only trading counterpart to the household sector as a whole, so the outside wealth effect can be inferred from the impact of monetary policy on government finances. More importantly, this result implies that we can use standard estimation techniques to identify the fiscal response to a monetary shock and discipline the ability of the model to generate quantitatively meaningful wealth effects.

We find that when constrained to match the estimated fiscal response, the standard RANK model generates a substantially weaker output response to monetary shocks than when fiscal backing is determined by a standard Taylor rule that restricts monetary shocks to follow an AR(1) process. Equivalently, the standard Taylor equilibrium requires a (passive) fiscal response that is counterfactually large. Our results can be made consistent with a Taylor equilibrium by allowing a more general specification of the monetary shock. We can then use both monetary and fiscal data to discipline the parameters of the interest rate rule. In this context, the presence of heterogeneity and risk becomes particularly relevant, as these forces can compensate for the missing fiscal response.

To quantify the importance of the channels present in the model, we decompose the response of output by sequentially adding time-varying risk and private debt to the standard RANK model. We find that time-varying risk accounts for more than 50% of the output response, while private debt accounts for roughly 20%, and the interaction between the two accounts for 10%.

Literature review. Wealth effects have a long tradition in monetary economics. Pigou (1943) relied on a wealth effect to argue that full employment could be reached even in a liquidity trap. Kalecki (1944) argued that these effects apply only to government liabilities, as inside assets cancel out in the aggregate, while

[5] See Caramp (2021) for the role of fiscal policy in the monetary transmission mechanism.

Tobin highlighted the role of private assets and high-MPC borrowers.[6] Recently, wealth effects have regained relevance. In an influential paper, Kaplan et al. (2018) build a quantitative HANK model and find only a minor role for the standard intertemporal-substitution channel, leading the way to a more important role for wealth effects. Much of the literature has focused on the role of heterogeneous marginal propensities to consume (MPCs) in settings with idiosyncratic income risk. Instead, our focus is on aggregate risk and private debt.

Our work is closely related to two strands of literature. First, it relates to the analytical HANK literature, such as Werning (2015), Debortoli and Galí (2017), and Bilbiie (2018). While this literature focuses primarily on how the cyclicality of income interacts with differences in MPCs, we focus instead on how heterogeneous asset positions interact with differences in MPCs. We see these two channels as mostly complementary: even though Cloyne et al. (2020) does not find significant differences in income sensitivity across borrowers and savers, Patterson (2019) finds a positive covariance between MPCs and the sensitivity of earnings to GDP across different demographic groups, suggesting that the income-sensitivity channel is operative for a different cut of the data. We share with Eggertsson and Krugman (2012) and Benigno et al. (2020) the emphasis on private debt, but they abstract from a precautionary motive and focus instead on the implications of deleveraging. Iacoviello (2005) also considers a monetary economy with private debt but focuses instead on the role of housing as collateral. Our work is also related to Auclert (2019), which studies the redistribution channel of monetary policy arising from portfolio heterogeneity. Our paper emphasizes the redistribution channel in the context of a general equilibrium setting with aggregate risk.

The paper is also closely related to work on how monetary policy affects the economy through changes in asset prices, including models with sticky prices, such as Caballero and Simsek (2020), and models with financial frictions, such as Brunnermeier and Sannikov (2016) and Drechsler et al. (2018). In recent contributions, Kekre and Lenel (2020) consider the role of the marginal propensity to take risk in determining the risk premium and shaping the response of the econ-

[6] Tobin (1982) describes the role of inside assets: "The gross amount of these 'inside' assets was and is orders of magnitude larger than the net amount of the base. Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and debtors. But if the spending propensity were systematically greater for debtors, even by a small amount, the Pigou effect would be swamped by this Fisher effect."

omy to monetary policy, and Campbell et al. (2020) use a habit model to study the role of monetary policy in determining bond and equity premia. Our model highlights instead the role of heterogeneous MPCs, positive private liquidity, and disaster risk in an analytical framework that preserves the tractability of standard New Keynesian models.

Finally, a recent literature studies rare disasters and business cycles. Gabaix (2011) and Gourio (2012) consider a real business cycle model with rare disasters, while Andreasen (2012) and Isoré and Szczerbowicz (2017) allow for sticky prices. They focus on the effect of changes in disaster probability while we study monetary shocks in an analytical HANK model with rare disasters.

2 An Analytical Rare Disasters HANK Model

In this section, we consider an analytical HANK model with two main ingredients: the possibility of rare disasters and positive private liquidity. First, we describe the non-linear model and later consider a log-linear approximation around a stochastic stationary equilibrium.

2.1 The Model

Environment. Time is continuous and denoted by $t \in \mathbb{R}_+$. The economy is populated by households, firms, and a government. There are two types of households, *borrowers* and *savers*, who differ in their discount rates. A mass $0 \leq \mu_b < 1$ of households are borrowers and a mass $\mu_s = 1 - \mu_b$ are savers. Households can borrow or lend at a riskless rate, but they are subject to a borrowing constraint.

Firms can produce final or intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as their only input and face Rotemberg pricing adjustment costs.[7] Intermediate-goods producers are subject to an aggregate productivity shock: with Poisson intensity $\lambda_t \geq 0$, they receive a shock that permanently reduces their productivity. This shock is meant to capture the possibility of rare disasters: low-probability, large drops in productivity

[7] Rotemberg adjustment costs simplify the derivations, but they are not essential for our results.

and output, as in the work of Barro (2006, 2009). We say that periods that predate the realization of the shock are in the *no-disaster state*, and periods that follow the shock are in the *disaster state*. The disaster state is absorbing, and there are no further shocks after the disaster is realized. Assuming an absorbing disaster state simplifies the presentation, but it can be easily relaxed, as shown in Appendix C.2.[8]

The government sets fiscal policy, comprising a sales tax on intermediate-goods producers and transfers to borrowers and savers, and monetary policy, specified by an interest rate rule subject to a sequence of monetary shocks. We assume that the government issues long-term nominal bonds that pay exponentially decaying coupons, where the coupon in period t is given by $e^{-\psi_L t}$. The rate of decay ψ_L is inversely related to the bond's duration, where a perpetuity corresponds to $\psi_L = 0$ and the limit $\psi_L \to \infty$ corresponds to the case of short-term bonds. We denote by $Q_{L,t}$ the nominal price of the bond in the no-disaster state and by $Q^*_{L,t}$ the price of the bond in the disaster state, where the star superscript is used throughout the paper to denote variables in the disaster state.

Households' problem. Households face a portfolio problem where they choose how much to invest in short-term and long-term bonds. In this section, we assume that borrowers issue only short-term risk-free bonds and the government issues only long-term bonds. We study the case of defaultable long-term household debt in Section 5. The nominal return on the long-term bond is given by

$$dR_{L,t} = \left[\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L\right] dt + \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

where $\mathcal{N}_t$ is a Poisson process with arrival rate λ_t.

Let $B_{j,t} = B^S_{j,t} + B^L_{j,t}$ denote the total value of bonds (in real terms) held by a type-j household, $j \in \{b, s\}$, that is, the sum of short-term ($B^S_{j,t}$) and long-term ($B^L_{j,t}$) bonds. The problem of a household of type j is to choose consumption $C_{j,t}$, labor supply $N_{j,t}$, and long-term bonds $B^L_{j,t}$, given an initial real value of bonds $B_{j,t}$,

[8] Allowing for partial recovery after a disaster, as in Barro et al. (2013) and Gourio (2012), introduces dynamics in the disaster state, but it does not change the main implications for the no-disaster state, which is our focus.

to solve the following problem:

$$V_{j,t}(B_{j,t}) = \max_{[C_{j,z},N_{j,z},B^L_{j,z}]_{z\geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_j(z-t)} \left(\frac{C_{j,z}^{1-\sigma}}{1-\sigma} - \frac{N_{j,z}^{1+\phi}}{1+\phi} \right) dz + e^{-\rho_j(t^*-t)} V^*_{j,t^*}(B^*_{j,t^*}) \right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + \frac{W_t}{P_t} N_{j,t} + \Pi_{j,t} + \widetilde{T}_{j,t} - C_{j,t} \right] dt + B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} d\mathcal{N}_t,$$

and the borrowing constraints

$$B_{j,t} \geq -\overline{D}_P \quad \text{and} \quad B^L_{j,t} \geq 0,$$

where $\rho_b > \rho_s > 0$, W_t is the nominal wage, P_t is the price level, $\Pi_{j,t}$ denotes real profits from corporate holdings, $\widetilde{T}_{j,t}$ denotes government transfers, and $r_{L,t} \equiv \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t$ is the excess return on long-term bonds conditional on no disasters. The random (stopping) time t^* represents the period in which the aggregate shock hits the economy. $V^*_{j,t^*}(\cdot)$ and B^*_{j,t^*} denote, respectively, the value function and the real value of bonds in the disaster state. The non-negativity constraint on $B^L_{j,t}$ captures the assumption that only the government can issue long-term bonds.

We assume that $B_{s,0} > 0$ and $B_{b,0} = -\overline{D}_P$. For sufficiently large ρ_b, borrowers are constrained in all periods. We also assume that $\Pi_{b,t} = 0$, that is, firms are entirely owned by savers.[9]

The labor supply is determined by the standard condition:

$$\frac{W_t}{P_t} = N^\phi_{j,t} C^\sigma_{j,t}.$$

The Euler equation for short-term bonds, if $B_{j,t} > -\overline{D}_P$, is given by

$$\frac{\dot{C}_{j,t}}{C_{j,t}} = \sigma^{-1}(i_t - \pi_t - \rho_j) + \frac{\lambda_t}{\sigma} \left[\left(\frac{C_{j,t}}{C^*_{j,t}} \right)^\sigma - 1 \right], \tag{1}$$

where $C^*_{j,t}$ is the consumption of household j in the disaster state. The first term

[9] Alternatively, we could have assumed that households can trade shares of the firms. In a steady state, borrowers would choose to sell their shares, and savers would entirely own the firms.

captures the usual intertemporal-substitution force present in RANK models. The second term captures the *precautionary savings motive* generated by the disaster risk, and it is analogous to the precautionary motive that emerges in HANK models with idiosyncratic risk.

The Euler equation for long-term bonds, if $B^L_{j,t} > 0$, is given by

$$r_{L,t} = \underbrace{\lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma}}_{\substack{\text{price of} \\ \text{disaster risk}}} \underbrace{\frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}}_{\substack{\text{quantity of} \\ \text{risk}}}. \tag{2}$$

This expression captures a risk premium on long-term bonds, which pins down the level of long-term interest rates in equilibrium. The premium on long-term bonds is given by the product of the *price of disaster risk*, the compensation for a unit exposure to the risk factor, and the *quantity of risk*, the loss the asset suffers conditional on switching to the disaster state.

Firms' problem. Intermediate-goods producers are indexed by $i \in [0,1]$ and operate in monopolistically competitive markets. Final good producers are price takers and combine intermediate goods to produce the final good. Their demand for variety i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, and the equilibrium price level is given $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

Intermediate-goods producers operate the linear technology $Y_{i,t} = A_t N_{i,t}$. Productivity in the no-disaster state is given by $A_t = A$, and productivity in the disaster state is given by $A_t = A^*$, where $0 < A^* < A$. Intermediate-goods producers choose the *rate-of-change* of prices $\pi_{i,t} = \dot{P}_{i,t}/P_{i,t}$, given the initial price $P_{i,0}$, to maximize the expected discounted value of real (after-tax) profits subject to Rotemberg quadratic adjustment costs:

$$Q_{i,t}(P_{i,t}) = \max_{[\pi_{i,z}]_{z\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_z}{\eta_t} \left((1-\tau)\frac{P_{i,z}}{P_z} Y_{i,z} - \frac{W_z}{P_z}\frac{Y_{i,z}}{A} - \frac{\varphi}{2}\pi^2_{i,t}\right) dz + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t^*}(P_{i,t^*})\right], \tag{3}$$

subject to the demand $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, where η_t denotes the stochastic discount factor (SDF) that is relevant to firms and $Q^*_{i,t}(P_i)$ denotes the firms' value function in the disaster state. Note that the price $P_{i,t}$ is a state variable in the firms' problem and $\pi_{i,t}$ is a control variable. The parameter φ controls the magnitude of the pricing adjustment costs. We assume that these costs are rebated to households, so they do not represent real resource costs. Moreover, as firms are owned by savers, we assume that firms discount profits using the SDF $\eta_t = e^{-\rho_s t} C_{s,t}^{-\sigma}$.

Combining the first-order condition and the envelope condition for problem (3), we obtain the non-linear New Keynesian Phillips curve:

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t} \right) \pi_t - \varphi^{-1}(\epsilon - 1) \left(\frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_t} \frac{1}{A} - (1 - \tau) \right) Y_t, \tag{4}$$

assuming a symmetric initial condition $P_{i,0} = P_0$, for all $i \in [0, 1]$.

Time-varying risk. To capture the effect of monetary policy on the market price of risk, as documented e.g. by Gertler and Karadi (2015) and Hanson and Stein (2015), we assume that monetary shocks affect directly the probability of disasters: $\lambda_t = \lambda(i_t - r_n)$, for a given function $\lambda(\cdot)$. Importantly, we assume that $\lambda(\cdot)$ is an increasing function, such that a contractionary monetary shock raises the probability of a disaster. This is consistent with recent evidence by Schularick et al. (2021) on the effects of monetary policy on the probability of a financial crisis.

Motivated by this evidence, we provide a new micro-foundation for the relationship between monetary policy and asset prices, where monetary shocks endogenously affect the probability of a crisis and, ultimately, the market price of risk. In Appendix B, we extend our model to incorporate banks that borrow short-term from savers to lend long-term to firms. Given this maturity mismatch, a contractionary monetary shock weakens the balance sheet of banks. As in Gertler et al. (2020), a weaker balance sheet makes banks more vulnerable to *bank runs*, so monetary policy affects the probability of crises through its impact on banks' balance sheet positions. In particular, we show that the sensitivity of λ_t to monetary shocks depends on features of the economy, such as bank's leverage, the maturity of bank loans, and even the persistence of monetary shocks.[10] Moreover, we show that the (linearized) model with banks and endogenous disasters is essentially identical to the model without financial intermediaries but with a sensitivity that depends on

[10] This implies that the sensitivity is not invariant to economic policy. This dependence does not affect our positive analysis, but it is potentially relevant when considering normative questions.

the financial sector's characteristics in the stationary equilibrium. For this reason, we describe the details of the micro-foundation in Appendix B, and here we simply assume that $\lambda(\cdot)$ depends on monetary policy.

Government. The government's flow budget constraint in the no-disaster state is given by

$$\dot{D}_{G,t} = (i_t - \pi_t + r_{L,t})D_{G,t} + \sum_{j\in\{b,s\}} \mu_j \widetilde{T}_{j,t} - \tau Y_t,$$

and the No-Ponzi condition $\lim_{t\to\infty} \mathbb{E}_0[\eta_t D_{G,t}] \leq 0$, $D_{G,t}$ denotes the real value of government debt and $D_{G,0} = \overline{D}_G$ is given. We assume that government transfers to borrowers are determined by the policy rule $\widetilde{T}_{b,t} = \widetilde{T}_b(Y_t)$, where transfers depend on aggregate output and the elasticity of $\widetilde{T}_b(\cdot)$ determines the cyclicality of government transfers to borrowers.

In the no-disaster state, monetary policy is determined by the policy rule

$$i_t = r_n + \phi_\pi \pi_t + u_t, \tag{5}$$

where $\phi_\pi > 1$, u_t is a monetary shock, and r_n denotes the real rate when $\pi_t = u_t = 0$ at all periods. We assume that in the disaster state there are no monetary shocks, that is, $i_t^* = r_n^* + \phi_\pi \pi_t^*$. By abstracting from the policy response after a disaster, we isolate the impact of changes in monetary policy during "normal times."

Market clearing. The market-clearing conditions for goods, labor, and bonds are given by

$$\sum_{j\in\{b,s\}} \mu_j C_{j,t} = Y_t, \quad \sum_{j\in\{b,s\}} \mu_j N_{j,t} = N_t, \quad \sum_{j\in\{b,s\}} \mu_j B_{j,t}^L = D_{G,t}, \quad \sum_{j\in\{b,s\}} \mu_j B_{j,t}^S = 0.$$

2.2 Equilibrium dynamics

Stationary equilibrium. We define a stationary equilibrium as an equilibrium in which all variables are constant in each aggregate state. In particular, the economy will be in a stationary equilibrium in the absence of monetary shocks, that is, $u_t = 0$ for all $t \geq 0$. Since variables are constant in each state, we drop time subscripts and write, for instance, $C_{j,t} = C_j$ and $C_{j,t}^* = C_j^*$. For ease of exposition, we follow Bilbiie

(2018) and focus on a *symmetric* stationary equilibrium, where $\widetilde{T}_b$ implements the same consumption level for each household, and discuss the general case $C_b \neq C_s$ in the appendix.

The natural interest rate, the real rate in the stationary equilibrium, is given by

$$r_n = \rho_s - \lambda \left[\left(\frac{C_s}{C_s^*} \right)^{\sigma} - 1 \right],$$

where $0 < C_s^* < C_s$ and, with a slight abuse of notation, $\lambda = \lambda(0) > 0$ is the disaster intensity when $i_t = r_n$. The presence of a precautionary motive depresses the natural interest rate relative to the one that would prevail in a non-stochastic economy, and the magnitude depends on the extent to which savers can self-insure. In particular, holding everything else constant, a higher level of private debt $\overline{D}_P$ implies a weaker precautionary motive and a higher natural interest rate.

From Equation (2), we obtain the term spread, the difference between the yield on the long-term bond and the short-term rate,

$$i_L - r_n = \lambda \left(\frac{C_s}{C_s^*} \right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L},$$

where i_L is the yield on the long-term bond in the stationary equilibrium.[11] We show in Appendix C.2.2 that the term spread $i_L - r_n$ is strictly positive. Thus, our model generates an upward-sloping yield curve, where the yield on the long-term bond exceeds the natural (short-term) rate, consistent with the data.[12]

Log-linear dynamics. Following the practice in the literature on monetary policy, we focus on a log-linear approximation of the equilibrium conditions. However, instead of linearizing around the non-stochastic steady state, we linearize the equilibrium conditions around the (stochastic) symmetric stationary equilibrium described above. Formally, we perturb the allocation around the economy where $u_t = 0$ and $\lambda > 0$, while the standard approach would perturb around the economy where $u_t = \lambda_t = 0$. This enables us to capture the effects of (time-varying)

[11] Note that the yield on the bond is given by $i_{L,t} = Q_{L,t}^{-1} - \psi_L$ and, in a stationary equilibrium, the expected excess return conditional on no disaster r_L equals the term spread $i_L - r_n$.

[12] The mechanism behind the upward-sloping yield curve is related to the lack of precautionary savings in the disaster state. We would obtain similar results by introducing expropriation and inflation in a disaster, as in Barro (2006).

precautionary savings and risk premia in a linear setting.[13]

Let lower-case variables denote log-deviations from the stationary equilibrium, e.g., $c_{j,t} \equiv \log C_{j,t}/C_j$ and $n_{j,t} \equiv \log N_{j,t}/N_j$. Borrowers' consumption is given by

$$c_{b,t} = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - (i_t - \pi_t - r_n)\overline{d}_P, \tag{6}$$

where $1-\alpha \equiv \frac{WN}{PY}$ is the labor share in the stationary equilibrium, $T_{j,t} \equiv \frac{\widetilde{T}_{j,t}-\widetilde{T}_j}{Y}$, and $\overline{d}_P \equiv \frac{\overline{D}_P}{Y}$. Using the fact that transfers satisfy $T_{b,t} = T_b'(Y)y_t$, and solving for the real wage, we obtain

$$c_{b,t} = \chi_y y_t - \chi_r \overline{d}_P(i_t - \pi_t - r_n), \tag{7}$$

where $\chi_y \equiv \frac{T_b'(Y)+(1-\alpha)(1+\phi)(1+\phi^{-1}\sigma)}{1+(1-\alpha)\phi^{-1}\sigma}$ and $\chi_r \equiv \frac{1}{1+(1-\alpha)\phi^{-1}\sigma}$. The coefficient χ_y controls the cyclicality of income inequality and has been extensively studied by the literature on analytical HANK models. We focus throughout the paper on the case in which $0 < \chi_y < \mu_b^{-1}$, such that the consumption of both agents increases with y_t.[14] The second term is not present in the commonly studied case of zero private liquidity, $\overline{d}_P = 0$, and it captures the impact of monetary policy on the consumption of constrained agents that is not directly mediated by aggregate output y_t. This term plays an important role in the analysis that follows.

Next, consider the savers' problem. Recall that we assume that the disaster probability depends on the interest rate, $\lambda_t = \lambda(i_t - r_n)$. In our linearized setting, the only relevant parameter is the semi-elasticity of the disaster probability with respect to monetary shocks, $\epsilon_\lambda \equiv \lambda'(0)/\lambda(0)$. We focus on the case in which $\epsilon_\lambda \geq 0$, with $\epsilon_\lambda = 0$ corresponding to a benchmark with constant probability.[15] Then, the savers' Euler equation is given by

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma c_{s,t} + \chi_d \epsilon_\lambda (i_t - r_n), \tag{8}$$

[13] This method differs from the procedure considered by Fernández-Villaverde and Levintal (2018) or Coeurdacier et al. (2011), as we linearize around a stochastic steady state of an economy with no monetary shocks, instead of the stochastic steady state of the economy with both shocks.

[14] The role of χ_y, including the case where $\chi_y > \mu_b^{-1}$, was originally considered by Bilbiie (2008).

[15] See Appendix B for the micro-foundation of this relationship and how ϵ_λ depends on deeper structural parameters.

where $\chi_d \equiv \frac{\lambda}{\sigma}\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right]$ is a parameter capturing the strength of the precautionary motive in the stationary equilibrium. Importantly, time-varying disaster risk introduces a new precautionary savings channel for savers, which ultimately shapes the impact of changes in nominal rates on households' consumption.

Combining condition (7) for borrowers' consumption, equation (8) for savers' Euler equation, and the market-clearing condition for goods, we obtain the evolution of aggregate output. Proposition 1 characterizes the dynamics of aggregate output and inflation. All proofs are provided in Appendix A.

Proposition 1 (Aggregate dynamics)**.** *The dynamics of output and inflation is described by the conditions:*

i. ***Aggregate Euler equation:***

$$\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t, \tag{9}$$

where $\tilde{\sigma}^{-1} \equiv \frac{(1-\mu_b)\sigma^{-1} - \mu_b \chi_r \overline{d}_P r_n}{1-\mu_b \chi_y}$, $\delta \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b \chi_r \overline{d}_P \kappa}{1-\mu_b \chi_y}$ *and* $v_t \equiv \frac{\mu_b \chi_r \overline{d}_P}{1-\mu_b \chi_y}(\rho(i_t - r_n) - \dot{i}_t) + \frac{1-\mu_b}{1-\mu_b \chi_y}\chi_d \epsilon_\lambda (i_t - r_n)$.

ii. ***New Keynesian Phillips curve:***

$$\dot{\pi}_t = \rho \pi_t - \kappa y_t, \tag{10}$$

where $\rho \equiv \rho_s + \lambda$ *and* $\kappa \equiv \varphi^{-1}(\epsilon - 1)(1-\tau)(\phi + \sigma)Y$.

Condition (9) represents the *aggregate Euler equation* for this economy. The aggregate Euler equation has three terms. The first term, the product of the (aggregate) elasticity of intertemporal substitution (EIS) and the real interest rate, corresponds to the one present in RANK models. The dependence of the aggregate EIS on the cyclicality of inequality is well-known in the literature, while the result that private liquidity may reduce $\tilde{\sigma}^{-1}$ is, to the best of our knowledge, new.[16]

The second term, δy_t, captures how the impact of real interest rate changes can be compounded or discounted in equilibrium. The sign of δ and, therefore, whether the economy exhibits compounding or discounting, depends on two forces. With disaster risk but in the absence of private debt, so that $\lambda > 0$ and $\overline{d}_p = 0$,

[16]In our calibration, we obtain $\tilde{\sigma}^{-1} > 0$, but most of our results do not rely on this.

we obtain $\delta > 0$. This corresponds to the discounted Euler equation of McKay et al. (2017), where aggregate disaster risk plays the role of idiosyncratic income risk. In contrast, if $\lambda = 0$ but $\overline{d}_P > 0$, we get compounding, that is, $\delta < 0$. As a contractionary monetary shock depresses the economy and reduces inflation in all periods, it increases the real burden of debt for borrowers, amplifying the effect of the monetary shock. This amplification translates into a compounded response of output to future interest rate changes. More generally, the aggregate Euler equation (9) can feature compounding or discounting. In particular, if λ is sufficiently small, we can have that savers' consumption satisfies a discounted Euler equation while the aggregate Euler equation features compounding.

The third term in the aggregate Euler equation, v_t, captures a direct effect of monetary policy on households that is not mediated by the intertemporal substitution motive or changes in aggregate demand. First, in an economy with household debt, monetary policy directly affects borrowers' disposable income, the high-MPC agents in this economy. Second, the time-varying component of the disaster risk directly impacts the savers' precautionary savings motive. Therefore, monetary policy has real effects even in the absence of intertemporal-substitution forces.

Finally, Proposition 1 derives the New Keynesian Phillips curve. The linearized Phillips curve coincides with the one obtained from models with Calvo pricing. As in a textbook New Keynesian model, inflation is given by the present discounted value of future output gaps, $\pi_t = \kappa \int_t^\infty e^{-\rho(s-t)} y_s ds$. One distinction relative to the standard formulation is that future output gaps are not discounted by the natural rate r_n but by a higher rate $\rho > r_n$. This is a consequence of the riskiness of the firm's value, so the appropriate discount rate incorporates an adjustment for risk.

Asset prices. The response of asset prices to monetary policy depends crucially on the behavior of the price of disaster risk. In its log-linear form, the price of disaster risk is given by

$$p_{d,t} \equiv \sigma c_{s,t} + \epsilon_\lambda (i_t - r_n). \tag{11}$$

Note that this expression has two terms. The first term captures the change in the effective size of the shock, represented by the drop in the savers' marginal utility of consumption if the disaster shock is realized. The second term represents the change in the disaster probability after a monetary shock.

Given the price of risk, we can price any financial asset in this economy. For example, the (linearized) price of the long-term bond in $t = 0$ is given by

$$q_{L,0} = -\underbrace{\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n)dt}_{\text{path of nominal interest rates}} - \underbrace{\int_0^\infty e^{-(\rho+\psi_L)t} r_L p_{d,t} dt}_{\text{term premium}}. \tag{12}$$

The yield on the long-term bond, expressed as deviations from the stationary equilibrium, is given by $-Q_L^{-1} q_{L,0}$, which can be decomposed into two terms: the path of nominal interest rates, as in the expectations hypothesis, and a *term premium*, capturing variations in the compensation for holding long-term bonds. The term premium depends on the price of risk, $p_{d,t}$, and the asset-specific loading r_L. Because the term premium responds to monetary shocks, the expectation hypothesis does not hold in this economy. This is important since the term premium accounts for the bulk of the response of long-term rates to monetary policy in the data.

The pricing condition for stocks is analogous to the one for bonds:

$$q_{S,0} = \underbrace{\frac{Y}{Q_S}\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)\right]dt}_{\text{dividends}} - \underbrace{\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_S p_{d,t}\right]dt}_{\text{discount rate}}, \tag{13}$$

where $r_S \equiv \lambda \left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_S - Q_S^*}{Q_S}$ is the (conditional) equity premium in the stationary equilibrium and Q_S is the value of a claim on firms' profits. This expression shows that the valuation of assets responds to changes in monetary policy through two channels: a *dividend channel*, capturing changes in firms' profits, and a *discount rate channel*, capturing changes in real interest rates and risk premia. Note that the risk premia depends on the price of risk, $p_{d,t}$, like in the expression for the long-term bond, but it has a loading r_S rather than r_L, capturing the different exposure to risk of the two assets.

2.3 The aggregate intertemporal budget constraint

By combining the intertemporal budget constraint for all households, we obtain the economy's aggregate intertemporal budget constraint:

$$\mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right] = Q_{L,0}D_{g,0} + \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left((1-\tau)Y_t + \widetilde{T}_t\right) dt\right],$$

where $C_t \equiv \sum_{j\in\{b,s\}} \mu_j C_{j,t}$, $\widetilde{T}_t \equiv \sum_{j\in\{b,s\}} \mu_j \widetilde{T}_{j,t}$, and we used that $\sum_{j\in\{b,s\}} \mu_j \left[\frac{W_t}{P_t} N_{j,t} + \Pi_{j,t}\right] = (1-\tau)Y_t$. This expression states that the present discounted value of *aggregate* consumption has to be equal to the initial value of assets in *positive net supply* (i.e. government bonds but not private debt) plus the present discounted value of *aggregate* disposable income (net of taxes and transfers).

Let $Q_{C,0} \equiv \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0} C_t dt\right]$ and $Q_{Y,0} \equiv \mathbb{E}_0\left[\int_0^\infty \frac{\eta_t}{\eta_0}\left((1-\tau)Y_t + \widetilde{T}_t\right) dt\right]$. Note that $Q_{C,0}$ and $Q_{Y,0}$ can be interpreted as the price of claims to the stream of consumption and disposable income, respectively. Thus, letting $q_{C,0}$ and $q_{Y,0}$ denote the log-linear approximations around the stationary equilibrium, we have

$$q_{C,0} = \frac{Y}{Q_C}\int_0^\infty e^{-\rho t} c_t dt - \int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_C p_{d,t}\right] dt,$$
$$q_{Y,0} = \frac{Y}{Q_Y}\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t\right] dt - \int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_Y p_{d,t}\right] dt,$$

where $r_C = \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_C - Q_C^*}{Q_C}$ and $r_Y = \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_Y - Q_Y^*}{Q_Y}$, and we used that $C = Y$. Note that these expressions have the same structure as the expression we derived for stocks: a first term capturing the change in "dividends," and a second term reflecting the change in the asset-specific discount rate, where the loading on the price of disaster risk depends on each asset's exposure to the disaster shock. Thus, we get that the log-linear approximation of the aggregate intertemporal budget constraint can be written as

$$\int_0^\infty e^{-\rho t} c_t dt - \frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_C p_{d,t}\right] dt = \overline{d}_g q_{L,0} +$$
$$\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t\right] dt - \frac{Q_Y}{Y}\int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + r_Y p_{d,t}\right] dt.$$

This expression reveals an important and often overlooked property of the effect of changes in discount rates on households' budget constraints. Changes in the discount rate have two effects. First, they generate a revaluation of households' assets. When the discount rate increases, the present discounted value of income streams (e.g. dividends, coupons, wages) decreases, implying a negative wealth effect. However, there is also an opposing effect. A higher discount rate *reduces* the cost of the consumption bundle, generating a positive wealth effect. Thus, the overall effect depends on which one of these two forces dominates. It turns out that the answer depends on the government's portfolio.

Lemma 1. *The net effect of changes in the discount rate on the households' aggregate budget constraint is given by*

$$\frac{Q_Y}{Y}\int_0^\infty e^{-\rho t}\left[i_t-\pi_t-r_n+r_Y p_{d,t}\right]dt-\frac{Q_C}{Y}\int_0^\infty e^{-\rho t}\left[i_t-\pi_t-r_n+r_C p_{d,t}\right]dt=$$
$$\overline{d}_G\int_0^\infty e^{-\rho t}\left[i_t-\pi_t-r_n+r_L p_{d,t}\right]dt,$$

where $\overline{d}_G\equiv\frac{D_G}{Y}$ *is the public debt-to-GDP ratio. If government debt is short-term, i.e.* $\psi_L\to\infty$, *then* $r_L=0$ *and the households' aggregate budget constraint is independent of the price of disaster risk,* $p_{d,t}$.

Lemma 1 provides a novel insight on how discount rates affect the households' aggregate wealth. Intuitively, the result reflects that, in a closed economy, the government is the only counterpart of the private sector as a whole. Thus, in the aggregate, the net effect of changes in the discount rate is given by the change in the value of the assets in positive net supply, i.e. government bonds. Using this result, we can express the aggregate intertemporal budget constraint as

$$\int_0^\infty e^{-\rho t}\left[\mu_b c_{b,t}+(1-\mu_b)c_{s,t}\right]dt=\Omega_0, \tag{14}$$

where Ω_0 denotes the outside wealth effect, and it is given by

$$\Omega_0\equiv\overline{d}_G q_{L,0}+\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t+T_t+\overline{d}_G(i_t-\pi_t-r_n+r_L p_{d,t})\right]dt. \tag{15}$$

Note that a simple rearrangement of (14) and (15) gives the government's intertemporal budget constraint. By writing it this way, we make explicit the role of the *out-*

side wealth effect Ω_0, which captures the revaluation of assets in positive net supply: stocks, human wealth, and government bonds. This is in contrast with the effect of the revaluation of assets in zero net supply, such as private debt, which we refer to as the *inside wealth effect*. Interestingly, while the value of stocks and human wealth also incorporate a risk premium and, therefore, are affected by changes in the price of risk, only in the presence of long-term government bonds does the price of risk *directly* affect the households' wealth. This is once again the manifestation that only the mismatch in exposure between the household sector as a whole and the government (in terms of both maturity and risk) matters for the determination of the change in the value of the economy's wealth.

3 Monetary Policy and Wealth Effects

In this section, we study how households' balance sheets determine the impact of monetary policy on the dynamics of the economy. The main result presents a decomposition that identifies the contribution of the different forces of the model to the aggregate dynamics of the economy. In particular, we isolate the role of intertemporal substitution, precautionary savings, and wealth effects in the transmission of monetary shocks. To derive this decomposition, we proceed in two steps. First, we express the evolution of output and inflation in terms of equilibrium policy variables, that is, the path of nominal interest rates $\{i_t\}$ and the corresponding fiscal backing $\{T_t\}$. Second, we derive an *implementability result* that shows how to map the path of policy variables to the underlying monetary shock u_t in the interest rate rule (5).

3.1 The dynamic system

We can express output and inflation in terms of policy variables by solving the system of differential equations described in Proposition 1:

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \delta & -\tilde{\sigma}^{-1} \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \nu_t \\ 0 \end{bmatrix}, \tag{16}$$

where $\nu_t \equiv \tilde{\sigma}^{-1}(i_t - r_n) + v_t$ depends only on the path of the nominal interest rate. The eigenvalues of the system are given by

$$\overline{\omega} = \frac{\rho + \delta + \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}, \qquad \underline{\omega} = \frac{\rho + \delta - \sqrt{(\rho+\delta)^2 + 4(\tilde{\sigma}^{-1}\kappa - \rho\delta)}}{2}.$$

The following assumption, which we will assume holds for all subsequent analysis, guarantees that the eigenvalues are real-valued and that they have opposite signs, that is, $\overline{\omega} > 0$ and $\underline{\omega} < 0$.

Assumption 1. *The following condition holds: $\rho\delta < \tilde{\sigma}^{-1}\kappa$.*

Assumption 1 implies that the system lacks exactly one boundary condition. Next, we show that the missing boundary condition can be provided by an intertemporal budget constraint.

From equation (14), we have that the aggregate intertemporal budget constraint is a necessary equilibrium condition. The next lemma establishes the sufficiency of the aggregate intertemporal budget constraint for pinning down the equilibrium. That is, it shows that if $[y_t, \pi_t]_0^\infty$ satisfies system (16) and the aggregate intertemporal budget constraint (in its log-linear form), then we can determine the value of consumption and labor supply for each household, wages, and prices such that all equilibrium conditions are satisfied.

Lemma 2. *Suppose that, given a path for the nominal interest rate $[i_t]_0^\infty$, $[y_t, \pi_t]_0^\infty$ satisfy system (16) and the aggregate intertemporal budget constraint (14). Then, $[y_t, \pi_t]_0^\infty$ can be supported as part of a competitive equilibrium.*

Therefore, the equilibrium dynamics can be characterized as the solution to the dynamic system (16), subject to the boundary condition (14).

3.2 Intertemporal substitution, risk and wealth effects

The next proposition characterizes the output response to a sequence of monetary policy shocks for a given value of the outside wealth effect Ω_0. We provide a full characterization of Ω_0 in Section 3.3. For ease of exposition, we focus on the case of exponentially decaying nominal interest rates; that is, we assume $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where ψ_m determines the persistence of the path of interest rates.

Proposition 2 (Aggregate output in D-HANK). *Suppose that* $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. *The path of aggregate output is then given by*

$$y_t = \underbrace{\sigma^{-1}\hat{y}_t}_{\text{ISE}} + \underbrace{\chi_d \epsilon_\lambda \hat{y}_t}_{\substack{\text{time-varying} \\ \text{risk}}} + \underbrace{\frac{\mu_b \chi_r}{1-\mu_b}\tilde{\psi}_m \overline{d}_P \hat{y}_t}_{\text{inside wealth effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0}_{\substack{\text{GE multiplier}\times \\ \text{outside wealth effect}}}, \tag{17}$$

where $\tilde{\psi}_m \equiv \rho - r_n + \psi_m$, *and* $\hat{y}_t$ *is given by*

$$\hat{y}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{(\rho - \underline{\omega})\,e^{\underline{\omega}t} - (\rho + \psi_m)\,e^{-\psi_m t}}{(\overline{\omega} + \psi_m)\,(\underline{\omega} + \psi_m)}(i_0 - r_n), \tag{18}$$

and satisfies $\int_0^\infty e^{-\rho t}\hat{y}_t dt = 0$, $\frac{\partial \hat{y}_0}{\partial i_0} < 0$.

Proposition 2 shows that output can be decomposed into four terms: an intertemporal-substitution effect (ISE), a time-varying risk channel, a revaluation of inside assets, and a revaluation of outside assets. Note that the first three terms have a common structure. First, there is $\hat{y}_t$, which is uniquely determined by the path of the nominal interest rate. Figure 1 (left) shows its dynamics after a contractionary monetary shock (we discuss the calibration in Section 4.1). After a contractionary monetary shock, $\hat{y}_t$ decreases on impact, but it eventually increases above its long-run level. Notably, the present value of $\hat{y}_t$ is equal to zero. This implies that in the absence of outside wealth effects, monetary policy affects only the *timing* of output, but not its present value. Moreover, as can be seen from Figure 1 (left), $\hat{y}_t$ responds more strongly to interest rate shocks in economies where inequality is more counter-cyclical, i.e. when χ_y is higher.[17] Second, there is the channel specific strengths, given by σ^{-1}, $\chi_d\epsilon_\lambda$ and $\frac{\mu_b\chi_r}{1-\mu_b}\tilde{\psi}_m\overline{d}_P$. Figure 1 (middle) shows the dynamics of each of the first three channels of transmission in (17). In our calibration, the time-varying channel has the largest impact on output, while the ISE has a significantly smaller effect. Finally, there is the outside wealth effect, plotted in Figure 1 (right), which is mediated by a GE multiplier that captures a general equilibrium amplification mechanism. Next, we consider each one of the channels separately.

The first term is the ISE, which captures the equilibrium implications of the intertemporal substitution channel. Similar in logic to the substitution effect in intro-

[17] This amplification lies at the heart of the mechanism in analytical HANK models with zero private liquidity and no aggregate risk.

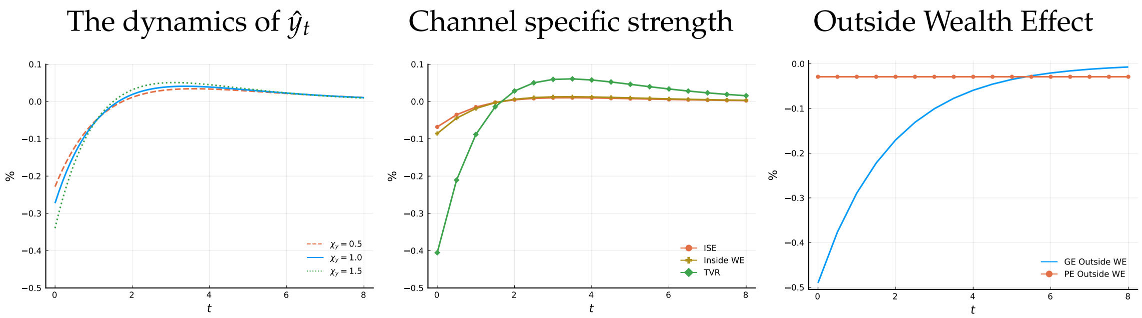

Figure 1: Aggregate output decomposition in HANK

ductory microeconomics, an increase in nominal interest rates reduces consumption today while it increases future consumption. In this sense, the intertemporal-substitution channel of monetary policy operates simply by shifting demand over time, and it is ineffective in the absence of an intertemporal-substitution motive; that is, we obtain $\sigma^{-1}\hat{y}_t = 0$ in all periods if $\sigma^{-1} = 0$.

The second term captures the role of time-varying risk. Given Ω_0, time-varying risk amplifies the response of output in a way that is similar to that of the ISE, and the magnitude of the amplification depends crucially on the strength of the precautionary motive, as captured by $\chi_d = \frac{\lambda}{\sigma}\left[\left(\frac{C_s}{C_s^*}\right)^{\sigma} - 1\right]$, and the degree of time-varying risk, as captured by ϵ_λ. Consider a contractionary monetary shock. If $\epsilon_\lambda > 0$, the increase in the nominal interest rate increases the probability of a disaster shock, which increases savers' precautionary motive. Thus, aggregate demand decreases. As the nominal interest rate reverts to its long-run level, the probability of disaster decreases, and savers' demand increases above its long-run target. Thus, we have that the present discounted value of the time-varying risk term is zero, i.e. $\int_0^\infty e^{-\rho t}\chi_d\epsilon_\lambda\hat{y}_t dt = 0$.

The third term corresponds to the inside wealth effect, and it is present only in economies with positive private debt and heterogeneous MPCs. The inside wealth effect is analogous to the ISE and the time-varying risk in many respects, as it operates by shifting demand over time, and it satisfies $\int_0^\infty e^{-\rho t}\chi_r\tilde{\psi}_m\bar{d}_P\hat{y}_t dt = 0$. A key distinction is that the strength of the inside wealth effect depends on the persistence of the monetary shock, and it is approximately equal to zero when the monetary shock is permanent, $\psi_m = 0$.[18] An important implication of this result

[18]The difference between $\tilde{\psi}_m$ and ψ_m is quantitatively small. The extra term $\rho - r_n$ reflects movements in the precautionary motive due to consumption fluctuations in the no-disaster state. Be-

is that the effectiveness of monetary policy depends on the persistence of monetary shocks. For instance, by promising to keep interest rates low for a very long period of time, the monetary authority increases the persistence of the shock and, therefore, reduces the importance of inside wealth effects and the overall output response. To understand this result, note that an increase in interest rates has a negative impact on borrowers and a positive impact on savers. When the shock is temporary, the impact of the change in interest rates is initially larger on borrowers, as savers respond less strongly to the change in wealth to smooth consumption. If the shock is permanent, however, there is no reason to smooth the shock. In this case, the savers' response coincides with the borrowers' response, and the inside wealth effect is zero. Thus, it is the *variability* of interest rates rather than the average level that matters for the inside wealth effect.

The last term in expression (17) plays a crucial role, as the outside wealth effect determines the average level of output. Holding everything else constant, the impact of a wealth effect Ω_0 on consumption would be simply $\rho\Omega_0$, as households attempt to smooth the impact of the change in wealth over time. However, the response of initial consumption is amplified in general equilibrium, as a positive wealth effect generates inflation, which reduces real interest rates and shifts consumption to the present. Figure 1 (right) plots the outside wealth effect. Monetary shocks usually have a small effect on households' wealth. However, the GE multiplier can significantly amplify the effect of the change in outside wealth on the initial level of output. In our calibration, the GE multiplier in period 0 is more than 15, amplifying the impact of the wealth effect in general equilibrium.[19]

Inflation. The next proposition characterizes the response of inflation to monetary policy shocks in the context of our heterogeneous-agent economy.

Proposition 3 (Inflation in D-HANK)**.** *Suppose $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$. The path of inflation is given by*

$$\pi_t = \sigma^{-1}\hat{\pi}_t + \chi_d \epsilon_\lambda \hat{\pi}_t + \frac{\mu_b \chi_r}{1-\mu_b}\tilde{\psi}_m \bar{d}_P \hat{\pi}_t + \kappa e^{\underline{\omega} t}\Omega_0, \tag{19}$$

cause the effect of the disaster is much larger than these fluctuations, the impact of these changes in consumption on the precautionary motive is small and can be safely ignored.

[19]Notably, the GE multiplier in period 0 is decreasing in the discounting parameter δ. This implies that the precautionary savings motive *dampens* the effect of the outside wealth effect, while positive private debt reduces the value of δ, which *increases* the effect of the outside wealth effect.

where $\hat{\pi}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_m t})}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}(i_0 - r_n)$ *and* $\hat{\pi}_0 = 0$.

Inflation can be analogously decomposed into four terms. The first three terms capture the impact of the ISE, the time-varying risk, and the inside wealth effect, while the last term captures the impact of the outside wealth effect. Because $\hat{\pi}_0 = 0$, the first three terms are initially zero. This implies that initial inflation is determined entirely by the outside wealth effect, a consequence of the forward-looking nature of the New Keynesian Phillips curve.

3.3 Outside Wealth Effects

We consider next the determination of the outside wealth effect Ω_0. The outside wealth effect depends on the path of output, inflation and the price of risk, as well as the initial price of government bonds:

$$\Omega_0 = \overline{d}_G q_{L,0} + \int_0^\infty e^{-\rho t}\left[(1-\tau)y_t + T_t + \overline{d}_G(i_t - \pi_t - r_n + r_L p_{d,t})\right]dt. \tag{20}$$

But output, inflation, the price of risk and the price of the long-term bond in turn depend on the outside wealth effect,

$$y_t = \chi\hat{y}_t + (\overline{\omega} - \delta)e^{\underline{\omega}t}\Omega_0, \qquad \pi_t = \chi\hat{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0, \tag{21}$$

$$p_{d,t} = \hat{p}_{d,t} + \chi_{p_d,\Omega}\Omega_0, \qquad q_{L,0} = \hat{q}_{L,0} + \chi_{q_L,\Omega}e^{\underline{\omega}t}\Omega_0, \tag{22}$$

where χ, $\chi_{p_d,\Omega}$ and $\chi_{q_L,\Omega}$ are constants defined in the appendix, and $\hat{p}_{d,t}$ and $\hat{q}_{L,0}$ collect the terms that are a function only of $[i_t]_0^\infty$. This simultaneity reflects the fact that spending decisions depend on the level of asset prices, as shown by (21), and that asset prices react to the level of aggregate demand, as shown by equation (22). By combining these expressions, we can express Ω_0 in terms of policy variables, that is, the path of nominal interest rates, i_t, and the fiscal backing to the monetary shock, T_t. In particular, we can express Ω_0 as follows:

$$\Omega_0 = \underbrace{(1-\epsilon_\Omega)\Omega_0}_{\substack{\text{aggregate demand}\\ \text{effect}}} + \underbrace{\overline{d}_G\hat{q}_{L,0} + \int_0^\infty e^{-\rho t}\left[T_t + \overline{d}_G(i_t - \chi\hat{\pi}_t - r_n + r_L\hat{p}_{d,t})\right]dt,}_{\text{direct effect}}$$

where ϵ_Ω is a constant defined in the appendix. The first term captures the impact of aggregate demand on the valuation of stocks, bonds, and human wealth, while the second term captures the impact of changes in monetary and fiscal variables that are not mediated by aggregate demand. Assumption 2 guarantees that outside wealth reacts less than one-to-one to aggregate demand.[20]

Assumption 2. *The parameters of the model are such that $\epsilon_\Omega \in (0, 1)$.*

The next proposition shows that the outside wealth effect can be expressed as the product of a *multiplier* and an *autonomous term*, that is, a term that does not depend directly on Ω_0.

Proposition 4. *Suppose Assumption 2 holds. The outside wealth effect is then given by*

$$\Omega_0 = \frac{1}{\epsilon_\Omega}\left[\bar{d}_G \hat{q}_{L,0} + \int_0^\infty e^{-\rho t}\left[T_t + \bar{d}_G(i_t - \chi\hat{\pi}_t - r_n + r_L \hat{p}_{d,t})\right] dt\right]. \tag{23}$$

Proposition 4 introduces an important relationship between the model-implied revaluation of assets in positive net supply, Ω_0, and the equilibrium path of policy variables. For example, expression (23) shows that, in the absence of any fiscal backing ($T_t = 0$) and government debt ($\bar{d}_G = 0$), the outside wealth effect is zero.[21] Monetary policy still affects the value of stocks and human wealth, as can be seen in (13), but the reduction in the value of households' assets is exactly offset by the reduction in the value of households' liabilities (in the form of consumption), as discussed in Section 2.3. Under Assumption 2, the aggregate demand effect cannot sustain a positive value of Ω_0 in the absence of a direct effect of policy variables.

By incorporating fiscal data into the analysis, this relationship provides a way to discipline the model's economic forces. One can estimate the fiscal response to a monetary shock in the data and introduce the estimated values into expression (23) to obtain the model's prediction for Ω_0. We follow this approach in Section 4.

[20] Assumption 2 implies that either the primary surplus or the cost of servicing the debt respond to economic activity, as captured by Ω_0, so essentially monetary policy has fiscal consequences.

[21] Adding capital does not qualitatively affect this result, as wealth effects from capital ownership are analogous to wealth effects from claims on profits. See Caramp (2021) for the details.

3.4 Implementability condition

The results in (17) and (19) express output and inflation in terms of the path of nominal interest rates and the outside wealth effect Ω_0, while equation (23) gives Ω_0 in terms of the underlying fiscal backing T_t. In combination, these results demonstrate how the policy variables (i_t, T_t) affect output and inflation. However, both the nominal interest rate and the associated fiscal backing are endogenous variables and depend on the monetary policy rule (5). The next proposition shows how we can determine the monetary rule that implements a particular equilibrium path of nominal interest rates and fiscal backing by appropriately choosing the exogenous process for the monetary shock u_t.

Proposition 5 (Implementability)**.** *Let y_t be given by (17) and π_t be given by (19), for a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $\psi_m \neq -\underline{\omega}$, and the associated fiscal backing T_t. Let $[\bar{i}_t, \overline{y}_t, \overline{\pi}_t]_0^\infty$ denote the (bounded) solution to the system comprising the Taylor rule (5), the aggregate Euler equation (9), and the New Keynesian Phillips curve (10), and suppose the monetary shock u_t is given by*

$$u_t = \vartheta e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega} t}. \tag{24}$$

Then, there exists parameters ϑ and θ such that $\bar{i}_t = i_t$, $\overline{y}_t = y_t$, and $\overline{\pi}_t = \pi_t$.

Proposition 5 shows that the process for the monetary shock uniquely pins down (i_t, T_t), so one can equivalently express the solution either in terms of equilibrium policy variable or in terms of the underlying process for u_t. The formulation in equation (24) generalizes the process for monetary shocks frequently used in the literature, where the parameter θ is usually set to zero. While ϑ simply scales the shock such that the initial nominal interest rate equals a given i_0, θ pins down the outside wealth effect Ω_0 and the underlying fiscal backing. An important feature of specification (24) is that the path of the nominal interest rate is the same for any value of θ, so the parameter θ affects only Ω_0.[22]

The extra degree of freedom given by the parameter θ will be important to discipline the outside wealth effect empirically. If we impose $\theta = 0$, we obtain

[22]Note that the sign of the effect of a monetary shock on nominal interest rates depends on ψ_m. If $\psi_m < |\underline{\omega}|$, a contractionary shock increases nominal rates, while it reduces nominal rates if $\psi_m > |\underline{\omega}|$. Thus, the nominal interest rate do not react to a monetary shock when $\psi_m = |\underline{\omega}|$.

the standard process $u_t = e^{-\psi_m t} u_0$ for some innovation u_0. This assumption determines a particular value of the fiscal backing that may be inconsistent with its empirical counterpart, which implies that the outside wealth effect implied by the model will also be counterfactual. By considering the generalized process (24), the model will be able to simultaneously match the persistence of the equilibrium interest rate and the corresponding fiscal backing.

4 The Quantitative Importance of Wealth Effects

In this section, we study the quantitative importance of wealth effects in the transmission of monetary shocks. We calibrate the model to match key unconditional and conditional moments, including asset-pricing dynamics and the fiscal response to a monetary shock. We find that household heterogeneity and time-varying risk are the predominant channels of transmission of monetary policy.

4.1 Calibration

The parameter values are chosen as follows. The discount rate of savers is chosen to match a natural interest rate of $r_n = 1\%$. We assume a Frisch elasticity of one, $\phi = 1$, and set the elasticity of substitution between intermediate goods to $\epsilon = 6$, common values adopted in the literature. The fraction of borrowers is set to $\mu_b = 30\%$, and the parameter $\overline{d}_P$ is chosen to match a household debt-to-disposable income ratio of 1 (consistent with the U.S. Financial Accounts). The parameter $\overline{d}_G$ is chosen to match a public debt-to-GDP ratio of 66%, and we assume a duration of five years, consistent with the historical average for the United States. The tax rate is set to $\tau = 0.27$ and the parameter $T_b'(Y)$ is chosen such that $\chi_y = 1$, which requires countercyclical transfers to balance the procyclical wage income. A value of $\chi_y = 1$ is consistent with the evidence in Cloyne et al. (2020) that the net income of mortgagors and non-mortgagors reacts similarly to monetary shocks. The pricing cost parameter φ is chosen such that κ coincides with its corresponding value under Calvo pricing and an average period between price adjustments of three quarters. The half-life of the monetary shock is set to three and a half months to roughly match what we estimate in the data, and we set $\phi_\pi = 1.5$.

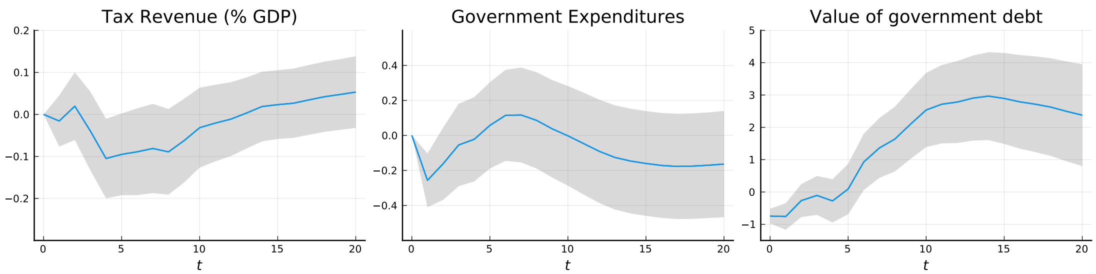

Figure 2: Estimated fiscal response to a monetary policy shock

Note: IRFs computed from a VAR identified by a recursiveness assumption, as in Christiano et al. (1999). Variables included: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, federal funds rate, 5-year constant maturity rate and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The real value of government debt and the 5-year rate are ordered last, and the fed funds rate is ordered third to last. Gray areas are bootstrapped 95% confidence bands. See Appendix D for the details.

We calibrate the disaster risk parameters in two steps. For the stationary equilibrium, we choose a calibration mostly based on the parameters adopted by Barro (2009). We set λ (the steady-state disaster intensity) to match an annual disaster probability of 1.7%, and A^* to match a drop in output of $1 - \frac{Y}{Y^*} = 0.39$.[23] The risk-aversion coefficient is set to $\sigma = 4$, a value within the range of reasonable values according to Mehra and Prescott (1985), but substantially larger than $\sigma = 1$, a value often adopted in macroeconomic models. Our calibration implies an equity premium in the stationary equilibrium of 6.1%, in line with the observed equity premium of 6.5%. Moreover, by setting $\sigma = 4$ we obtain a micro EIS of $\sigma^{-1} = 0.25$, in the ballpark of an EIS of 0.1 as recently estimated by Best et al. (2020). We discuss the calibration of ϵ_λ, which determines the elasticity of asset prices to monetary shocks, in the next subsection.

For the policy variables, we estimate a standard VAR augmented to incorporate fiscal variables and compute empirical IRFs applying the recursiveness assumption of Christiano et al. (1999). From the estimation, we obtain the path of monetary and fiscal variables: the path of the nominal interest rate, the change in the initial value of government bonds, and the path of fiscal transfers. We provide the details of the estimation in Appendix D. Figure 2 shows the dynamics of fiscal variables in the estimated VAR in response to a contractionary monetary

[23] As discussed in Barro (2006), it is not appropriate to calibrate A^*/A to the average magnitude of a disaster, given that empirically the size of a disaster is stochastic. We instead calibrate A^*/A to match $\mathbb{E}[(C_s/C_s^*)^\sigma]$ using the empirical distribution of disasters reported in Barro (2009).

shock. Government revenues fall in response to the contractionary shock, while government expenditures fall on impact and then turn positive, likely driven by the automatic stabilizer mechanisms embedded in the government accounts. The present value of interest payments increases by 69 bps and the initial value of government debt drops by 50 bps.[24] In contrast, the present value of transfers T_t drops by 12 bps.[25] Moreover, we cannot, at the 95% confidence level, reject the possibility that the present discounted value of the primary surplus does not change in response to monetary shocks and that the increase in interest payments is entirely compensated by the initial reaction in the value of government bonds.

4.2 Asset-pricing implications of time-varying risk

Recall that the price of the long-term government bond is given by

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}(i_t - r_n + r_L p_{d,t})dt,$$

where $p_{d,t} = \sigma c_{s,t} + \epsilon_\lambda(i_t - r_n)$ is the price of the disaster risk. We use this expression and calibrate ϵ_λ to match the initial response of the 5-year yield on government bonds. Consistent with Gertler and Karadi (2015) and our own estimates reported in Appendix D, we find that a 100 bps increase in the nominal interest rate leads to an increase in the 5-year yield of roughly 20 bps. This procedure leads to a calibration of ϵ_λ of 2.25, which implies an annual increase in the probability of disaster of roughly 95 bps after a 100 bps increase in the nominal interest rate. Figure 3 shows the response of the yield on the long bond and the contributions of the path of future interest rates and the term premium. We find that the bulk of the reaction of the 5-year yield reflects movements in the term premium, a finding that is consistent with the evidence.

The model is also able to capture the responses of asset prices that were not di-

[24] The present discounted value of interest payments is calculated as $\sum_{t=0}^{\mathcal{T}}\left(\frac{1-\lambda}{1+\rho_s}\right)^{\frac{t}{4}}\left[\overline{d}_t^g(\hat{i}_{L,t} - \hat{\pi}_t)\right]$, where $\mathcal{T}$ is the truncation period, $\hat{i}_{L,t}$ is the IRF of the 5-year rate estimated in the data, and $\hat{\pi}_t$ is the IRF of inflation. We choose $\mathcal{T} = 60$ quarters, when the main macroeconomic variables, including government debt, are back to their pre-shock values. Other present value calculations follow a similar logic.

[25] In the data, expenditures also include the response of government consumption and investment. When run separately, however, we cannot reject the possibility that the sum of these two components is equal to zero in response to monetary shocks.

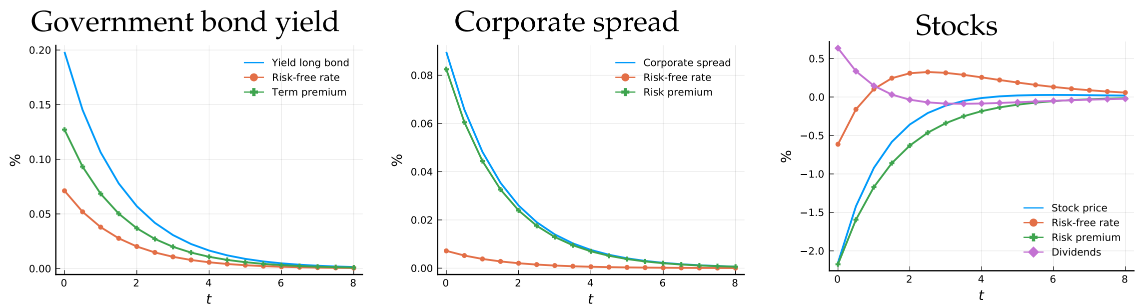

Figure 3: Asset-pricing response to monetary shocks with time-varying risk.

rectly targeted in the calibration. Consider first the response of the *corporate spread*, the difference between the yield on a corporate bond and the yield on a government bond (without risk of default) with the same promised cash flow. This corresponds to how the GZ spread is computed in the data by Gilchrist and Zakrajšek (2012). Let $e^{-\psi_F t}$ denote the coupon paid by the corporate bond. We assume that the monetary shock is too small to trigger a corporate default, but the corporate bond defaults if a disaster occurs, where lenders recover the amount $1 - \zeta_F$ in case of default. We calibrated ψ_F and ζ_F to match a duration of 6.5 years and a credit spread of 200 bps in the stationary equilibrium, which is consistent with the estimates reported by Gilchrist and Zakrajšek (2012). Note that the calibration targets the *unconditional* level of the credit spread. We evaluate the model on its ability to generate an empirically plausible *conditional* response to monetary shocks.

The price of the corporate bond can be computed analogously to the computation of the long-term government bond:

$$q_{F,0} = -\int_0^\infty e^{-(\rho+\psi_F)t}(i_t - r_n)dt - \int_0^\infty e^{-(\rho+\psi_F)t}\left[\lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F} p_{d,t}\right]dt,$$

where Q_F and Q_F^* denote the price of the corporate bond in the stationary equilibrium in the no-disaster and disaster states, respectively. Given the price of the corporate bond, we can compute the corporate spread. Figure 3 shows that the corporate spread responds to monetary shocks by 8.9 bps. We introduce the excess bond premium (EBP) in our VAR and find an increase in the EBP of 6.5 bps and an upper bound of the confidence interval of 10.9 bps, consistent with the model's prediction. Thus, even though this was not a targeted moment, time-varying risk is able to produce quantitatively plausible movements in the corporate spread.

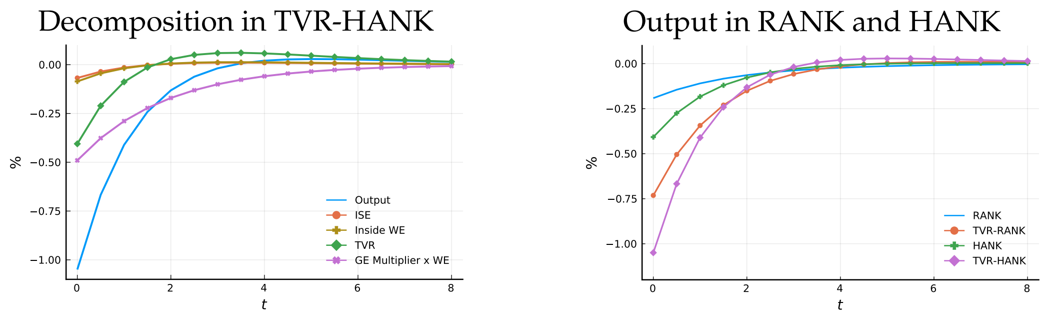

Figure 4: Output in RANK and HANK.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

Another moment that is not targeted by the calibration is the response of stocks to monetary shocks. We find a substantial response of stocks to changes in interest rates, which is explained mostly by movements in the risk premium. In contrast to the empirical evidence, we find a *positive* response of dividends to a contractionary monetary shock. This is the result of the well-known feature of sticky-prices models that profits are strongly countercyclical. This counterfactual prediction could be easily solved by introducing some form of wage stickiness. Despite the positive response of dividends, the model generates a decline in stocks of 2.15% in response to a 100 bps increase in interest rates, which is smaller than the point estimate of Bernanke and Kuttner (2005) but is still within their confidence interval.[26] Fixing the degree of countercyclicality of profits would likely bring the response of stocks closer to their point estimate.

4.3 Wealth effects in the monetary transmission mechanism

Figure 4 (left) presents the response of output and its components to a monetary shock in the New Keynesian model with heterogeneous agents and time-varying risk. We find that output reacts by -1.05% to a 100 bp increase in the nominal interest rate, which is consistent with the empirical estimates of e.g. Miranda-Agrippino and Ricco (2021). In terms of its components, time-varying risk (TVR) and the outside wealth effect are the two main components determining the output dynamics, representing 39% and 47% of the output response, respectively. In

[26]We follow standard practice in the asset-pricing literature and report the response of a levered claim on firms' profits, using a debt-to-equity ratio of 0.5, as in Barro (2006).

contrast, the ISE accounts for only 6.5% of the output response, indicating that intertemporal substitution plays only a minor role in the monetary transmission mechanism.

These findings stand in sharp contrast to the dynamics in the absence of heterogeneity and time-varying risk. Figure 4 (right) plots the response of output for different combinations of heterogeneity ($\mu_b > 0$ and $\mu_b = 0$) and time-varying risk ($\epsilon_\lambda > 0$ and $\epsilon_\lambda = 0$). By shutting down the two channels, denoted by "RANK" in the figure, the initial response of output would be -0.14%, a more than a sevenfold reduction in the impact of monetary policy. There are two reasons for this result. First, our calibration of $\sigma = 4$ implies an EIS that is one fourth of the standard calibration. This significantly reduces the quantitative importance of the ISE, even if the intertemporal substitution channel represents a large fraction of the output response in the RANK model. Second, our estimate of the fiscal response is substantially lower than the one implied by a standard Taylor equilibrium that imposes an AR(1) process for the monetary shock. We discuss the role of fiscal backing and the implications for the New Keynesian model in Section 4.5 below.

Figure 4 (right) also plots the response of output when there is household heterogeneity but not time-varying risk ("HANK" in the figure), and the response of output when there is time-varying risk but not household heterogeneity ("TVR-RANK" in the figure). We find that heterogeneity increases the response of output by 22 bps while time-varying risk increases it by 54 bps. Notably, by combining both features, we get an increase in the response of output of 86 bps, which is 10 bps larger than the sum of the individual effects. Thus, heterogeneity and time-varying risk reinforce each other. In terms of the fraction of the response of output that can be attributed to each channel, we find that 20.5% can be attributed to household heterogeneity, 51.5% corresponds to time-varying risk, and 9.7% is the amplification effect of heterogeneity together with time-varying risk (which is around 50% larger than the contribution of the ISE), while the remainder represents the channels in the RANK model.

Finally, time-varying risk is essential for properly capturing the heterogeneous response of borrowers and savers to monetary policy. Figure 5 shows that borrowers are disproportionately affected by monetary shocks. However, the magnitude of the relative response of borrowers and savers is too large in the economy without time-varying risk. The drop in borrowers' consumption is 7 times greater than

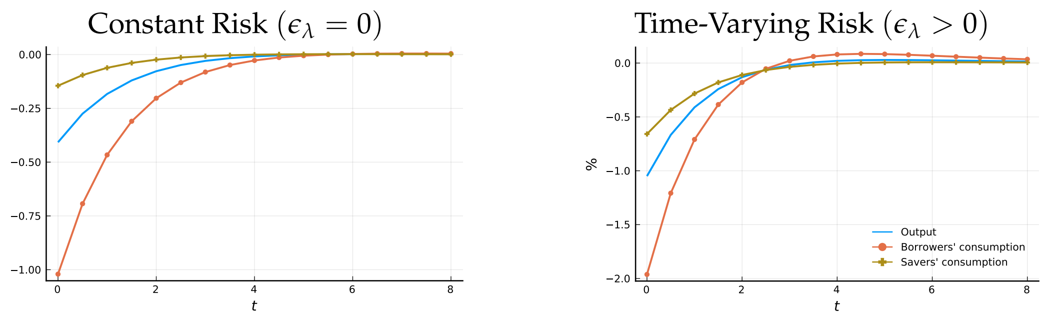

Figure 5: Consumption of borrowers and savers with constant risk and time-varying risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1.

the decline in savers' consumption with a constant disaster probability, while it is 3 times greater in the economy with time-varying risk. Cloyne et al. (2020) estimate a relative peak response of mortgagors and homeowners of roughly 3.6. Therefore, allowing for time-varying risk is also important if we want to capture the heterogeneous impact of monetary policy.

4.4 The limitations of the constant disaster risk model

Consider the response of asset prices to a monetary shock in an economy that features constant disaster risk (i.e. $\lambda > 0$ but $\epsilon_\lambda = 0$). Figure 6 (left) shows that the yield on the long bond increases by 6.5 bps, which implies a decline of the value of the bond of 32 bps (given a 5-year duration), less than half of the response estimated by the VAR in Section 4.1. Moreover, movements in the long bond yield are almost entirely explained by the path of nominal interest rates, while the term premium is indistinguishable from zero. This stands in sharp contrast to the evidence reported in Gertler and Karadi (2015) and Hanson and Stein (2015). Similarly, it can be shown that most of the response of stocks in the model is explained by movements in interest rates instead of changes in risk premia, a finding that is inconsistent with the evidence documented in e.g. Bernanke and Kuttner (2005).

Figure 6 (right) shows how the presence of constant disaster risk affects the response of output to monetary shocks for the HANK and RANK economies. We find that risk has only a minor impact on the response of output. Aggregate risk increases the value of the discounting parameter δ, which reduces the GE multi-

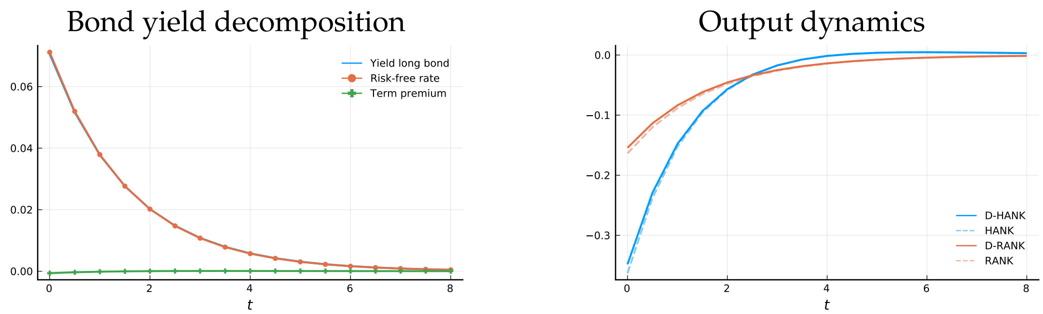

Figure 6: Long-term bond yields and output for economies with and without risk.

Note: In both plots, the path of the nominal interest rate is given by $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, where $i_0 - r_n$ equals 100 bps, and the fiscal backing corresponds to the value estimated in Section 4.1. D-HANK and D-RANK correspond to heterogeneous-agent and representative agent economies with constant disaster risk (i.e. $\lambda > 0$ and $\epsilon_\lambda = 0$). HANK and RANK correspond to economies with no disaster risk (i.e. $\lambda = 0$).

plier and dampens the initial impact of the monetary shock. Given that the term premium barely moves, disaster risk plays only a small role in determining the outside wealth effect. In contrast, the important role of heterogeneity can be seen by comparing the response of the D-HANK and D-RANK economies.

Therefore, while introducing a constant disaster probability allows the model to capture important *unconditional* asset-pricing moments, such as the (average) risk premium or the upward-sloping yield curve, the model is unable to match key *conditional* moments, in particular, the response of asset prices to monetary policy. The limitations of the model with constant disaster probability in matching conditional asset-pricing moments were recognized early on in the literature, leading to an assessment of the implications of *time-varying disaster risk*, as in Gabaix (2012) and Gourio (2012). This justifies our focus on time-varying disaster risk and how it affects the asset-pricing response to monetary shocks and, ultimately, its impact on real economic variables.

4.5 The role of fiscal backing and the EIS

We have found that time-varying risk and heterogeneity substantially amplify the impact of monetary policy on the economy. To properly assess the importance of these two channels, however, it was crucial to control for the implicit fiscal backing, as discussed in Section 3.3.

Figure 7 illustrates this point. In the three panels, we show the impact of a

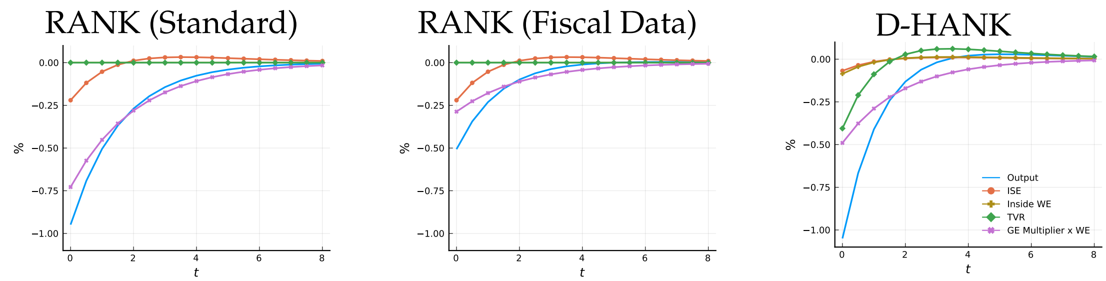

Figure 7: Output in RANK vs D-HANK with time-varying risk.

Note: The first two panels show output in RANK ($\mu_b = \lambda = 0$) with unit EIS ($\sigma^{-1} = 1$). In the left panel, fiscal backing is determined by a Taylor rule, while in the middle panel fiscal backing corresponds to the value estimated in the data. The right panel corresponds to the D-HANK economy with time-varying risk and the estimated fiscal backing.

monetary shock that leads to an increase in nominal interest rates on impact of 100 bps. In the left panel, we consider a RANK economy ($\mu_b = \lambda = 0$) with the standard value for the EIS ($\sigma^{-1} = 1$) and fiscal backing implicitly determined by a Taylor rule with a monetary shock that follows a standard AR(1) process, corresponding to the textbook New Keynesian model. In the middle panel, we consider the same economy but the fiscal backing is set to the value estimated in the data, corresponding to a Taylor equilibrium with a monetary shock that follows the more general specification from equation (24). The right panel shows our D-HANK model with time-varying risk and the calibrated value of the EIS, $\sigma^{-1} = 0.25$.

The response of the textbook economy is only slightly smaller than that of our D-HANK economy despite the lack of time-varying risk or heterogeneous agents. An important reason for this is the difference in the value of the (implicit) fiscal backing, which is almost ten times larger in the textbook economy compared with the one we estimated in the data. When the fiscal backing is the same as in the data, the response of output drops by almost half. The EIS also plays an important role. Even with fiscal backing directly from the data, the response of output is still significant, only slightly less than that in our D-RANK with time-varying risk (see Figure 4). But this same response comes from very different channels. In the RANK economy, the ISE accounts for roughly 40% of the output response, while in our D-RANK the ISE accounts for less than 7% of that response.

These results suggest that the quantitative success of the RANK model is likely the result of a counterfactually large fiscal backing in response to monetary shocks and a strong intertemporal-substitution channel, which compensate for missing heterogeneous agents and risk channels. Once we discipline the fiscal backing with data and calibrate the EIS to the estimates obtained from microdata, our model suggests that heterogeneous agents and, in particular, time-varying risk are crucial for generating quantitatively plausible output dynamics. However, it is important to note that our model made several simplifications to incorporate indebted agents and time-varying aggregate risk without sacrificing the tractability of standard macro models. A natural extension would be to incorporate these channels into a medium-sized DSGE model to better assess the quantitative properties of the New Keynesian model.

5 The Effect of Risk and Maturity of Household Debt

We have assumed so far that households borrow using short-term riskless debt. In practice, however, most household debt takes the form of long-term risky debt. In this case, the effect of monetary policy on borrowers depends on how the term spread and credit spread, the compensation for holding interest rate and default risk, respond to changes in the short-term interest rate. In this section, we extend the baseline model to allow for default risk and long-term maturities on household debt and show how these two features affect the transmission of monetary policy shocks to the real economy.

5.1 The model with long-term risky household debt

We describe next the model with long-term risky household debt. We highlight the main differences with the model described in Section 2 and present a detailed description in Appendix C. Households issue long-term debt that promises to pay exponentially decaying coupons given by $e^{-\psi_P t}$ at period $t \geq 0$, where $\psi_P \geq 0$. Importantly, households cannot commit to always repay their debts. In response to a large shock, i.e. the occurrence of a disaster, households default and lenders receive a fraction $1 - \zeta_P$ of the promised coupons, where $0 \leq \zeta_P \leq 1$. We assume that fluctuations in the no-disaster state are small enough such that they do not trigger a default. Thus, households default only in the disaster state.

We denote the price of household debt in the no-disaster (disaster) state by $Q_{P,t}$ ($Q^*_{P,t}$), so the nominal return on household debt is given by

$$dR_{P,t} = \left[\frac{1}{Q_{P,t}} + \frac{\dot{Q}_{P,t}}{Q_{P,t}} - \psi_P\right] dt + \frac{Q^*_{P,t} - Q_{P,t}}{Q_{P,t}} d\mathcal{N}_t,$$

where $i_{P,t} \equiv \frac{1}{Q_{P,t}} - \psi_P$ is the yield on the bond. In a stationary equilibrium, the spread between the interest rate on household debt and the short-term interest rate controlled by the central bank is given by

$$r_P = \lambda \left(\frac{C_s}{C^*_s}\right)^{\sigma} \frac{Q^*_P - Q_P}{Q_P}.$$

Note that the interest rate on household debt incorporates both a credit and a term spread.[27] We assume that households can borrow up to $\overline{D}_{P,t} = Q_{P,t}\overline{F}$, which effectively puts a limit on the face value of household debt $\overline{F}$.[28] In a log-linear approximation of the economy around a zero-inflation stationary equilibrium, borrowers are constrained at all periods, and their consumption is given by

$$c_{b,t} = (1-\alpha)(w_t - p_t + n_{b,t}) + T_{b,t} - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P. \tag{25}$$

Equation (25) generalizes the expression for borrowers' consumption given in Section 2. Monetary policy affects borrowers indirectly through its effect on the yield on household debt $i_{P,t}$. If we assume that debt is short-term, $\psi_P \to \infty$, and riskless, $\zeta_P = 0$, we obtain $i_{P,t} = i_t$ and the expression above boils down to equation (6). At the other extreme, we have the case of a perpetuity, $\psi_P = 0$. In this case, households simply pay the coupon every period and there is no need to issue new debt. Therefore, they are completely insulated from movements in nominal interest rates.

[27] Let $i^{ND}_{P,t}$ denote the yield on a non-defaultable bond with coupons decaying at rate ψ_p. The term spread corresponds to $i^{ND}_{P,t} - i_t$ and the credit spread to $i_{P,t} - i^{ND}_{P,t}$, so $r_P = (i^{ND}_P - r_n) + (i_P - i^{ND}_P)$.

[28] This formulation guarantees that, after an increase in nominal rates, the value of household debt and the borrowing limit decline by the same amount. This specification of the borrowing constraint, combined with the assumption of impatient borrowers, guarantees that borrowers are constrained at all periods.

The price of household debt evolves according to

$$\dot{q}_{P,t} = (\rho + \psi_P) q_{P,t} + i_t - r_n + r_P p_{d,t}.$$

The price of the bond depends on the future path of short-term interest rates and the risk premium. Quantitatively, the fluctuations in the risk premium are dominated by the time-varying risk component, while the term $\sigma c_{s,t}$ gives a negligible contribution, as shown in Figure 6. Motivated by this fact, we assume that $r_P \sigma c_{s,t}$ is negligible in a first-order approximation, such that we can write the price of the bond as follows[29]

$$q_{P,t} = -\frac{1 + r_P \epsilon_\lambda}{\rho + \psi_P + \psi_m}(i_t - r_n), \tag{26}$$

where ψ_m is the decaying rate of nominal interest rates.

Combining the behavior of borrowers' consumption with savers' Euler equation (8) and the Phillips curve (10), we can derive the response of aggregate output to monetary shocks. In particular, we can extend the decomposition in Proposition 2 to the case of long-term risky debt.

Proposition 6 (Aggregate output with long-term risky debt)**.** *Suppose that $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and $r_P \sigma = \mathcal{O}(i_0 - r_n)$. The path of aggregate output is then given by*

$$y_t = \underbrace{\sigma^{-1}\hat{y}_t}_{\textit{ISE}} + \underbrace{\chi_d \epsilon_\lambda \hat{y}_t}_{\substack{\textit{time-varying}\\ \textit{risk}}} + \underbrace{\frac{\mu_b \chi_r \overline{d}_P}{1-\mu_b}\frac{\psi_P(1 + r_P\epsilon_\lambda)}{\rho + \psi_P + \psi_m}\tilde{\psi}_m \hat{y}_t}_{\textit{inside wealth effect}} + \underbrace{(\rho - \underline{\omega})e^{\underline{\omega} t}\Omega_0,}_{\substack{\textit{GE multiplier}\times\\ \textit{outside wealth effect}}}. \tag{27}$$

Proposition 6 shows that default risk and debt maturity have opposite effects on the magnitude of the inside wealth effect. For instance, in the case of short-term debt, the term $\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}$ simplifies to $1 + r_P\epsilon_\lambda > 1$, so the inside wealth effect is amplified relative to the case of riskless debt. The interest rate on household debt now moves in response to changes in the short-term interest rate as well as changes in the risk premium. In contrast, the inside wealth effect is dampened for long-term bonds. In the limit case of a perpetuity, $\psi_P = 0$, the inside wealth goes to zero. Given that households do not issue new debt, they are not affected by the

[29] Formally, we assume that the parameter $r_P\sigma$ is of the same order as the (small) monetary shock, $r_P\sigma = \mathcal{O}(i_0 - r_n)$. Therefore, the term $r_P\sigma c_{s,t}$ is second-order in $i_0 - r_n$ and it can be ignored in a first-order approximation. The solution in the absence of this assumption is available upon request.

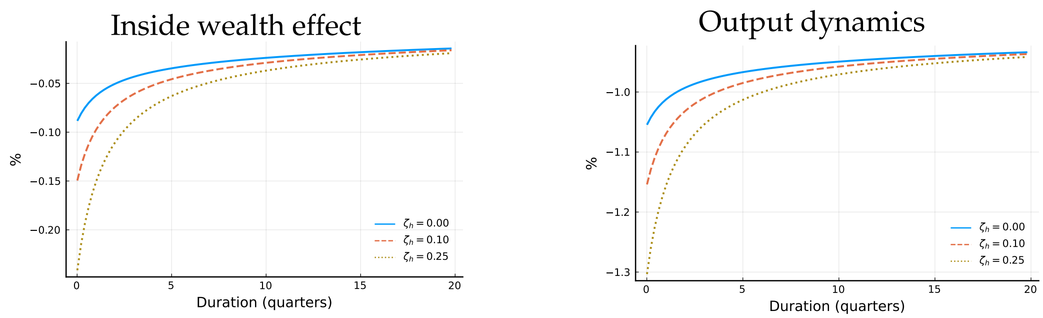

Figure 8: Inside wealth effect and output as a function of duration and haircut ζ_P.

change in interest rates, which eliminates the (inside) wealth effect.

5.2 Quantitative implications

We consider next the quantitative implications of default risk and maturity on household debt. As shown in Proposition 6, these two features have opposing effects on the response of output to monetary policy. To assess the quantitative impact of risk and maturity, we show in Figure 8 the inside wealth effect (left panel) and aggregate output (right panel) as a function of the duration of household debt for different values of the haircut parameter ζ_P. Greenwald et al. (2021) estimate the duration of mortgage debt as 5.2 years, the duration of student debt as 4.50, and the duration of consumer debt as 1.0 year. Therefore, we focus on values of duration up to five years in Figure 8. We consider three different values for the haircut parameter: riskless debt ($\zeta_P = 0$); risky debt with a spread in the stationary equilibrium of roughly 4.0% with a 5-year duration ($\zeta_P = 0.10$); risky debt with a spread of 5.0% with a 5-year duration ($\zeta_P = 0.25$).

Default risk substantially amplifies the effect of monetary policy on output when debt is short term. The inside wealth effect is almost three times larger in the case of $\zeta_P = 0.25$ compared to $\zeta_P = 0.0$, which corresponds to an increase in the initial response of output of almost 25%. However, this effect is strongly attenuated when household debt is long term. For even relatively small values of duration, the inside wealth effect is smaller than in the case of short-term riskless debt. For instance, in the case of a five-year duration, the response of output is roughly 10% smaller than the response in the case of short-term riskless debt.

It is important to note that even though the inside wealth effect is substantially dampened with long-term debt, the presence of household debt still generates amplification through its impact on the compounding parameter δ, as shown in Proposition 1. The response of output when household debt is zero is roughly 35% smaller than in the economy with (positive) riskless debt, a much larger drop relative to the one caused by introducing long-term bonds.

6 Conclusion

In this paper, we provide a novel unified framework to analyze the role of heterogeneity and risk in a tractable linearized New Keynesian model. The methods introduced can be applied beyond the current model. For instance, they can be applied to a full quantitative HANK model with idiosyncratic risk, extending the results of Ahn et al. (2018) to allow for time-varying risk premia. Alternatively, one could introduce a richer capital structure for firms and study the pass-through of monetary policy to households and firms. These methods may enable us to bridge the gap between the extensive existing work on heterogeneous agents and monetary policy and the emerging literature on the role of asset prices in the transmission of monetary shocks.

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Appendix: For Online Publication

A Proofs

Proof of Proposition 1. Consider first the New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t \frac{\eta_t^*}{\eta_t}\right)\pi_t - (\epsilon - 1)\varphi^{-1}\left(\frac{\epsilon}{\epsilon - 1}\frac{W}{PA}e^{w_t - p_t} - (1-\tau)\right)Ye^{y_t}.$$

Linearizing the above expression, and using $\frac{\epsilon}{\epsilon-1}\frac{W}{P}\frac{1}{A} = (1-\tau)$, we obtain

$$\dot{\pi}_t = \left(r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\right)\pi_t - \varphi^{-1}(\epsilon - 1)(1-\tau)(w_t - p_t).$$

Using the expression for $w_t - p_t$, we obtain $\dot{\pi}_t = (\rho_s + \lambda)\pi_t - \kappa y_t$, where $\kappa \equiv \varphi^{-1}(\epsilon - 1)(1-\tau)(\phi+\sigma)Y$ and we used that $r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} = \rho_s + \lambda$.

Consider next the generalized Euler equation. From the market-clearing condition for goods and borrowers' consumption, we obtain $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \pi_t - r_n)$. Combining this condition with the Phillips Curve and savers' Euler equation, and using the fact that $r_n = \rho - \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}$, we obtain $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi - r_n) + \delta y_t + v_t$, where the constants $\tilde{\sigma}^{-1}$, δ, and v_t are defined in the proposition.

□

Proof of Lemma 1. First, note that from the intertemporal budget constraint in the stationary equilibrium, we have $\frac{Q_Y}{Y} - \frac{Q_C}{Y} = \overline{d}_G$. Next, we have

$$r_Y\frac{Q_Y}{Y} - r_C\frac{Q_C}{Y} = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{(Q_Y - Q_Y^*) - (Q_C - Q_C^*)}{Y} = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{Q_L - Q_L^*}{Q_L}\overline{d}_G = r_L\overline{d}_G,$$

where $Q_L = \frac{1}{i_L+\psi_L}$ and $Q_L^* = \frac{1}{i_L^*+\psi_L}$. Then, $r_L = \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}\frac{r_n^* - r_n}{r_n^*+\psi_L+\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma}} \to 0$ as $\psi_L \to \infty$.

□

Proof of Lemma 2. Suppose $[y_t, \pi_t]_0^\infty$ satisfies system (16) and the intertemporal budget constraint (14) in the no-disaster state. We will show that $[y_t, \pi_t]_0^\infty$ can be supported as an equilibrium. Consider first the disaster state. The savers' budget constraint implies $T_{s,t} = -\rho_s b_s b_{s,t^*}$. All the remaining variables are equal to zero in the disaster state.

Consider now the no-disaster state. The real wage is given by $w_t - p_t = (\phi+\sigma)y_t$. Borrowers' consumption is given by $c_{b,t} = \chi_y y_t - \chi_r\overline{d}_P(i_t - \pi_t - r - n)$, while savers' consumption are given by $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \pi_t - r_n)$, and the labor supply is given by $n_{j,t} = \phi^{-1}(w_t - p_t) - \phi^{-1}\sigma c_{j,t}$.

By construction, the market-clearing condition for goods and labor are all satisfied. Because y_t satisfies the aggregate Euler equation, the savers' Euler equation is also satisfied. Because π_t satisfies the New Keynesian Phillips curve, the optimality condition for firms is satisfied. Bond holdings by savers and government debt evolve according to

$$b_s \dot{b}_{s,t} = r_n b_s b_{s,t} + (1-\alpha)(w_t - p_t + n_{s,t}) + T_{s,t} + \frac{(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)}{1-\mu_b} + \\ (i - \pi_t - r_n)b_s + (r_{L,t} - r_L)b_s^L + r_L b_s^L b_{s,t}^L - c_{s,t},$$

$$\overline{d}_G \dot{d}_{G,t} = \overline{d}_G(r_n + r_L)d_{G,t} + T_t - \tau y_t + (i_t - \pi_t - r_n + r_{L,t} - r_L)\overline{d}_G,$$

where $b_{s,0} = \frac{b_s^L}{b_s} q_{L,0}$ and $d_{G,0} = \overline{d}_G q_{L,0}$. The value of $c_{b,t}$ is such that the flow budget constraint for borrowers also holds.

Aggregating the budget constraint of borrowers and savers and using the market clearing condition for goods and labor, we obtain

$$(1-\mu_b)b_s \dot{b}_{s,t} = r_n(1-\mu_b)b_s b_{s,t} + T_t - \tau y_t + (i - \pi_t - r_n)\left[(1-\mu_b)b_s - \mu_b \overline{d}_P\right] + \\ (r_{L,t} - r_L + r_L b_{s,t}^L)(1-\mu_b)b_s^L.$$

Note that $b_s b_{s,t} = b_s^S b_{s,t}^S + b_s^L b_{s,t}^L$. We set $b_{s,t}^S = 0$, so the market for short-term bonds clear at all periods. It remains to show that the market for long-term bonds also clears. Subtracting the government's flow budget constraint from the condition above, we obtain

$$(1-\mu_b)b_s^L \dot{b}_{s,t}^L - \overline{d}_G \dot{d}_{G,t} = (r_n + r_L)((1-\mu_b)b_s^L b_{s,t}^L - \overline{d}_G d_{G,t}),$$

using $b_s b_{s,t} = b_{s,t}^L b_{s,t}^L$ and $(1-\mu_b)b_s - \mu_b \overline{d}_P = (1-\mu_b)b_s^L = \overline{d}_G$. Integrating this expression, we obtain $(1-\mu_b)b_s^L b_{s,t}^L - \overline{d}_G d_{G,t} = e^{(r_n + r_L)t}\left[(1-\mu_b)b_s^L b_{s,0}^L - \overline{d}_G d_{G,0}\right] = 0$, where the equality uses the market clearing condition in period 0. Therefore, the market clearing condition for long-term bonds is satisfied in all periods. The only condition that remains to be checked is the No-Ponzi condition for the government or, equivalently, the aggregate intertemporal budget constraint. Because condition (14) is satisfied, the No-Ponzi condition for the government is also satisfied. □

Proof of Propositions 2 and 3. We can write dynamic system (16) in matrix form as $\dot{Z}_t = AZ_t + B\nu_t$, where $B = [1, 0]'$. Applying the spectral decomposition to matrix A, we obtain $A = V\Omega V^{-1}$ where $V = \begin{bmatrix} \frac{\rho - \overline{\omega}}{\kappa} & \frac{\rho - \underline{\omega}}{\kappa} \\ 1 & 1 \end{bmatrix}$, $V^{-1} = \frac{\kappa}{\overline{\omega} - \underline{\omega}} \begin{bmatrix} -1 & \frac{\rho - \underline{\omega}}{\kappa} \\ 1 & -\frac{\rho - \overline{\omega}}{\kappa} \end{bmatrix}$, and $\Omega = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix}$.

Decoupling the system, we obtain $\dot{z}_t = \Omega z_t + b\nu_t$, where $z_t = V^{-1}Z_t$ and $b = V^{-1}B$.

Solving the equation with a positive eigenvalue forward and the one with a negative eigenvalue backward, and rotating the system back to the original coordinates, we obtain

$$y_t = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{11}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{12}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz$$
$$\pi_t = V_{22}\left(V^{21}y_0 + V^{22}\pi_0\right)e^{\underline{\omega}t} - V_{21}V^{11}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + V_{22}V^{21}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz,$$

where $V^{i,j}$ is the (i,j) entry of matrix V^{-1}. Integrating $e^{-\rho t}y_t$ and using the intertemporal budget constraint,

$$\Omega_0 = V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right)\frac{1}{\rho-\underline{\omega}} - \frac{1}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt + \frac{1}{\rho-\underline{\omega}}V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt.$$

Rearranging the above expression, we obtain

$$V_{12}\left(V^{21}y_0 + V^{22}\pi_0\right) = (\rho-\underline{\omega})\Omega_0 + \frac{\rho-\underline{\omega}}{\rho-\overline{\omega}}V_{11}V^{11}\int_0^\infty\left(e^{-\overline{\omega}t} - e^{-\rho t}\right)\nu_t dt - V_{12}V^{21}\int_0^\infty e^{-\rho t}\nu_t dt.$$

Output is then given by $y_t = \tilde{y}_t + (\rho-\underline{\omega})e^{\underline{\omega}t}\Omega_0$, where $\tilde{y}_t = -\frac{\overline{\omega}-\rho}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\overline{\omega}-\delta}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\rho-\underline{\omega}}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$. Inflation is given by $\pi_t = \tilde{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\tilde{\pi}_t = \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_t^\infty e^{-\overline{\omega}(z-t)}\nu_z dz + \frac{\kappa}{\overline{\omega}-\underline{\omega}}\int_0^t e^{\underline{\omega}(t-z)}\nu_z dz - \frac{\kappa}{\overline{\omega}-\underline{\omega}}e^{\underline{\omega}t}\int_0^\infty e^{-\overline{\omega}z}\nu_z dz$.

If $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, then $\dot{i}_t = -\psi_m(i_t - r_n)$. This allows us to write $\tilde{y}_t = \sigma^{-1}\hat{y}_t + \chi_p\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r\overline{d}_p}{1-\mu_b}\tilde{\psi}_m\hat{y}_t$ and $\tilde{\pi}_t = \sigma^{-1}\hat{\pi}_t + \chi_p\epsilon_\lambda\hat{\pi}_t + \frac{\mu_b\chi_r\overline{d}_p}{1-\mu_b}\tilde{\psi}_m\hat{\pi}_t$, where $\tilde{\psi}_m \equiv \rho - r_n + \psi_m$, $\hat{y}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\left[-\frac{\psi_m+\rho}{(\psi_m+\underline{\omega})(\psi_m+\overline{\omega})}e^{-\psi_m t} + \frac{\rho-\underline{\omega}}{(\psi_m+\overline{\omega})(\psi_m+\underline{\omega})}e^{\underline{\omega}t}\right](i_0 - r_n)$, and $\hat{\pi}_t = \frac{1-\mu_b}{1-\mu_b\chi_y}\frac{\kappa(e^{\underline{\omega}t}-e^{-\psi_m t})}{(\underline{\omega}+\psi_m)(\overline{\omega}+\psi_m)}(i_0 - r_n)$. Note that $\int_0^\infty e^{-\rho t}\hat{y}_t dt = 0$, and $\frac{\partial\hat{y}_0}{\partial i_0} = -\frac{1-\mu_b}{1-\mu_b\chi_y}\frac{1}{\psi_m+\overline{\omega}} < 0$. Moreover, $\hat{\pi}_0 = 0$.

□

Proof of Proposition 4. From Propositions 2 and 3, we have $y_t = \chi\hat{y}_t + (\overline{\omega}-\delta)e^{\underline{\omega}t}\Omega_0$, and $\pi_t = \chi\hat{\pi}_t + \kappa e^{\underline{\omega}t}\Omega_0$, where $\chi \equiv \sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\tilde{\psi}_m\overline{d}_P$. Moreover, we can rewrite the price of disaster risk as $p_{d,t} = \hat{p}_{d,t} + \chi_{p_d,\Omega}e^{\underline{\omega}t}\Omega_0$, where $\hat{p}_{d,t} \equiv \sigma\frac{1-\mu_b\chi_y}{1-\mu_b}\chi\hat{y}_t + \sigma\frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \chi\hat{\pi}_t - r_n) + \epsilon_\lambda(i_t - r_n)$ and $\chi_{p_d,\Omega} \equiv \sigma\left(\frac{1-\mu_b\chi_y}{1-\mu_b}(\rho-\underline{\omega}) - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\kappa\right)$. Finally, the price of the long-term bond in period 0 can be written as $q_{L,0} = \hat{q}_{L,0} + \chi_{q_L,\Omega}\Omega_0$, where $\hat{q}_{L,0} \equiv -\int_0^\infty e^{-(\rho+\psi_L)t}\left[i_t - r_n + r_L\hat{p}_{d,t}\right]$ and $\chi_{q_L,\Omega} \equiv -\frac{r_L}{\rho+\psi_L-\underline{\omega}}\chi_{p_d,\Omega}$. Introducing these expressions into (14), and using that $\int_0^\infty e^{-\rho t}y_t dt = \Omega_0$, we obtain

$$\Omega_0 = (1-\chi_\Omega)\Omega_0 + \overline{d}_G\hat{q}_{L,0} + \int_0^\infty e^{-\rho t}\left[T_t + \overline{d}_G(i_t - \chi\hat{\pi}_t - r_n + r_L\hat{p}_{d,t})\right]dt,$$

where $\chi_\Omega \equiv \tau + \frac{\kappa \bar{d}_G}{\rho - \underline{\omega}} - \frac{\psi_L r_L \bar{d}_G}{(\rho-\underline{\omega})(\rho+\psi_L-\underline{\omega})}\chi_{p_d,\Omega}$.

$\square$

Proof of Proposition 5. We divide this proof in three steps. First, we derive the condition for local uniqueness of the solution under the policy rule (5). Second, we derive the path of $[y_t, \pi_t, i_t]_0^\infty$ for a given path of monetary shocks. Third, we show how to implement a given path of nominal interest rates $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$ and a given value of Ω_0, which maps to a given value of fiscal backing $\int_0^\infty e^{-\rho t} T_t dt$.

Equilibrium determinacy. First, note that we can write ν_t as $\nu_t = \tilde{\sigma}^{-1}\left[1 + \frac{1-\mu_b}{1-\mu_b\chi_y}\tilde{\sigma}\chi_d\epsilon_\lambda\right]\phi_\pi\pi_t + \frac{\mu_b\chi_r\bar{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa y_t + \tilde{\nu}_t$, where $\tilde{\nu}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_d\epsilon_\lambda\right]u_t - \frac{\mu_b\chi_r\bar{d}_P}{1-\mu_b\chi_y}((r_n-\rho)u_t + \dot{u}_t)$.

The dynamic system for y_t and π_t can now be written as

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi) \\ -\kappa & \rho \end{bmatrix}\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\tilde{\nu}_t,$$

where $\tilde{\delta} \equiv \delta + \frac{\mu_b\chi_r\bar{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa$ and $\tilde{\phi}_\pi \equiv \left[1 + \frac{1-\mu_b}{1-\mu_b\chi_y}\tilde{\sigma}\chi_d\epsilon_\lambda\right]\phi_\pi$. The eigenvalues of the system are given by $\overline{\omega}_T = \frac{\rho+\tilde{\delta}+\sqrt{(\rho+\tilde{\delta})^2+4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa-\rho\tilde{\delta})}}{2}$ and $\underline{\omega}_T = \frac{\rho+\tilde{\delta}-\sqrt{(\rho+\tilde{\delta})^2+4(\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa-\rho\tilde{\delta})}}{2}$. The system has a unique bounded solution if both eigenvalues have positive real parts. A necessary condition for the eigenvalues to have positive real parts is

$$\rho + \delta + \frac{\mu_b\chi_r\bar{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa > 0 \iff \phi_\pi > -(\rho+\delta)\left(\frac{\mu_b\chi_r\bar{d}_P\kappa}{1-\mu_b\chi_y}\right)^{-1} = 1 - \left(\rho + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\right)\frac{1-\mu_b\chi_y}{\mu_b\chi_r\bar{d}_P\kappa}.$$

If the condition above is violated, then the real part of $\underline{\omega}_T$ is negative. Another necessary condition for the eigenvalues to have positive real parts is

$$\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi)\kappa < \rho\left[\delta + \frac{\mu_b\chi_r\bar{d}_P}{1-\mu_b\chi_y}\phi_\pi\kappa\right] \iff \phi_\pi > 1 - \frac{\chi_d\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_d\epsilon_\lambda + \sigma^{-1}}.$$

If this condition is violated, then the eigenvalues are real-valued and $\underline{\omega}_T < 0$. This establishes the necessity of the condition

$$\phi_\pi > \max\left\{1 - \frac{\chi_d\epsilon_\lambda + \frac{\rho\lambda}{\kappa}\frac{1-\mu_b\chi_y}{1-\mu_b}\left(\frac{C_s}{C_s^*}\right)^\sigma}{\chi_d\epsilon_\lambda + \sigma^{-1}}, 1 - \left(\rho + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma\right)\frac{1-\mu_b\chi_y}{\mu_b\chi_r\bar{d}_P\kappa}\right\}.$$

If $\mu_b\chi_y < 1$, then $\phi_\pi > 1$ is sufficient to guarantee the local uniqueness of the solution.

Solution to the dynamic system. The dynamic system for $[y_t, \pi_t]_{t=0}^{\infty}$ is given by

$$\begin{bmatrix} \dot{y}_t \\ \dot{\pi}_t \end{bmatrix} = \begin{bmatrix} \tilde{\delta} & -\tilde{\sigma}^{-1}(1-\tilde{\phi}_\pi) \\ -\kappa & \rho \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tilde{v}_t.$$

In matrix form, the system is given by $\dot{\tilde{Z}}_t = \tilde{A}\tilde{Z}_t + B\tilde{v}_t$, where $B = [1,0]'$. Applying the spectral decomposition to matrix $\tilde{A}$, we obtain $\tilde{A} = \tilde{V}\Omega_T\tilde{V}^{-1}$ where $\tilde{V} = \begin{bmatrix} \frac{\rho-\overline{\omega}_T}{\kappa} & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & 1 \end{bmatrix}$, $\tilde{V}^{-1} = \frac{\kappa}{\overline{\omega}_T-\underline{\omega}_T}\begin{bmatrix} -1 & \frac{\rho-\underline{\omega}_T}{\kappa} \\ 1 & -\frac{\rho-\overline{\omega}_T}{\kappa} \end{bmatrix}$, and $\Omega_T = \begin{bmatrix} \overline{\omega}_T & 0 \\ 0 & \underline{\omega}_T \end{bmatrix}$. Decoupling the system, we obtain $\dot{\tilde{z}}_t = \Omega_T\tilde{z}_t + \tilde{b}\tilde{v}_t$, where $\tilde{z}_t = \tilde{V}^{-1}\tilde{Z}_t$ and $\tilde{b} = \tilde{V}^{-1}B$. Solving the system forward, and rotating the system back to the original coordinates, we obtain

$$\begin{aligned} y_t &= -\tilde{V}_{11}\tilde{V}^{11}\int_t^{\infty} e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz - \tilde{V}_{12}\tilde{V}^{21}\int_t^{\infty} e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz \\ \pi_t &= -\tilde{V}_{21}\tilde{V}^{11}\int_t^{\infty} e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz - \tilde{V}_{22}\tilde{V}^{21}\int_t^{\infty} e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz. \end{aligned}$$

We rewrite the above expression as follows:

$$\begin{aligned} y_t &= -\frac{\overline{\omega}_T-\rho}{\overline{\omega}_T-\underline{\omega}_T}\int_t^{\infty} e^{-\overline{\omega}_T(z-t)}\tilde{v}_z dz + \frac{\underline{\omega}_T-\rho}{\overline{\omega}_T-\underline{\omega}_T}\int_t^{\infty} e^{-\underline{\omega}_T(z-t)}\tilde{v}_z dz \\ \pi_t &= -\frac{\kappa}{\overline{\omega}_T-\underline{\omega}_T}\int_t^{\infty}\left(e^{-\underline{\omega}_T(z-t)} - e^{-\overline{\omega}_T(z-t)}\right)\tilde{v}_z dz, \end{aligned}$$

where $\tilde{v}_t \equiv \left[\frac{1-\mu_b}{1-\mu_b\chi_y}\sigma^{-1} + \frac{1-\mu_b}{1-\mu_b\chi_y}\chi_d\epsilon_\lambda\right]u_t - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}((r_n-\rho)u_t + \dot{u}_t)$. Using that $u_t = e^{-\psi_m t}u_0$, we obtain

$$\begin{aligned} y_t &= -\frac{\rho+\psi_m}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_t \\ \pi_t &= -\frac{\kappa}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_t, \end{aligned}$$

where $(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m) = \tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi-1) + (\tilde{\delta}+\psi_m)(\rho+\psi_m) > 0$.

The wealth effect is given by $\Omega_0 = -\frac{1}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)u_0$.

The nominal interest rate is given by $i_t = r_n + \frac{(\delta+\psi_m)(\rho+\psi_m)-\tilde{\sigma}^{-1}\kappa}{(\tilde{\delta}+\psi_m)(\rho+\psi_m)+\tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi-1)}u_t$. Note that if $\psi_m = -\underline{\omega} > 0$, then $i_t - r_n$, using the fact that $\overline{\omega}\underline{\omega} = \rho\delta - \tilde{\sigma}^{-1}\kappa$ and $\overline{\omega} + \underline{\omega} = \rho + \delta$. Despite the zero interest rate, the impact on output and inflation is non-zero. In particular, the outside wealth effect is given by $\Omega_0 = -\frac{1}{(\overline{\omega}_T-\underline{\omega})(\underline{\omega}_T-\underline{\omega})}\left(\frac{1-\mu_b}{1-\mu_b\chi_y}(\sigma^{-1}+\chi_d\epsilon_\lambda) + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}(\rho - r_n - \underline{\omega})\right)u_0$.

Implementability condition. Suppose $u_t = \vartheta e^{-\psi_m t}(i_0 - r_n) + \theta e^{\underline{\omega} t}$ and denote by $(\bar{i}_t, \overline{y}_t, \overline{\pi}_t)$ the value of the nominal interest rate, output, and inflation under the Taylor rule. Given the linearity of the system, the solution will be sum of the solutions for $u_{1,t} = \vartheta e^{-\psi_m t}(i_0 - r_n)$ and $u_{2,t} = \theta e^{\underline{\omega} t}$. After some algebra, we get that the nominal interest rate is given by $\bar{i}_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, using the fact that the nominal interest rate is zero under $u_{2,t}$, and choosing $\vartheta = \frac{(\tilde{\delta}+\psi_m)(\rho+\psi_m)+\tilde{\sigma}^{-1}\kappa(\tilde{\phi}_\pi - 1)}{(\delta+\psi_m)(\rho+\psi_m)-\tilde{\sigma}^{-1}\kappa} = \frac{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}{(\overline{\omega}+\psi_m)(\underline{\omega}+\psi_m)}$.

The outside wealth effect, $\overline{\Omega}_0 = \int_0^\infty e^{-\rho t}\overline{y}_t dt$, is given by

$$\overline{\Omega}_0 = -\frac{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\right)}{(\overline{\omega}_T + \psi_m)(\underline{\omega}_T + \psi_m)}\vartheta(i_0 - r_n) - \frac{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b\chi_y}(\rho - r_n - \underline{\omega})\right)}{(\overline{\omega}_T - \underline{\omega})(\underline{\omega}_T - \underline{\omega})}\theta$$

To implement a $\overline{\Omega}_0 = \Omega_0$, we must choose $\theta = \frac{(\overline{\omega}_T + |\underline{\omega}|)(\underline{\omega}_T + |\underline{\omega}|)}{\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1}+\chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(\rho - r_n + |\underline{\omega}|)\right)}(\Omega_0^{AR(1)} - \Omega_0)$, where $\Omega_0^{AR(1)} = -\frac{1}{(\overline{\omega}_T+\psi_m)(\underline{\omega}_T+\psi_m)}\frac{1-\mu_b}{1-\mu_b\chi_y}\left(\sigma^{-1} + \chi_d\epsilon_\lambda + \frac{\mu_b\chi_r}{1-\mu_b}\tilde{\psi}_m\right)\vartheta(i_0 - r_n)$.

Given the process for u_t and the values of ϑ and θ, output can be written as $\overline{y}_t = \sigma^{-1}\hat{y}_t + \chi_d\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\tilde{\psi}_m\hat{y}_t + (\overline{\omega} - \delta)e^{\underline{\omega} t}\Omega_0$, which coincides with (17), where we used $\rho - \underline{\omega} = \overline{\omega} - \delta$. This result also implies that $\overline{\pi}_t = \pi_t$, as $\overline{\pi}_t = \kappa\int_0^\infty e^{-\rho t}\overline{y}_t dt = \kappa\int_0^\infty e^{-\rho t} y_t dt = \pi_t$.

Fiscal transfers. Given that y_t and π_t are proportional to u_t, we can write $i_t - r_n - \pi_t = \chi_{r,u}u_t$, $p_{d,t} = \chi_{p,u}u_t$, and $q_{L,0} = \chi_{q_L,u}u_0$. Rearranging the aggregate intertemporal budget constraint, we obtain the present discounted value of transfers: $\int_0^\infty e^{-\rho t}\mu_s T_{s,t} dt = (\tau - \mu_b T_b'(Y))\Omega_0 - \overline{d}_G\chi_{q_L,u}(\vartheta(i_0 - r_n) + \theta) - \overline{d}_g(\chi_{r,u} + r_L\chi_{p,u})\left[\frac{\vartheta(i_0-r_n)}{\rho+\psi_m} + \frac{\theta}{\rho-\underline{\omega}}\right]$. □

Proof of Proposition 6. From the market clearing condition for goods, we obtain the consumption of savers: $c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\left(\frac{\psi_P}{i_P+\psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P$. Assuming exponentially decaying interest rates, we can write the expression above as

$$c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\left[\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}(i_t - r_n) - \pi_t\right]. \tag{A.1}$$

The Euler equation for savers can be written as

$$\dot{c}_{s,t} = \sigma^{-1}(i_t - \pi_t - r_n) + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma c_{s,t} + \chi_d\epsilon_\lambda(i_t - r_n). \tag{A.2}$$

Combining equations (A.1) and (A.2), and using equation (C.8), we obtain

$$\dot{y}_t = \left[\frac{1-\mu_b}{1-\chi_y\mu_b}\sigma^{-1} - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}r_n\right](i_t - \pi_t - r_n) + \left[\lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\kappa\right] y_t$$
$$+ \left[\frac{1-\mu_b}{1-\chi_y\mu_b}\chi_d\epsilon_\lambda + \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\left(r_n + \frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}(\rho - r_n + \psi_m)\right)\right](i_t - r_n),$$

We can then write the aggregate Euler equation as $\dot{y}_t = \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + \delta y_t + v_t$, where $\tilde{\sigma}^{-1} \equiv \frac{1-\mu_b}{1-\chi_y\mu_b}\sigma^{-1} - \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}r_n$, $\delta \equiv \lambda\left(\frac{C_s}{C_s^*}\right)^{\sigma} - \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}\kappa$, and $v_t \equiv \frac{\mu_b\chi_r\overline{d}_P}{1-\chi_y\mu_b}\Big[r_n + \frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}$ $(\rho - r_n + \psi_m)\Big](i_t - r_n) + \frac{1-\mu_b}{1-\chi_y\mu_b}\chi_d\epsilon_\lambda(i_t - r_n)$. Therefore, output is given by

$$y_t = \sigma^{-1}\hat{y}_t + \chi_d\epsilon_\lambda\hat{y}_t + \frac{\mu_b\chi_r}{1-\mu_b}\frac{\psi_P(1+r_P\epsilon_\lambda)}{\rho+\psi_P+\psi_m}\tilde{\psi}_m\overline{d}_P\hat{y}_t + (\rho - \underline{\omega})e^{\underline{\omega}t}\Omega_0,$$

where $\tilde{\psi}_m \equiv \psi_m + \rho - r_n$. □

B Monetary Policy and Endogenous Disasters

In this appendix, we consider an extension of the model in Section 2 where the probability of a disaster is endogenous and responds to changes in monetary policy. Building on the work of Gertler et al. (2020), we introduce financial intermediaries which are exposed to runs that lead to financial disentermediation and large economic losses. To avoid repetition, we describe only the main differences with respect to the baseline model.

B.1 Model

Firms' capital structure. Firms' pricing decision is analogous to the one described in Section 2. Instead of being entirely equity financed, firms are now financed by a combination of equity, which is held by savers, and bank loans. Firms promise to pay an exponentially decaying coupon $e^{-\psi_F t}$ to banks at date $t \geq 0$. Firms may default on their debt. With Poisson intensity λ_F, debt is restructured and banks suffer a haircut ζ_F on corporate loans, where ζ_F is drawn from a distribution $G_F(\cdot)$ with support $[\underline{\zeta}_F, \overline{\zeta}_F]$. Default events are aggregate shocks, as they affect all firms at the same time.[1]

[1]Alternatively, we could assume that a fraction ζ_F of firms default on their debts in a default event, which would deliver similar results.

The return on bank loans is given by

$$dR_{F,t} = \left[\frac{1}{Q_{F,t}} + \frac{\dot{Q}_{F,t}}{Q_{F,t}} - \psi_F\right] dt + \frac{(1-\zeta_F)Q^{\dagger}_{F,t} - Q_{F,t}}{Q_{F,t}} d\mathcal{N}_{F,t},$$

where $Q_{F,t}$ denotes the price of corporate debt prior to default and $Q^{\dagger}_{F,t}$ is the price of corporate debt (with unit face value) after a default event. Let $D_{F,t}$ denote the total value of corporate loans outstanding in nominal terms and $F_{F,t} \equiv \frac{D_{F,t}}{Q_{F,t}}$ the face value of debt. Corporations maintain fix the face value, that is, $F_{F,t} = F_F$. This requires a debt repayment in the amount $F_{F,t} - \psi_F D_{F,t} = i_{F,t} D_{F,t}$ every period, where $i_{F,t} = Q^{-1}_{F,t} - \psi_F$. Firms also issue new debt after a default event to keep the face value constant after the shock.

Banks' problem. Banks issue deposits, which pay interest rate $i_{D,t}$, and choose how much to lend to corporations. Banks only pay dividends at the terminal date, which arrives with Poisson intensity λ_b. Every period a mass λ_b of new banks are created, which start with funds $\tilde{B}_{b,t}$ given by savers, so the total mass of bankers is kept fixed.

The banker's problem is given by

$$V_{b,t}(B_b) = \max_{B^C_b} \mathbb{E}_t \left[\frac{\eta_T}{\eta_t} B_{b,T}\right]$$

subject to

$$dB_{b,t} = \left[(i_{D,t} - \pi_t)B_{b,t} + r_{F,t}B^F_{b,t}\right] dt + (B^{\dagger}_{b,t} - B_{b,t})d\mathcal{N}_t,$$

where T is a stopping time with arrival rate λ_b, $r_{F,t} \equiv \frac{1}{Q_{F,t}} + \frac{\dot{Q}_{F,t}}{Q_{F,t}} - \psi_F - i_{D,t}$ denotes the spread with respect to the deposit rate, and $B^{\dagger}_{b,t}$ denotes the bank's net worth after the default event:

$$B^{\dagger}_{b,t} = \max\left\{B_{b,t} + B^F_{b,t}\frac{(1-\zeta_F)Q^{\dagger}_{F,t} - Q_{F,t}}{Q_{F,t}}, 0\right\}.$$

In equilibrium, banks operate leveraged ($B^F_{b,t} > B_{b,t}$), so there is a chance that they will not have enough resources to fully repay depositors in a default event. In this case, which we refer to as a *bank failure*, equity holders get zero, depositors suffer a loss, and the bank is ultimately liquidated. We follow Gertler et al. (2020) and assume that the financial disintermediation caused by bank failures leads to real economic losses, which we capture by a drop in firms' productivity from A to $A^* < A$.

Bank runs. Bank failures can be caused by self-fulfilling reasons, as they create the possibility of *bank runs*: if depositors decide to keep funding the bank, $Q^{\dagger}_{F,t}$ is high enough for banks to fully repay depositors; if depositors decide to run, then banks sell their loans at a fire sale price $Q^{\dagger}_{F,t} = Q^{*}_{F,t}$, which could trigger losses to depositors, justifying the decision to run. To focus on the implications of bank runs, we make the following assumptions: i) in the absence of a run, the default event is not enough to cause banks to fail; ii) for a small enough haircut ζ_F, banks do not fail even if depositors decide to run; iii) banks are immediately recapitalized after a default event that does not trigger a bank run. The first two assumptions guarantee that banks fail only after large enough shocks which are followed by a run. The third assumption allows us to abstract from the implications of small shocks to bank net worth, focusing only on crises triggered by runs. We state the assumptions in terms of parametric restrictions below.

Given the assumptions above, the price of a corporate loan does not change if there is a default event but no runs, so $Q^{\dagger}_{F,t} = Q_{F,t}$ in this case. In the case of a run, banks sell the loans at a discount, that is, we assume that $Q^{*}_{F,t} = (1-\zeta_d)Q_{F,t}$, where $0 < \zeta_d < 1$ represents the fire-sale discount. Moreover, there exists a threshold $\zeta^{*}_{F,t}$ such that the probability of a run is zero if $\zeta_F \leq \zeta^{*}_{F,t}$. If $\zeta_F > \zeta^{*}_{F,t}$, then a run occurs depending on the realization of a sunspot variable. We follow Gertler et al. (2020) and assume a constant sunspot probability $0 < \varkappa < 1$. The probability of a run, conditional on a default event, is then given by $(1 - G_F(\zeta^{*}_{F,t}))\varkappa$, where $\zeta^{*}_{F,t}$ satisfies the condition

$$\frac{B^F_{b,t}}{B_{b,t}} \frac{(1-\zeta^{*}_{F,t})Q^{*}_{F,t} - Q_{F,t}}{Q_{F,t}} = -1 \Rightarrow \zeta^{*}_{F,t} = \frac{\frac{B_{b,t}}{B^F_{b,t}} - \zeta_d}{1-\zeta_d},$$

where ζ_r must be sufficiently small such that $\zeta^{*}_{F,t} > 0$. Moreover, $\overline{\zeta}_F < \frac{\overline{B}_{b,t}}{\overline{B}^F_{b,t}}$, so there is no bank failure in the absence of a run. Note that the probability of a run is endogenous and increasing in bank's leverage $\frac{B^F_{b,t}}{B_{b,t}}$.

Banks' optimality condition. The HJB equation is given by

$$0 = \max_{B^F_b} -(i_t - \pi_t)V_{b,t} + \frac{\partial V_{b,t}}{\partial t} + \frac{\partial V_{b,t}}{\partial B_b}\left[(i_{D,t} - \pi_t)B_{b,t} + r_{F,t}B^F_{b,t}\right] + \lambda_b\left[B_{b,t} - V_{b,t}\right] +$$
$$\lambda_F \int_{\underline{\zeta}_F}^{\overline{\zeta}_F} \left[V^{nr}_{b,t} - V_{b,t}\right] dG_F(\zeta_F) + \lambda_F \varkappa \int_{\zeta^{*}_{F,t}}^{\overline{\zeta}_F} \left[\frac{\eta^{*}_t}{\eta_t} V^{*}_{b,t} - V^{nr}_{b,t}\right] dG_F(\zeta_F),$$

where $V^{nr}_{b,t} = V_{b,t}(B_{b,t}(1 + \mathcal{I}_t(\zeta_F)) - \zeta_F B^F_{b,t})$ is the value function conditional on a no-run default event, and $\mathcal{I}_t(\zeta_F)$ represents the capital injection that happens at this period; $V^{*}_{b,t}$ is the value function conditional on a bank run.

Let's guess-and-verify that the value function is given by $V_{b,t} = \xi_{b,t} B_{b,t}$, and the value function conditional on a run is simply $V_{b,t}^{*} = 0$. The HJB equation can then be written as

$$0 = \max_{B_b^F} \dot{\xi}_{b,t} + \xi_{b,t} \left[i_{D,t} - i_t + r_{F,t} \frac{B_{b,t}^F}{B_{b,t}} \right] + \lambda_b \left[1 - \xi_{b,t} \right] + \lambda_c \xi_{b,t} \int_{\underline{\zeta}_F}^{\overline{\zeta}_F} \left[-\zeta_F \frac{B_{b,t}^F}{B_{b,t}} + \mathcal{I}_t(\zeta_F) \right] dG_F(\zeta_F)$$
$$- \lambda_F \xi_b \varkappa \int_{\zeta_{F,t}^{*}}^{\overline{\zeta}_F} \left[1 + \mathcal{I}_t(\zeta_F) - \zeta_F \frac{B_{b,t}^F}{B_{b,t}} \right] dG_F(\zeta_F).$$

We assume that banks internalize that they are fully recapitalized in the case of default that does not trigger a run, that is, they internalize that $B_{b,t} I_t(\zeta_F) = \zeta_F B_{b,t}^F$. From the first-order condition with respect to B_t^F, we obtain $r_{F,t} = 0$. Note that, because losses on loans are passed to depositors at the margin in the case of a bank failure, the spread $r_{F,t}$ does not incorporate the effect of losses due to bank runs. The HJB can then be written as

$$\dot{\xi}_{b,t} = \lambda_b \left[\xi_{b,t} - 1 \right] - \xi_{b,t} \left[i_{D,t} - i_t + \lambda_F \varkappa (1 - G_F(\zeta_{F,t}^{*})) \right].$$

Law of motion of banks' net worth. Let $\overline{B}_{b,t}$ denote the aggregate net worth of the banking sector and $\overline{B}_{b,t}^F$ the aggregate holdings of bank loans. The law of motion of $\overline{B}_{b,t}$ is given by

$$d\overline{B}_{b,t} = \left[(i_{D,t} - \pi_t) \overline{B}_{b,t} + \lambda_b \left(\tilde{B}_{b,t} - \overline{B}_{b,t} \right) \right] dt - \iota_t \overline{B}_{b,t} d\mathcal{N}_t,$$

where ι_t is an indicator that is equal to one in the event of a bank run and zero otherwise.

Deposit rates and savers' behavior. For simplicity, we assume that the deposit rate is proportional to the short-term interest rate, $i_{D,t} = \kappa_D i_t$, where the parameter κ_D captures the pass-through of the policy rate to the deposit rate. Note that the spread between the deposit rate and the short-term interest rate responds to changes in monetary policy, as the spread is given by $i_{D,t} - i_t = (\kappa_D - 1) i_t$.

Deposits are provided by savers, so they are ultimately the owners of corporations, as they own bank equity and deposits that finance lending to firms. Savers' behavior will then be the same as in the setup of Section 2, where we assumed that savers are the sole owners of firms.

B.2 Stationary equilibrium

We consider next a stationary equilibrium. The economy after a bank run coincides with the disaster state described in Section 2, where the probability of a disaster is given by

$$\lambda \equiv \lambda_F \varkappa \left(1 - G_F(\zeta_F^*)\right),$$

where $\zeta_F^* = \frac{1}{1-\zeta_d}\left(\frac{B_b}{B_b^F} - \zeta_d\right)$. The interest rate on deposits is $i_D = \kappa_D r_n$ and the price of corporate debt is given by $Q_F = \frac{1}{i_D+\psi_F}$. The value of corporate loans is $\overline{B}_b^F = F_F Q_F$. The net worth of banks in the stationary equilibrium is given by $\overline{B}_b = \frac{\lambda_b \tilde{B}_b}{\lambda_b - i_D}$. We assume that $\tilde{B}_b = \chi_b \overline{B}_b^F$, $0 < \chi_b < 1$, which allow us to solve for bank's leverage:

$$\frac{\overline{B}_b^F}{\overline{B}_b} = \frac{\lambda_b - i_D}{\lambda_b \chi_b},$$

where we assume that $\lambda_b > \frac{i_D}{1-\chi_b}$, such that $\frac{\overline{B}_b^F}{\overline{B}_b} > 1$. To guarantee that $\zeta_F^* > 0$, the fire-sale parameter must satisfy $\zeta_d < \frac{\lambda_b \chi_b}{\lambda_B - i_D}$. To guarantee there is no bank failure in the absence of a run, the upper-bound of the support for ζ_F must satisfy $\overline{\zeta}_F < \frac{\lambda_b \chi_b}{\lambda_B - i_D}$.

B.3 Log-linear dynamics

In the presence of monetary shocks, banks' leverage will be time-varying and so it will be the probability of disaster. The probability of disaster $\lambda_t = \lambda_F \chi (1 - G_F(\zeta_{F,t}^*))$ and the threshold $\zeta_{F,t}^*$, up to first order, are given by

$$\hat{\lambda}_t = -\frac{g_F(\zeta_F^*)}{1 - G_F(\zeta_F^*)} \hat{\zeta}_{F,t}^*, \qquad \hat{\zeta}_{F,t}^* = \frac{1}{1-\zeta_d} \frac{\overline{B}_b}{\overline{B}_b^F} (b_{b,t} - b_{b,t}^F),$$

where $\hat{\lambda}_t \equiv \frac{\lambda_t - \lambda}{\lambda}$, $\hat{\zeta}_{F,t}^* = \zeta_{F,t}^* - \zeta_F^*$, $b_{b,t} \equiv \frac{\overline{B}_{b,t} - \overline{B}_b}{\overline{B}_b}$ and $b_{b,t}^F \equiv \frac{\overline{B}_{b,t}^F - \overline{B}_b^F}{\overline{B}_b^F}$. Combining the previous two equations, we obtain

$$\hat{\lambda}_t = \frac{g_F(\zeta_F^*)}{1 - G_F(\zeta_F^*)} \frac{1}{1-\zeta_d} \frac{\overline{B}_b}{\overline{B}_b^F} (b_{b,t}^F - b_{b,t}).$$

In equilibrium, $B_{b,t}^F$ equals the real value of corporate debt $\overline{D}_{F,t}/P_t$. Therefore, banks' leverage is given by $\frac{B_{b,t}^F}{B_{b,t}} = \frac{D_{F,t}}{\overline{B}_{b,t}^n}$, where $D_{F,t}$ is the nominal value of corporate debt and $\overline{B}_{b,t}^n$ is the nominal value of banks' net worth. Given the assumption that corporations keep the face value of debt fixed, we have that $b_{b,t}^F - b_{b,t} = q_{F,t} - b_{b,t}^n$, where $q_{F,t} = \frac{Q_{F,t} - Q_F}{Q_F}$ is the change in the nominal price of corporate debt and $b^n_{b,t} = \frac{\overline{B}^N_{b,t} - \overline{B}^N_b}{\overline{B}^N_b}$ is the change in the nominal value of banks' net worth. From the pricing condition for loans, we have that

$$-\frac{1}{Q_F} q_{F,t} + \dot{q}_{F,t} = \kappa_D(i_t - r_n) \Rightarrow q_{F,t} = -\frac{\kappa_D(i_t - r_n)}{i_F + \psi_F + \psi_m}.$$

The law of motion of $b^n_{b,t}$ is given by $\dot{b}^n_{b,t} = (i_D - \lambda_b) b^n_{b,t} + \kappa_D(i_t - r_n)$, and the initial value $b_{b,0}$ is given by $b^n_{b,0} = \frac{\overline{B}^F_b}{\overline{B}_b} q_{F,0}$. Integrating the law of motion of $b^n_{b,t}$ and assuming that nominal interest rates are exponentially decaying, $i_t - r_n = e^{-\psi_m t}(i_0 - r_n)$, we obtain

$$b^n_{b,t} = -e^{-(\lambda_b - i_D)t} \frac{\overline{B}^F_b}{\overline{B}_b} \frac{\kappa_D(i_0 - r_n)}{i_F + \psi_F + \psi_m} + \frac{e^{-\psi_m t} - e^{-(\lambda_b - i_D)t}}{\lambda_b - i_D - \psi_m} \kappa_D(i_0 - r_n).$$

Leverage is then given by

$$q_{F,t} - b^n_{b,t} = \left(\frac{\overline{B}^F_b}{\overline{B}_b} - 1 \right) \frac{\kappa_D(i_t - r_n)}{i_F + \psi_F + \psi_m} + \varphi_t,$$

where $\varphi_t \equiv \left[\frac{1}{i_F + \psi_F + \psi_m} \frac{\overline{B}^F_b}{\overline{B}_b} + \frac{1}{\lambda_b - i_D - \psi_m} \right] \left(e^{-(\lambda_b - i_D)t} - e^{-\psi_m t} \right) \kappa_D(i_0 - r_n)$.

The leverage of the banking sector is then given by two terms. First, a term that is proportional to the nominal interest rate. Therefore, this term is initially positive after a contractionary monetary shock and decays at the rate ψ_m. The second term φ_t controls the *speed of convergence* of leverage, which can be potentially different from the decay rate of interest rates. These properties imply that φ_t does not affect leverage on impact or in the long run, but it affects how fast leverage converges back to the stationary equilibrium. For instance, if $\varphi_t > 0$ for $t > 0$, then leverage decays more slowly than the monetary shock ψ_m, and leverage decays faster if $\varphi_t < 0$.

Given the expression for leverage, we obtain the probability of a disaster

$$\hat{\lambda}_t = \epsilon_\lambda(i_t - r_n) + \epsilon^\varphi_\lambda \varphi_t, \tag{B.1}$$

where $\epsilon^\varphi_\lambda \equiv \epsilon_\lambda \left[\left(\frac{\overline{B}^F_b}{\overline{B}_b} - 1 \right) \frac{\kappa_D}{i_F + \psi_F + \psi_m} \right]^{-1}$ and

$$\epsilon_\lambda \equiv \frac{g_F(\zeta^*_F)}{1 - G_F(\zeta^*_F)} \frac{1}{1 - \zeta_d} \left(1 - \frac{\overline{B}_b}{\overline{B}^F_b} \right) \frac{\kappa_D}{i_F + \psi_F + \psi_m}. \tag{B.2}$$

A convenient special case. Expression (B.1) generalizes the expression for the disaster probability given in Section 2, as it includes the additional term $\epsilon_\lambda^\varphi \varphi_t$. This term can be positive or negative, for $t > 0$, depending on parameters, and it is equal to zero if

$$\lambda_b = (\psi_m + i_D)\left(1 - \frac{\overline{B}_b}{\overline{B}_b^F}\right) - \psi_F \frac{\overline{B}_b}{\overline{B}_b^F},$$

which after using $\frac{\overline{B}_b}{\overline{B}_b^F} = \frac{\lambda_b \chi_b}{\lambda_b - i_D}$, we obtain

$$\lambda_b = \frac{(1-\chi_b)(\psi_m + i_D) + i_D - \psi_F \chi_b + \sqrt{\Delta_\lambda}}{2},$$

where $\Delta_\lambda \equiv \left((1-\chi_b)(\psi_m + i_D) + i_D - \psi_F \chi_F\right)^2 - 4(\psi_m + i_D) i_D$. We assume that ψ_m is large enough such that $\lambda_b > \frac{i_D}{1-\chi_b}$ and $\frac{\overline{B}_b^F}{\overline{B}_b} > 1$.

In this case, the probability of a disaster simplifies to the expression given in Section 2:

$$\hat{\lambda}_t = \epsilon_\lambda (i_t - r_n).$$

In this model with endogenous disasters, we are able to pin down the determinants of ϵ_λ using expression (B.2). For instance, the probability of disaster is increasing in financial intermediaries' leverage in the stationary equilibrium, the fire-sale parameter ζ_d, the pass-through of nominal interest rates to deposits κ_D, and the maturity of corporate debt.

C Derivations

C.1 The non-linear model

We consider the case where household debt is risky and long-term, as in Section 5. The case of short-term riskless debt discussed in Section 2 corresponds to a particular special case of this more general formulation.

Households' problem. The household problem is given by

$$V_{j,t}(B_j) = \max_{[C_{j,z}, N_{j,z}, B_{j,z}^L, B_{j,z}^P]_{z \geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho_j (z-t)} \left(\frac{C_{j,z}^{1-\sigma}}{1-\sigma} - \frac{N_{j,z}^{1+\phi}}{1+\phi} \right) dz + e^{-\rho_j (t^* - t)} V_{j,t^*}^*(B_{j,t^*}^*) \right],$$

subject to the flow budget constraint

$$dB_{j,t} = \left[(i_t - \pi_t) B_{j,t} + r_{L,t} B^L_{j,t} + r_{P,t} B^P_{j,t} + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \widetilde{T}_{j,t} - C_{j,t} \right] dt + \left[B^L_{j,t} \frac{Q^*_{L,t} - Q_{L,t}}{Q_{L,t}} + B^P_{j,t} \frac{Q^*_{P,t} - Q_{P,t}}{Q_{P,t}} \right] d\mathcal{N}_t,$$

and the borrowing constraint and no-negativity constraint for long-term bonds

$$B_{j,t} \geq -\overline{D}_{P,t}, \qquad B^S_{j,t} \geq 0, \qquad B^L_{j,t} \geq 0,$$

given the initial condition $B_{j,t} = B_j \geq -\overline{D}_{P,t}$, where $D_{P,t} = Q_{P,t}\overline{F}$ is the debt limit and $r_{k,t} \equiv \frac{1}{Q_{k,t}} + \frac{\dot{Q}_{k,t}}{Q_{k,t}} - \psi_k - i_t$ is the excess return on the long-term bond of type $k \in \{L, P\}$, conditional on no disasters.

The first-order conditions with respect to consumption and labor are given by

$$C^{-\sigma}_{j,t} = \frac{\partial V_{j,t}}{\partial B}, \qquad N^{\phi}_{j,t} = \frac{\partial V_{j,t}}{\partial B} \frac{W_{j,t}}{P_t}.$$

Writing the non-negativity condition on $B^S_{j,t}$ as $B^L_{j,t} + B^P_{j,t} \leq B_{j,t}$, and denoting the Lagrange multiplier on the non-negativity constraint on short-term and long-term non-defaultable bonds by $\mu^S_{j,t}$ and $\mu^L_{j,t}$, respectively, we obtain the first-order condition for the two types of long-term bonds:

$$\frac{\partial V_{j,t}}{\partial B} r_{L,t} = \lambda_t \frac{\partial V^*_{j,t}}{\partial B} \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}} - \mu^L_{j,t} + \mu^S_{j,t}, \qquad \frac{\partial V_{j,t}}{\partial B} r_{P,t} = \lambda_t \frac{\partial V^*_{j,t}}{\partial B} \frac{Q_{P,t} - Q^*_{P,t}}{Q_{P,t}} + \mu^S_{j,t}.$$

The solution is also subject to the state-constraint boundary condition[2]

$$\frac{\partial V_{j,t}(-\overline{D}_P)}{\partial B} \geq \left(-(i_{P,t} - \pi_t)\overline{D}_P + \frac{W_{j,t}}{P_t} N_{j,t} + \frac{\Pi_{j,t}}{P_t} + \widetilde{T}_{j,t} \right)^{-\sigma},$$

where $i_{P,t} = Q^{-1}_{P,t} - \psi_P$.

Combining the first-order conditions for consumption and labor, we obtain the standard labor-supply condition $\frac{W_t}{P_t} = N^{\phi}_{j,t} C^{\sigma}_{j,t}$.

Abstracting from the non-negativity constraints, the first-order conditions for long-

[2] See Achdou et al. (2017) for a discussion of state-constraint boundary conditions in the context of continuous-time savings problems with borrowing constraints.

term bonds can be written as

$$r_{L,t} = \lambda_t \left(\frac{C^*_{j,t}}{C_{j,t}}\right)^{-\sigma} \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}, \qquad r_{P,t} = \lambda_t \left(\frac{C^*_{j,t}}{C_{j,t}}\right)^{-\sigma} \frac{Q_{P,t} - Q^*_{P,t}}{Q_{P,t}}. \tag{C.1}$$

The envelope condition with respect to $B_{j,t}$ for an unconstrained household is given by

$$\rho_j \frac{\partial V_{j,t}}{\partial B} = (i_t - \pi_t)\frac{\partial V_{j,t}}{\partial B} + \frac{\mathbb{E}_t[d\left(\frac{\partial V_{j,t}}{\partial B}\right)]}{dt}.$$

Combining this expression with the first-order condition for consumption, we obtain

$$\frac{\mathbb{E}_t[dC^{-\sigma}_{j,t}]}{dt} = -(i_t - \pi_t - \rho_j)C^{-\sigma}_{j,t}.$$

Expanding the expectation of the marginal utility, we obtain the Euler equation for savers

$$\frac{\dot{C}_{s,t}}{C_{s,t}} = \sigma^{-1}(i_t - \pi_t - \rho_s) + \frac{\lambda_t}{\sigma}\left[\left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma} - 1\right]. \tag{C.2}$$

Firms' problem. Final goods are produced according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}}\right)^{\frac{\epsilon-1}{\epsilon}}$. The solution to final-good producers problem is a demand for variety i given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$. The price level is given by $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$.

The intermediate-goods producers' problem is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t\left[\int_t^{t^*} \frac{\eta_s}{\eta_t}\left((1-\tau)\frac{P_{i,s}}{P_s}Y_{i,s} - \frac{W_s}{P_s}\frac{Y_{i,s}}{A_s} - \frac{\varphi}{2}\pi_s^2(j)\right) ds + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t^*}(P_{i,t^*})\right],$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t}P_{i,t}$, given $P_{i,t} = P_i$. The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no disaster is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t}\right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i}\pi_{i,t}P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{C.3}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon)(1-\tau)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2}\pi_{i,t}P_{i,t} + \frac{\partial Q_{j,t}}{\partial P_i}\pi_{i,t} - (i_t - \pi_t)\frac{\partial Q_{i,t}}{\partial P_i} + \lambda_t\frac{\eta_t^*}{\eta_t}\left(\frac{\partial Q_{i,t}^*}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i}\right). \quad \text{(C.4)}$$

Multiplying the expression by $P_{i,t}$ and using Equations (C.3) and (C.4), we obtain

$$0 = \left((1-\epsilon)(1-\tau)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi\dot{\pi}_t - (i_t - \pi_t)\varphi\pi_{i,t} + \lambda_t\frac{\eta_t^*}{\eta_t}\varphi(\pi_{i,t}^* - \pi_{i,t}).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = \left(i_t - \pi_t + \lambda_t\frac{\eta^*}{\eta_t}\right)\pi_t - (\epsilon-1)\varphi^{-1}\left(\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_t}\frac{1}{A} - (1-\tau)\right)Y_t,$$

where we have assumed that $P_{i,t} = P_t$ for all $i \in [0,1]$ and that $\pi_t^* = 0$.

C.2 Stationary equilibrium

Introducing recurrent shocks. Suppose aggregate productivity follows $\frac{dA_t}{A_t} = -\zeta d\mathcal{N}_t$, given $A_0 = A$ and $0 < \zeta < 1$, where $\mathcal{N}_t$ is a Poisson process with arrival rate $\lambda_t = \lambda(i_t - \rho; A_t)$.[3] The setting discussed in the paper corresponds to the case where $\lambda(i_t - \rho; A) > 0$ and $\lambda(i_t - \rho; A_t) = 0$ for $A_t < A$.

C.2.1 Consumption and natural interest rate

Consumption determination in a stationary equilibrium. In a stationary equilibrium, all variables are independent of calendar time and depend only on aggregate productivity A_t, so they are constant between the realization of disasters. We can then write, for instance, consumption and output as $C_{j,t} = \mathbf{C}_j(A_t)$ and $Y_t = \mathbf{Y}(A_t)$.

Imposing $\dot{C}_{s,t} = 0$ in the Euler equation (C.2), we obtain the natural rate $\mathbf{r}_n(A)$

$$\mathbf{r}_n(A) = \rho_s - \lambda(A)\left[\left(\frac{\mathbf{C}_s(A)}{\mathbf{C}_s(A(1-\zeta))}\right)^{\sigma} - 1\right],$$

where we abuse notation and write $\lambda(A)$ instead $\lambda(0; A)$ in a stationary equilibrium.

[3] The process can be easily generalized to allow for trend growth, $dA_t = gA_t dt - \zeta A_t d\mathcal{N}_t$. The expressions in the text apply to this case as well after all variables are properly detrended.

The consumption of borrowers satisfies the condition

$$\mathbf{C}_b(A) = \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} \mathbf{C}_b^{-\frac{\sigma}{\phi}}(A) + \mathbf{T}_b(A) - \mathbf{r}_n(A)\overline{D}_P,$$

where $\mathbf{T}_b(A)$ represents the level of transfers as a function of productivity, and we used the labor supply condition to solve for N_b and the fact that the real wage is given by $\frac{W_t}{P_t} = A_t(1-\tau)(1-\epsilon^{-1})$.[4] The consumption of savers satisfies the condition

$$\mathbf{C}_s(A) = \left[A(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1+\phi}{\phi}} \mathbf{C}_s^{-\frac{\sigma}{\phi}}(A) + \frac{1-(1-\tau)(1-\epsilon^{-1})}{1-\mu_b}\mathbf{Y}(A) - \frac{\mu_b \mathbf{T}_b(A)}{1-\mu_b} + \frac{\mathbf{r}_n(A)\mu_b}{1-\mu_b}\overline{D}_P.$$

Given that $\mathbf{Y}(A) = \sum_{j\in\{b,s\}} \mu_j \mathbf{C}_j(A)$, the above conditions provide a pair of functional equations that determine $\mathbf{C}_j(A)$.

Symmetric stationary equilibrium. In a symmetric equilibrium $\mathbf{C}_s(A) = \mathbf{C}_b(A) = \overline{\mathbf{Y}}(A)$, hence $\overline{\mathbf{Y}}(A) = A^{\frac{1+\phi}{\sigma+\phi}} \left[(1-\tau)(1-\epsilon^{-1})\right]^{\frac{1}{\sigma+\phi}}$. This equilibrium requires that $\mathbf{T}_b(A)$ satisfies $\mathbf{T}_b(A) = \left[1-(1-\tau)(1-\epsilon^{-1})\right]\overline{\mathbf{Y}}(A) + \left(\rho_s - \lambda(A)\left[\left(\frac{\overline{\mathbf{Y}}(A)}{\overline{\mathbf{Y}}(A(1-\zeta))}\right)^{\sigma} - 1\right]\right)\overline{D}_P$. Moreover, in a symmetric stationary equilibrium, the real interest rate is given by $\mathbf{r}_n(A) = \rho_s - \lambda(A)\left[(1-\zeta)^{-\sigma\frac{1+\phi}{\sigma+\phi}} - 1\right]$. These results hold for both the case of non-recurrent shocks, where $\lambda(A) > 0$ and $\lambda(A^*) = 0$, and the case of recurrent shocks, where $\lambda(A)$ is independent of productivity level A.

C.2.2 Risk premia

Equity premium. The value of the intermediate-goods firms is given by $Q_{S,t} = \mathbb{E}_t\left[\int_t^\infty \frac{\eta_s}{\eta_t}\Pi_s ds\right]$, where $\Pi_t = (1-\tau)Y_t - \frac{W_t}{P_t}N_t$. From this expression we get the expected return on the firm:

$$\frac{\Pi_t}{Q_{S,t}}dt + \frac{\mathbb{E}_t[dQ_{S,t}]}{Q_{S,t}} = \left[\underbrace{(i_t - \pi_t)}_{\text{interest rate}} + \underbrace{\lambda_t \frac{(C_{s,t}^*)^{-\sigma} - C_{s,t}^{-\sigma}}{C_{s,t}^{-\sigma}} \frac{Q_{S,t} - Q_{S,t}^*}{Q_{S,t}}}_{\text{risk premium}}\right] dt. \tag{C.5}$$

In a stationary equilibrium, profits are given by $\Pi_t = \epsilon^{-1}(1-\tau)Y_t$. Using that $r_{n,t} +$

[4] For ease of exposition, we focus on the case of riskless household debt. The case of long-term risky household debt can be handled analogously.

$\lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma = \rho_s + \lambda_t$, we can write the dividend-yield as follows:

$$\frac{\Pi_t}{Q_{S,t}} = \frac{\rho_s + \lambda_t}{1 + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma \frac{Q^*_{S,t}}{\Pi^*_t}\frac{\Pi^*_t}{\Pi_t}},$$

Suppose $\lambda_t > 0$ in the no-disaster state and $\lambda_t = 0$ in the disaster state. Then, $\Pi^*_t = \rho_s Q^*_{S,t}$ and $\Pi_t / Q_{S,t}$ can be written as

$$\frac{\Pi_t}{Q_{S,t}} = \frac{\rho_s + \lambda_t}{1 + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma \frac{(1-\zeta_Y)}{\rho_s}},$$

where $1 - \zeta_Y \equiv \frac{Y^*}{Y}$. The (unlevered) equity premium can then be written as

$$\frac{\Pi_t}{Q_{S,t}} dt + \lambda \frac{Q^*_{S,t} - Q_{S,t}}{Q_{S,t}} - r_n = \lambda \left((1-\zeta_Y)^{-\sigma} - 1\right) \left(1 - \frac{(\rho_s + \lambda_t)(1-\zeta_Y)}{\rho_s + \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^\sigma}\right),$$

using the fact that $\Pi^*_t / \Pi_t = 1 - \zeta_Y$.

Term spread. From the first-order condition (C.1), the excess return on the long-term bonds satisfies

$$\frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} - \psi_L - i_t = \lambda_t \left(\frac{C_t}{C^*_t}\right)^\sigma \frac{Q_{L,t} - Q^*_{L,t}}{Q_{L,t}}. \tag{C.6}$$

Let $i_{L,t}$ denote the yield on the long-term bond, then $i_{L,t}$ satisfies

$$Q_{L,t} = \int_t^\infty e^{-(i_{L,s}+\psi_L)(s-t)} ds \Rightarrow \frac{1}{Q_{L,t}} + \frac{\dot{Q}_{L,t}}{Q_{L,t}} = i_{L,t} + \psi_L.$$

We consider next a stationary equilibrium with non-recurrent shocks. Combining the previous two expressions, we obtain that the term spread, the difference between the long and short interest rate, is given by $i_L - r_n = \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma \frac{Q_L - Q^*_L}{Q_L}$. The price of the long-term bond in the disaster state is given by $Q^*_L = \frac{1}{i^*_L + \psi_L}$, where $i^*_L = r^*_n$ is the yield on the long-term bond in the disaster state. We can then express the term spread as

$$i_L - r_n = \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma \frac{r^*_n - r_n}{r^*_n + \psi_L + \lambda \left(\frac{C_s}{C^*_s}\right)^\sigma}.$$

Note that $i_L > r_n$ and the difference is decreasing in ψ_L, so the yield increases with the bond duration.

Corporate bond premium. We can also price a *corporate bond*, which is a defatutable bond. Let $Q_{F,t}$ denote the value of a bond that pays off coupons $e^{-\psi_F t}$ in nominal terms in the absence of default. We assume that monetary shocks are too small to trigger default, so there is no default in the no-disaster state, and that the bond suffers a loss $1 - \zeta_F$ conditional on a disaster.

A derivation analogous to the one for government bonds shows that the yield on the corporate bond, which is given by $i_{F,t} = Q_{F,t}^{-1} - \psi_F$, can be expressed as follows in a stationary equilibrium

$$i_F - r_n = \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_F - Q_F^*}{Q_F}.$$

The value of the corporate bond in the disaster state is given by $Q_F^* = \frac{1-\zeta_F}{r_n^* + \psi_F}$. In the stationary equilibrium, the value of the corporate bond in the no-disaster state is given by

$$Q_F = \frac{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_F^*}{\psi_F + \rho_s + \lambda}.$$

The yield on the corporate bond is given by $i_{F,t} = Q_{F,t}^{-1} - \psi_F$. The *corporate spread*, the difference between the yield on the corporate bond and a government bond with the same coupons, is given by in the stationary equilibrium

$$r_F = \frac{\psi_F + \rho_s + \lambda}{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_F^*} - \frac{\psi_F + \rho_s + \lambda}{1 + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} Q_F^*/(1-\zeta_F)}.$$

C.3 Log-linear dynamics

C.3.1 Consumption of borrowers and savers

Labor supply and market clearing. The labor supply condition can be written as

$$w_t - p_t = \phi n_{j,t} + \sigma c_{j,t}.$$

Log-linearizing the market-clearing conditions for consumption and labor, we obtain

$$\mu_b^c c_{b,t} + (1-\mu_b^c) c_{s,t} = y_t, \qquad \mu_b^n n_{b,t} + (1-\mu_b^n) n_{s,t} = n_t,$$

where $\mu_b^c \equiv \frac{\mu_b C_b}{Y}$ and $\mu_b^n \equiv \frac{\mu_b N_b}{N}$. Equating both agents' labor-supply, we obtain

$$n_{s,t} = n_{b,t} + \phi^{-1}\sigma(1-\mu_b^c)^{-1}(c_{b,t} - y_t),$$

where we used the market-clearing condition for goods to eliminate $c_{s,t}$. Plugging the above expression into the market-clearing condition for labor, we obtain

$$n_{b,t} = (1+\phi^{-1}\sigma)y_t - \phi^{-1}\sigma c_{b,t} + \phi^{-1}\sigma \frac{\mu_b^c - \mu_b^n}{1-\mu_b^c}(y_t - c_{b,t}).$$

The real wage is then given by

$$w_t - p_t = (\phi+\sigma)y_t + \sigma \frac{\mu_b^c - \mu_b^n}{1-\mu_b^c}(y_t - c_{b,t}).$$

Borrowers' consumption. Since we assume that borrowers are sufficiently impatient, such that the borrowing constraint is always binding, their consumption is given by

$$C_{b,t} = \frac{W_t}{P_t}N_{b,t} + \widetilde{T}_{b,t} - (i_{P,t} - \pi_t)\overline{D}_{P,t}.$$

Linearizing this expression, we obtain

$$c_{b,t} = \frac{WN_b}{PC_b}(w_t - p_t + n_{b,t}) + T_{b,t} - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P,$$

where $T_{b,t} \equiv \frac{\widetilde{T}_{b,t} - \widetilde{T}_b}{C_b}$, and $\overline{d}_P \equiv \frac{\overline{D}_P}{C_b}$, and we used that $q_{P,t} = -\frac{i_{P,t} - i_P}{i_P + \psi_P}$. Using the expression for the real wage and labor supply, we obtain

$$c_{b,t} = \frac{WN_b}{PC_b}\left[(1+\phi^{-1})(\phi+\sigma)y_t - \phi^{-1}\sigma c_{b,t} + (1+\phi^{-1})\sigma\frac{\mu_b^c - \mu_b^n}{1-\mu_b^c}(y_t - c_{b,t})\right] + T_{b,t} - \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P.$$

Solving for $c_{b,t}$, and using $T_{b,t} = T_b'(\overline{Y})y_t$, we obtain

$$c_{b,t} = \chi_y y_t - \chi_r \left(\frac{\psi_P}{i_P + \psi_P}(i_{P,t} - i_P) - \pi_t\right)\overline{d}_P,$$

where $\chi_y \equiv \frac{T_b'(\overline{Y}) + (1-\alpha)(1+\phi^{-1})(\phi+\sigma)}{1+(1-\alpha)\phi^{-1}\sigma}$ and $\chi_r \equiv \frac{1}{1+(1-\alpha)\phi^{-1}\sigma}$. The expressions in the paper are obtained by imposing $C_b = C_s = Y$, so $\mu_b^c = \mu_b^n = \mu_b$, and $1-\alpha \equiv \frac{WN}{PY}$.

Savers' consumption. From the borrowers' consumption and market clearing for goods, we obtain

$$c_{s,t} = \frac{1-\mu_b\chi_y}{1-\mu_b}y_t + \frac{\mu_b\chi_r\overline{d}_P}{1-\mu_b}(i_t - \pi_t - r_n). \tag{C.7}$$

C.3.2 Asset pricing

Price of risk. The price of risk is given by $P_{d,t} \equiv \lambda_t \left(\frac{C_{s,t}}{C^*_{s,t}}\right)^{\sigma}$. Approximating around the stationary equilibrium, we get

$$p_{d,t} = \sigma c_{s,t} + \epsilon_\lambda (i_t - r_n).$$

Stocks. Rearranging and log-linearizing expression (C.5) around the stationary equilibrium, we obtain

$$\dot{q}_{S,t} = \rho q_{S,t} + (i_t - \pi_t - r_n) + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_S - Q_S^*}{Q_S} p_{d,t} - \frac{Y}{Q_S}\left((1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)\right).$$

Solving this equation forward, we obtain

$$q_{S,0} = \frac{Y}{Q_S}\int_0^\infty e^{-\rho t}\left[(1-\tau)y_t - (1-\alpha)(w_t - p_t + n_t)\right]dt - \int_0^\infty e^{-\rho t}\left[i_t - \pi_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_S - Q_S^*}{Q_S} p_{d,t})\right]dt.$$

Long-term bonds. Rearranging expression (C.6) and log-linearizing around the stationary equilibrium, we obtain

$$\dot{q}_{L,t} = (\rho + \psi_L) q_{L,t} + i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n)).$$

Solving the above equation forward, we obtain

$$q_{L,0} = -\int_0^\infty e^{-(\rho+\psi_L)t}\left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n))\right]dt.$$

Using that $Q_L^{-1} = i_L + \psi_L$, the yield on the long-term bond can then be written as

$$i_{L,0} - i_L = (i_L + \psi_L)\int_0^\infty e^{-(\rho+\psi_L)t}\left[i_t - r_n + \lambda \left(\frac{C_s}{C_s^*}\right)^{\sigma} \frac{Q_L - Q_L^*}{Q_L}(\sigma c_{s,t} + \epsilon_\lambda (i_t - r_n))\right]dt.$$

Household debt. The price of the household debt evolves according to

$$\dot{q}_{P,t} = (\rho + \psi_P) q_{P,t} + i_t - r_n + r_P p_{d,t}.$$

Under the assumption that $r_P\sigma = \mathcal{O}(i_0 - r_n)$, we can write the price of the bond as

$$\dot{q}_{P,t} = (\rho + \psi_P)q_{P,t} + i_t - r_n + r_P\epsilon_\lambda(i_t - r_n) \Rightarrow q_{P,t} = -\frac{1 + r_P\epsilon_\lambda}{\rho + \psi_P + \psi_m}(i_t - r_n). \tag{C.8}$$

Corporate bonds. The linearized price of the corporate bond is given by an expression analogous to the one for government bonds:

$$q_{F,0} = -\int_0^\infty e^{-(\rho+\psi_F)t}\left[i_t - r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right]dt.$$

The yield on the corporate bond is $i_{F,0} - i_F = -Q_F^{-1}q_{F,0}$, which can be written as

$$i_{F,0} - i_F = (i_F + \psi_F)\int_0^\infty e^{-(\rho+\psi_F)t}\left[i_t - r_n + \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))\right]dt,$$

using the fact that $Q_F^{-1} = i_F + \psi_F$.

The corporate spread is $r_{F,0} = i_{F,0} - \bar{i}_{F,0}$, where $\bar{i}_{F,0}$ is the yield of a government bond with the same coupons as the corporate bond:

$$r_{F,0} = r_F\int_0^\infty e^{-(\rho+\psi_F)t}(i_t - r_n)dt + \left[(i_F + \psi_F)(i_F - i) - (\bar{i}_F + \psi_F)(\bar{i}_F - i)\right] \int_0^\infty e^{-(\rho+\psi_F)t}(\sigma c_{s,t} + \epsilon_\lambda(i_t - r_n))dt,$$

where $i_F - i = \lambda\left(\frac{C_s}{C_s^*}\right)^\sigma \frac{Q_F - Q_F^*}{Q_F}$ is the difference between the corporate bond yield and the short-term nominal rate in the stationary equilibrium and $\bar{i}_F - i$ is the corresponding object for a bond without default risk.

D Estimation of Fiscal Response to a Monetary Shock

We estimate the empirical IRFs using a VAR identified by a recursiveness assumption, as in Christiano et al. (1999), extended to include fiscal variables. The variables included are: real GDP per capita, CPI inflation, real consumption per capita, real investment per capita, capacity utilization, hours worked per capita, real wages, tax revenues over GDP, government expenditures per capita, the federal funds rate, the 5-year constant maturity rate, and the real value of government debt per capita. We estimate a four-lag VAR using quarterly data for the period 1962:1-2007:3. The identification assumption of the monetary shock is as follows: the only variables that react contemporaneously to the monetary shock

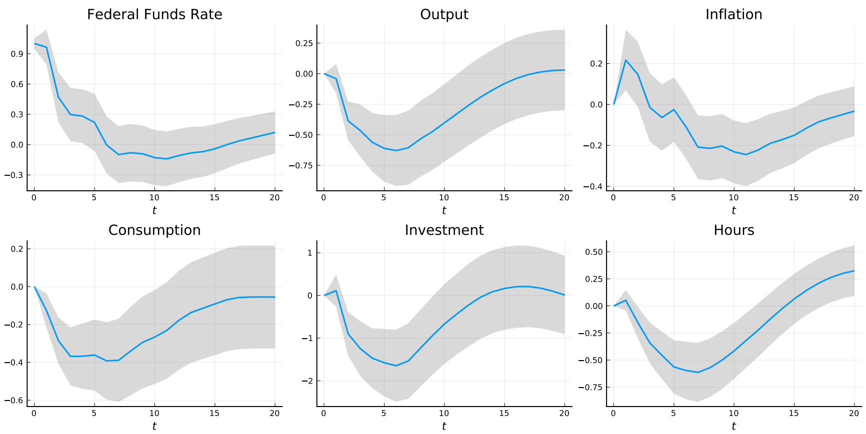

Figure D.1: Estimated IRFs.

are the federal funds rate, the 5-year rate and the value of government debt. All other variables, including tax revenues and expenditures, react with a lag of one quarter.

Data sources. The data sources are: **Nominal GDP:** BEA Table 1.1.5 Line 1; **Real GDP:** BEA Table 1.1.3 Line 1, **Consumption Durable:** BEA Table 1.1.3 Line 4; **Consumption Non Durable:** BEA Table 1.1.3 Line 5; **Consumption Services:** BEA Table 1.1.3 Line 6; **Private Investment:** BEA Table 1.1.3 Line 7; **GDP Deflator:** BEA Table 1.1.9 Line 1; **Capacity Utilization:** FRED CUMFNS; **Hours Worked:** FRED HOANBS; **Nominal Hourly Compensation:** FRED COMPNFB; **Civilian Labor Force:** FRED CNP16OV; **Nominal Revenues:** BEA Table 3.1 Line 1; **Nominal Expenditures:** BEA Table 3.1 Line 21; **Nominal Transfers:** BEA Table 3.1 Line 22; **Nominal Gov't Investment:** BEA Table 3.1 Line 39; **Nominal Consumption of Net Capital:** BEA Table 3.1 Line 42; **Effective Federal Funds Rate (FF):** FRED FEDFUNDS; **5-Year Treasury Constant Maturity Rate:** FRED DGS5; **Market Value of Government Debt:** Hall et al. (2018).

All the variables are obtained from standard sources, except for the real value of debt, which we construct from the series provided by Hall et al. (2018). We transform the series into quarterly frequency by keeping the market value of debt in the first month of the quarter. This choice is meant to avoid capturing changes in the market value of debt arising from changes in the *quantity* of debt after a monetary shock instead of changes in *prices*.

VAR estimation. Figure D.1 shows the results. As is standard in the literature, we find that a contractionary monetary shock increases the federal funds rate and reduces output and inflation on impact. Moreover, the contractionary monetary shock reduces consumption, investment, and hours worked.

	(1) Revenues	(2) Interest Payments	(3) Transfers & Expenditures	(4) Debt in T	(5) Initial Debt	(1) - (2) - (3) + (4) - (5) Residual
Data	-26	68.88	-12.09	2.91	-49.74	30.13
	[-72.89,20.89]	[30.01,107.75]	[-48.74,24.56]	[-12.79,18.62]	[-68.03,-31.46]	[-4.74,65]

Table D.1: The impact on fiscal variables of a monetary policy shock

Note: Calculations correspond to a a 100 bps unanticipated interest rate increase. Confidence interval at 95% level.

The Government's Intertemporal Budget Constraint. The fiscal response in the model corresponds to the present discounted value of transfers over an infinite horizon, that is, $\sum_{t=0}^{\infty} \tilde{\beta}^t T_t$, where $\tilde{\beta} = \frac{1-\lambda}{1+\rho_s}$. We next consider its empirical counterpart. First, we calculate a truncated intertemporal budget constraint from period zero to $\mathcal{T}$:

$$\underbrace{b_y b_0}_{\substack{\text{debt}\\\text{revaluation}}} = \sum_{t=0}^{\mathcal{T}} \tilde{\beta}^t \left[\underbrace{\tau y_t + \tau_t}_{\text{tax revenue}} - \underbrace{\tilde{\beta}^{-1} b_y (i^m_{t-1} - \pi_t - r^n)}_{\text{interest payments}} \right] - \underbrace{T_{0,\mathcal{T}} + \tilde{\beta}^{\mathcal{T}} b_y b_{\mathcal{T}}}_{\substack{\text{other transfers/expenditures}\\\text{\& final debt}}} \tag{D.1}$$

The right-hand side of (D.1) is the present value of the impact of a monetary shock on fiscal accounts. The first term represents the change in revenues that results from the real effects of monetary shocks. The second term represents the change in interest payments on government debt that results from change in nominal rates. The last two terms are adjustments in transfers and other government expenditures, and the final debt position at period $\mathcal{T}$, respectively. In particular, $T_{0,\mathcal{T}}$ represents the present discounted value of transfers from period 0 through $\mathcal{T}$. Provided that $\mathcal{T}$ is large enough, such that (y_t, τ_t, i_t) have essentially converged to the steady state, then the value of debt at the terminal date, $b_{\mathcal{T}}$, equals (minus) the present discounted value of transfers and other expenditures from period $\mathcal{T}$ onward. Hence, the last two terms combined can be interpreted as the present discounted value of fiscal transfers from zero to infinity. Finally, the left-hand side represents the revaluation effect of the *initial* stock of government debt.

Table D.1 shows the impact on the fiscal accounts of a monetary policy shock, both in the data and in the estimated model. We first apply equation (D.1) to the data and check whether the difference between the left-hand side and the right-hand side is different from zero. The residual is calculated as

Residual = Revenues - Interest Payments - Transfers + Debt in $\mathcal{T}$ - Initial Debt

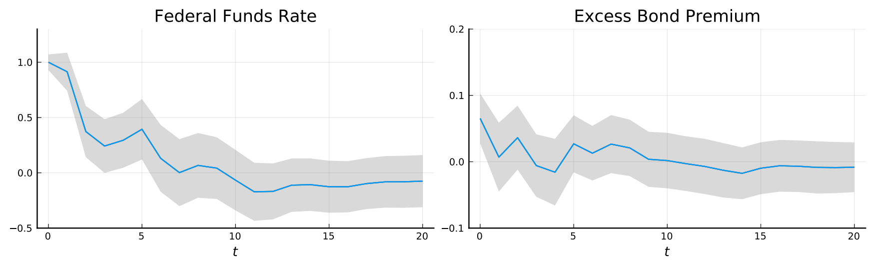

Figure D.2: IRFs for the federal funds rate and excess bond premium.

We truncate the calculations to quarter 60, that is, $\mathcal{T} = 60$ (15 years) in equation (D.1). The results reported in Table D.1 imply that we cannot reject the possibility that the residual is zero and, therefore, we cannot reject the possibility that the intertemporal budget constraint of the government is satisfied in our estimation.

The adjustment of the fiscal accounts in the data corresponds to the patterns we observed in Figure 2. The response of initial debt is quantitatively important, and it accounts for the bulk of the adjustment in the fiscal accounts.

EBP. To estimate the response of the corporate spread in the data, we add the EBP measure of Gilchrist and Zakrajšek (2012) into our VAR (ordered after the fed funds rate). Since the EBP is only available starting in 1973, we reduce our sample period to 1973:1-2007:7. The estimated IRFs are in line with those obtained for the longer sample. We find a significant increase of the EBP on impact, of 6.5 bps, in line with the estimates in the literature.

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