The Pass-through of Productivity Shocks to Wages and the Cyclical Competition for Workers

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Abstract

Using French matched employer-employee data, I document that after positive firm-level productivity shocks, the wage of stayers rises and job-to-job transitions fall whereas after positive sectoral productivity shocks wages rise significantly more, and job-to-job transitions rise. To explain this difference, I build a search model in which firms use dynamic wage contracts to attract and retain workers, subject to two-sided limited commitment and imperfect information. After positive firm-level shocks, firms increase wages to reduce the quit rate but only by a limited amount because workers are risk-averse and value insurance against shocks. After positive sectoral shocks, the competition for workers heats up and workers become more likely to switch jobs. In response, firms increase wages more aggressively to retain workers. I find that it is optimal for firms to pass-through sectoral productivity shocks more in high-productivity matches, and that the degree of firm commitment is a critical determinant of the cyclicality of earnings risk. Firing costs play a new role when contracts are endogenous. Lowering them reduces the commitment power of firms and makes income risk larger and counter-cyclical.

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1 Introduction

How do firms and workers share risk when profits fluctuate? Do firms absorb productivity shocks into profits or pass it through to the wage of workers? The idea that firms provide some form of insurance to workers through wage contracts is old (Knight, 1921) and is supported by recent evidence showing that the pass-through of firm-level productivity shocks to wages is small (Guiso, Pistaferri and Schivardi, 2005). In contrast, firms pass-through a much higher fraction of sectoral productivity shocks to wages (Carlsson, Messina and Skans, 2016). In this sense, the data suggests that firms provide relatively more insurance against firm-level productivity shocks than against sectoral shocks.

There is essentially no existing work that simultaneously accounts for the patterns of pass-through of both firm-level and sectoral productivity shocks documented in the data. A literature studies risk-sharing between firms and workers but has focused so far on firm-level and worker-level productivity shocks (Balke and Lamadon, 2022). A large macroeconomic literature studies the response of wages to sectoral or aggregate productivity shocks (e.g. Moscarini and Postel-Vinay, 2013) but in models with risk-neutral workers, thus overlooking the risk-sharing problem between firms and workers.

This paper builds a model that generates these patterns of pass-through as a result of optimal contracts between firms and risk-averse workers. The key idea in this model is that the insurance provided by firms is limited by a competition for workers whose intensity varies following sectoral productivity shocks. Workers are less likely to leave their current firms when they receive a higher wage, so an effective employee retention strategy requires to pay workers relatively more when productivity is high and workers generate a lot of profits. The optimal contract balances this retention strategy with the worker demand for insurance. After sectoral productivity shocks, the intensity of the competition for workers varies and firms face further incentives to change wages. In booms, retaining workers requires large wage increases whereas in downturns, firms can reduce wages further without losing all their workers. I derive new analytical formulas for the pass-through highlighting the critical role of this cyclical competition for workers and of the worker risk aversion. I use the model to quantify how much insurance firms provide to workers over the cycle and evaluate policies that influence the risk faced by workers. In particular, I show that reducing firing costs increases the pass-through of productivity shocks to wages, especially in downturns.

I start by documenting using matched employer-employee data from France between 2008 and 2019 the pass-through of firm-level and sectoral productivity shocks to the wage of stayers. I find, consistent with existing literature, that wages respond a lot more to sectoral shocks than to firm-level shocks. After a 1% increase in productivity, wages increase by 0.04% when shocks are firm-level and by 0.18% when they are sectoral. Since the competition for workers is central to my model, I also measure job-to-job transition rates and find that they respond very differently to firm-level and sectoral shocks. After a 1% increase in productivity, job-to-job transitions fall by about 0.02 percentage points when shocks are firm-level and rise by 0.04 percentage points when they are sectoral.

To understand these facts, this paper builds an equilibrium model of the labor market with risk-averse workers and dynamic wage contracts. Workers can switch job but face search frictions. Contracts are subject to limited commitment on the side of workers and firms as in Thomas and Worrall (1988), and there is imperfect information about the worker search decision and preference for job mobility. Firms experience firm-level and sectoral productivity shocks, and have fixed differences in productivity. In a baseline version of the model, I assume that workers are hand-to-mouth and I later relax this assumption by introducing trades in risk-free bonds. My model builds on Menzio and Shi (2011) and Balke and Lamadon (2022).

The model captures a trade-off between retaining workers relatively more when they are most productive, and providing insurance to workers. Workers apply for better jobs every period, and due to search frictions leave their current employers with positive probability. After an increase in firm-level productivity, firms become eager to retain workers who have become more profitable and thus choose to pay them more in order to lower the quit rate. After a fall in productivity, workers generate less profits and it is optimal for firms to pay them less despite an increase in the quit rate. However, workers are risk averse and value stable wages. Firms therefore balance the benefits of indexing wages on productivity to optimize worker retention, against the benefits of providing insurance to workers against shocks.

In sharp contrast, sectoral productivity shocks not only affect the profitability of a match but also the intensity of the competition for workers. After positive sectoral shocks, new entrants post more vacancies and workers would become more likely to find another job if their wage remained constant. In response, employers increase wages more aggressively not just because their workers are more profitable but also because they are more likely to quit. As a result, wages increase significantly and job-to-job transitions increase. Similarly, after negative sectoral shocks the competition for workers cools down and firms can reduce wages further without fear of seeing all their workers quit.

In order to characterize this trade-off between worker retention and insurance, I first derive a new analytical formula for the pass-through of firm-level productivity shocks to wages. Specifically, I compute the impulse response of wages to a mean-reverting shock.

I derive this result in a continuous time version of the model using recent methods from Sannikov (2008) to write the contract recursively, and I introduce a novel approximation in the search efficiency of employed workers. I find that the pass-through is back-loaded, that it falls with the worker risk aversion coefficient and increases with the level of a retention elasticity, defined as the percentage point change in the worker job-to-job transition rate induced by a 1 percent increase in the present value of wages. The retention elasticity is an endogenous variable that summarizes the influence that firms exert on worker mobility decisions. When it is large, increasing wages is an effective strategy to retain workers and thus it is optimal for the pass-through to be high. The pass-through formula is reminiscent of the Chetty-Baily statistic for optimal unemployment insurance (Baily, 1978, Chetty, 2006), highlighting a common structure behind their problem and mine.

A similar formula for sectoral productivity shocks shows how cyclical variations in the competition for workers influence dynamic contracts. The pass-through of sectoral shocks is larger than the pass-through of firm-level shocks when two conditions are satisfied: the firm value is positive, and the retention elasticity is pro-cyclical. In booms, workers receive more offers from other firms trying to poach them. The pass-through formula shows that current employers respond to this increase in the competition for workers only if it is worth it, that is if the firm value from the current match is positive. Furthermore, firms only respond if the retention elasticity is pro-cyclical, which is true in general unless workers become so likely to leave in a boom that increasing their wage no longer affects their job-to-job transition rate. I find that high-wage workers are those for which this elasticity is most likely to be pro-cyclical, an observation that turns out to be important to understand what workers are most exposed to sectoral productivity shocks.

Wages also tend to respond more to negative productivity shock when they are sectoral than firm-level. In my model, the growth rate of wages is positive on average because new hires start with a relatively low wage. After a negative productivity shock, most workers thus do not experience wage cuts but a lower wage growth. The response of wages to sectoral shocks is amplified because firms can reduce wage growth even more when the competition for workers cools down.

With these intuitions in mind, I bring the model to the data to quantify the pass-through of sectoral shocks to wages and the cyclicality of income risk across workers. I calibrate the model using moments on firm and sectoral productivity shocks as well as labor market flows estimated on the matched employer-employee data. I focus on moments that are informative about the retention elasticity. I calibrate the degree of commitment power of firms using estimates of firing costs, which are large in France. I find that the model accounts well for the differential response of wages and job-to-job transitions to firm-level and sectoral productivity shocks, which are not targeted in the calibration. Remarkably, the model generates about 40% of the cross-sectional dispersion in wage growth observed in the data, which is significantly more than what observable worker characteristics can explain.

With this quantitative model, I first measure the direct contribution of sectoral productivity shocks to earnings risk. I find that sectoral shocks account on average for 10% of the risk borne by workers in the model, compared to 40% for firm-level productivity shocks and 50% for the risk of being matched to a low-productivity firm. Sectoral productivity shocks disproportionately affect workers in high productivity matches whereas firm-level shocks are much more important for workers in low productivity matches. To understand this heterogeneity, it is useful to remember that the retention elasticity is the critical determinant of the pass-through. The retention elasticity is more cyclical for high wage workers, who tend to work in high-productivity firms. As a result, workers in high-productivity matches are more exposed to sectoral productivity shocks than they are to firm-level shocks. The opposite is true for workers in low-productivity matches.

I then study the cyclicality of income risk by evaluating whether the pass-through of firm-level productivity shocks changes over the cycle. I find that in my baseline calibration the pass-through of firm-level shocks is roughly constant over the cycle, which is consistent with my estimates from the data. High firing costs are critical for this result because they imply that firms have a significant amount of commitment power. Even if firms make losses, they would still rather keep workers at their pre-existing wage than walk-away from the deal and pay the firing costs. I compute a counterfactual in which I reduce firing costs to a much lower level consistent with the U.S. The pass-through of firm-level shocks increases significantly and becomes counter-cyclical: it is higher in downturns than in booms by 15%. Because workers benefit from less insurance, welfare falls by 1%. This result shows that firing costs can increase welfare when contracts are endogenous and firms have limited commitment. This is in stark contrast with previous work on firing costs, which emphasized their ambiguous effects on firing and hiring (Bentolila and Bertola, 1990) and their perverse effect on the reallocation of workers towards more productive firms (Hopenhayn and Rogerson, 1993).

Finally, I show that the impact of sectoral productivity shocks is especially strong for workers who transit through unemployment because they lose the insurance from wage contracts. Continuously employed workers experience a smooth adjustment in their wages after a sectoral shock, while the wage of new hires out of unemployment changes much more drastically, consistent with existing work on new hires and incumbent workers (Rudanko, 2009, Kudlyak, 2014, Basu and House, 2016). As a result, when a worker loses her job during a recession she starts her new job at a much lower wage. This differential exposure of continuously employed workers and workers who experience unemployment spells generates fluctuations in income inequality over the cycle. In a recession, the income distribution expands at the bottom because new hires start with a much lower wage relative to existing workers. In a boom, the bottom of the income distribution shifts up and income inequality falls. These changes are substantial: the cross-sectional dispersion in log wages is 10% larger in downturns than in booms.

An important assumption that I make in the baseline model, similar to most of the literature, is that workers have no access to financial markets and consume their wage. In reality, workers have access to alternative forms of insurance, such as credit card debt, that can interact with the insurance provided by firms through wage contracts. To illustrate this point, I characterize a 2-period version of the model in which workers can trade risk-free bonds. I find that firms actively make workers borrow because it allows them to backload wages more to retain workers while smoothing consumption at the same time. This means that access to financial markets increases the ability of firms to retain workers and makes job-to-job transitions fall, a result consistent with Stevens (2004). When they set the wage of workers, and effectively pin down how much they borrow, firms take into account that borrowing is risky because workers might end up in the future with a lot of debt and very little income to pay for it, for example if they become unemployed. This precautionary savings motive is the new force that limits the degree of backloading in wage contracts. When trades in risk-free bonds are private information to workers, firms also use the pass-through of productivity shocks to wages to manipulate the risk that workers face and influence their savings decision. In ongoing work, I extend this problem to a dynamic setting and evaluate whether the degree of wage backloading and the pass-through change significantly when workers have access to realistic asset markets.

Related literature My paper relates to the literature studying how firms compete for workers over the business cycle. These models have been used to explain the cyclicality of labor market flows (Menzio and Shi, 2011, Moscarini and Postel-Vinay, 2016b, Schaal, 2017, Carrillo-Tudela, Clymo and Coles, 2021), to study the reallocation of workers towards more productive firms (Moscarini and Postel-Vinay, 2013, Coles and Mortensen, 2016, Lise and Robin, 2017, Acabbi, Alati and Mazzone, 2022) and to create a theory of the Phillips curve consistent with recent evidence on wage growth and worker mobility (Moscarini and Postel-Vinay, 2022). I study how firms insure workers against risk from and over cycles generated by sectoral productivity shocks. To do this I depart from most of the literature by introducing risk-averse workers[1] and I consider a rich contracting environment. I build on the directed search model of Menzio and Shi (2011) because Block Recursivity makes it tractable to study sectoral shocks.

This paper also relates to the literature studying how firms and workers share risks that make profits fluctuate. The most closely related paper in this literature is Balke and Lamadon (2022) who use a dynamic contracting model with directed search to evaluate whether firms insure workers from firm-level and worker-level productivity shocks. My paper differ in that it focuses on the risk that workers face over and from the cycle. I also focus on different contracting frictions. In particular I assume that the preference of workers for job mobility is private information, that firms have limited commitment (and in an extension that workers can trade risk-free bonds). My model is consistent with recent evidence that job-to-job transitions are an important driver of wage growth over the cycle (Moscarini and Postel-Vinay, 2016a, 2017, Karahan, Michaels, Pugsley, Sahin and Schuh, 2017) and can talk to the large empirical literature studying the pass-through of productivity shocks to wages (e.g. Guvenen, Schulhofer-Wohl, Song and Yogo, 2017, Guiso and Pistaferri, 2019, Chan, Salgado and Xu, 2020). In particular, my pass-through formulas show the critical role of the retention elasticity, a parameter that has been estimated in Kline, Petkova, Williams and Zidar (2019) and Dube, Giuliano and Leonard (2019).

My analytical results add to the literature studying the properties of dynamic wage contracts with job-to-job mobility. Building on the work of Burdett and Mortensen (1998), these papers sought to characterize the optimal hiring and retention policy of firms and the implications for wage dispersion in equilibrium. Burdett and Coles (2003) study dynamic wage contracts with risk-averse workers and random search in a model with constant productivity within matches. Shi (2009) later extends this analysis to a model with directed search. I prolong this work by characterizing the transmission of various productivity shocks to wages in a similar environment. I do it with a continuous time formulation of this problem, using recent methods introduced by Sannikov (2008) to write the contract recursively in a stochastic environment, and using a new first-order approximation in the degree of job-to-job mobility. Finally, I study the implications of risk-free bonds with and without hidden trade and show that it alters the allocation and trade-off substantially. In doing so I extend work by Stevens (2004) who studied optimal wage contracts when financial markets are complete.

[1] One recent exception is Acabbi et al. (2022) who use a model with risk-averse workers similar to mine. They focus on the persistent effects of recessions when workers have heterogeneous human capital.

Layout The paper starts in section 2 with motivating evidence on the response of wages and job-to-job mobility to firm-level and sectoral productivity shocks. Section 3 presents the model environment, and I characterize the contract in section 4. Section 5 brings the model to the data and quantify the risk faced by workers from and over the cycle. In section 6, I consider trade in risk-free bonds. Proofs are in the appendix.

2 Motivating evidence

I start by I documenting using matched employer-employee data that wages and job-to-job transitions respond very differently to firm-level and setcoral productivity shocks. I will use these facts as testable implications of my model.

2.1 Matched employer-employee data from France

I use administrative data from France between 2008 and 2019 to discipline my analysis. I combine annual data on firm balance sheet with a panel of worker from social security data containing 1/12th of the French labor force. Using administrative data is critical for my analysis because I estimate the response of wages and job-to-job mobility decisions at the individual level to changes in firm and sectoral productivity.

I focus on a sample of workers with relatively strong attachment to labor markets and for which I can measure job-to-job mobility accurately. Specifically, I only keep in the sample workers with permanent full time contracts, and prime age workers (25-55 years old). I focus on private sector jobs in for-profit firms with at least 3 employees. Appendix A provides more details on the sample selection and data construction and summary statistics on the population of interest. I end up with about 530,000 workers and 130,000 firms per year.

I measure labor productivity using value added per worker, controlling for the cost of capital. I measure the cost of capital as the product of tangible assets and interest rates plus depreciation rates, where interest rates are estimated from the balance sheet data and depreciation rates are estimated at the annual-sector level using national accounts data. I model labor productivity y_{jst} at firm j in sector s and at time t as

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst}$$

where a_t is an aggregate component, z_{st} a sectoral component and x_{jst} a firm-level component. I first residualize $\log y_{jst}$ on time dummies to extract the common component and on firm age dummies to control for the life cycle of firms, which is not in the model. I then measure the sectoral component $\log z_{st}$ as the average productivity across firms within a sector and compute the firm component $\log x_{jst}$ as the residual.

I measure wages as annual labor earnings divided by the number of days worked. Given that I consider a sample of relatively stable workers, changes in hours within the day are unlikely to be large. Labor earnings are net of payroll taxes but before income taxes and they include all types of compensations, including bonuses and payment in kinds, but excludes stock options. Unlike in the U.S., medical insurance is not an important part of pay in France. I residualize the log of labor earnings on observable worker and firm characteristics, such as occupation, industry or location dummies, a gender dummy, a polynomial in experience and dummies for firm age.

I then compute the growth rate of earnings and productivity and remove outliers at the bottom and top 0.5% of the distribution each year.

2.2 The differential response of wages and job-to-job transitions to firm and sectoral productivity shocks

I measure the response of wages and job-to-job mobility as the percent change in wages, and the percentage point change in job-to-job transition rate after a 100% increase in productivity. These responses are estimated using standard estimators from Guiso et al. (2005).

Define the growth rate of residualized wages for worker i in firm j sector s and between year $t-1$ and t as $\Delta \log w_{ijst}$ and define the growth rate of firm and sectoral productivity as $\Delta \log x_{jst}$ and $\Delta \log z_{st}$.

The response of wages to firm-level and sectoral productivity shocks are defined as

$$\theta^{w,y} = \frac{\mathrm{Cov}(\Delta \log w_{ijst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})}{\mathrm{Cov}(\Delta \log y_{jst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})} \tag{1}$$

where $y \in \{x, z\}$ denotes firm-level or sectoral productivity and where $\sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau}$ is the 3-year cumulative sum of productivity growth. In a model with permanent productivity shocks and static pass-through as in Guiso et al. (2005), this estimator recovers the true pass-through of productivity shocks to wages. It can be computed from a regression of wage growth on productivity growth, using the 3-year cumulative sum as an instrument for productivity growth. This instrument allows to filter out the effect of transitory changes in productivity, which I am going to interpret as measurement errors. In my model the pass-through is not static and shocks not permanent so these coefficients

	Wages	job-to-job transition rate
Firm productivity shock	4.6% (0.61%)	- 1.7pp (0.89pp)
Sectoral productivity shock	18.5% (3.5%)	4.0pp (7.5pp)

Note: response of wages growth and job-to-job transition rates to growth in firm and sectoral productivity, estimated using equation (1). Standard errors are shown in parenthesis and are estimated by Block Bootstrap in which firms are re-sampled. Number of observations is approximately 5530,000 workers per year for 12 years.

Table 1: Estimated response of wages and job-to-job transitions to productivity shocks in the data

do not measure the pass-through directly. I will therefore treat them as auxiliary statistics used to compare my model with the data. I find it useful to report these statistics since they have been extensively documented in the literature.

I measure the response of job-to-job transitions using an indicator variable $\mathrm{J2J}_{ijst}$ equal to 1 if worker i leaves firm j during a job-to-job transition in year t. I then recover $\theta^{\mathrm{J2J},x}$ and $\theta^{\mathrm{J2J},z}$ using similar estimators than (1).

Estimation results are shown in table 1 with standard errors in parenthesis. I find that wages and job-to-job transitions respond very differently to firm-level and sectoral productivity shocks. Wages respond almost 4 times more to sectoral productivity shocks than to firm-level shocks, while job-to-job transitions fall after a positive firm-level shock and increase after a positive sectoral shock. The model that I use in this paper captures this differential response.

3 Model

I now present a model of business cycles with frictional labor markets and job-to-job mobility with directed search, dynamic wage contracts subject to rich contracting frictions and firm-level and sectoral productivity shocks. The model builds on Menzio and Shi (2011) and is closely related to Balke and Lamadon (2022), even though it considers different types of shocks and different contracting frictions.

3.1 Environment

Time is discrete and runs forever at interval Δt. In the quantitative analysis in section 5 I set $\Delta t = 1$ quarter and when I characterize the contract in section 4 I take the continuous time limit $\Delta t \to 0$. The setup in continuous time can be found in the appendix.

Agents This is a small open economy model of a sector.

A continuum of ex-ante homogeneous workers can either be employed or unemployed. Workers have no access to financial markets so they consume their wage w when employed, and home production b when unemployed. I relax this assumption in section 7. They have utility function $u(w)$ with discount rate β.

Firm are owned by outside investors with discount rate β. They produce a unique homogeneous good using the constant returns to scale production function $\overline{x}\exp(x_t + z_t)$ with firm specific shocks x_t and aggregate shocks z_t following mean reverting processes

$$x_t = (1-\alpha_x)x_{t-1} + \sigma_x \nu_{xt} \quad \text{and} \quad z_t = (1-\alpha_z)z_{t-1} + \sigma_z \nu_{zt}$$

where ν_{xt} and ν_{zt} are iid innovations with standard normal distribution. $\overline{x}$ is firm average productivity drawn at the start of the match, independently across firms and constant over time. $1-\alpha_x$ and $1-\alpha_z$ parametrize the persistence of shocks.

Timing Each period,

a) Firm-level shocks x_t and sectoral productivity shocks z_t are realized

b) Firms produce and pay current wage; workers consume

c) Job mobility phase: preference shocks ξ_t are realized; employed and unemployed workers apply for jobs; firms post vacancies; new matches are formed and new contracts are signed

d) Quits and exogenous separations into unemployment occur

Search and matching Search is directed: there is a continuum of labor markets indexed by the promise value to a worker denoted v. Workers choose in which labor market v to apply, and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets.

Firms post vacancies in these labor markets, and learn about their productivity x and $\overline{x}$ only after they match.

In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u(v,z) + \kappa\phi_e(v,z), \phi_f(v,z))$ turns workers searching for a job and vacancies into matches. $\phi_u(v,z)$ and $\phi_e(v,z)$ denote mass of unemployed and employed workers searching for a job while $\phi_f(v,z)$ denotes the mass of vacancies posted by firms. κ is the search intensity of employed workers relative to unemployed workers.

In the notation above, I use the result that only sectoral productivity z is an aggregate state in this economy so that policies need not be indexed by time or other aggregate states. This is a consequence of Block Recursivity (Menzio and Shi (2010, 2011)).

Define the job finding rate $\tilde{\lambda}_w(v,z)$ as the probability that an unemployed worker finds a job, and the vacancy fulling rate $\tilde{\lambda}_f(v,z)$ as the probability that a vacancy finds a worker

$$\tilde{\lambda}_w(\phi_e(v,z),\phi_u(v,z),\phi_f(v,z)) \equiv \frac{\mathcal{M}(\kappa\phi_e(v,z)+\phi_u(v,z),\phi_f(v,z))}{\kappa\phi_e(v,z)+\phi_u(v,z)}$$

$$\tilde{\lambda}_f(\phi_e(v,z),\phi_u(v,z),\phi_f(v,z)) \equiv \frac{\mathcal{M}(\kappa\phi_e(v,z)+\phi_u(v,z),\phi_f(v,z))}{\phi_f(v,z)}$$

In equilibrium, we can rewrite the matching probabilities as functions of v and z

$$\lambda_w(v,z) \equiv \tilde{\lambda}_w(\phi_e(v,z),\phi_u(v,z),\phi_f(v,z) \quad \text{and} \quad \lambda_f(v,z) \equiv \tilde{\lambda}_f(\phi_e(v,z),\phi_u(v,z),\phi_f(v,z))$$

I assume that the probability that a worker finds a job is at most 1, i.e. $\lambda_w(v,z) \leq 1$.

Unemployed workers Unemployed workers consume endowment b and choose in which labor market v to apply.

Given $\lambda_w(v,z)$, the value of unemployment workers satisfies

$$U(z_t) = u(b) + \beta \max_{\hat{v}} \left[\lambda_w(v,z_t)v + (1-\lambda_w(\hat{v},z_t))\mathbb{E}_{z_{t+1}}\left[U(z_{t+1})|z_t\right]\right]$$

Workers choose in which labor market to apply v. Applying to a high-v labor market brings a higher conditional on a match, but matches occur with lower probability because $\lambda_w(v,z_t)$ decreases in v in equilibrium. All unemployed workers apply to the same labor market because they share the same state z_t. Denote their search policy $v_u(z_t)$.

Employed workers Workers also search for new jobs while employed. They decide in which labor market v to apply and receive preference shocks ξ_t if they are successful at finding a job, irrespective of where they find a job. This preference shock is iid over time and across workers with distribution $\mathcal{N}(0,\sigma_\xi^2)$. It is meant to capture non-monetary reasons why workers change jobs in a tractable way, and will help the quantitative model match the large number of job transitions with negative wage changes.

Matches also separate into unemployment for two reasons. First, with exogenous probability δ a match is dissolved. Second, workers can quit voluntarily.

Contracts When they are matched, firms and employed workers sign wage contracts, defined as history-dependent paths for wages. The contract is subject to limited commitment by workers and firms, and imperfect information about the worker search decisions and preferences about job mobility.

Specifically, workers cannot commit not to take a job offer they would benefit from if they receive one, nor could they commit not to quit into unemployment if they are better off doing so. Both the choice of labor market v in which they search and the preference shocks ξ they would receive if they changed jobs are private information to workers. This leads to a moral hazard problem with both hidden action and hidden state. Productivity shocks x and z are public information.

When I describe the contract, I use the time subscript t to denote tenure, with $t = 0$ the period where they match and $t = 1$ the first period of production. For simplicity of notations, I abstract from randomization.

An important restriction that I make on contracts is that they cannot be conditioned on reported offers received by workers. This prevents incumbent firms from making counter-offers to their workers when they receive another job offer. This assumption on contract incompleteness is standard in the literature, and sometimes justified on the ground that outside offers are not observable to the incumbent firm. I make this restriction to keep the problem tractable and focus in this paper on the strategy that firms follow to preempt workers from getting an offer in the first place.

The worker reports to firms a value for the preference shock, denoted $\hat{\xi}_t$, just after the true preference shock ξ_t is realized. The contract specifies a wage w for each history of productivity x, z, each history of reported preference shocks $\hat{\xi}$ and conditional on the firm average productivity $\overline{x}$ as long as the match continues

$$w_t(\{\overline{x}, x_s, z_s, \hat{\xi}_{s-1}; 1 \leq s \leq t\})$$

where wages w_t do depend on contemporaneous reported preference shocks $\hat{\xi}_t$ because they are paid before preference shocks are realized.

Given the contract w, workers choose a search strategy v, a reporting strategy $\hat{\xi}$ and a quit strategy q for each history of productivity x, z and preference shocks ξ and for each average firm productivity $\overline{x}$

$$v_t(\{\overline{x}, x_s, z_s, \xi_s; 1 \leq s \leq t\}), \quad \hat{\xi}_t(\{\overline{x}, x_s, z_s, \xi_s; 1 \leq s \leq t\}) \quad q_t(\{\overline{x}, x_s, z_s, \xi_s; 1 \leq s \leq t\})$$

to maximize their utility.

Firms also have limited commitment. I assume that they cannot sign contracts giving them a value lower than $-\Phi$ after any history, where Φ is a parameter that measures the degree of firm commitment and that I will calibrate using estimates of firing costs. If firms were to sign such contracts, they would rather walk away ex-post by laying off workers and paying the firing cost. I justify this assumption in more details in section 5 where I discuss the consequences of reducing firing costs. When $\Phi = 0$, firms have no commitment at all, whereas firms have full commitment when $\Phi = \infty$.

Note that this contract can achieve efficient separations by setting the wage sufficiently low after some histories and letting the worker quit on her own.

Denote the state for productivity as $s_t = \{\overline{x}, x_t, z_t\}$ with corresponding history s^t.

Define the worker value after history $\{s^t, \xi^{t-1}\}$ given contract w and given search, reporting and quit strategies $v, \hat{\xi}, q$ as

$$\begin{aligned} V_t(s^t, \xi^{t-1}; w)[v, \hat{\xi}, q] &= u(w_t(s^t, \hat{\xi}^{t-1}(s^{t-1}, \xi^{t-1}))) \\ &+\beta \mathbb{E}_{\xi_t}\left[\kappa \lambda_w(v_t(s^t, \xi^t), z_t)\left(v_t(s^t, \xi^t) + \xi_t\right) + (1 - \kappa\lambda_w(v_t(s^t, \xi^t), z_t)) W_{t+1}(s^t, \xi^t; w)[v, \hat{\xi}, q]\right] \end{aligned}$$

where $W_{t+1}(s^t, \xi^t; w)[v, \hat{\xi}, q]$ is the worker expected continuation value after history $\{s^t, \xi^t\}$

$$\begin{aligned} W_{t+1}(s^t, \xi^t; w)[v, \hat{\xi}, q] &= \left(\delta + (1-\delta) q(s^t, \xi^t)\right) \mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] \\ &+(1-\delta)(1 - q(s^t, \xi^t)) \mathbb{E}_{x_{t+1}, z_{t+1}}\left[V_{t+1}(s^{t+1}, \xi^t; w)[v, \hat{\xi}, q] | x_t, z_t\right] \end{aligned}$$

In these expressions, expectations over ξ_t are unconditional because preference shocks are i.i.d. The value of a worker depends on wages today and the continuation value, which depends on the probability of a job-to-job transition, the value of being unemployed and the value of continuing within the same job at $t + 1$.

The value of the worker at $t = 0$ from contract w given strategies $v, \hat{\xi}, q$ is

$$V_0(z_0; w)[v, \hat{\xi}, q] = \mathbb{E}_{x_1, \overline{x}, z_1}\left[V_1(s^1; w)[v, \xi, q] | z_0\right]$$

where I dropped ξ^0 from the arguments of V_1 because $\xi^0 = \emptyset$.

Define the firm value after history $\{s^t, \xi^{t-1}\}$ given contract w and for given search, reporting and quit strategies $v, \hat{\xi}, q$ as

$$\begin{aligned} F_t(s^t, \xi^{t-1}; w)[v, \hat{\xi}, q] &= \overline{x} \exp(x_t + z_t) - w_t(s^t, \hat{\xi}^{t-1}(s^{t-1}, \xi^{t-1})) \\ &+\beta \mathbb{E}_{\xi_t}\left[(1 - \kappa\lambda_w(v_t(s^t, \xi^t), z_t))(1-\delta)(1 - q(s^t, \xi^t)) \mathbb{E}_{x_{t+1} z_{t+1}}\left[F_{t+1}(s^{t+1}, \xi^t; w)[v, \hat{\xi}, q] | x_t, z_t\right]\right] \end{aligned}$$

We are now ready to define an optimal contract. I say that a contract is incentive compatible given $\hat{\xi}$ if search and quit decisions are optimal for workers

$$v, q = \arg\max_{\tilde{v},\tilde{q}} \; V_0(z_0; w)[\tilde{v}, \hat{\xi}, \tilde{q}]$$

By the revelation principle, I focus on contracts in which workers reveal their preference shocks truthfully, i.e. $\hat{\xi}_t(s^t, \xi^t) = \xi_t$ for all t. I say that an incentive compatible contract is truth-telling if

$$\max_{v,q} V_0(z_0; w)[v, \xi, q] \geq \max_{v,q} V_0(z_0; w)[v, \hat{\xi}, q] \quad \forall \hat{\xi}$$

Given a promised value to workers V_0, sector productivity z_0, the value of unemployment $U(z)$ and the job finding rate $\lambda(v, z)$, the optimal contract solves

$$\Pi(V_0, z_0) = \max_{w} \; \mathbb{E}_{x_1, \overline{x}, z_1} \left[F_1(s^1; w)[v, \xi, q] | z_0 \right]$$

subject to

$$\begin{aligned}
&\text{(PK)}: && V_0 \leq V_0(z_0; w)[v, \xi, q] \\
&\text{(IC)}: && v, \xi, q = \arg\max\nolimits_{\tilde{v},\tilde{\xi},\tilde{q}} \; V_0(z_0; w)[\tilde{v}, \tilde{\xi}, \tilde{q}] \\
&\text{(PC-F)}: && F_t(s^t, \xi^{t-1}; w)[v, \xi, q] \geq -\Phi \qquad \forall s^t, \xi^{t-1}
\end{aligned}$$

The optimal contract maximizes the present value of profits subject to a promise keeping constraint, an incentive compatibility constraint and a participation constraint of firms. $\Pi(V_0, z_0)$ denotes the value of a firm matched with a worker when sector productivity was z_0. The firm draws its average productivity $\overline{x}$ from distribution $\log \overline{x} \sim \mathcal{N}(0, \sigma_{\overline{x}}^2)$, and the initial value of its productivity x_1 from distribution $x_1 \sim \mathcal{N}(0, \sigma_x^2)$. The (IC) constraint states that the contract is truth-telling and incentive compatible.

Free entry Firms are subject to a free entry condition. They have to pay a unit cost for posting a vacancy k_1 and if they are successful at matching a worker, firms have to pay a training cost k_2. Training costs and vacancy posting costs are important quantitatively because they have different implications for the job finding rate. The vacancy posting cost influences how many vacancies are posting in a labor market, whereas the training costs also influences which labor markets are active.

The free entry condition states

$$-k_1 + \lambda_f(v, z_0)\beta \left(\Pi(v, z_0) - k_2 \right) \leq 0$$

with equality for each active market v. This condition implies that the vacancy filling rate

$\lambda_f(v, z)$ is increasing in v in equilibrium, and from the matching function this implies that the job finding rate $\lambda_w(v, z)$ is decreasing in v.

3.2 Recursive formulation of the contract

I now state the optimal contract recursively as is standard in the literature using the worker's promised value as a state variable. Preference shocks of workers ξ make this step not standard compared to existing models (Balke and Lamadon (2022), Menzio and Shi (2011)) but lemma 1 shows that it is easy to handle.

I prove that in any truth-telling contract ($\hat{\xi}_t(s^t, \xi^t) = \xi_t$), the worker expected continuation value $W_{t+1}(s^t, \xi^t; w)[v, \xi, q]$ after any history $\{s^t, \xi^t\}$ is independent of current preference shock ξ_t. Critical to this result are the assumptions that preference shocks ξ_t are i.i.d. and are realized after wages are paid.

Lemma 1. *An incentive compatible contract w is truth-telling if and only if the worker expected continuation value is independent of current preference shocks*

$$W_{t+1}(s^t, \xi^t; w)[v, \xi, q] = W_{t+1}(s^t, \xi^{t-1}; w)[v, \xi, q] \quad \forall \xi^{t-1}, \xi_t$$

where $v, q = \arg\max_{\tilde{v}, \tilde{q}} \; V_0(z_0; w)[\tilde{v}, \xi, \tilde{q}]$.

Proof. See appendix B.1. The intuition is that firms cannot price discriminate between workers with different preference shocks ξ_t when they are i.i.d. because all workers evaluate future paths for ξ with the same conditional expectation. □

Lemma 1 shows that in any contract in which workers reveal their preference shocks truthfully, expected continuation values cannot depend on current preference shocks. This means that firms have no incentives to make continuation values and wages depend on preference shocks, since it does not help them retain workers and it only generates risk. With this result in hand, it is standard to write the contracting problem recursively.

The value from a match $\Pi(v, z_0)$ in labor market v given z_0 solves

$$\begin{aligned} \Pi(v, z_0) = \quad & \max_{V_1(s_1)} \mathbb{E}_{\overline{x}, x_1, z_1} \left[F(V_1(s_1), s_1) | z_0 \right] \\ \text{s.t.} \quad & \mathbb{E}_{\overline{x}, x_1, z_1} \left[V_1(s_1) | z_0 \right] = v \end{aligned}$$

where the value of a firm $F(V, s_t)$ satisfies

$$\begin{aligned} F(V, s_t) = \quad & \max_{w, V(s_{t+1})} \overline{x} \exp(x_t + z_t) - w \\ & + \beta \left(1 - \mathbb{E}_{\xi_t} \left[\kappa \lambda_w(v(\xi_t), z_t) \right] \right) (1 - \delta)(1 - q) \mathbb{E}_{x_{t+1}, z_{t+1}} \left[F(V(s_{t+1}), s_{t+1}) | x_t, z_t \right] \end{aligned}$$

subject to

$$
\begin{array}{ll}
\text{(PK)}: & V \leq u(w) + \beta \left(W_{t+1} + \mathbb{E}_{\xi_t} \left[\kappa \lambda_w(v(\xi_t), z_t) \left(v(\xi_t) + \xi_t - W_{t+1} \right) \right] \right) \\
\text{(IC-v)}: & v(\xi_t) = \arg\max_{\tilde{v}} \; \lambda_w(\tilde{v}, z_t) \left(\tilde{v} + \xi_t - W_{t+1} \right) \\
\text{(IC-q)}: & q = 1 \quad \text{if} \quad \mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] \geq \mathbb{E}_{x_{t+1}, z_{t+1}} \left[V(s_{t+1}) | x_t, z_t \right] \\
\text{(PC-F)}: & F(V(s_{t+1}), s_{t+1}) \geq -\Phi
\end{array}
$$

where $W_{t+1} = (\delta + (1-\delta)q)\, \mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] + (1-\delta)(1-q)\mathbb{E}_{x_{t+1}, z_{t+1}} \left[V(s_{t+1}) | x_t, z_t \right]$.

The choice variables are now the current wage, and the continuation values contingent on the realization of productivity shocks tomorrow.

The first constraint (PK) is the promise keeping constraint, stating that the value the worker gets from that contract either through wage or future values must meet the value she was promised initially.

The second constraint (IC-v) is the incentive compatibility constraint for the search strategy v of workers. It defines the search strategy that workers choose as a function of the preference shock ξ_t and the expected continuation value W_{t+1} at the current job. Workers with high preference shocks ξ_t apply in labor markets with low values v and high job finding rate $\lambda(v, z)$ because they really value getting a new job. Conversely, workers with low ξ_t apply in labor markets with high values v.

The third constraint (IC-q) is the incentive compatibility constraint for quits into unemployment. It states that workers will quit if they are better off unemployed than employed.

The last constraint (PC-F) is the participation constraint of firms, stating that they cannot be better off walking away from the contract and laying off workers after any history.

When they compute the probability that a match continues and the continuation value of workers, firms take expectations over the realization of the preference shock ξ_t. They know that workers with a high preference shocks ξ_t are more likely to leave but cannot use future wages to retain these workers because of lemma 1.

By changing the continuation value of workers at $t+1$, $V(s_{t+1})$, firms influence not only future profits but also the quit and search decisions of workers q and v today and therefore the retention probability $\left(1 - \mathbb{E}_{\xi_t} \left[\kappa \lambda_w(v(\xi_t), z_t) \right]\right) (1-\delta)(1-q)$. This leads to a trade-off in which promising workers more value in the future reduces expected profits but increases the probability that the match continues. In the next section, I characterize the optimal choice of firms, how it translates into changes in wages and how it depends on the aggregate state variable z_t.

3.3 Definition of an equilibrium

A recursive equilibrium is a set of value functions, policies and matching rates for each labor market v such that

- the firm policies and worker search policy satisfy the optimal contract
- the free entry condition is satisfied
- the job finding and vacancy filling rates are consistent with the matching function

Define the retention probability as

$$p(V_t, s_t) = \left(1 - \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(V_t, s_t, \xi_t), z_t)\right]\right)(1-\delta)(1-q(V_t, s_t))$$

and denote the distribution of unemployed and employed workers as $D_t^u, D_t(V, \overline{x}, x)$.

The resulting equilibrium outcome path is a sequence of distributions such that the distribution of employed workers satisfies

$$\begin{aligned} D_t(V_t,\overline{x},x_t) = &\int_{V_{t-1}}\int_{x_{t-1}} D_{t-1}(V_{t-1},\overline{x},x_{t-1})p(V_{t-1},s_{t-1})\pi_x(x_t|x_{t-1})\mathbb{1}\left\{V_t = V(V_{t-1},s_{t-1},s_t)\right\}dx_{t-1}dV_{t-1} \\ &+D^u_{t-1}\lambda(v_u(z_{t-1}),z_{t-1})\pi_{x_1}(x_t)\pi_{\overline{x}}(\overline{x})\mathbb{1}\left\{V_t = V_1(v_u(z_{t-1}),z_{t-1},s_t)\right\} \\ &+\int_{V_{t-1}}\int_{x_{t-1}}\int_{\overline{x}}\int_{\xi_{t-1}} D_{t-1}(V_{t-1},\overline{x},x_{t-1})\kappa\lambda(v(V_{t-1},s_{t-1},\xi_{t-1}),z_{t-1}) \times \\ &\qquad \times \pi_\xi(\xi_{t-1})\pi_{\overline{x}}(\overline{x})\pi_{x_1}(x_t)\mathbb{1}\left\{V_t = V_1(v(V_{t-1},s_{t-1},\xi_{t-1}),z_{t-1},s_t)\right\}d\xi_{t-1}d\overline{x}dx_{t-1}dV_{t-1} \end{aligned}$$

and the distribution of unemployed workers

$$\begin{aligned} D^u_t = &D^u_{t-1}\left(1-\lambda(v_u(z_{t-1}),z_{t-1})\right) + \int_{V_{t-1}}\int_{x_{t-1}}\int_{\overline{x}}\left(1-\mathbb{E}_{\xi_{t-1}}\left[\kappa\lambda_w(v(V_{t-1},s_{t-1},\xi_{t-1}),z_{t-1})\right]\right) \times \\ &\qquad \times \left(\delta + (1-\delta)q(V_{t-1},s_{t-1})\right)D_{t-1}(V_{t-1},\overline{x},x_t)d\overline{x}dx_{t-1}dV_{t-1} \end{aligned}$$

where $\pi_x(x_t|x_{t-1})$ is the probability of x_1 given x_{t-1}, $\pi_{x_1}(x_t)$ is the probability that firm productivity is x_t during the first period of production and $\pi_{\overline{x}}(\overline{x})$ is the probability that fixed productivity is $\overline{x}$. The first term contributing to $D_t(V_t, \overline{x}, x_t)$ represents stayers, the second represents new hires from unemployment and the third new hires from employment. The first term contributing to $D_t(V_t, \overline{x}, x_t)$ represents unemployed workers who did not find a job, and the second term represents quits and exogenous separations.

These distributions are functions of time, not of sectoral productivity, because they depend on the entire history of shocks.

4 Characterizing the optimal contract

Before bringing this model to the data, I characterize optimal wage contracts. My main focus is on understanding the effects of job-to-job mobility on wage growth for new hires and on the pass-through of firm-level and sectoral productivity shocks to wages. For this reason I strip down the model to a simpler version in order to obtain clean analytical formulas. Specifically, I assume that firms have full commitment power ($\Phi \to \infty$) and have the same fixed heterogeneity $\overline{x} = 1$ and that workers cannot quit or fall into unemployment ($\delta = q = 0$) and have no preference shocks ($\sigma_\xi = 0$).

In order to derive analytical solutions, I use a continuous time formulation of the problem ($\Delta t \to 0$)[2]. The optimal contracting problem becomes

$$F(V_t, x_t, z_t) = \max_{w, \Delta_x, \Delta_z} \quad \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \int_0^t \kappa\lambda_w(v_s, z_s)ds\right)(\exp(x_t + z_t) - w_t)dt\right]$$

subject to

$$\begin{aligned}
\text{(PK)}: \quad & dV_t = (rV_t - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t))\,dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\
\text{(IC-v)}: \quad & v_t = v(V_t, z_t) \\
& dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \\
& dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}
\end{aligned}$$

where $r = 1/\beta - 1$ and the expectation is taken over the path of productivity x, z. As before, the search decision $v(V_t, z_t)$ solves the static problem

$$v(V_t, z_t) \in \arg\max_v \ \lambda_w(v, z_t)(v - V_t) \tag{2}$$

The policy functions in discrete time were $V(x_{t+1}, z_{t+1})$ whereas in continuous time they are the pass-through variables Δ_x, Δ_z, which measure how the worker value responds to firm-level and sectoral productivity shocks. The law of motion of the worker's value dV_t can be understood as a first-order approximation of the policy function in discrete time, evaluated at current state, $V(V_t, x_t, z_t, x_{t+1}, z_{t+1})$.

It will be convenient later to introduce the differential operator

$$\mathcal{D}f(x,z) \equiv -\alpha_x x f_x(x,z) - \alpha_z z f_z(x,z) + \frac{\sigma_x^2}{2} f_{xx}(x,z) + \frac{\sigma_z^2}{2} f_{zz}(x,z)$$

that I will use to define a function $f(x,z)$. For example, the present value of output

[2] The optimal contract in continuous time is derived from primitives in appendix B.2.1.

$g(x,z)$ satisfies $rg(x,z) = \exp(x+z) + \mathcal{D}g(x,z)$. The term $\mathcal{D}g(x,z)$ is an adjustment due to the mean-reversion and the volatility of shocks.

Approximate solution for $\kappa \to 0$ The optimal contract does not admit a closed-form solution because this problem is non-linear. To build some intuitions, I introduce a novel approximation that allows me to derive an analytical solution to this problem. Specifically, I take a first-order approximation of the contract in the search efficiency κ of employed workers. This approximation will also be useful when I characterize the pass-through of firm-level and sectoral productivity shocks in sections 4.2 and 4.3.

The optimal contract under this approximation is shown in the next lemma.

Lemma 2. *To first-order in κ, the firm value is given by*

$$F(V,x,z) = g(x,z) - h(V) - \kappa\ell(V,x,z)$$

where $g(x,z) = (\exp(x+z) + \mathcal{D}g(x,z))/r$ is the present value of output and $h(V) = u^{-1}(rV)/r$ is the cost of providing value V to workers with constant wage. $\ell(V,x,z)$ represents the cost of job-to-job mobility for firms and satisfies

$$r\ell(V,x,z) = \underbrace{\lambda_w(v(V,z),z)}_{\text{Prob. of J2J transition}} \left[\underbrace{g(x,z) - h(V)}_{\text{Lost profits}} - \underbrace{[v(V,z) - V]\, h'(V)}_{\text{Gains from job mobility}} \right] + \mathcal{D}\ell(V,x,z)$$

Proof. See appendix B.2.3. □

Lemma 2 describes the firm value given productivity x,z and a promised value to workers V. It depends on the profits generated by the firm, captured by $g(x,z)$, and the cost of paying wages to the worker $h(V)$. This cost is measured at constant wages because when there is no worker mobility ($\kappa = 0$), it is optimal for firms to provide perfect consumption smoothing to agents. The value of the firm also depends on the cost of job-to-job mobility because firms lose their profits when workers change jobs. However, firms might benefit from job-to-job mobility because it makes it easier to hire workers in the first place. In particular, the second term in $\ell(V,x,z)$ shows that firms benefit from job-to-job mobility because it allows them to provide worker with the same promised value V and pay them less. When workers switch jobs, they incur a utility gain of $v(V,z) - V$, which can be translated into units of output using $h'(V)$. The firm can then provide some of the promised value V through future job transitions, instead of future wages. For example, consulting jobs are often seen as a stepping stone for a career in management. As a result, consulting firms might find it easier to hire workers in the first place because they offer a high option value of future employment opportunities.

Even if firms benefit ex-ante from job mobility because it makes it easier to hire workers, ex-post it is often in the firm's interest to retain them. To see why, it is useful to compare the optimal search decision of workers with hidden information from (2)

$$\lambda_w(v, z_t) + \frac{\partial \lambda_w(v, z_t)}{\partial v}(v - V_t) = 0$$

to the search decision with full information when firms control the search decision

$$\lambda_w(v^{\mathrm{FI}}, z_t) + \frac{\partial \lambda_w(v^{\mathrm{FI}}, z_t)}{\partial v^{\mathrm{FI}}}\left(v^{\mathrm{FI}} - \left[V_t^{\mathrm{FI}} + F(V_t^{\mathrm{FI}}, x, z)u'(w_t^{\mathrm{FI}})\right]\right) = 0$$

The first term in these expressions measure the benefit of applying to a market with higher value v, whereas the second term measures the cost. With full information, firms take into account both the worker and firm values when they compute the surplus from job-to-job transitions $v^{\mathrm{FI}} - \left[V_t^{\mathrm{FI}} + F(V_t^{\mathrm{FI}}, x, z)u'(w_t^{\mathrm{FI}})\right]$. By contrast, with hidden information workers only take into their own gains $v - V_t$. Intuitively, both the worker and the firm want the worker to find a better job, but the worker wants it more. This asymmetry is the reason why firms use wage contracts to influence the mobility choices of workers.

The retention elasticity A critical determinant of the retention strategy of firms, and thereby wages, is the retention elasticity. I define it as the percentage point change in the retention probability induced by a 1% increase in the present value of wages

$$\epsilon(V, x, z) \equiv \underbrace{\frac{\partial\left(1 - \kappa\lambda_w(v(V, z), z)\right)}{\partial v(V, z)}}_{\text{Change in retention probability}} \times \underbrace{\frac{\partial v(V, z)}{\partial V}}_{\substack{\text{Change in} \\ \text{search decision}}} \times \underbrace{\frac{w(V, x, z)u'(w(V, x, z))}{r + \kappa\lambda(v(V, z), z)}}_{\substack{\text{Change in worker value after} \\ \text{1\% increase in PV of wages}}} \geq 0 \quad (3)$$

This elasticity is positive in equilibrium: paying workers more makes them search in labor markets with a higher value v and a lower job finding rate $\lambda_w(v, z)$. It is an endogenous variable that depends on the assumption of directed search, the costs that firms pay to hire workers and the average productivity of new entrants among other things. I find it to be a convenient and intuitive statistic to describe the trade-off that firms face because it measures the extent to which firms can influence worker mobility.

4.1 The tenure-profile of wages

I first establish that the wage of new hires grows until the firm valueor the retention elasticity reach 0.

The first proposition characterizes the path of wages for a worker on the job from the optimality condition of the firm.

Proposition 1. *The path of wages satisfies*

$$dw_t = (r + \kappa\lambda(v(V_t, z_t), z_t))\, F(V_t, x_t, z_t)\, \frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)} dt + 0 \times dB_{xt} + 0 \times dB_{zt}$$

where $\gamma(w) \equiv -wu''(w)/u'(w)$ *is the coefficient of relative risk aversion.*

Proof. See appendix B.2.3. This condition is derived by combining the optimality condition with respect to the wage w_t with the envelope condition. □

Proposition 1 is a generalization of lemma 3.2 in Shi (2009) to an environment with productivity shocks, and it is the continuous time analogue of proposition 2 in Balke and Lamadon (2022). This equation states wages are strictly increasing as long as both the firm value $F(V_t, x_t, z_t)$ and the retention elasticity $\epsilon(V_t, x_t, z_t)$ are strictly positive.

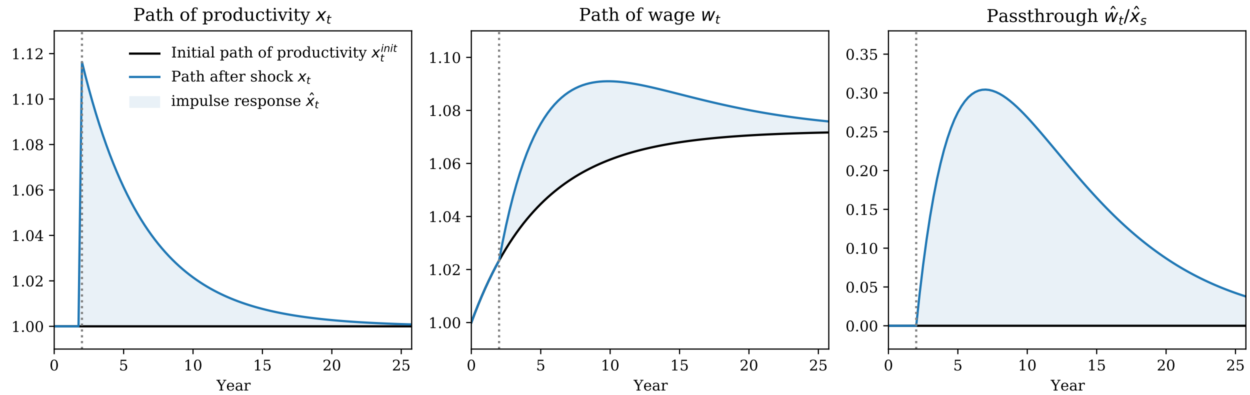
The intuitions behind this equation are well established. If the firm value $F(V_t, x_t, z_t)$ is positive, it is optimal for firms to increase the wage of workers to induce them to stay. The optimal strategy to do so is to backload wages, i.e. increase them with tenure. The wage continues to growth until the elasticity $\epsilon(V_t, x_t, z_t)$ reaches 0 or until the firm makes no profit anymore and is indifferent between retaining workers or letting them go. When the elasticity of inter-temporal substitution $1/\gamma$ is low, wages are less backloaded because workers dislike changes in consumption over time.

This equation shows that the wage increases over time if the worker starts at a value v such that the firm makes positive profits. From the free entry condition,

$$-k_1 + \lambda_f(v, z_t)\,[F(v, x_0, z_t) - k_2] \leq 0$$

it is easy to see that $F(v, x_0, z_t) = k_2 + k_1/\lambda_f(v, z_t) > 0$ for all active labor markets. Therefore, the wage of new hires grows over time. This observation will be important when I discuss the differential response of negative sectoral productivity shocks to wages in section 4.3.

Figure 1: Impulse responses from firm productivity shock



4.2 The pass-through of firm-level shocks

I now characterize the pass-through to firm-level productivity shocks to the wage and value of stayers. I first compute an approximate formula for the impulse response of wages w_t following a 1% increase in firm productivity and then derive the exact response of the worker value V_t. In both cases, I rely in the approximation $\kappa \to 0$.

The starting point to compute the pass-through to wages is proposition 1. Clearly, wages do not respond on impact to a change in productivity because the coefficient on the Brownian motion B_{xt} is zero. However, changes in productivity alter the path of wages through a change in the firm value $F(V, x, z)$ in the drift (the retention elasticity in the drift is constant conditional on w from equation (3)). An increase in productivity x raises the firm value $F(V, x, z)$ and thus increases the growth rate of wages going forward.

Solving for the path of wages in response to a productivity shock is difficult because this problem is non-linear. For this reason, I rely on my approximation as $\kappa \to 0$. In this case, the path of wages becomes

$$dw_t = (rg(x_t, z_t) - w_t) \frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)} dt$$

Consider a worker with some initial path for productivity and wages $x^{\text{init}}, w^{\text{init}}$. I then simulate a small unanticipated shock to productivity $\hat{x}_0$ at time $t = 0$ and compute the impulse response relative to this initial path: $\hat{x}_t \equiv x_t - x_t^{\text{init}}$ and $\hat{w}_t \equiv w_t - w_t^{\text{init}}$, where t here denotes the time since the shock occurred.

Given the law of motion of productivity, the impulse response of x_t is

$$\hat{x}_t = \exp(-\alpha_x t)\, \hat{x}_0 \quad \text{for } t \geq 0$$

The paths of firm productivity before and after the shock are shown in the left panel of figure 1. In the middle panel is shown the paths of wages. I use the example of a new hire whose wage is increasing over time. After the shock, wage growth accelerates and eventually overshoots before falling back to its stationary level. On the right panel, I show the pass-through of productivity shocks to wages, defined as the impulse response of wages normalized by the initial shock to firm productivity.

For a small shock, we can write the change in the wage as

$$\hat{w}_t \approx \int_0^t \left(rg_x(x_s, z_s)\hat{x}_s - \hat{w}_t\right) \frac{\epsilon_0}{\gamma_0} dt \tag{4}$$

where $\epsilon_0 \equiv \epsilon(V_0, x_0, z_0)$ and $\gamma_0 \equiv \gamma(w_0)$ denote the retention elasticity and the risk aversion coefficient at the time of the shock. This equation is an approximation of the true wage response because I abstract from changes in the ratio ϵ/γ over time.

In appendix B.2.3, I show that equation (4) can be solved in closed-form for $\hat{w}_t$ given the path for $\hat{x}_t$. The pass-through is then given by

$$\frac{\hat{w}_t}{\hat{x}_0} \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_x} g_x(x_0, z_0) \frac{\epsilon_0}{\gamma_0} \left[\exp\left(-\alpha_x t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0} t\right)\right] \geq 0 \tag{5}$$

This expression shows that the pass-through is back-loaded: wages do not respond on impact, and then changes for $t > 0$. When shocks are mean-reverting ($\alpha_x > 0$), the pass-through converges back to 0. When the shock is permanent ($\alpha_x = 0$), the pass-through converges towards $rg_x(x_0, z_0)$ and the worker eventually absorbs the entire shock. The speed and magnitude of the pass-through depends on the retention elasticity ϵ_0, the curvature aversion γ_0 and the persistence of shocks ρ_x. The term $g_x(x_0, z_0)$ captures how a change in productivity affects the present value of output.

This equation captures the trade-off that firms face between worker retention and insurance. A persistent increase in productivity makes firms eager to retain workers because profits go up. The more persistent this increase in productivity, the more firms want to retain workers. In order to retain workers, firms increase their wage to make their value go up and their job-to-job transition rate fall.

However, firms only increase the wage if it actually makes workers less likely to leave. In particular, a high retention elasticity $\epsilon(V, x, z)$ means that an increase in the wage leads to a sharp fall in the worker mobility rate. In this case, increasing the wage is a very effective strategy to retain workers and the pass-through is high.

Firms therefore want to pay workers more when productivity is high, and less when productivity is low. These ups and downs in wages generate risk for workers, who are risk averse. Furthermore, wage increases after positive productivity shocks are backloaded and therefore wages are not constant over time. Because of risk aversion and preference for smooth consumption over time, workers must be compensated for the pass-through. The degree of worker aversion to time and state varying wages increases in γ, so the optimal pass-through falls in γ.

I now provide a formula for the pass-through into the worker value Δ_x. Unlike the wage pass-through, this formula is exact but it still relies on the approximation $\kappa \to 0$.

Proposition 2. *To first-order in κ, the pass-through of a firm productivity shock x to the value of stayers satisfies*

$$\Delta_x(V,x,z) = (r+\alpha_x)^{-1}\left[g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)\right]$$

where $w(V) = u^{-1}(rV)$ is the wage when $\kappa = 0$.

Proof. See appendix B.2.3. To first-order in κ, the optimality condition for the value pass-through is $\Delta_x = -\ell_{Vx}(V,x,z)/h''(V)$. Rearranging the terms gives the result. □

To understand proposition 2, it is instructive to examine the relation between the value pass-through Δ_{xt} and the wage pass-through $\hat{w}_t/\hat{x}_0$ from equation (5). The pass-through to the present value of wages, defined as $\widehat{PV(w)} \equiv \int_s^\infty \exp(-rt)\hat{w}_t dt$, is given by $(r+\alpha_x)^{-1} g_x(x_0,z_0)\epsilon_0/\gamma_0$. This shows that the change in the worker value is only induced by a change in the present value of wages that the worker will receive.

Relation to the Chetty-Baily formula for optimal unemployment insurance In appendix B.3, I show that the pass-through formula is reminiscent of the Chetty-Baily formula for optimal unemployment insurance (Baily, 1978, Chetty, 2006). In this literature, the planner wants to insure workers against unemployment risk but is wary that smoothing consumption too much will prevent workers from searching for a job. The optimal degree of insurance thus depends on the worker risk aversion coefficient, and the elasticity of the job finding rate with respect to unemployment benefits. In both this problem and mine, how much insurance workers receive influences the probability that the worker finds a new job. It is therefore not too surprising that the optimal policies are similar despite the problems being completely different.

4.3 The differential pass-through of sectoral shocks

I now turn to the pass-through of sectoral productivity shocks and show that it differs from that of firm-level shocks because of changes in the intensity of the competition for workers. I first derive the pass-through to wages, and then to the worker value.

I derive the pass-through of sectoral productivity shocks to wages using similar steps than in section 4.2 and detailed derivations are given in appendix B.2.3. The pass-through of wages is given by

$$\frac{\hat{w}_t}{\hat{z}_0} \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_z}\left(\underbrace{g_z(x_0,z_0)\frac{\epsilon_0}{\gamma_0}}_{\substack{\text{Pass-through of}\\ \text{firm shocks}}} + \underbrace{F(V_0,x_0,z_0)\frac{\epsilon_{z0}}{\gamma_0}}_{\substack{\text{Differential}\\ \text{pass-through}}}\right)\left[\exp\left(-\alpha_z t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right] \tag{6}$$

where $\epsilon_{z0} \equiv \partial\epsilon(V_0,x_0,z_0)/\partial z$ is the cyclicality of the retention elasticity evaluated at $t=0$. The key difference between sectoral and firm-level productivity shocks is that sectoral productivity z enters directly as an argument of the job finding rate $\lambda(v,z)$.

Equation (6) shows that the pass-through of sectoral productivity shocks to wages exceeds that of firm-level shocks if the shock persistence is the same, $\alpha_x = \alpha_z$, and two conditions are met: the firm value is positive $F(V_0,x_0,z_0) > 0$ and the retention elasticity is pro-cyclical $\epsilon_z(V_0,x_0,z_0) > 0$. I now explain the intuitions that these terms capture.

After a positive sectoral productivity shocks, workers receive more outside offers and incumbent firms thus have an additional incentive to increase the wage of workers to retain them. However, firms will only only the wage of workers if they are worth fighting for, that is if the firm value is positive $F(V_s,x_s,z_s) > 0$. If the match did not generate any value to the firm, there is no reason to increase the wage when workers receive more offers from other firms.

After a negative productivity shock, firms reduce the wage because they become less eager to retain workers. The pass-through of productivity shocks is still amplified with sectoral shocks when profits are positive. In this case, wages were growing before the shock from proposition 1. After the shock, workers do not experience wages cuts but lower wage growth. When the competition for workers also cools down as a result of the shock, firms can reduce wage growth even more because workers have become very unlikely to leave. To summarize, workers experience faster wage growth in a boom and in a recession they mostly suffer through missing wage growth. This asymmetric response is illustrated in figure 2, which shows the effect of positive and negative firm-level and sectoral productivity shocks on wages when the firm value is positive $F(V_0,x_0,z_0) > 0$. In the figure I set $\alpha_x = \alpha_z$ and let the shocks be of the same size $\hat{x}_0 = \hat{z}_0 = 10\%$. The

Figure 2: Impulse responses from firm and sectoral productivity shocks

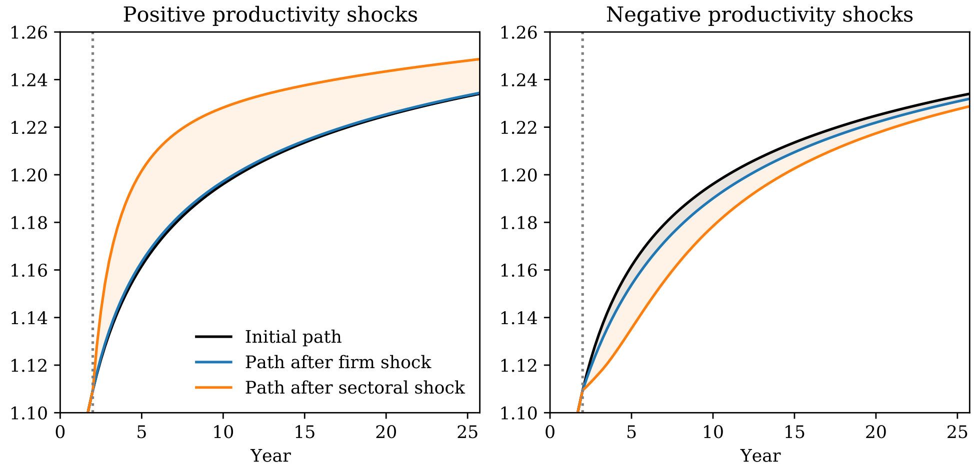
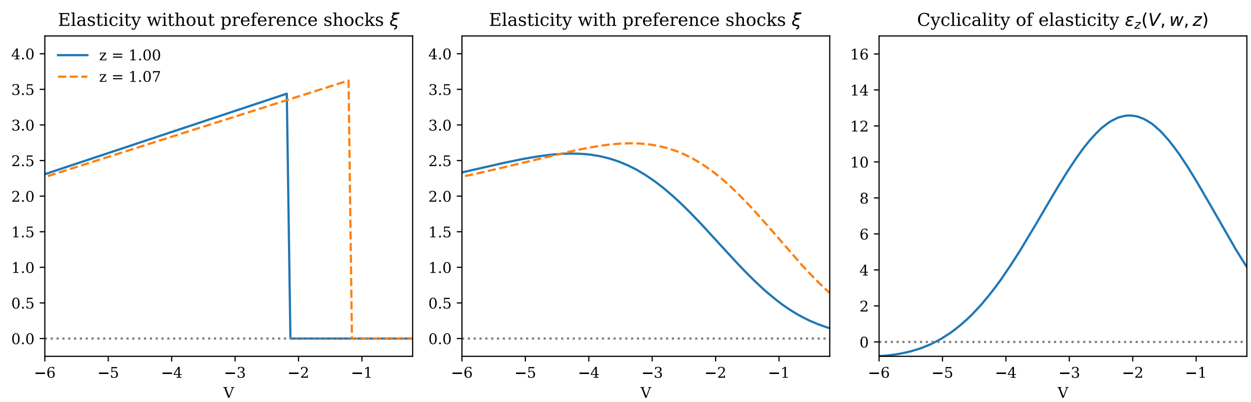


Figure 3: Level and cyclicality of the retention elasticity



pass-through of firm-level shocks here is almost 0 because the retention elasticity is close to 0 in this specific example.

I now turn to the second condition that must be satisfied for sectoral shocks to have a higher pass-through: the pro-cyclicality of the retention elasticity $\epsilon_z(V_0, x_0, z_0) > 0$. The cyclicality of the retention elasticity and the cyclicality of job-to-job transitions are closely related but do not always move in the same direction. Consider for example a worker with a relatively low wage. This worker receives more job offers in boom than usual and therefore has a pro-cyclical job-to-job transition rate. However, in boom this worker can become so likely to leave that increasing the wage marginally does not affect her job-to-job transition rate as much as before and therefore her retention elasticity is counter-cyclical. Consider now a worker with a relatively high wage. This worker had almost no chance of changing job in normal time because she was too expensive to get poached.

In boom, she suddenly receives more outside offers and both her job-to-job transition rate and her retention elasticity increase sharply. This worker has a pro-cyclical job-to-job transition rate and a pro-cyclical elasticity. The difference between low-wage workers and high-wage workers is illustrated in figure 3, which shows the retention elasticity without preference shocks in the left panel, the retention elasticity with them in the middle panel and the cyclicality in the right panel. It is easy to verify that workers with high wages have the most pro-cyclical elasticity while workers with low wages have a mildly pro-cyclical elasticity but a higher elasticity. This heterogeneity will turn out to be important when I evaluate the effects of sectoral productivity shocks on workers in section 5.2.

I now derive the pass-through of sectoral productivity shocks to the worker value Δ_z.

Proposition 3. *To first-order in κ, the pass-through of a sectoral productivity shock z to the value of stayers satisfies*

$$(r+\alpha_z)\,\Delta_z(V,x,z) = \Big[\underbrace{g_z(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))} + F(V,x,z)\frac{\epsilon_z(V,x,z)}{\gamma(w(V))}}_{\text{Pass-through to present value of wages}}\Big]u'(w(V))$$

$$+ \underbrace{\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)}_{\text{Change in expected gains from J2J transitions}} + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z) \equiv \frac{\partial \epsilon(V,x,z)}{\partial z}$ *and* $\lambda_{wz}(v(V,z),z) \equiv \frac{\partial \lambda_w(v,z)}{\partial z}|_{v=v(V,z)}$.

Proof. See appendix B.2.3. □

Proposition 3 shows how changes in the intensity of the competition for workers impact workers after sectoral shocks. First, the worker value changes due to the larger response of wages. Second, sectoral productivity shocks impacts the worker value through changes in the probability of finding a job. In booms, the competition for workers heats up and workers are more likely to find a new job so that $\lambda_{wz}(v(V,z),z) > 0$. When they switch, they gain value $v(V,z) - V$. Workers therefore benefit in booms because it increases the probability of a job-to-job transition. In downturns, the opposite is true.

5 Quantitative analysis

The previous section established that the retention elasticity is a critical determinant of the pass-through of productivity shocks to wages. In this section, I calibrate the quantitative model using administrative data from France with a focus on moments that are informative about this elasticity. I then use it to evaluate how much insurance firms provide to workers over the cycle.

5.1 Calibration

I quantify the model using the matched employer-employee data from France between 2008 and 2019.

Calibration strategy I set some parameters externally. Specifically, I use a CRRA utility function with coefficient γ to 1.5 following Balke and Lamadon (2022) and the discount factor to match an annual interest rate of 4%.

I use a Cobb-Douglas matching function

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu}\phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). I calibrate B to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the equilibrium job finding rate in my model.

I calibrate the degree of firm commitment Φ using estimates of firing costs. It is standard in the literature to justify firm commitment on the ground that firms have reputation concerns. This justification however only holds for relatively large and well-known firms. Instead, I argue that firing costs are a better proxy for firm commitment power. In my model firms only want to walk away from the contract if their value falls below the cost of firing the worker. Layoffs are tightly regulated in France and can lead to lawsuits and large compensations for workers. The parameter Φ captures the expected cost of a layoff, including severance payments and penalties that firms pay when layoffs are challenged in court. I calibrate the firing cost to account for approximately 6 months of labor earnings, $\Phi = 2$, following a methodology from Bentolila and Bertola (1990) that I update with recent data from the International Labor Organization (see appendix A.5 for details).

I normalize the search efficiency of employed workers κ to 1 and set the vacancy posting cost to be equal to 2 months of labor earnings, $k_1 = 0.66$[3].

Other model parameters are calibrated by matching moments in the data and in the model. Specifically, I simulate a panel of workers across different industries in the model and estimate the exact same set of moments in the model and in the data.

[3] I am working on ways to discipline these two parameters.

Section 4 showed that the critical determinants of the pass-through are the persistence of the productivity processes and the job mobility decision of workers. For this reason, I target several moments that inform these two aspects.

For productivity, I target the variance of productivity growth as well as the autocorrelogram to estimate its persistence. In the data I cannot identify the persistence of sectoral shocks with enough precision, and therefore I set $\alpha_x = \alpha_z$. In order to capture the amount of cross-sectional dispersion in productivity, I target the correlation between the average profit share of firms, defined as $1 - w/zx$, and average wages. The fact that firms paying higher wages do not necessarily have lower profits is indicative that they are also more productive. I add transitory measurement errors to firm and sectoral productivity, and recover their variances from the calibration.

To calibrate labor market transitions, I measure average flows of workers in and out of non-employment, as well as across jobs. Since the focus of this paper are wage contracts, and the separation rate is critical for the path of wages, I focus on non-employment rather than unemployment. I target the number of months spent out of employment to calibrate the job finding rate, and I measure the separation rate directly using flows into non-employment. I also target the average job-to-job mobility rate defined as the fraction of jobs ending up in a job-to-job transition each year.

Section 4 showed that the retention elasticity was a critical determinant of the pass-through. This elasticity measures the response of worker mobility to a change in the wage, so it is critical that my model captures well the reasons why workers decide to change jobs and how wage changes affect these decisions. For this reason, I target the share of job-to-job transitions with a positive wage change. It is well known that with heterogeneous productivity across firms some workers experience wage cuts when they change jobs because they expect higher wage growth in the future. In my quantitative exercise I find this to be insufficient to account for the large number of transitions with negative wage changes and therefore I calibrate the volatility of preference shocks v_ξ to match this moment.

The moments used in the paper are described in table 2. I find that about 6.5% of workers change job every year, and 5.5% of jobs end up in a separation into non-employment. Workers spend on average 14 months non-employed, which leads to a quarterly job finding rate of 20%. Remarkably, only 51% of workers changing jobs experience a positive change change when they do so, highlighting the critical importance of preference shocks for France.

Moments	Data	Model
Average duration unemployed in months	14.3 (0.063)	13.4
Average annual separation rate into non-employment	5.5% (0.071%)	5.4%
Average annual job-to-job transition rate	6.6% (0.10%)	7.0%
Share of job-to-job transitions with a positive wage change	51% (0.39%)	59%
Correlation between average profit share and wages	0.051 (0.0022)	- 0.024
Variance of firm productivity growth	0.089 (0.00068)	0.089
Variance of sector productivity growth	0.0032 (0.00035)	0.0032
1st order auto-correlation of firm productivity growth	- 0.24 (.0023)	- 0.21
2nd order auto-correlation of firm productivity growth	- 0.046 (0.0020)	- 0.08
1st order auto-correlation of sector productivity growth	- 0.13 (0.053)	- 0.0044
2nd order auto-correlation of sector productivity growth	- 0.14 (0.058)	- 0.13

Table 2: Moments: data vs. model

Parameter values Parameters values are shown in table 3. I find that training costs k_2 are high, equivalent to about 1 year of wages. This is to be put in perspective with a long average match duration of about 8 years. The model requires a low value of home production b to rationalize the low job finding rate of 20% per quarter.

Productivity shocks are moderately persistent, with a quarterly AR-1 coefficient of 0.93, which is equivalent to about 0.75 annually. Firm-level shocks are four times more volatile than sectoral productivity shocks, but also more subject to measurement errors.

Does the model account for the differential pass-through? As validation of the model, I assess whether it matches the differential response of wages and job-to-job mobility to firm-level and sectoral shocks that I measure in the data.

Table 4 shows estimates of the response of wages and job-to-job mobility to firm-level and sectoral shocks. The first column repeats the data estimates from table 1 while the second column reports the same estimates from the model.

The model accounts for the larger response of wages to sectoral productivity shocks than to firm level shocks, as well as the differential response of job-to-job transitions to firm and sectoral shocks. Wages respond 3.2 times more to sectoral shocks than firm level shocks in the model (14/4.4), compared to 3.8 times more in the data (18/4.7). The model overstates the response of job-to-job transitions to productivity shocks but get the

Parameters	Value	Parameters	Value
Risk aversion coefficient γ	1.5	Discount factor β	0.99
Firing cost Φ	2	Elasticity of matching function ν	0.5
Search efficiency on the job κ	1	Vacancy posting cost k_1	0.66
Training cost k_2	4	Exogenous separation rate δ	0.015
Matching efficiency B	0.26	Value of home production b	0.4
Volatility of preference shocks σ_ξ	1	Dispersion in fixed productivity σ_{x_0}	0.3
Persistence of firm productivity $1-\alpha_x$	0.93	Persistence of sectoral productivity $1-\alpha_z$	0.93
Volatility of firm productivity σ_x	0.14	Volatility of sectoral productivity σ_z	0.035
Volatility of firm meas. errors σ_x^{meas}	0.15	Volatility of sectoral meas. errors σ_z^{meas}	0.005

Table 3: Model parameters

Moments	Data	Model
Response of wages to firm-level productivity shock $\theta^{w,x}$	4.7%	4.4%
Response of wages to sectoral productivity shock $\theta^{w,z}$	18.5%	14.0%
Response of job-to-job transitions to firm-level productivity shock $\theta^{\text{J2J},x}$	- 1.7 pp	- 2.5 pp
Response of job-to-job transitions to sectoral productivity shock $\theta^{\text{J2J},z}$	4.0 pp	8.1 pp

Table 4: Differential response of wages and job-to-job transitions to shocks

difference in sign right.

From section 4, we already know that the pass-through of productivity shocks to wages is in general larger for sectoral shocks than firm-level shocks. In order to understand why the model also gets the differential response of job-to-job transition rates right, it is useful to consider the quit probability of an individual worker $\kappa\mathbb{E}_{\xi_t}\left[\lambda_w(v(W_{t+1},\xi_t,z_t),z_t)\right]$. Totally differentiating this probability with respect to firm productivity x gives

$$\left.\frac{d\text{J2J}}{dx}\right|_{\text{total}} = \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial \text{ PV of wages}}}_{-\text{ret elasticity}<0} \times \underbrace{\frac{\partial \text{ PV of wages}}{\partial x}}_{\text{passthrough to x}>0} < 0$$

After a positive firm-level shocks, the job-to-job transition rate falls because firms increase wages precisely to reduce worker turnover.

After a positive sectoral productivity shock, not only are firms retaining workers by

	Data	Model
Cross-sectional dispersion in annual wage growth	0.178	0.0693

Table 5: Cross-sectional dispersion in wage growth - data vs. model

increasing the wage but new entrants also try to hire workers by posting more vacancies. This new effects shows up in the job-to-job transition rate of an individual worker because sector productivity z_t enters directly as an argument. Totally differentiating this probability with respect to z gives

$$\left.\frac{d\text{J2J}}{dz}\right|_{\text{total}} = \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial \text{ PV of wages}}}_{-\text{ret elasticity}<0} \times \underbrace{\frac{\partial \text{ PV of wages}}{\partial z}}_{\text{passthrough to z}>0} + \underbrace{\frac{\partial \text{ J2J transition rate}}{\partial z}}_{\text{change in worker outside option}>0} \lesseqgtr 0$$

In general, the response of job-to-job transitions to a sectoral shock is ambiguous. In the quantitative model, I find that the second term dominates and the total response of job-to-job transitions is positive, which is consistent with the data.

Cross-sectional dispersion in earnings growth The model generates a lot of dispersion in earnings growth from productivity shocks and from the tenure profile of wages. Table 5 shows the cross-sectional dispersion of annual earnings wage growth for job stayers in the data and in the model. The model accounts for almost 40% of the cross-sectional dispersion in the data, which is remarkable compared to what observable worker characteristics can account fro. For example, in appendix A.3 I show that the $R2$ from a regression of wage growth on detailed worker characteristics, such as occupation, industry, location or demographics, is only 1.1%. The vast majority of the dispersion in wage growth comes from the tenure profile of wages but some of it also comes from firm-level and sectoral productivity shocks as I show next.

5.2 Decomposing the sources of risk for workers

I use the quantitative model to decompose the sources of risk faced by workers. I distinguish several sources of risk:

- risk on the job from firm-level shocks x
- risk on the job from sectoral productivity shocks z
- risk from matching with a firm with high or low fixed productivity x_0

	Baseline	1. constant z	2. constant x	3. constant x_0
Standard deviation of quarterly wage growth	0.0291	0.0286	0.0269	0.0262
Difference relative to benchmark	n.a.	- 0.0004	- 0.0021	- 0.0028
Percent of total risk (1+2+3)	n.a.	8.11%	39.29%	52.60%

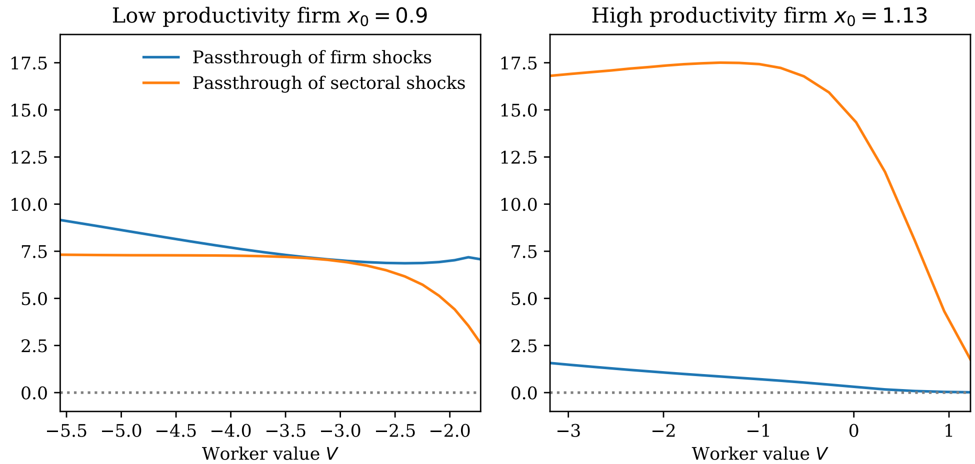
Table 6: Wage risk decomposition

I simulate the path of wages for a worker using the equilibrium policy functions. In each simulation I turn one source of risk off but keep the policy functions computed with all the risk on. I then compare the volatility of wage growth with that of a worker exposed to all sources of risk together.

Table 6 shows the standard deviation of wage growth across different simulations. I find that the largest sources of risk faced by workers are firm level shocks x and the ex-ante uncertainty about fixed match productivity x_0. Together, they account for almost 92% of the wage uncertainty faced by workers on the job.

Workers in high-productivity firms are disproportionately affected by the cycle The model predict that workers with different states (V, x, z) are differently exposed to firm-level and sectoral productivity shocks. In particular, workers in low-productivity matches experience a high pass-through of firm-level shocks but a relatively low pass-through of sectoral shocks, whereas workers in high productivity matches are more exposed to sectoral shocks than they are to firm-level shocks. Figure 4 shows the pass-through of firm and sectoral shocks for a match with relatively low fixed productivity $\overline{x} = 0.9$ and for a match with relatively high fixed productivity $\overline{x} = 1.13$. To understand why there is no amplification of the pass-through for workers in low productivity matches ($\overline{x} = 0.9$), it is useful to remember from section 4.3 that the pass-through of sectoral shocks is amplified when firms have positive values and the retention elasticity is pro-cyclical. Workers in low-productivity matches generate little profits for firms, and their retention elasticity is only mildly pro-cyclical. By contrast, workers in high productivity matches ($\overline{x} = 1.13$) generate a lot of profits and have high wages. As a result, they are very unlikely to get poached in bad time, but quite likely in good time and their elasticity is strongly pro-cyclical. Intuitively, after a positive sectoral productivity shocks high-productivity firms realize that their workers are now going to get poached and that it must increase their wage in order to retain them. This result shows that workers in different match quality are subject to very different types of risk: workers in low productivity firms are more exposed to firm-level shocks whereas workers in high productivity firms are more

Figure 4: Pass-through from firm and sectoral productivity shocks or different $\overline{x}$



impacted by the cycle.

5.3 Firm commitment and counter-cyclical income risk

The cyclicality of income risk depends on the degree of firm commitment.

As in Thomas and Worrall (1988), firm limited commitment implies that after large negative shocks wages sometimes have to fall sharply to prevent firms from walking away from the deal. Specifically, remember that contracts cannot be such that

$$F(V, s_t) \geq -\Phi$$

where Φ captures the degree of commitment of firms, and is calibrated using estimates of firing costs. This constraint is more likely to bind when the worker value V is high and firm and sectoral productivity x, z are low.

The competition for workers makes it more likely that this constraint binds because wages have a tendency to grow until the firm value $F(V, s_t)$ becomes 0, from equation 1. This is especially true in boom when the competition for workers heats up. During a downturn following a boom, firms suddenly become much less productive and still pay relatively high wages. This is when the limited commitment constraint is most likely to be binding.

I first assess whether in my quantitative model calibrated for France the limited commitment constraint of firms is likely to be binding. This depends on the size of firing costs, the volatility of productivity shocks and any parameter governing how fasts profits fall towards 0. I report in table 7 the fraction of firms hitting this constraint each year. I find

	Firms at constraint	$\theta^{w,z}$	$\theta^{w,x}$ in booms	$\theta^{w,x}$ in downturns
Data	n.a.	18.5%	4.6%	4.8%
Baseline model ($\Phi = 2$)	0.64%	14.3%	4.5%	4.4%
Low firing costs ($\Phi = 0.33$)	4.73%	18.3%	6.9%	8.0%

Table 7: Firing costs and the cyclicality of income risk

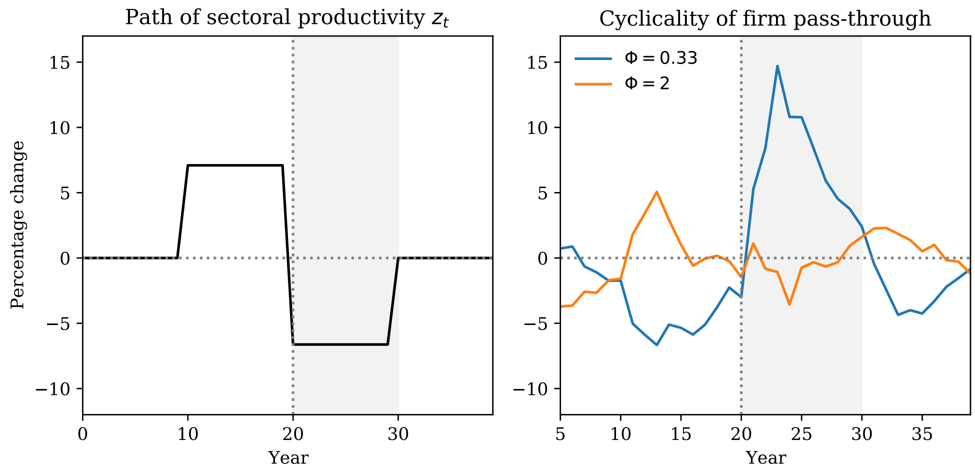
that only 0.64% of firms hit this constraint, which means that firing costs are sufficiently large in France that firms effectively have almost full commitment. Table 7 shows that as a result the pass-through of firm-level shocks is roughly constant in booms vs. downturns, which is consistent with my estimates from the data.

Counter-cyclical income risk with lower firm commitment Φ I now reduce the degree of firm commitment Φ using estimates of firing costs for the U.S. Effectively, I ask what would happen if a country like France with high firing costs were to implement labor market liberalization policies lowering firing costs to a level similar to the U.S.? In appendix A.5, I document that the U.S. has much lower firing costs and that a reasonable estimates is 1 month of monthly wage, equivalent in the model to $\Phi = 0.33$.

When firing costs are reduced, I find that firms are a lot more likely to hit their participation constraint. Specifically, this constraint is binding for almost 5% of firms each quarter. As a result, the pass-through of sectoral shocks $\theta^{w,z}$ and firm-level shocks $\theta^{w,x}$ rises. More importantly, the pass-through of firm-level shocks $\theta^{w,x}$ becomes counter-cyclical: it is 16% (8/6.9) larger in downturns than in booms. This makes income risk counter-cyclical. The reason for this is that in downturns sectoral productivity is low relative to wages. As a result, profit margins are depressed and firms are more likely to hit the constraint after a negative firm-level shock. Insurance contracts are critical for this result because they imply that wages are relatively more stable than productivity. They do fall in downturns but not quite as much as profits.

The state dependence of the pass-through of firm-level shocks is more complex than suggested in table 7 because it depends on the entire history of sectoral productivity shocks. In particular, the pass-through is at its maximum at the beginning of a downturn after a boom because productivity is low but wages remain high due to the intense competition for workers during the boom in past periods. This state dependence is illustrated in the simulations of figure 5. I simulate a panel of workers in a single industry with a path for sectoral productivity shown in the left panel: the economy is initially at the

Figure 5: Cyclicality of the pass-through with high and low firing costs



stationary level, then enters a boom (sectoral productivity increases by 2 standard deviations) for 10 years and finally a downturn for 10 years before returning to its stationary level. The right panel shows the pass-through of firm-level shocks to wages relative to the average pass-through with high firing costs ($\Phi = 2$) and low firing costs ($\Phi = 0.33$). When firing costs are low ($\Phi = 0.33$), the pass-through of firm-level shocks falls during the boom at $t = 10$ and then jumps at the beginning of the downturn at $t = 20$. The increase in downturn is larger than the fall in boom because the economy is initially in a state with relatively high wages due to the competition for workers in boom.

Novel implications for firing costs These results point to a novel role of firing costs when wage contracts are endogenous: they enhance firm commitment power and improve insurance for workers. As a result, I find that welfare falls by 1% when we introduce lower firing cost because workers are more exposed to shocks.

It is noteworthy that firing costs protect workers against large wage cuts more than they protect them against the risk of becoming unemployed. Indeed, the separation rate rises only marginally from 5.4% to 5.6% when I reduce firing costs. In my model, most workers keep their job after a large negative shock by experiencing large negative wage cuts. With firing costs, these workers enjoy a more stable wage. The small increase in the separation rate arises because the joint value of a match depends on the ability of firms to provide insurance to workers. If firms cannot insure workers through employment when firing costs are reduced, the joint value of matches falls and matches with low productivity are more likely to be efficiently terminated.

The results in this section stand in contrast to existing work on firing costs, which tend to find that they are detrimental to welfare. For example, Bentolila and Bertola (1990) study the effects of firing costs on the separation rate and the hiring rate. They find that firing costs could account for the poor employment dynamics among European countries after the oil shocks of the 1970s. Hopenhayn and Rogerson (1993) show that taxes on job destruction prevent the reallocation of workers towards more productive firms.

My results show that firing costs can improve welfare through their effect on insurance. The fact that firing costs are most useful at the start of recessions is useful for a concrete implementation of firing costs because other effects studied in the literature might also have a cyclical component. For example, the reallocation of workers towards more productive firms might be particularly important during recoveries when new businesses start growing. If this is true, an optimal firing cost strategy would be to implement firing costs permanently and relax them when the economy recovers from a recession because the benefits in terms of insurance are smaller whereas the costs in terms of reallocation are larger.

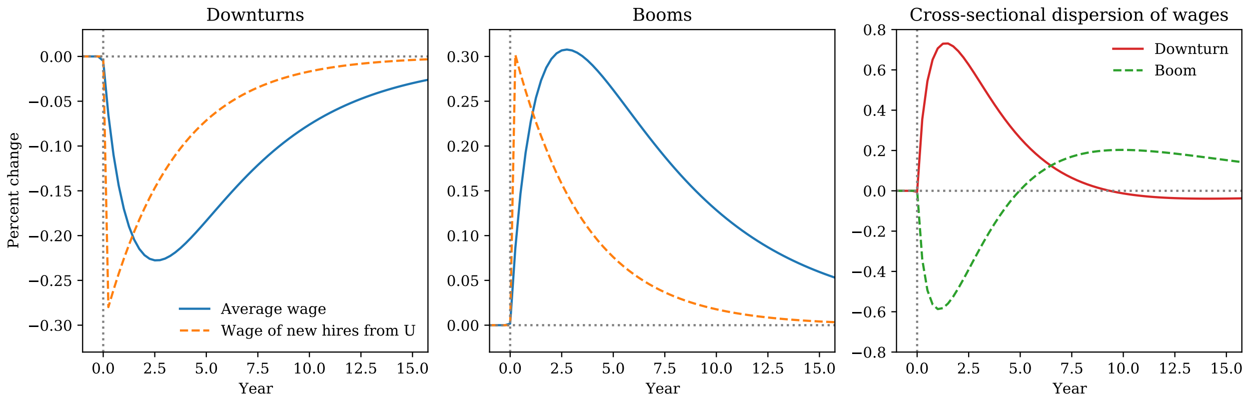
My model was not meant to capture all the effects of firing costs on welfare. In particular, I assume that firms only pay firing costs when they unilaterally walk away from a match, and not when workers quit or when separations are exogenous. In practice, firing costs do not apply when workers quit or under specific economic conditions, but the mapping between reality and the model is complex so I leave a more detailed analysis of firing costs when these competing forces are at play for future work.

5.4 Income inequality over the cycle

Sectoral fluctuations in productivity are not only a source of risk for employed workers, but also for workers transitioning through unemployment. In a downturn, these workers start with a much lower wage compared to workers who kept their job and experience a much more progressive fall in wages due to the insurance provided by contracts.

Figure 6 illustrates this differential response of wages for newly hired workers out of unemployment and for continuously employed workers. The left panel shows the percent change of wages for new workers out of unemployment and the average wage after a 1% negative and mean-reverting shock to sectoral productivity. The wage of workers out of unemployment falls on impact by about 0.3%, and then gradually recovers following the path of productivity. The wage of continuously employed on the other hand only falls gradually, which is reflected in the trajectory of the average wage. The average wage eventually becomes even lower than the wage of new hires out of unemployment after 2 years because the pass-through is backloaded. This differential dynamic of wages leads to

Figure 6: Income inequality over the cycle



an increase in income inequality during downturns, as illustrated by the right panel. The cross-sectional standard deviation of log wages among employed workers rises by about 0.75% in downturn because a large mass of new hires start with relatively low wages, while the wage of continuously employed worker adjusts slowly. After 9 years, wage inequality has reverted back to its pre-shock level and overshoots slightly as the wage of new hires from unemployment has recovered faster than the average wage.

The middle panel shows the response of wages to a 1% mean-reverting positive sectoral productivity shocks. The dynamics is similar except that average wages increase faster because the pass-through of sectoral shocks is larger for positive shocks, and because job switchers experience more job transitions with positive wage growth. As a result, the cross-sectional dispersion of log wages falls initially but by a smaller amount and for a much shorter period of time. After 5 years income inequality overshoots because the wage of current workers is significantly higher than the wage of new hires, and income inequality remains high for a significant period of time.

Figure 6 shows the surprising result that both downturns and booms amplify income inequality but for different reasons. Downturns amplify inequality because workers going through unemployment initially experience a much sharper fall in wages than continuously employed workers. Booms amplify inequality because continuously employed workers experience large wage increases due to the increased competition by poachers and these wages increases are very persistent.

In order to quantify this point, I compute the cross-sectional standard deviation of log wages in steady state when sectoral productivity shocks are turned off, and in the ergodic distribution when productivity shocks are present. The simulation results, shown in table 8, show that sectoral productivity shocks increase the cross-sectional dispersion in log wages from 0.178 to 0.187, a 5% increase. This shows that the cycle, not just negative

	No sectoral shocks	With sectoral shocks	in downturn	in boom
Cross-sectional sd. of log wages	0.178	0.187	0.198	0.179

Table 8: Sectoral shocks as a driver of income inequality

sectoral shocks, amplifies income inequality. The last two columns report the dispersion of log wages in downturns (sectoral productivity below average), and in booms (sectoral productivity above average). Income inequality in downturns is 10% larger than in booms.

To summarize, changes in income inequality over the cycle reflect a differential exposure of continuously employed workers and new hires from unemployment to sectoral shocks. Unemployed workers suffer most from downturns because they do not benefit from the insurance provided by employers and start new jobs with a much lower wage. This result shows that changes in income inequality over the cycle are alarming because they signal that some workers are particularly affected by shocks.

6 Extension: introducing risk-free bonds

An important assumption I made so far in this paper is that workers do not have access to financial markets and consume their wage. In this section I show that when workers have access to risk-free bonds, the wage contracts offered by firms changes significantly: they feature much more wage backloading and face a new trade-off between worker retention and precautionary savings. When trades in risk-free bonds are private information, workers borrow more than what firms would like so firms increase the pass-through of productivity shocks to wages in order to make workers save for precautionary reasons.

6.1 Environment

I present a 2-period version of the model without preference shocks.

The timing works as follows: in period 1, workers are matched to firms and sign wage contracts given some exogenous promised value V_0. They receive a wage and produce. In period 2, firm and sectoral productivity shocks x and z are realized. Workers can lose their job with exogenous probability δ and if they do not lose their job, they search for a new job. Workers then produce and receive a wage from their employer. Workers can trade risk-free bonds when they receive their wage in period 1 and these bonds are due in period 2 when wages are paid.

Workers can trade risk-free bonds a at rate $R = 1/\beta$. They are not able to default on this asset. I assume first that firms observe and control this choice, and will relax this assumption in section 6.3. Firms can pay severance payments τ to worker in the event of a separation into unemployment. I assume that if the firm commits to a level of severance payment but does not fulfill its promise ex-post, it is subject to the firing cost Φ.

In the 2-period model, labor markets are indexed by the wage that workers receive from poachers $\tilde{w}_2$ and by the asset held by workers a. The expected profit from posting a vacancy in market $(\tilde{w}_2, a)$ is

$$\Pi(\tilde{w}_2, a) = -k + q(\tilde{w}_2, a)\,[x_0 z_2 - \tilde{w}_2]$$

where x_0 is the firm productivity of a new entrant. The free entry condition $\Pi(\tilde{w}_2, a) = 0$ implies that the vacancy filling rate $q(\tilde{w}_2, a) = q(\tilde{w}_2)$ is independent of assets, and from the matching function so is the job finding rate $\lambda(\tilde{w}_2)$.

Given the wage contract $\{w_1, w_{xz}\}$, severance payment τ and assets a, the value of the worker satisfies

$$\begin{aligned} &V = u(w_1 - a) + \delta\beta u(b + aR + \tau) \\ &+(1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[\max_{\tilde{w}}(1-\kappa\lambda(\tilde{w}, z_2))u(w_{xz} + Ra) + \kappa\lambda(\tilde{w}, z_2)u(\tilde{w} + Ra)|x_1, z_1\right] \end{aligned} \tag{7}$$

The worker search policy $\tilde{w}_{xz} = \tilde{w}_2(w_{xz}, z_2, a)$ satisfies the optimality condition

$$\lambda_w(\tilde{w}, z_2)\,[u(\tilde{w} + Ra) - u(w_{xz} + Ra)] + \lambda(\tilde{w}, z_2)u'(\tilde{w} + Ra) = 0 \tag{8}$$

The optimal contract solves

$$\begin{aligned} \max_{w_1, w_{xz}, \tau, a} \quad & x_1 z_1 - w_1 - \beta\delta\tau + (1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[(1-\kappa\lambda(\tilde{w}_{xz}, z_2))\,(x_2 z_2 - w_{xz})\right] \\ \text{s.t.} \quad & V_0 = u(w_1 - a) + \delta\beta u(b + Ra + \tau) \\ & \quad +(1-\delta)\beta\mathbb{E}_{x_2,z_2}\left[(1-\kappa\lambda(\tilde{w}_{xz}, z_2))u(w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u(\tilde{w}_{xz} + Ra)|x_1, z_1\right] \\ & \tilde{w}_{xz} = \tilde{w}_2(w_{xz}, z_2, a) \\ & x_2 z_2 - w_{xz} \geq -\Phi \\ & -\tau \geq -\Phi \end{aligned} \tag{9}$$

I characterize the optimal contract in section 6.2 and discuss the implications of hidden trade in section 6.3.

6.2 Firms use debt to backload wages even more

I first show that introducing risk-free bonds enables firms to backload wages even more. When firms have limited commitment however, the extent to which firms can backload wages depends a precautionary savings motives.

To build some intuition for the results in this section, it is useful to remember the trade-off that firms faced when workers had no access to financial markets. Firms backload wages to prevent workers from leaving for another job, because it makes them apply for jobs with a lower job finding rate. However, when workers have no access to financial markets backloading wages implies backloading consumption too, and firms must compensate risk-averse workers for this. As a result, firms backload wages but only partially as illustrated by proposition 1. Trade in risk-free bonds allows firms to backload wages and smooth consumption over time by making the worker borrow. As a result, the firm is able to retain workers at minimal cost and the job-to-job mobility rate falls drastically. Said differently, risk-free bonds expand the set of instrument that firms can use to retain workers: a debt that workers must repay even if they change job.

I show in this section that the extend to which firms can use bonds is limited by a precautionary savings motive of workers. Workers face the risk of becoming unemployed before receiving their wage, or receiving a low wage tomorrow because of a negative productivity shocks. Because of this workers might not want to borrow too much today in anticipations of expected future income because this income is uncertain. If firms have full commitment power they can provide insurance to the worker against these risk but if commitment is limited, as in my baseline model, the worker can only borrow to some extend and the contract implies partial backloading.

In order to make these points as transparent as possible, I assume in this sub-section that utility is CARA $u(c) = -\exp(-\gamma c)/\gamma$. This assumption implies from condition (8) that the search policy $\tilde{w}(x_2, z_2, a) = \tilde{w}(x_2, z_2)$ is independent of assets a. I will relax this assumption in the next sub-section when I discuss the implications of hidden trades.

Combining the optimality conditions of the contracting problem gives

$$\frac{u'(b + Ra + \tau)}{u'(w_1 - a)} = 1 + \mu_U$$

$$\frac{u'(w_{xz} + Ra)}{u'(w_1 - a)} = 1 + \frac{\kappa \partial_{w(x,z)}[\lambda(\tilde{w}_{xz}, z_2)]\,(x_2 z_2 - w_{xz}) + \mu_{xz}}{1 - \kappa\lambda(\tilde{w}_{xz}, z_2)}$$

$$\begin{aligned} u'(w_1 - a) &= \delta u'(b + Ra + \tau) \\ &+ (1-\delta)\mathbb{E}_{x_2, z_2}\left[(1 - \kappa\lambda(\tilde{w}_{xz}, z_2))u'((w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u'(\tilde{w}_{xz} + Ra)|x_1, z_1\right] \end{aligned} \tag{10}$$

where μ_{xz} and $\delta\beta\mu_U$ are the Lagrange multipliers of the firm participation constraints. The first equation is the optimality condition for severance payments τ. The second equation is the optimal wage growth condition. The third condition is the optimal saving condition, which here turns out to be the worker's Euler equation.

Full backloading with firm commitment Consider first the case with full commitment ($\Phi \to \infty$) as a benchmark. The two participation constraints of the firm drop out so $\mu_{xz} = \mu_U = 0$.

The optimality condition for severance payments shows that the firm insure the worker against unemployment risk since $c_1 \equiv w_1 - a = b + Ra + \tau = c_U$.

Combining the three optimality conditions, we find that

$$w_{xz} \geq x_2 z_2$$

so that workers get at least the value of output tomorrow. This is an extreme form of backloading in which firms pay more than the value of output to workers tomorrow irrespective of their promised value V_0 or wage at w_1. The wage at $t = 1$ is given by the promise keeping constraint and depends on V_0. It can even be negative, meaning that workers have to pay an upfront fee to firms. Because wages are so backloaded, the job-to-job transition rate falls drastically.

Consumption satisfies

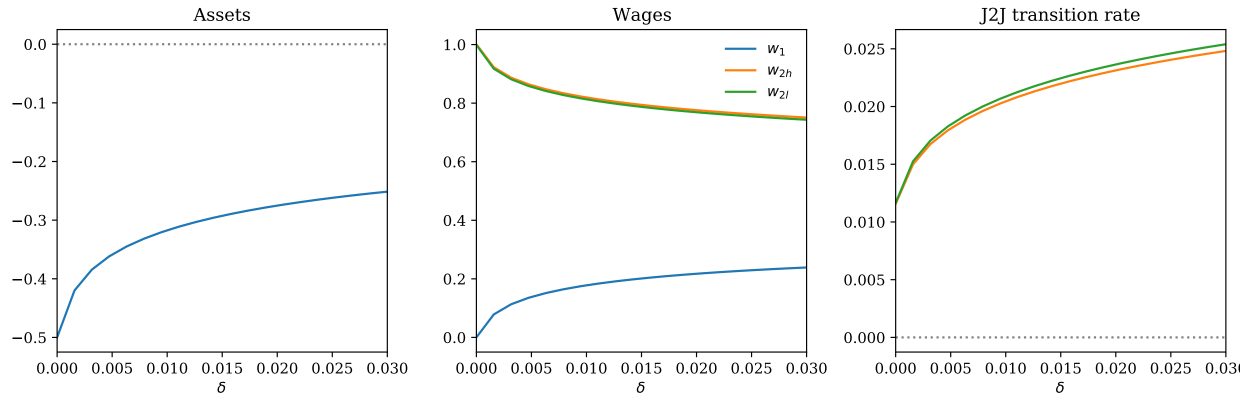
$$c_{xz} \leq c_1 = c_U \leq \tilde{c}_{xz}$$

where $\tilde{c}_{xz}$ is the consumption of a worker in state (x_2, z_2) after a job-to-job transition.

This benchmark result is closely related to Stevens (2004) who studies optimal wage contracts with complete financial markets. Remarkably, risk-free bonds are almost sufficient here to achieve the same results even though there are many sources of risk (e.g. unemployment, productivity shocks). This is because firms are able to insure workers against these risks. The only difference is that the firm is not able to insure workers against the upside risk of finding another job. Workers always get a higher wage when they changes jobs so their consumption must jump. The firm provides some form of limited insurance by making the worker borrow at $t = 1$ and backloading wages at $t = 2$. This is the reason why wages here are sometimes larger than output at $t = 2$.

Partial backloading with limited firm commitment Consider now the case with limited firm commitment ($\Phi < \infty$). In this case firms can only insure workers against negative events such as unemployment risk and negative productivity shocks if it is in their in-

Figure 7: Optimal contract with risk-free bonds: comparative statics with respect to δ



Note: illustrative example with two productivity states x_h and x_l.

terests ex-post. In the limit case with $\Phi = 0$, firms cannot commit to make losses after adverse shocks so that $\tau = 0$ and $w_{xz} \leq x_2 z_2$.

As a result, workers dislike entering period 2 with too much debt because in the event of a negative shock or unemployment shock they will have very little income to repay it and will as a result consume very little. This prevents firms from backloading wages too much because it would make the worker either borrow or backload consumption too much at $t = 1$. Precautionary savings motives prevent workers from borrowing too much in anticipation of future income because this income is uncertain.

Figure 7 illustrates how the path of consumption changes as the exogenous separation rate δ rises from 0 to 3% when Φ is set to 0. When the risk of separation into unemployment increases, the optimal contract implies less borrowing since workers face the risk of having to repay the debt while unemployed. As a result, wages become less backloaded and the job-to-job transition rate increases.

6.3 Hidden trade

I have assumed so far that firms could observe and thus control the asset choice of agents. I now assess whether the allocation changes when agents can privately access financial markets, i.e. they are hidden trades.

I solve the problem with hidden trade using the first-order approach following Werning (2001) and Abraham and Pavoni (2008).

With hidden trade, the worker privately chooses in which labor market to apply $\tilde{w}_2$ and how much assets to hold a to maximize her present value (7). Taking the first-order condition with respect to asset a gives the Euler equation (10). The optimal contract now solves the problem (9) with the Euler equation (10) as an additional constraint.

With CARA utility the relaxed problem without hidden trade (9) solves the problem with hidden trade. To see this, note that the optimality condition with respect to assets a in the relaxed problem is the agent's Euler equation. Therefore the solution to the relaxed problem is also feasible in the problem with hidden trade, and since we can always do better in a relaxed problem it is also the solution with hidden trade. Intuitively, with CARA utility there are no wealth effects in that the level of assets does not influence the worker's search decision. As a result, given a choice for v, the worker and firm preferences towards savings are aligned.

For a general utility function there are profitable joint deviations for the worker. To understand this, it is useful to consider the optimal choice of assets of the firm in the relaxed problem (9) for a general $u(c)$ when firms control the level of assets directly. The optimality condition for a becomes, using the envelope theorem,

$$\begin{aligned} u'(w_1 - a)\left(1 + \kappa \partial_a \lambda(\tilde{w}_{xz}, z_2)(1-\delta)\beta \mathbb{E}_{x_2,z_2}\left[(x_2 z_2 - w_{xz})\,|x_1, z_1\right]\right) &= \delta u'(b + Ra + \tau) \\ +(1-\delta)\mathbb{E}_{x_2,z_2}\left[(1 - \kappa\lambda(\tilde{w}_{xz}, z_2))u'((w_{xz} + Ra) + \kappa\lambda(\tilde{w}_{xz}, z_2)u'(\tilde{w}_{xz} + Ra)|x_1, z_1\right] \end{aligned}$$

The firm takes into account that assets influence the search decision of workers, and therefore their job-to-job transition rate $\partial_a \lambda(\tilde{w}_{xz}, z_2) \neq 0$. When firms make positive profits, they alter the level of assets in a way that reduces the job-to-job transition rate so as to retain workers. From the optimal search condition (8), we can show by comparative static that workers with more assets search for jobs with a higher wage $\partial_a \tilde{w}_2(w_{xz}, a, z) > 0$. Since the job finding rate $\lambda(w, z_2)$ is decreasing in w, firms increase the worker's savings in order to reduce their job-to-job transition rate. Intuitively, workers enter period 2 with some assets a. The lower this level of asset, the higher her marginal utility of consumption. With low assets a, worker would rather apply for jobs that are easier to get and deliver a smaller increase in consumption, than jobs that are difficult to get. Conversely, if the worker enters with a high level of assets she is willing to apply for jobs that she is unlikely to get but delivers a high payoff if she gets it. Another way to see this result is that applying for a job is like buying a lottery ticket, and the worker is more willing to enter a risky lottery if her marginal utility is low (her asset level is high) than otherwise. Because of this, firms make workers borrow less and enter period 2 with a relatively low level of debt.

What would workers choose with hidden trade? In the allocation without private trade workers are borrowing constrained: relative to firms, they would prefer to borrow more at $t = 1$ to increase consumption. The joint deviation is therefore borrow more at $t = 1$, enter period $t = 2$ with more debt and apply to a labor market $\tilde{w}$ with a lower wage and higher job finding rate. In response, firms backload wages more to induce workers to apply in labor markets with a lower job finding rate. They also increase the pass-through of productivity shocks so as to increase the precautionary savings motive of workers, and make them save more at $t = 1$.

Future directions I am working on a dynamic version of this problem that I will use to evaluate whether wages are much more backloaded in optimal contracts when workers face a realistic amount of risks and have access to frictional financial markets, such as trades in risk-free bonds subject to a borrowing constraint. I will also use the dynamic model to study whether workers use bonds to smooth wages by borrowing when they receive a low income today. The 2-period model cannot be used to answer this question because trades in risk-free bonds are decided at $t = 1$ before shocks are realized.

7 Conclusion

I ask in this paper whether firms insure workers against risk over and from cycles generated by sectoral productivity shocks. I build a model with dynamic wage contracts consistent with the response of wages and job-to-job mobility to firm-level and sectoral productivity shocks estimated using administrative data. I find that workers in high-productivity firms are disproportionately affected by sectoral shocks and that the commitment power of firms is critical for the cyclicality of income risk. Because firing costs influence firm commitment, lowering them increases the risk faced by workers and makes it counter-cyclical. Finally, income inequality rises in downturns because newly hired workers from unemployment do not benefit from the same degree of insurance than continuously employed workers.

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Appendix

Contents

A Data appendix

A.1 Sample construction

I use administrative data provided by the CASD in France between 2008 and 2019. My analysis relies on main two main files:

a) the panel version of the "DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) "FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms ("Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level ("UL"), and several legal units can belong to the same firm. Since I want to measure job-to-job transitions across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the "Entreprise profilée" ("EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

Definition of sectors I use detailed information about a firm industry to define a sector. Specifically, I use the NAF Rev 2. APE 2 classification, which contains about 81 sectors, including 24 sectors within manufacturing. Examples of sectors are pharmaceutical industry, retail trade or restaurants. My dataset features 6,761 workers and 1,751 per sector on average.

One advantage of using industry classifications instead of occupation or location to define a labor market is that industry is reported at the firm level, and it is therefore easy to aggregate and compute productivity at the sector level. By contrast, several employees within the same firm might belong to different occupations or live in different locations. The main drawback of using sectors is that workers might change sector when they change jobs more than they change occupation or location. I am working on a model extension with imperfect labor mobility across sectors to assess whether my results are robust to this modification.

The industry classification is available at different levels of aggregation. I choose an intermediate definition for two reasons. First, with more aggregated definition (e.g. manufacturing), changes in the sector productivity are likely affecting the entire economy. It is then more difficult to justify in my model that the interest rate is independent of sector productivity. With a more granular classification, idiosyncratic shocks to sector productivity are more likely to cancel out in the aggregate. At the other extreme, with the most granular definition most sectors are made of a few firms only and estimates of sector productivity (the average across firms) become contaminated by firm-level changes in productivity. In this case estimates of pass-through against sectoral productivity shocks are biased because they reflect the response of wages to firm-level productivity shocks. I run simulations to ensure that with the classification that I use (APE 2) and given the volatility of firm and sectoral productivity and the size of firms in my sample, this small-sample bias is negligible.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.1}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. In ongoing work I investigate how my results change with alternative assumptions about productivity; for example one in which sectoral productivity is a function of aggregate productivity but with different loading coefficients. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying job-to-job transitions is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job transition. A job-to-job transition occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as job-to-job transitions if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of job-to-job transitions with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make a job-to-job transition experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job transitions with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

	Number per year	Average duration in sample in years
Workers	532,005	7.89
Firms	129,576	8.16

Table A.1: Sample description

Avg. age	Shale male	Avg. firm size (firm obs)	Avg. firm size (worker obs)
38.4	66%	66.7	8999

Table A.2: Sample characteristics

When a worker separates from their previous jobs and does not make a job-to-job transition, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. transition from private sector to public sector), I do not count it either as a job-to-job transition nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.

Summary statistics I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years, for firms with at least 3 employees (in the panel or not) and I keep sectors with at least 20 employees and 3 firms per year. I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

A.2 Estimation of standard errors by Block bootstrap

The estimation of the moments used in the quantitative analysis is done in several steps, and for this reason I estimate standard errors by bootstrap. I sample firms with replacement and keep all the years and workers associated with a firm if it is sampled. I then create 1,000 samples and then apply my estimation procedure in each of them, including rezidualizing productivity, removing outliers, computing sectoral productivity or estimating the pass-through. The estimates that I report and their standard errors are the average estimates across bootstrap samples and the standard deviation across sample.

A.3 Additional moments

Pass-through estimates in boom vs. downturns Table A.3 reports the pass-through of firm-level and sectoral productivity shocks to wages estimated separately in booms, and in downturns. I define booms as periods in which sector-productivity in level is higher than the mean, and downturns as the complement. The estimates for the pass-through of firm shocks are meant to capture

	Booms	Downturns
Pass-through of firm-level shocks	4.63% (1.00%)	4.88% (0.75%)
Pass-through of sectoral shocks	16.81% (4.70%)	20.1% (5.41%)

Table A.3: Pass-through estimates in boom vs. downturns

	p1	p5	p10	p25	p50	p75	p90	p95	p99	mean	s.d.
Real wage growth	- 0.57	- 0.25	- 0.13	- 0.038	0.011	0.071	0.18	0.28	0.58	0.015	0.18

Table A.4: Pass-through estimates in boom vs. downturns

how idiosyncratic risk faced by workers vary over the cycle. The results show that the pass-through in booms and in downturns are not significantly different from one another, which is consistent with my model calibrated for France.

Dispersion in earnings growth Table X describes the distribution of real annual wage growth in the data for workers continuously employed at the same firm between year $t-1$ and year t. The mean annual wage growth is 1.5% and the distribution is remarkably symmetric. The standard deviation is 0.18, the vast majority of which cannot be accounted for by observable worker characterized. Specifically, I estimate

$$\Delta \log w_{ijst} = \alpha + X'\beta + \epsilon_{ijst}$$

where X is a vector of worker characteristics, including a polynomial in experience (age minus 20), dummies for gender and firm as well as dummies for occupation (4-digit), industry (4-digit) and commuting zones. The R^2 from this regression is only 0.011.

A.4 Aggregate shocks

In this paper I focus on the behavior of sectoral $\log z_{st}$ and firm productivity $\log x_{jst}$ because I want to isolate the effects of the cyclical competition from workers from that of time-varying price of risk. The assumption underlying this approach is that sectoral productivity shocks are diversifiable for firm owners because sectors are sufficiently small. Changes in aggregate productivity $\log a_t$ on the other hand cannot be diversified and will therefore influence both the cyclicality of the competition for workers and the ability of firms to provide insurance against these shocks. This distinction is especially important in the context of wage contracts because we know that there is perfect risk-sharing if workers and firms are both equally risk-averse and the contract is not subject to any friction.

Sectoral shocks also have the advantage that there is more data we can use to estimate these moments. For example in my data I have one time series of 12 years for aggregate shocks, but a panel of 81 sectors for sectoral shocks. I am working on extending my sample to the 1990s in order to get better estimates for aggregate and sectoral shocks. Doing so is difficult because there is an important break in the firm dataset in 2008 due to changes in the survey methodology.

	Response of wages (OLS)	Response of job-to-job transitions (OLS)	Variance of shocks
Firm shocks	0.011 (0.002)	- 0.0031 (0.0025)	0.0887 (0.0007)
Sectoral shocks	0.043 (0.015)	0.034 (0.028)	0.0032 (0.0004)
Aggregate shocks	0.138 (0.017)	0.321 (0.034)	0.00042 (0.00002)

Table A.5: The response of wages and job-to-job transitions to firm, sectoral and aggregate shocks

Table A.5 shows the response of wages and job-to-job transitions to firm, sectoral and aggregate productivity shocks estimated using equation A.1 as well as estimates of the variance of these different shocks. Aggregate shocks have an even higher pass-through than sectoral shocks, and an even more cyclical response of job-to-job transitions to shocks. This supports the idea that studying sectoral shocks is informative about the nature of aggregate cycles, since the coefficients move in the same direction relative to firm-level shocks, but that they are also different, since the response is much larger to aggregate shocks. The difference in the cyclicality of job-to-job transitions with respect to sectoral and aggregate shocks might also be informative for other line of research, such as the literature on unemployment volatility (Shimer (2005)).

Table A.5 also shows that the variance of sectoral shocks is about 10 times larger than the variance of aggregate shocks, which suggest that sectoral shocks might be a larger source of risk than aggregate shocks.

A.5 Estimates of firing costs

I update estimates on firing costs from Bentolila and Bertola (1990) using data from the International Labor Organization to discipline the degree of commitment of firms Φ. In appendix C, they define firing costs as

$$\Phi = N + (1 - p_a)SP + p_a \left[(1 - p_u)(SP + LC) + p_u(UP + LC)\right]$$

where N represents pay during the notice period, SP is the severance payment, LC are legal costs and UP are dismissal costs if the layoff is deemed unjustified in court. p_a is the probability that the layoff is brought to court, and p_u the probability that courts rule in favor of workers. This firing costs is evidently difficult to estimate since we do not have precise information about all of these elements, especially the probabilities p_a and p_u or legal fees.

Table A.6 reports information about layoff costs from the International Labor Organization, as well as the estimates that I use in this paper. I report estimates for a worker with an average of 8 years of tenure, which is the average tenure in my sample ($1/(\text{J2J} + \text{EU})$).

I find that firing costs account for 4.2 months in France and 0.3 months in the U.S. on average. The average wage in the model is 1.08 per quarter so my estimates for firing costs is 1.5 and 0.11. These likely represent lower bounds since severance payments are sometimes increased at the industry level, and the probability I use most likely understate the chances that workers win in court nowadays. For this reason I use estimates of $\Phi = 2$ and $\Phi = 0.33$ for France and the U.S. Bentolila and Bertola (1990) report much higher estimates for France, of 8.2 months for the 1960s and 11 months for the 1970-80s because they use much higher severance payments for these periods.

The stringency of layoff restrictions in France relative to the U.S. is consistent with indicators

	France	United States
Notice period N (in months of pay)	2	0
Severance pay SP (in months of pay)	2	0
Redress costs UP	10	10 (max is \$50K to \$300K)
Probability of going to court p_a (from Bentolila and Bertola (1990))	0.05	0.05
Probability worker wins p_u (from Bentolila and Bertola (1990))	0.25	0.25
Legal costs LC (from Bentolila and Bertola (1990))	2	2
Estimates of firing costs	4.2	0.3

Table A.6: Estimates of firing costs in France and the U.S.

published by the Organization of Economic Co-operation and Development. The measure of firing costs published by the OECD is more comprehensive than the one I use, but it is more difficult to translate it into model parameters because it is an index.

B Model appendix

B.1 Proofs for quantitative model

I prove Lemma 1.

Proof. Only if: I prove that an incentive compatible contract is truth-telling if and only if after any history $\{s^t, \xi^t\}$, expected continuation values W_{t+1} cannot depend on the realization of the preference shock today ξ_t.

Assume, by way of contradiction, that $W_{t+1}(s^t, (\xi^{t-1}, \xi_t))[\xi] < W_{t+1}(s^t, (\xi^{t-1}, \xi'_t))[\xi']$ for some preference shocks $\xi_t \neq \xi'_t$. Since the contract is incentive compatible, the value of a worker after history $\{s^t, \xi^{t-1}\}$ is

$$V_t(s^t, \xi^{t-1}; w)[\xi] = u(w_t(s^t, \xi^{t-1})) + \beta \mathbb{E}_{\xi_t} \left[\max_v \ \kappa\lambda_w(v, z_t)\,(v + \xi_t) + (1 - \kappa\lambda_w(v, z_t))W_{t+1}(s^t, \xi^t; w)[\xi] \right]$$

Now consider the following deviation: report history ξ^{t-1} up to time $t-1$ and then reports ξ'_t. This deviation gives a higher worker value at time t since

$$W_{t+1}(s^t, \xi^t; w)[\xi'] = W_{t+1}(s^t, \xi'^t; w)[\xi']$$

In words, the continuation value of a worker following reporting strategy ξ' after history ξ^t is the same than for a worker reporting strategy ξ' after history ξ'^t. This equality follows form the fact that expectations over ξ_s for $s > t$ are unconditional ξ^t. Since workers have the same history up to t, they also have the same flow utility up to t and therefore this deviation is profitable for workers at $t = 0$. This contradicts the assumption that the contract was truth-telling.

If: assume that $W_{t+1}(s^t, \xi^t)[v, \xi, q] = W_{t+1}(s^t, \xi^{t-1})[v, \xi, q]$. Then for all histories ξ^{t-1} and after any realization ξ^t the worker is indifferent between reporting the truth ξ or deviating to any other reporting strategy $\hat{\xi}$. Therefore, reporting the truth ξ is a truthful strategy. $\square$

When preference shocks are not i.i.d., it is possible to construct contracts in which the expected continuation values W_{t+1} vary with preference shocks by varying the path of wage payments in the future. A worker with a persistently high preference shock today will be more likely to leave for another job tomorrow and therefore will value more wage payments that are front-loaded compared to a worker with persistently low preference shocks.

B.2 Continuous time model

B.2.1 Environment

I describe the environment of the simple version of the model studied in section 4.

Workers have utility $u(w)$ and have no accesss to financial markets. Their discount rate is r.

Firms are owned by risk-neutral investors with discount rate r. Output $\exp(x+z)$ is produced within matches with firm productivity x and sectoral productivity z following

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \quad \text{and} \quad dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}$$

where B_{xt} and B_{zt} are Brownian motions.

Search is directed so wokers apply for jobs in labor markets indexed by the present value that a worker would get v. The job finding probability follows a Poisson process with intensity $\kappa\lambda_w(v,z)$,

$$P(\tau > t) = \exp\left(-\kappa \int_0^t \lambda_w(v_s, z_s) ds\right)$$

where τ denotes the stopping time describing when the worker finds another job.

Contracts Contracts specify a wage for each history of firm and sectoral productivity shocks

$$w_t(\{x_s, z_s; 0 \le s \le t\})$$

After signing the contract, the worker chooses a job search strategy

$$v_t(\{x_s, z_s; 0 \le s \le t\})$$

to maximize her expected utility.

Worker value function Given contract w and search strategy v, the value of a worker is

$$V(w)[v] = \mathbb{E}\left[\int_0^\tau e^{-rt} u(w_t) dt + e^{-r\tau} v_\tau\right]$$

where the expectation is taken over the paths of (B_{xt}, B_{zt}, τ^v). We can rewrite this as

$$\begin{aligned} V_0(w)[v] &= \mathbb{E}\left[\int_0^\infty e^{-rt} \left(\mathbb{1}(\tau^v > t) u(w_t) + \mathbb{1}(\tau^v = t) v_t \right) dt \right] \\ &= \mathbb{E}\left[\int_0^\infty e^{-rt} \left(P(\tau^v > t) u(w_t) + P(\tau^v = t) v_t \right) dt \right] \\ &= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa \int_0^t \lambda_w(v_s, z_s) ds \right) \left(u(w_t) + \kappa \lambda_w(v_t, z_t) v_t \right) dt \right] \end{aligned}$$

where the second line uses the fact that B_{xt}, B_{zt}, τ^v are mutually independent and the last line uses the definition of τ. The expectation in the last line is taken over the paths of (B_{xt}, B_{zt}). In general, the value of a worker at time t is

$$V_t(w)[v] = \mathbb{E}_t\left[\int_t^\infty \exp\left(-r(h-t) - \kappa \int_t^h \lambda_w(v_s, z_s) ds \right) \left(u(w_h) + \kappa \lambda_w(v_h, z_h) v_h \right) dh \right]$$

Therefore, the value of a contract for a worker is defined as

$$V = \max_v \; V_0(w)[v]$$

Optimal contract Given contract w and search strategy v, the value of a firm is

$$\begin{aligned} F_0(w)[v] &= \mathbb{E}\left[\int_0^\tau e^{-rt} \left(\exp(x_t + z_t) - w_t \right) dt \right] \\ &= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa \int_0^t \lambda_w(v_s, z_s) ds \right) \left(\exp(x_t + z_t) - w_t \right) dt \right] \end{aligned}$$

Given the job finding rate $\lambda(v, z)$ and the processes for productivity x and z, the optimal contract solves

$$\begin{aligned} F_0(V_0, x_0, z_0) = \max_w \quad & \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa \int_0^t \lambda_w(v_s, z_s) ds \right) \left(\exp(x_t + z_t) - w_t \right) dt \right] \\ \text{s.t.} \quad & V_0 = \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa \int_0^t \lambda_w(v_s, z_s) ds \right) \left(u(w_t) + \kappa \lambda_w(v_t, z_t) v_t \right) dt \right] \\ & v \in \arg\max_v \; V(w)[v] \end{aligned}$$

B.2.2 Recursive formulation of the contract

In this section I use methods introduced in Sannikov (2008) to write this contract recursively. For simplicity of notations I write $V_t(w)[v] = V_t^v$.

I first derive the law of motion of the worker promised value V_t^v for any contract and strategy.

Lemma 3. *Given a contract w and a search strategy v, the worker value V_t^v satisfies*

$$dV_t^v = \left(rV_t^v - u(w_t) - \kappa \lambda_w(v_t, z_t)(v_t - V_t^v) \right) dt + \Delta_{xt} \sigma_x dB_{xt} + \Delta_{zt} \sigma_z dB_{zt}$$

for some stochastic processes Δ_{xt}, Δ_{zt}.

Proof. Define the process H_t^v as

$$H_t^v = \int_0^t R_h^v \left(u(w_h) + \kappa \lambda_w(v_h, z_h) v_h \right) dh + R_t^v V_t^v \tag{A.2}$$

where $R_t^v \equiv \exp(-rt - \kappa \int_0^t \lambda_w(v_s, z_s) ds)$ is the effective discount rate. Notice that

$$\begin{aligned}\mathbb{E}\left[H_t^v\right] = & \ \mathbb{E}\left[\int_0^t R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right] + \mathbb{E}\left[\mathbb{E}_t\left[\int_t^\infty R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right]\right] \\ = & \ \mathbb{E}\left[\int_0^\infty R_h^v \left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right) dh\right] = H_0^v\end{aligned}$$

so H_t^v is a Martingale with respect to the filtration generated by x and z. By the Martingale representation theorem, there exist processes Δ_{xt}, Δ_{zt} such that

$$dH_t^v = \Delta_{xt} R_t^v \sigma_x dB_{xt} + \Delta_{zt} R_t^v \sigma_z dB_{zt}$$

Now using Ito's lemma on equation A.2 we find

$$dH_t^v = R_t^v \left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right) dt - R_t^v \left(r + \kappa\lambda(v_t, z_t)\right) V_t^v dt + R_t^v dV_t^v$$

Combining the two expressions for dH_t^v gives

$$dV_t^v = \left(rV_t^v - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v)\right) dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt}$$

This concludes the proof. □

The next lemma characterizes incentive compatible strategies v in terms of the worker continuation value V_t^v.

Lemma 4. *A strategy v is incentive compatible if*

$$v_t \in \arg\max_v \ \lambda_w(v, z_t)\left(v - V_t^v\right)$$

Proof. Let v be an incentive compatible search strategy. We show that deviations are not profitable at any t. Assume that the worker deviates to an alternative strategy $\hat{v}$ until time t. Define $H_t^{\hat{v}}$ the process corresponding to this deviation,

$$H_t^{\hat{v}} = \int_0^t R_h^{\hat{v}} \left(u(w_h) + \kappa\lambda_w(\hat{v}_h, z_h)\hat{v}_h\right) dh + R_t^{\hat{v}} V_t^v \tag{A.3}$$

where the continuation value at time t is V_t^v because the worker follows the recommended strategy thereafter.

Note that $H_0^{\hat{v}} = H_0^v$. We want the process H_t^v to be a martingale under v and a super-martingale under any alternative strategy $\hat{v}$ so that $\mathbb{E}[H_t^v] = H_0^v = H_0^{\hat{v}} \geq \mathbb{E}[H_t^{\hat{v}}]$. This ensures that the worker will never choose to deviate from search strategy v since this would lower her expected utility $\mathbb{E}[H_t^{\hat{v}}]$. Using the law of motion for V from lemma 3 and equation A.3 we get

$$\begin{aligned} dH_t^{\hat{v}} &= R_t^{\hat{v}} \left(u(w_t) + \kappa\lambda_w(\hat{v}_t, z_t)\hat{v}_t\right) dt - R_t^{\hat{v}} \left(r + \kappa\lambda_w(\hat{v}_t, z_t)\right) V_t^v dt + R_t^{\hat{v}} dV_t^v \\ &= R_t^{\hat{v}} \left(\kappa\lambda_w(\hat{v}_t, z_t)(\hat{v}_t - V_t^v) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v)\right) dt + R_t^{\hat{v}} \Delta_{xt}\sigma_x dB_{xt} + R_t^{\hat{v}} \Delta_{zt}\sigma_z dB_{zt}\end{aligned}$$

$H_t^{\hat{v}}$ is a super-martingale if and only its drift is negative, i.e.

$$\lambda_w(v_t, z_t)\left(v_t - V_t\right) \geq \lambda_w(\hat{v}_t, z_t)\left(\hat{v}_t - V_t\right) \quad \text{for all } \hat{v}_t$$

This can be written as

$$v_t \in \arg\max_v \ \lambda_w(v, z_t)\left(v - V_t^v\right)$$

This concludes the proof. $\square$

Using lemmas 3 and 4 we can rewrite the optimal contracting problem as

$$F(V_t, x_t, z_t) = \max_{w,\Delta_x,\Delta_z} \quad \mathbb{E}\left[\int_0^\infty e^{-rt-\int_0^t \kappa\lambda_w(v_s,z_s)ds}(\exp(x_t+z_t)-w_t)dt\right]$$

subject to

$$\begin{array}{ll}
(\text{PK}): & dV_t = (rV_t - u(w_t) - \kappa\lambda_w(v_t,z_t)(v_t - V_t))\,dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\
(\text{IC-v}): & v_t \in \arg\max_v \ \lambda_w(v,z_t)\,(v - V_t) \\
& dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \\
& dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}
\end{array}$$

B.2.3 Proofs for section 4

It will be useful to write the HJB corresponding to the optimal contract

$$\begin{aligned}
(r+\kappa\lambda_w(v(V,z),z))F(V,x,z) \quad &= \max_{w,\Delta_x,\Delta_z} \exp(x+z) - w \\
&+ (rV - u(w) - \kappa\lambda_w(v(V,z),z)(v(V,z)-V))\,F_V(V,x,z) \\
&-\alpha_x x F_x(V,x,z) - \alpha_z z F_z(V,x,z) \\
&+\sigma_x^2\left[\tfrac{1}{2}\Delta_x^2 F_{VV}(V,x,z) + \tfrac{1}{2}F_{xx}(V,x,z) + \Delta_x F_{Vx}(V,x,z)\right] \\
&+\sigma_z^2\left[\tfrac{1}{2}\Delta_z^2 F_{VV}(V,x,z) + \tfrac{1}{2}F_{zz}(V,x,z) + \Delta_z F_{Vz}(V,x,z)\right]
\end{aligned}$$

Proposition 1 I first derive the optimal path of wages since it does not require my approximation in the degree of search efficiency $\kappa \to 0$.

Proof. The optimality conditions of the HJB with respect to w is

$$w(V,x,z) = (u')^{-1}\left(-\frac{1}{F_V(V,x,z)}\right) \tag{A.4}$$

and with respect to , Δ_x and , Δ_z are

$$\Delta_x(V,x,z) = -\frac{F_{Vx}(V,x,z)}{F_{VV}(V,x,z)}, \quad \Delta_z(V,x,z) = -\frac{F_{Vz}(V,x,z)}{F_{VV}(V,x,z)} \tag{A.5}$$

Applying Ito's lemma on the optimality condition for w gives

$$dw_t \quad = \quad -\tfrac{w_t u'(w_t)}{\gamma(w_t)}\, dF_V(V_t, X_t) \tag{A.6}$$

and applying Ito's lemma on $F_V(V,x,z)$ gives

$$\begin{aligned}
dF_V = \quad & (\mu_V F_{VV} + \mu_x F_{Vx} + \mu_z F_{Vz})\,dt \\
& + \left(\tfrac{1}{2}\left(\Delta_x^2\sigma_x^2 + \Delta_z^2\sigma_z^2\right)F_{VVV} + \sigma_x^2\left(\tfrac{1}{2}F_{Vxx} + \Delta_x F_{VVx}\right) + \sigma_z^2\left(\tfrac{1}{2}F_{Vzz} + \Delta_z F_{VVz}\right)\right)dt
\end{aligned} \tag{A.7}$$

where we used optimality condition for $\Delta_x(V,X)$ to get ride of the diffusion terms. The terms μ_V, μ_x and μ_z denote the drift of V, x and z.

Differentiate the HJB equation with respect to V gives

$$\begin{aligned}\kappa\tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}F &= \mu_V F_{VV} + \mu_x F_{Vx} + \mu_z F_{Vz} \\ &+\tfrac{1}{2}\left(\Delta_x^2\sigma_x^2 + \Delta_z^2\sigma_z^2\right)F_{VVV} + \sigma_x^2\left(\tfrac{1}{2}F_{Vxx} + \Delta_x F_{VVx}\right) + \sigma_z^2\left(\tfrac{1}{2}F_{Vzz} + \Delta_z F_{VVz}\right)\end{aligned}$$

where we used the envelope theorem to get $\partial_V\lambda_w(v(V,z),z)(v(V,z)-V) = -\lambda_w(v(V,z),z)$. Combining this expression with A.6 and A.7 gives

$$dw_t = -\frac{w_t u'(w_t)}{\gamma(w_t)}\,F(V_t,x_t,z_t)\,\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial V}dt + 0\times dB_{xt} + 0\times dB_{zt}$$

Rewriting this expression using the retention elasticity 3 gives the desired result. □

Lemma 2 Before solving the for firm value when $\kappa \to 0$, I solve it when $\kappa = 0$.

Lemma 5. *If $\kappa = 0$, wages are constant and the firm value is*

$$F(V,x,z) = g(x,z) - h(V)$$

where $g(x,z) = r^{-1}\left(\exp(x+z) + \mathcal{D}g(x,z)\right)$ is the present value of output and $h(V) = u^{-1}(rV)/r$ is the cost of providing a value V to workers. The policy functions are

$$w(V) = u^{-1}(rV), \quad \Delta_x = 0, \quad \Delta_z = 0$$

Proof. We prove this result by guess and verify. Conjecture that the firm value takes the form

$$F(V,x,z) = g(x,z) - h(V)$$

for two functions $g(x,z)$ and $h(V)$ to be determined. This implies that $\Delta_x = \Delta_z = 0$ and that $w(V,x,z) = w(V)$ from equations A.4 and A.5. Plugging this conjecture in the HJB together with $\kappa = 0$ gives two conditions that $g(x,z)$ and $h(V)$ must satisfy

$$\begin{aligned} rg(x,z) &= \exp(x+z) + \mathcal{D}g(x,z) \\ rh(V) &= w(V) + (rV - u(w(V)))\,h'(V)\end{aligned}$$

Since these equations are independent, this confirms our guess. It is straightforward to verify that $h(V) = u^{-1}(rV)/r$ and $w(V) = u^{-1}(rV)$ satisfy the second ODE. Finally, plug in the value of $w(V)$ in dV_t together with $\Delta_x = \Delta_z = 0$ to show that the worker value and therefore the wage are constant over time. □

I now prove lemma 2.

Proof. Consider a first-order Taylor expansion of $F(V,x,z)$ around $\kappa = 0$

$$F(V,x,z) = g(x,z) - h(V) + \kappa\partial_\kappa F(V,x,z;\kappa=0) + O(\kappa^2) \tag{A.8}$$

using lemma 5. Introduce the function

$$\ell(V,x,z) \equiv -\kappa^{-1}\left[F(V,x,z) - (g(x,z) - h(V))\right] \tag{A.9}$$

and therefore $\partial_\kappa F(V,x,z;\kappa=0) = \lim_{\kappa\to 0} \ell(V,x,z)$. The function $\ell(V,x,z)$ can be interpreted as the cost of retaining workers to the firm, scaled by the degree of worker mobility κ.

Plugging (A.9) in the HJB and using the optimality conditions A.4 and A.5 gives

$$\begin{aligned}
0 \;&= \exp(x+z) - w(V,x,z) \\
&+ \left(rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z)-V)\right)\left(-h'(V) - \kappa\ell_V(V,x,z)\right) \\
&-\alpha_x x g_x(x,z) - \alpha_z z g_z(x,z) \\
&+\tfrac{\sigma_x^2}{2}\tfrac{(g_{xx}(x,z)-\kappa\ell_{xx}(V,x,z))(h''(V)-\kappa\ell_{VV}(V,x,z))+\kappa^2\ell_{Vx}^2(V,x,z)}{h''(V)+\kappa\ell_{VV}(V,x,z)} \\
&+\tfrac{\sigma_z^2}{2}\tfrac{(g_{zz}(x,z)-\kappa\ell_{zz}(V,x,z))(h''(V)-\kappa\ell_{VV}(V,x,z))+\kappa^2\ell_{Vz}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)} \\
&-\left(r+\kappa\lambda_w(v(V,z),z)\right)\left(g(x,z)-h(V)-\kappa\ell(V,x,z)\right)
\end{aligned}$$

We now subtract $rg(X)$ and $rh(V)$ on both sides and use their definition from lemma 5 to get

$$\begin{aligned}
-(r+\kappa\lambda_w(v(V,z),z))\kappa\ell(V,x,z) \;&= \kappa\lambda_w(v(V,z),z)h(V) - \kappa\lambda_w(v(V,z),z)g(x,z) - [w(V,x,z)-w(V)] \\
&- \left(rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z)-V)\right)\kappa\ell_V(V,x,z) \\
&+\kappa\lambda_w(v(V,z),z)[v(V,z)-V]h'(V) + [u(w(V,x,z)) - u(w(V))]\,h'(V) \\
&-\kappa\alpha_x x\ell_x(V,x,z) - \kappa\alpha_z z\ell_z(V,x,z) \\
&-\tfrac{\sigma_x^2}{2}\tfrac{\kappa\ell_{xx}(V,x,z)(h''(V)-\kappa\ell_{VV}(V,x,z))-\kappa^2\ell_{Vx}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)} \\
&-\tfrac{\sigma_z^2}{2}\tfrac{\kappa\ell_z(V,x,z)(h''(V)-\kappa\ell_{VV}(V,x,z))-\kappa^2\ell_{Vz}^2(V,x,z)}{h''(V)-\kappa\ell_{VV}(V,x,z)}
\end{aligned} \tag{A.10}$$

where $w(V)$ is the wage policy when $\kappa = 0$.

Consider the term

$$\lim_{\kappa\to 0}\frac{w(V,x,z)-w(V)}{\kappa} = \lim_{\kappa\to 0}\frac{1}{\kappa}\left((u')^{-1}\left(\frac{1}{h'(V)-\kappa\ell_V(V,x,z)}\right) - (u')^{-1}\left(\frac{1}{h'(V)}\right)\right)$$

Take a Taylor expansion of the first term around $\kappa = 0$

$$(u')^{-1}\left(\frac{1}{h'(V)-\kappa\ell_V(V,x,z)}\right) = (u')^{-1}\left(\frac{1}{h'(V)}\right) + \kappa\frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))} + O(\kappa^2)$$

where we used $h'(V) = 1/u'(w(V))$. Therefore,

$$\lim_{\kappa\to 0}\frac{w(V,x,z)-w(V)}{\kappa} = \frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))}$$

Similarly, we get

$$\lim_{\kappa\to 0}\frac{u(w(V,x,z))-u(w(V))}{\kappa} = -\frac{\ell_V(V,x,z)}{h'(V)^2}\frac{u'(w(V))}{u''(w(V))}$$

Now divide equation A.10 by κ and take limit as $\kappa \to 0$

$$\begin{aligned}
r\ell(V,x,z) \;&= -\lambda_w(v(V,z),z)h(V) + \lambda_w(v(V,z),z)g(x,z) \\
&-\lambda_w(v(V,z),z)[v(V,z)-V]h'(V) \\
&-\alpha_x x\ell_x(V,x,z) - \alpha_z z\ell_z(V,x,z) \\
&+\tfrac{\sigma_x^2}{2}\ell_{xx}(V,x,z) + \tfrac{\sigma_z^2}{2}\ell_{zz}(V,x,z)
\end{aligned}$$

where we used $rV - u(w(V)) = 0$ from lemma 5. We can reformulate this equation using the differential operator introduced at the start of section 4 as

$$r\ell(V,x,z) = \lambda_w(v(V,z),z)\left[g(x,z) - h(V) - [v(V,z)-V]h'(V)\right] + \mathcal{D}\ell(V,x,z)$$

This concludes the proof. $\square$

Proposition 2

Proof. To first order in κ, the optimality condition with respect to Δ_x (A.5) in the HJB becomes

$$\Delta_x(V,x,z) = -\kappa\frac{\ell_{Vx}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vx}(V,x,z)$ from lemma 2. We have

$$r\ell_x(V,x,z) = \lambda_w(v(V,z),z)g_x(x,z) - \alpha_x\ell_x(V,x,z) + \mathcal{D}\ell_x(V,x,z)$$

and therefore

$$(r+\alpha_x)\,\ell_x(V,x,z) = \lambda_w(v(V,z),z)g_x(x,z) + \mathcal{D}\ell_x(V,x,z)$$

Now differentiate with respect to V to find

$$(r+\alpha_x)\,\ell_{Vx}(V,x,z) = \frac{\partial\lambda_w(v(V,z),z)}{\partial V}g_x(x,z) + \mathcal{D}\ell_{Vx}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial V}\frac{g_x(x,z)}{h''(V)} + \mathcal{D}\Delta_x(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (3) when $\kappa\to 0$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)$$

This concludes the proof. $\square$

Proposition 3

Proof. To first order in κ, the optimality condition with respect to Δ_z (A.5) in the HJB becomes

$$\Delta_z(V,x,z) = -\kappa\frac{\ell_{Vz}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vz}(V,x,z)$ from lemma 2. We have

$$\begin{aligned}(r+\alpha_z)\,\ell_z(V,x,z) = \lambda_w(v(V,z),z)g_z(x,z) + \tfrac{\partial\lambda_w(v(V,z),z)}{\partial z}\left(g(x,z)-h(V)\right)\\ -\lambda_{wz}(v(V,z),z)\left(v(V,z)-V\right)h'(V) + \mathcal{D}\ell_z(V,x,z)\end{aligned}$$

where I used the envelope condition of the worker search problem 2 for the third term. $\frac{\partial\lambda_w(v(V,z),z)}{\partial z}$ denotes the total derivative with respect to z whereas $\lambda_{wz}(v(V,z),z)$ denotes the partial derivative with respect to the second argument.

Differentiate with respect to V to find

$$(r+\alpha_z)\,\ell_{Vz}(V,x,z) = \tfrac{\partial\lambda_w(v(V,z),z)}{\partial V} g_z(x,z) + \tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\left(g(x,z)-h(V)\right) \\ -\lambda_{wz}(v(V,z),z)\left(v(V,z)-V\right)h''(V) + \mathcal{D}\ell_{Vz}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = -\kappa\tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}\tfrac{g_z(x,z)}{h''(V)} - \kappa\tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\tfrac{(g(x,z)-h(V))}{h''(V)} \\ +\kappa\lambda_{wz}(v(V,z),z)\left(v(V,z)-V\right) + \mathcal{D}\Delta_z(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (3) when $\kappa\to 0$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = \left[g_z(x,z)\tfrac{\epsilon(V,x,z)}{\gamma(w(V))} + \left(g(x,z)-h(V)\right)\tfrac{\epsilon_z(V,x,z)}{\gamma(w(V))}\right]u'(w(V)) \\ +\kappa\lambda_{wz}(v(V,z),z)\left(v(V,z)-V\right) + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z)\equiv\partial\epsilon(V,x,z)/\partial z$. Finally, use $\left(g(x,z)-h(V)\right)\epsilon_z(V,x,z) = F(V,x,z)\epsilon_z(V,x,z)$ to first order in κ to rewrite this expression as in the proposition. □

The pass-through of firm-level shocks to wages in section 4.2 Start from the path of wages in proposition 1 and take a first-order approximation in κ. This equation becomes

$$dw_t = \left(rg(x_t,z_t)-w_t\right)\,\frac{\epsilon(V_t,w_t,z_t)}{\gamma(w_t)}dt$$

where I wrote the retention elasticity as a function of the wage directly, and to first order in κ,

$$\epsilon(V,w,z)\equiv -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial v(V,z)}\times\frac{\partial v(V,z)}{\partial V}\times\frac{wu'(w)}{r}$$

To first order in x, we can rewrite the wage growth equation as

$$\hat{w}_t \approx \int_0^t\left(rg_x(x_0,z_0)\exp\left(-\alpha_z s\right)\hat{x}_0-\hat{w}_s\right)\,\frac{\epsilon(V_0,w_0,z_0)}{\gamma(w_0)}ds$$

where we used the definition of $\hat{x}_s$ and made the additional approximation that the ratio $\epsilon(V_0,w_0,z_0)/\gamma(w_0)$ was constant over time. We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0-\alpha_x}\frac{\epsilon_0}{\gamma_0}g_x(x_0,z_0)\left[\exp\left(-\alpha_x t\right)-\exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right]\hat{x}_0$$

where $\epsilon_0\equiv\epsilon(V_0,w_0,z_0)$ and $\gamma_0\equiv\gamma(w_0)$.

The pass-through of sectoral shocks to wages in section 4.3 Again start from the path of wages with $\kappa\to 0$,

$$dw_t = \left(rg(x_t,z_t)-w_t\right)\,\frac{\epsilon(V_t,w_t,z_t)}{\gamma(w_t)}dt$$

To first order in z, we can rewrite the wage growth equation as

$$\begin{aligned}\hat{w}_t \approx & \int_0^t \left(rg_z(x_0, z_0) \exp\left(-\alpha_z s\right) \hat{z}_0 - \hat{w}_s\right) \tfrac{\epsilon(V_0, w_0, z_0)}{\gamma(w_0)} ds \\ & + \int_0^t \left(rg(x_0, z_0) - w_0\right) \tfrac{\epsilon_z(V_0, w_0, z_0)}{\gamma(w_0)} \exp\left(-\alpha_z s\right) \hat{z}_0 ds\end{aligned}$$

where $\epsilon_z(V_0, w_0, z_0) \equiv \partial\epsilon(V_0, w_0, z_0)/\partial z$. This equation approximates the true wage response because we keep the ratio $\epsilon(V_0, w_0, z_0)/\gamma(w_0)$ and the firm value $rg(x_0, z_0) - w_0$ constant over time. The key difference with the pass-through from-firm level shocks derived in appendix B.2.3 is that sectoral productivity z enters directly as an input in the retention elasticity $\epsilon(V_0, w_0, z_0)$.

We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_z}\left(g_z(x_0, z_0)\frac{\epsilon_0}{\gamma_0} + F(V_0, x_0, z_0)\frac{\epsilon_{z0}}{\gamma_0}\right)\left[\exp\left(-\alpha_z t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right]\hat{z}_0$$

where $\epsilon_{z0} \equiv \partial\epsilon(V_0, w_0, z_0)/\partial z$.

Special case: no aggregate shocks and mean-reverting productivity To illustrate the formulas derived in section 4, I focus on a special case in which these formulas can be derived in closed form.

Assume that there are no aggregate shocks so we drop z from all equations, and assume that the production function is $a + x$ for some constant a with x following the process

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt}$$

In this case, we can solve for $g(x)$ by guess and verify as

$$g(x) = \frac{a}{r} + \frac{x}{r + \alpha_x}$$

so that $g_x(x) = 1/(r + \alpha_x)$. To solve for $\Delta_x(V, x, z)$ notice that the elasticity $\epsilon(V, x, z)$ is independent of x to first order in κ. Then, by guess and verify we get

$$\Delta_x(V) = (r + \alpha_x)^{-2}\frac{\epsilon(V)}{\gamma(w(V))}u'(w(V))$$

B.3 Relation to Chetty-Baily statistic

Equation 1 in Chetty (2006) is

$$\gamma\frac{\Delta c}{c}(b^*) \approx \epsilon_{D,b}$$

where $\Delta c/c(b^*)$ is the optimal change in consumption following an unemployment shock, $\epsilon_{D,b}$ is the elasticity of the duration of unemployment D with respect to unemployment benefits b and γ is the coefficient of relative risk aversion.

Note that the duration is defined as $D = 1/(1-p)$ where p is the probability that the worker stays unemployed (the "retention probability"). Therefore,

$$\epsilon_{D,b} = \frac{dD}{db}\frac{b}{D} = \frac{dp}{db}(1-p)^{-2}b(1-p) = \frac{dp}{db}\frac{b}{1-p} = \frac{\epsilon_{p,b}}{1-p}$$

where $\epsilon_{p,b}$ is the semi-elasticity of the retention probability (the probability that the worker remains unemployment) with respect to the unemployment benefit. Thus, the Chetty-Baily formula can be written as

$$\frac{\Delta c}{c} \approx \frac{\epsilon_{p,b}}{\gamma}(1-p)^{-1}$$