The Pro-Competitive Consequences of Trade in Frictional Labor Markets*

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What are the pro-competitive consequences of trade in frictional labor markets? This paper develops and estimates a dynamic general equilibrium trade model to show that the interplay between endogenously variable markups in product markets and frictions in labor markets has important implications for aggregate as well as distributional consequences of trade. In particular, I show that once markups are allowed to respond to trade liberalization, unemployment and residual wage inequality rise almost three times more than in a model with constant markups (in the steady state). The presence of labor market frictions makes the pro-competitive gains from trade liberalization negative.

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1 Introduction

A large body of literature explores the responses of markups to trade liberalization and whether they increase or decrease the gains from trade (Edmond et al., 2015; Arkolakis et al., 2019). The consequences of trade-induced changes in markups in frictional labor markets and their welfare implications are, however, less explored. This is an important question due to the rising importance of trade (restrictions), market power and markups, and labor market imperfections. This paper shows that the interplay between variable markups in product markets and frictions in labor markets has important implications for aggregate as well as distributional consequences of trade, which are less studied.\footnote{As Arkolakis et al. (2019), for example, mention, their study “has little to say about how variable markups may affect the distributional consequences of trade, alleviate misallocations between oligopolistic sectors, or worsen labor market distortions.”} In particular, a downward pressure on markups induced by foreign competition would increase firms’ responsiveness to their idiosyncratic shocks, thereby increasing unemployment and wage inequality in frictional labor markets.

I develop and estimate a dynamic general equilibrium model of international trade featuring endogenously variable markups to show that trade-induced changes in markups have important impacts on labor market outcomes in a frictional labor market. In particular, I show that once markups are allowed to respond to trade liberalization, unemployment and residual wage inequality rise almost three times more than in a model with constant markups (in the steady state). The surge in unemployment caused by variable markups tends to reduce the pro-competitive gains from trade in the steady state.\footnote{Following Arkolakis et al. (2019), by the pro-competitive gains from trade, I mean the differential welfare implications of trade when markups are variable and when they are constant.} Interestingly, I show that while abstracting from labor market frictions in my framework implies positive pro-competitive gains from trade liberalization, taking labor market frictions into account makes the pro-competitive gains from trade liberalization negative—that is, the steady-state welfare gains are smaller under variable markups than those under constant markups.

In the model, an endogenous measure of firms operate in a small open economy, each subject to idiosyncratic productivity shocks. Firms use intermediate inputs and hire (multiple) homogeneous workers to produce their differentiated products, subject to vacancy posting costs, firing costs, and matching frictions. These frictions create rents from worker-firm matches, and a standard bargaining game determines how they are divided up. Firms sell their products in oligopolistically competitive markets à la Atkeson and Burstein (2008), and therefore larger firms have more market power and charge higher markups, as is common in this class of models.

The link between openness, markups, and labor market outcomes in the model is attributable to trade liberalization increasing competition in product markets, thereby raising demand elasticities and putting downward pressure on markups.\footnote{Throughout this paper, “demand elasticity” refers to the price elasticity of demand.} This in turn increases the labor elasticities of...
revenue, which makes firms’ employment decisions more responsive to their idiosyncratic shocks. More responsiveness to shocks makes the firm size distribution more dispersed by shifting employment toward larger firms that pay higher wages because of labor market frictions. As jobs get more concentrated in high-paying firms, more individuals decide to search for jobs in the labor market, which in turn reduces the labor market tightness and increases unemployment. Furthermore, in a less tight labor market, it is more likely for firms to fill vacancies, which further increases firms’ responsiveness to idiosyncratic shocks, thereby reinforcing the effects on the size distribution and unemployment. In addition, a more dispersed firm size distribution, along with the fact that larger firms pay higher wages, raises residual wage inequality.

My focus on the response of demand elasticities to openness is supported by various studies in the empirical trade literature. To further motivate my key mechanism that links labor elasticities of revenue with firms’ employment responsiveness to shocks, Section 2 reports a variety of reduced-form suggestive evidence. In particular, I divide firms into different size bins and show that the estimated labor elasticities of revenue as well as the job turnover rates for all size bins rise after trade liberalization in Colombia; a rise in job turnover indicates that firms adjust their employment more frequently. Moreover, the size bins with a larger increase in labor elasticities of revenue also experience a larger rise in their job turnover rates.

I also demonstrate a positive cross-firm size correlation between estimated labor elasticities of revenue and rates of job turnover. In line with the literature (e.g., Haltiwanger et al. (2013) for US; Coşar et al. (2016) for Colombia; Dix-Carneiro et al. (2021) for Brazil), I show that larger firms provide more stable jobs—that is, they have lower job turnover rates. Moreover, I document that larger firms have lower labor elasticities of revenue. Hence, this paper provides a new explanation for the size-stability relationship: larger firms provide more stable jobs, partly because their labor elasticities of revenue are smaller, which implies less responsiveness to shocks.

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4The large literature that studies the impact of international trade on demand elasticities and markups in various countries suggests that import competition tends to increase demand elasticities and reduce markups. See, for example, Eslava et al. (2004) for Colombia; Roberts and Tybout (1996) for Mexico, Colombia, Chile, and Morocco; De Melo and Urata (1986) for Chile; Harrison (1994) for Cote d’Ivoire; Krishna and Mitra (1998) for India; Kim (2000) for Korea; Bottasso and Sembenedi (2001); Konings et al. (2005) for Bulgaria and Romania; Badinger (2007) for European countries; Levinsohn (1993) for Turkey; Edmond et al. (2015) for Taiwan; and Arkolakis et al. (2019), MacKenzie (2021), and Atrianfar and Firooz (2019). Some studies define markup as price over marginal cost, whereas others define it as total revenue over total variable costs. I use the former definition in this paper. De Loecker et al. (2016), who study trade liberalization in India, note that trade liberalization affects Indian firms’ markups through two competing forces: higher competition as well as lower input costs. They show that while firms charge higher markups after trade liberalization, markups fall conditional on input costs. Tybout (2003) reviews this literature and concludes that the price-cost margin falls by trade exposure. Eslava et al. (2004) estimate demand elasticities faced by Colombian manufacturing plants and show that demand elasticities fall by trade openness.

5“Job turnover” is defined as the sum of job creation (total number of jobs created) and job destruction (total number of jobs destroyed).

6This finding is in line with the literature documenting that larger firms charge higher markups; see, for example, Atkin et al. (2015) for Pakistan, De Loecker and Warzynski (2012) for Slovenia, De Loecker et al. (2016) for India, and Edmond et al. (2015) for Taiwan.
Using my quantitative model, I explore how variable markups can influence trade-induced changes in labor market outcomes in a frictional labor market, thereby affecting the pro-competitive gains from trade. To this end, I compare the counterfactual outcomes of my baseline, variable-markup model to those of a restricted version with constant markups. I first fit both models to the same data moments from Colombian plant-level data in the pre-trade liberalization episode. Importantly, both models have the same (ex post) trade elasticity and imply the same increase in export (and import) share of revenue in a counterfactual trade liberalization exercise. I show that taking variable markups into account increases trade-induced unemployment and residual wage inequality by almost three times. In particular, my counterfactual exercises show that, under variable (constant) markups, Colombia’s reduction in import tariffs in the early 1990s increases the unemployment rate and residual wage inequality in the steady state by 16.2% (5.8%) and 9% (3.8%), respectively. The counterfactual outcomes under variable markups are more consistent with changes in size distribution, unemployment, and residual wage inequality observed in Colombia after trade liberalization.7

The surge in unemployment in the variable-markup model reduces the measure of produced varieties, which increases the price index and puts downward pressure on welfare. In addition to this unemployment effect, variable markups in my model also affect the welfare consequences of trade through two other forces that are standard in the literature. On the one hand, Home producers’ markups fall and become less dispersed. Moreover, employment shifts toward larger, under-producing firms that charge higher markups. This effect, as in Edmond et al. (2015), tends to reduce misallocation arising from markups and to generate pro-competitive gains from trade liberalization (i.e., increasing the welfare consequences of trade relative to the constant-markup model).

On the other hand, however, as emphasized by Arkolakis et al. (2019), Foreign firms charge higher markups after trade liberalization, which tends to reduce the pro-competitive gains from trade. Overall, my counterfactual exercises imply that the pro-competitive “gains” from trade liberalization in a frictional labor market are quite negative: while Colombia’s trade liberalization in the 1990s raises its real income by 1.7% (in the steady state) under constant markups, accounting for variable markups reduces these gains to only 0.3%. That is, taking variable markups into account reduces the gains from trade liberalization by 80%.

To emphasize the role of search frictions in driving these negative pro-competitive gains from trade liberalization, it is important to show that this result is indeed due to labor market

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7The implications of the model are also broadly in line with what is documented in the literature. For instance, the literature documents that after adopting outward-oriented trade policies, several Latin American countries experienced higher (within-industry) job reallocations (Haltiwanger et al., 2004) and the associated negative consequences for workers, including higher unemployment rates (de Desarrollo, 2004) and higher wage inequality (Attanasio et al., 2004).
frictions, not to Foreign firms charging higher markups. To this end, I shut down search frictions in the labor market in both constant- and variable-markup models and perform the same trade liberalization counterfactual exercise. Both models again imply the same (ex post) trade elasticity and generate the same increase in the export share of revenue in the steady state, similar to the models with search frictions. Interestingly, in this world without labor market frictions, the pro-competitive gains from trade liberalization are positive: while steady-state real income rises by 2.5% in the variable-markup model, it rises by 2% in the constant-markup model. These positive pro-competitive gains from trade liberalization in a frictionless labor market are consistent with Edmond et al. (2015), as my model structure without search frictions is broadly in line with theirs.

We can therefore conclude that while abstracting from labor market frictions in my framework implies positive pro-competitive gains from trade liberalization, taking labor market frictions into account makes the pro-competitive gains from trade liberalization negative. That is, accounting for variable markups in a frictional labor market substantially reduces the gains from trade liberalization. In the language of Arkolakis et al. (2019), I show that the pro-competitive gains from trade liberalization can be even more “elusive” in the presence of labor market frictions.

This paper contributes to several strands of literature. First, this work contributes to the literature on the effects of trade openness in the presence of frictional labor markets (Davidson et al., 1999, 2008; Egger and Kreickemeier, 2009; Davis and Harrigan, 2011; Amiti and Davis, 2011; Fajgelbaum, 2013). More specifically, since the formulation in this paper uses random search and wage bargaining, this paper is relatively close to Helpman and Itskhoki (2010), Helpman et al. (2010), Felbermayr et al. (2011), Coşar et al. (2016), Helpman et al. (2017), Dix-Carneiro et al. (2021), and Ruggieri (2018). I contribute to this literature by exploring the impacts of trade-induced changes in markups in frictional labor markets. I show that variable markups substantially magnify the link between trade and labor market outcomes. By showing that variable markups can raise unemployment in frictional labor markets, which affects the welfare implications of trade, this paper also contributes to the literature quantifying the pro-competitive gains from trade (e.g., Edmond et al., 2015; Arkolakis et al., 2019; Gutiérrez, 2022). My paper complements these studies by focusing on how variable markups interact with search frictions in the labor market, which in turn has important aggregate and distributional consequences.\footnote{This paper complements the interesting study by Gutiérrez (2022); while I focus on unemployment and search frictions in the labor market and abstract from oligopsony in the labor market, Gutiérrez (2022) focuses on oligopsony and abstracts from unemployment and search frictions. Interestingly, he finds positive pro-competitive gains from trade in oligopsonistic labor markets.}

The theoretical link between international trade, demand elasticity, and labor market outcomes dates back (at least) to Rodrik (1997), who argues that globalization, through offshoring, makes labor demand more elastic, which in turn increases wage volatility. Slaughter (2001) documents that demand elasticity for production labor in most of the US manufacturing industries
between 1961 and 1991 has risen, partly as a result of international trade. De Loecker et al. (2020) state that the fall in worker flow in the US economy over the past few decades is consistent with the rise in average markups. Decker et al. (2020) document that the downward trend in job reallocation in the US is due to the lower responsiveness of firms to the shocks, which might be a result of globalization. I contribute to this literature by examining the quantitative importance of trade-induced changes in demand elasticities (i) in frictional labor markets and (ii) in a general equilibrium setting.

By quantifying that one main underlying mechanism that influences firms’ responsiveness to shocks and firm size distribution is changes in markups (and labor elasticities of revenue), this paper contributes to the literature that studies patterns and underlying forces of job turnover (e.g., Davis and Haltiwanger, 1992; Haltiwanger and Vodopivec, 2002; Coşar et al., 2016) as well as the vast literature on firm dynamics—for example, Jovanovic (1982), Hopenhayn (1992), Hopenhayn and Rogerson (1993), Ericson and Pakes (1995), Klette and Kortum (2004), Cooper et al. (2007), Luttmer (2007), Rossi-Hansberg and Wright (2007), Elsby and Michaels (2013), Coşar et al. (2016), Arkolakis (2016), and Decker et al. (2020), among others.

Finally, this paper contributes to the literature that quantifies labor or capital adjustment costs (e.g., Cooper and Haltiwanger, 2006; Cooper et al., 2007; Coşar et al., 2016; David and Venkateswaran, 2019). As pointed out by, for example, David and Venkateswaran (2019), adjustment costs would be overestimated if other sources of frictions or distortions are ignored. Using a structural model, this paper shows that imposing constant markups would overestimate the importance of labor market frictions by 11%: without markup adjustments, limited hiring and firing responses to idiosyncratic shocks are likely to be attributed entirely to input market frictions.

Although this paper studies the consequences of trade liberalization, the mechanism introduced here is not confined to international economics; any policy that enhances product market

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9The mechanism in Decker et al. (2020) is different from the one in this paper. Decker et al. (2020) argue that, as a result of globalization, firms may respond to shocks partly by adjusting the labor force in their production lines outside the US. As they cannot observe firms’ labor force working abroad, they say that a possible conclusion is that firms are less responsive to shocks. They also examine the link between import penetration and firms’ responsiveness and find a mixed result suggesting that, as they note, this is an important line of future research.

10The distinction between worker reallocation and job reallocation is emphasized in this literature. In both my model and data, I measure job reallocation, not worker reallocation. Davis and Haltiwanger (1992) report that a major fraction (35%-56%) of worker reallocation in the US is accounted for by job reallocation, and an even larger fraction is documented by Haltiwanger and Vodopivec (2002) for Estonia (about two-thirds). In this paper, I focus on the effect of demand elasticity on job (i.e., employment opportunities within firms) turnover, which leads to worker turnover. Since worker flow is not observed in my plant-level data, the only source of worker reallocation in my model is assumed to be job reallocation. Indeed, worker reallocation and job reallocation are equivalent in my model.

11David and Venkateswaran (2019) show that a large portion of capital misallocation is due to the capital tax (wedge), an exogenous object in their model, which is correlated with productivity. As a result, high productivity firms do not react much to productivity shocks. In my model, however, the (labor) wedge is an endogenous/variable object, sourced from variable markups. Moreover, this wedge reacts to trade-induced changes in product market competition.
competition would potentially influence demand elasticities faced by firms, which in turn influences labor market outcomes. Moreover, the insights provided in this paper are not limited to the labor market studies; changes in demand elasticities would influence not only the employment decisions of firms but also their other decision variables, such as investment.

The rest of this paper is organized as follows. Section 2 presents a simple theory to highlight the link between labor elasticities of revenue and firms’ responsiveness to shocks. Moreover, to support the essential mechanism of the paper, I provide some reduced-form suggestive evidence from the Colombian economy. Section 3 describes the environment, lays out the quantitative model, and defines the stationary equilibrium of the model. Section 4 discusses the calibration procedure, provides the intuition behind the moments targeted to learn about model parameters, and performs counterfactual exercises. Section 5 concludes.

2 The Main Mechanism

This section explains and provides suggestive evidence for the main mechanism introduced in this paper that links the labor elasticity of revenue with firms’ employment responsiveness to their shocks. First, I provide a simple model to show that firms’ employment responsiveness to shocks is a function of the labor elasticity of revenue. Then, I use Colombian manufacturing plant-level data to show that there is a positive association between the labor elasticity of revenue and job turnover.

2.1 A Simple Model

Although the full model developed in Section 3 is a dynamic general equilibrium trade model with labor market frictions, here, to fix the idea and show the main mechanism, I employ a static model with a competitive labor market. Consider an economy with price-setter firms that face the demand elasticity $\sigma$:

$$ q = Bp^{-\sigma}, $$

(1)

where $q$ is the quantity demanded, $p$ is the price charged by the firm, and $B$ is the aggregate demand shifter. Assume the following production function:

$$ q = \phi l^{\psi}, $$

(2)

where $\phi$ is firm’s productivity and $l$ is labor. To measure firms’ employment responsiveness to shocks in what follows, it is important to note that $\psi$ is the labor elasticity of output after all factors of production are optimized out and expressed in terms of labor.\(^{12}\)

\(^{12}\)Note that one can also introduce some fixed factors, such as capital, into the production function, assuming that they are fixed and do not respond to shocks. The measurement exercise in the next section takes into
Based on equations (1) and (2), one can write the firm’s revenue function as

\[ R = A^\zeta, \]

where \( A \equiv B^{\frac{1}{\phi}} \phi^{\frac{\sigma-1}{\sigma}} \) is the revenue productivity residual, or the firm’s revenue shock, and \( \zeta \equiv \psi\left(\frac{\sigma-1}{\sigma}\right) \) is the labor elasticity of revenue, again after all factors of production are optimized out and expressed in terms of labor. The labor elasticity of revenue consists of the labor elasticity of output \( \psi \) and the demand elasticity \( \sigma \). The labor elasticity of revenue rises with demand elasticity; the intuition is that for a firm that faces a more elastic demand curve, changes in labor (and therefore quantity) result in larger changes in revenue, since price is less responsive.

Using the first order condition of the firm’s profit maximization problem, one easily shows that

\[ \frac{dl/l}{dA/A} = 1 - \zeta, \]

where I call the left-hand side the firm’s employment responsiveness to revenue shocks \( A \) (responsiveness hereafter). Equation (4) implies that firms’ responsiveness increases with the labor elasticity of revenue.\(^{13}\) This is because as the labor elasticity of revenue rises, the marginal revenue product of labor changes more slowly with labor, and firms therefore respond more (in terms of employment) to shocks. The next section provides some suggestive evidence that firms with a higher labor elasticity of revenue are more responsive to their shocks and have higher job turnover rates.

An immediate implication of equation (4) is that when firms face a more elastic demand curve, they would become more responsive (in terms of employment) to their idiosyncratic shocks because price would be less responsive to changes in quantity.\(^{14}\) In the full model developed in Section 3, demand elasticity \( \sigma \) (and therefore, labor elasticity of revenue \( \zeta \)) is a firm-level endogenous equilibrium object that responds to the level of competition in the market. By influencing the degree of product market competition, trade liberalization affects the demand elasticities that firms face, which in turn changes firms’ responsiveness to shocks, thereby influencing various labor market outcomes. The next section documents that trade liberalization tends to increase both the labor elasticities of revenue and job turnover rates, and that firms that experience a larger rise in their labor elasticity of revenue after trade liberalization also experience a larger increase in job turnover rates.

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\(^{13}\) I assume \( \zeta < 1 \). Section 2.2 estimates the labor elasticity of revenue and shows this is indeed the case for manufacturing plants in Colombia.

\(^{14}\) Appendix A takes an alternative perspective and uses the labor demand function to relate output demand elasticity to firms’ responsiveness to shocks. To elaborate, instead of arguing that the labor elasticity of revenue is affected by changes in output demand elasticity, Appendix A shows that, as emphasized in the literature, labor demand elasticity responds to changes in output demand elasticity. These two perspectives are closely connected since the labor elasticity of revenue and labor demand elasticity are two sides of the same coin.
2.2 Suggestive Evidence

In this section, I use the Colombia Annual Manufacturing Survey to support the essential mechanism of this paper that links the labor elasticity of revenue to firms’ responsiveness to their shocks. The data cover all manufacturing plants with at least 10 employees, from the periods 1983 to 1991 and 1992 to 2012.\textsuperscript{15,16} This section presents three pieces of suggestive evidence. First, I show that larger firms have smaller labor elasticities of revenue and adjust their labor less frequently (i.e., they have lower job turnover rates). Hence, there is a positive association between job turnover and the labor elasticity of revenue across the firm size distribution. The second piece of suggestive evidence shows that trade liberalization in Colombia tends to increase both the labor elasticities of revenue and job turnover rates. Moreover, I show that firms that experience a larger rise in their labor elasticity of revenue after trade liberalization also experience a larger increase in job turnover rates. The last piece of evidence documents that firms with a higher labor elasticity of revenue are more responsive to their revenue shocks.\textsuperscript{17}

**Suggestive Evidence 1.** Larger firms have smaller labor elasticities of revenue and adjust their labor less frequently (i.e., they have lower job turnover rates). Hence, there is a positive association between the labor elasticity of revenue and job turnover across the firm size distribution.

To present this evidence, I divide plants into 40 employment bins\textsuperscript{18} and estimate the revenue function separately for each employment bin $j$:\textsuperscript{19}

$$
\log(R_{ijt}) = \zeta_j \log(l_{ijt}) + \log(A_{ijt}) + \varepsilon_{ijt},
$$

(5)

where $R_{ijt}$ and $l_{ijt}$ are, respectively, revenue and employment of plant $i$ in employment bin $j$ at time $t$. The term $A_{ijt}$ is the transmitted part of revenue shocks, while $\varepsilon_{ijt}$ are transitory i.i.d. shocks.\textsuperscript{20} The objects of interest are the labor elasticities of revenue $\zeta_j$.

There are six categories of workers employed at plants: unskilled workers, apprentices, foreign

\textsuperscript{15}This is a plant-level data set. In what follows, however, I use the words “firm” and “plant” interchangeably. Moreover, I winsorize the data at 1% and 99%.

\textsuperscript{16}Special thanks to Jim Tybout, who provided me with the 1983-1991 data. The data from 1992 to 2012 are from Colombia National Administrative Department of Statistics (DANE). Moreover, note that since the plant IDs before 1991 are not generated by DANE, I cannot link the plants before 1991 to those after 1992.

\textsuperscript{17}Since both the labor elasticity of revenue and firms’ responsiveness are endogenous variables, all the pieces of suggestive evidence presented in this section are correlations; that is, I do not claim to establish any causal link between the two in this section.

\textsuperscript{18}Firms are placed into employment bins based on their size in the initial year in the sample, and therefore firms do not change employment bins over time.

\textsuperscript{19}In supporting my mechanism, one advantage of estimating the labor elasticity of revenue instead of demand elasticity (or the labor elasticity of output) is that the former can be more credibly estimated without observing price and quantity separately, a common issue in estimating production or demand functions.

\textsuperscript{20}I call $A$ the “revenue shocks” since it may contain productivity as well as demand-side shocks. This is also called the “revenue productivity residual” and is estimated in the literature; see, for example, Gopinath et al. (2017), Decker et al. (2018), and Foster et al. (2017).
technicians, local technicians, skilled workers, and management staff. To control for worker heterogeneity/productivity to the extent possible, I use effective labor to measure plant-level employment throughout this paper. To measure effective labor, I first calculate the average wage of each type of worker across all plants and all time periods. Then, each type of worker is weighted by its average wage relative to the average wage of unskilled workers. The sum of these weighted workers is the plant-level effective labor.

In estimating the revenue function (5), I employ the control function approach in Levinsohn and Petrin (2003) to solve the simultaneity problem between revenue shocks and labor input; Appendix B reports the results using Ackerberg et al. (2015) methodology as a robustness check. In line with my structural model in Section 3, I treat labor as a dynamic input; that is, labor is a state variable since it is subject to adjustment costs. Finally, I use material usage as the proxy variable.

As discussed before, the revenue function (5) assumes that all factors of production except labor are static (i.e., freely adjustable) and are optimized out. As an alternative, Appendix B estimates the labor elasticities of revenue by including capital, which is assumed to be fixed, and all other factors of production except labor are assumed to be static and optimized out. Notice that estimating the labor elasticity of revenue that is informative for firms’ responsiveness to shocks needs at least one of the aforementioned assumptions about capital (and other production inputs). This is because if more than one production input is assumed to be dynamic, one needs to explicitly model the factor adjustment costs to be able to back out the labor elasticity of revenue. Under these two alternative assumptions, I estimate the labor elasticity of revenue $\zeta_j$, and the results are very similar.

I calculate the job turnover rate for each employment bin $j$ in year $t$ as

$$JT_{jt} = \frac{\sum_{i \in c} |l_{ijt} - l_{ijt-1}| + \sum_{i \in exit} l_{ijt-1}}{(L_{jt-1} + L_{jt})/2} \times 100,$$

where $i$ denotes plants, $c$ is the set of continuing plants, and $L_{jt}$ is total employment for employment bin $j$ at time $t$. The terms in the numerator measure the employment adjustment by continuing plants and exiting plants, respectively. I then take the average job turnover rate for

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21 Cooper and Haltiwanger (2006) use GMM to estimate the revenue function. Their estimation method is similar to the second stage of Levinsohn and Petrin (2003). Note that here I employ the control function approach to estimate the revenue function by writing the revenue shocks (which consist of both productivity and demand shocks) as a function of the proxy variable. The assumption here is monotonicity of the proxy variable in revenue shocks.

22 As a robustness check, I use energy consumption as the proxy variable and show that the results hold.

23 This is similar to the way that the capital adjustment literature, for example, Cooper and Haltiwanger (2006), estimates the capital elasticity of revenue: they assume all factors of production except capital are static.

24 For the fixed-capital assumption to seem more appropriate, I use the relatively short sample period 1983-1991 to estimate revenue functions.
Figure 1: Job Turnover and Labor Elasticity of Revenue across the Firm Size Distribution

(a) Job Turnover  
(b) Labor Elasticity of Revenue

Notes: Plants are divided into 40 employment bins, displayed by the horizontal axes. The shaded areas reflect the linear fitted lines along with their 95% confidence bands.

Each employment bin over the sample period.$^{25}$

Figure 1 reports the results. The left panel shows that larger firms have lower rates of job turnover. The right panel of this figure shows that the labor elasticity of revenue falls with firm size.$^{26}$ This paper links firms’ size and job stability through the labor elasticity of revenue. Through the lens of the quantitative model in Section 3, larger firms face lower demand elasticities and therefore have smaller labor elasticities of revenue. This makes larger firms less responsive to their idiosyncratic shocks.

**Suggestive Evidence 2.** Trade liberalization in Colombia tends to increase both the labor elasticities of revenue and job turnover rates. Moreover, plants with a larger rise in their labor elasticity of revenue experience a larger increase in their job turnover rates.

This suggestive evidence exploits variations in the labor elasticity of revenue and job turnover over time. To this end, I follow the methodology outlined above to estimate the labor elasticity of revenue separately for each of the 40 employment bins—once for the pre-trade liberalization episode (i.e., from 1983 to 1991) and once using the post-liberalization data (i.e., from 1992 to

$^{25}$Two points regarding the definition of the job turnover rate are worth mentioning. First, to ensure the robustness of results, Appendix B measures the job turnover rate net of employment change for each size bin by subtracting from the numerator the net employment change for each size bin. As Appendix B shows, all the results in this section are robust to that definition. Second, the aggregate job turnover rate defined in the literature (and also in Section 4 of this paper) includes in the numerator the job creation by entrants. However, since entrants do not belong to any size bins in year $t - 1$ (simply because they did not exist), they do not enter employment bins' job turnover rates.

$^{26}$This is true even after controlling for industry. As a robustness check, I divide plants in each three-digit ISIC industry into five employment bins based on their size and show that within each industry, the labor elasticity of revenue falls with size.
Moreover, for each employment bin, I take the average of within-bin job turnover in (6) over the pre- and post-liberalization episode.\footnote{For the pre-trade liberalization period, I divide plants into 40 employment bins based on their size in the initial year in the sample. I use the same size quantiles to divide plants in the post-liberalization episode, based on their size in the initial year. Plants’ assigned employment bins (in both pre- and post-liberalization episodes) are fixed and do not change over time. As explained before, using the six categories of labor reported in the pre-1991 data, labor is measured in effective units. The data after 1992, however, do not report the detailed information about the types of workers employed at each plant. Hence, to make the post-1992 labor units comparable to the pre-liberalization data, I proceed as follows. Using the pre-liberalization plant-level data, I fit effective labor to a polynomial function of total number of workers. Then, I use the coefficients from this regression to convert total number of workers observed in the post-liberalization data to effective labor units.}

Figure 2a plots changes in the labor elasticity of revenue and job turnover from pre- to post-trade liberalization episode for each employment bin, with 1 being the smallest and 40 being the largest size bin. Figure 2a shows three facts. First, within each size bin, both the labor elasticity of revenue and job turnover rate rise after trade liberalization in Colombia (i.e., changes are positive). Second, firms that experience a larger rise in the labor elasticity of revenue also experience a larger increase in their job turnover rates. Lastly, the largest plants (e.g., size bins 30 to 40) experience the largest increase in their labor elasticity of revenue, which is indeed consistent with a variety of variable-markup models in the literature: Conditional on the trade shock, larger firms experience a larger increase in their demand elasticity (and therefore in their labor elasticity of revenue) compared to smaller firms. This result is also in line with the quantitative model in Section 3, in which trade liberalization has a heterogeneous effect on the perceived demand elasticity across the firm size distribution.

A natural question is whether the observed increases in the labor elasticity of revenue and job turnover documented in Figure 2a are associated with trade liberalization in Colombia. To show that this is indeed the case, I employ a shift-share treatment approach (in the spirit of Kovak, 2013; Dix-Carneiro and Kovak, 2017; Felix, 2021, among others) by exploiting the industrial composition within each size bin as well as tariff variations across industries.\footnote{Colombia has different phases of trade liberalization. The largest drop in tariffs and non-tariff barriers happened in 1991. For more details on trade liberalization in Colombia, see Attanasio et al. (2004), Coşar et al. (2016), and Alessandria and Avila (2020), among others.} In particular, I construct the change in import competition exposure for each size bin \(j\) as

\[
\Delta\text{exposure}_j = -\sum_k s_{kj} \ln \left( \frac{1 + \tau_{k,\text{post}}}{1 + \tau_{k,\text{pre}}} \right),
\]

where \(s_{kj}\) denotes the employment share of three-digit ISIC industry \(k\) in employment bin \(j\), and \(\tau_{k,\text{pre}}\) and \(\tau_{k,\text{post}}\) are industry \(k\)’s average import tariffs in the pre- and post-liberalization episode, respectively. Intuitively, this measure is a weighted average of changes in tariffs for each size bin.

\footnote{As mentioned before, Appendix B shows that results are robust to defining the job turnover rate net of employment change for each size bin.} I note that while the aforementioned literature exploits industrial composition within geographic markets, I use industrial composition within size bins.
Figure 2

(a) Turnover vs. Labor Elas. of Rev.  (b) Turnover vs. Exposure  (c) Labor Elas. of Rev. vs. Exposure

Notes: Plants are divided into 40 employment bins. Changes in job turnover and labor elasticity of revenue are from pre- to post-trade liberalization episode. The shaded area reflects the linear fitted line along with its 95% confidence band.

where weights are industries’ employment shares in that bin. As Figures 2b and 2c show, firms that have a larger rise in import competition exposure experience a larger increase in both job turnover and labor elasticity of revenue. Therefore, we can conclude that the observed increase in labor elasticity of revenue and job turnover is indeed associated with trade liberalization in Colombia.

**Suggestive Evidence 3.** Plants with larger labor elasticities of revenue are more responsive (in terms of employment) to their revenue shocks.

To measure revenue shocks, since plants in different industries potentially face different shocks, I first divide all plants within each three-digit ISIC industry into five employment bins. I then follow the methodology explained above to estimate a revenue function analogous to (5) separately for each employment bin $j$ within industry $k$. This procedure estimates the labor elasticity of revenue $\hat{\zeta}_{jk}$ and the (transmitted part of) revenue shocks $\hat{A}_{ijkt}$ for firm $i$ in employment bin $j$ within industry $k$ at time $t$.

Next, I estimate firms’ employment responsiveness to their revenue shocks.$^{31}$ In the quantitative model developed in Section 3, in line with the firm dynamics literature, the employment policy function of a firm is a function of the revenue shock to the firm, last period employment of the firm (due to labor adjustment costs), and persistence of revenue shocks. I estimate the responsiveness of firms by running the following regression separately for each employment bin $j$ within industry $k$: $^{32}$

$$
\log(l_{ijkt}) = \alpha_{jk} + \beta_{jk} \log(\hat{A}_{ijkt}) + \theta_{jk} \log(l_{ijkt-1}) + D_t + \nu_{ijkt},
$$

$^{31}$Recall that the employment responsiveness to revenue shock $A$ for a firm with $l$ workers is defined as $\frac{dl/l}{dA/A}$.

$^{32}$Since I measure the persistence of revenue shocks at the size-sector level $jk$, I cannot include this in regression (7). I control for the persistence of the shocks in what follows.
Table 1: Firms Responsiveness to Shocks vs. Labor Elasticity of Revenue

<table>
<thead>
<tr>
<th></th>
<th>Responsiveness</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor Elasticity of Revenue</td>
<td>0.057*</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Shocks Persistence</td>
<td>0.090*</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Industry FE</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.475</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>82</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Plants within each three-digit ISIC industry are divided into five employment bins. Responsiveness, labor elasticity of revenue, and persistence of shocks are estimated separately for each employment bin-industry pair. Standard errors in parentheses. * $p < 0.1$

where $D_t$ are time dummies, and $\nu_{ijkt}$ is the error term. The coefficients $\beta_{jk}$ measure firms’ employment responsiveness to revenue shocks in employment bin $j$ within industry $k$.

After estimating the employment responsiveness to revenue shocks for each size-industry pair $jk$, I examine whether the estimated responsiveness $\hat{\beta}_{jk}$ is positively correlated with the estimated labor elasticities of revenue $\hat{\zeta}_{jk}$. To this end, I run the following regression:

$$\hat{\beta}_{jk} = \gamma_0 + \gamma_1 \hat{\zeta}_{jk} + \gamma_2 \rho_{jk} + D_k + \epsilon_{jk},$$

where $D_k$ are industry fixed effects, and $\rho_{jk}$ is the persistence of revenue shocks at the size-industry level $jk$. Note that, as suggested by the firm dynamics literature (e.g., Cooper and Haltiwanger, 2006; Cooper et al., 2007), in the presence of labor adjustment costs, firms’ responsiveness to shocks rises with shocks’ persistence. As a result, I also control for the persistence of shocks in the regression above. To estimate $\rho_{jk}$, I assume revenue shocks $A_{ijkt}$ follow an AR(1) process, which I estimate separately for each size-industry pair $jk$.

Table 1 reports the results. As expected, firms’ employment responsiveness to shocks is positively associated with the labor elasticity of revenue. Moreover, firms’ responsiveness is also positively correlated with the persistence of shocks.

This section provided some suggestive evidence linking the labor elasticity of revenue with firms’ employment responsiveness to their shocks. The next section develops a dynamic general

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33 Since I approximate a presumably highly non-linear employment policy function using a linear function, I assume the source of the error term $\nu_{ijkt}$ is the functional form assumption. Hence, I allow the error term to be heteroskedastic.

34 In principle, what determines the firms’ employment decisions is shocks to the revenue function rather than shocks to the production function. Measuring responsiveness using shocks to the revenue function is also done in the literature; see, for example, Decker et al. (2020).

35 I include only the size-sector pairs with a statistically significant estimate for the labor elasticity of revenue.
equilibrium trade model featuring endogenously variable markups to quantify how international competition, by affecting demand elasticities and therefore the labor elasticities of revenue, can influence aggregate and distributional consequences of trade liberalization in frictional labor markets.

3 The Model

3.1 Environment

There are two countries in the world, Home (H) and Foreign (F)—the rest of the world. Home is a small open economy. Product markets in H and F are segmented. There are three types of goods: intermediate goods (or manufacturing sector), a final good, and services (which are the model numeraire). Only intermediate goods are tradable. Final good producers bundle intermediate goods (both domestic and imported) to produce a homogeneous final good to sell to consumers as well as intermediate producers in a perfectly competitive market. Labor is not used in producing the final good. Homogeneous services are produced by perfectly competitive producers with a constant returns to scale production technology that produces one unit of services using one worker. Intermediate producers are subject to idiosyncratic productivity shocks and employ workers as well as the final good to produce a particular variety using a proprietary technology. Intermediate producers sell their goods to final good producers under an oligopolistic market structure. The labor market is frictional. There is a unit measure of infinitely lived risk-neutral homogeneous workers who own the firms.

3.2 Consumers and Final Good Producers

Final good producers make a homogeneous composite good out of a continuum of intermediate goods \( \nu \) and sell it to consumers as well as intermediate producers in a perfectly competitive market. The final good in country \( i \in \{H, F\} \) is produced according to the following commonly available constant elasticity of substitution (CES) technology:\footnote{The size of final good producers is indeterminate, since they use a constant returns to scale technology and sell their goods in a perfectly competitive market.}

\[
M_i = \left[ \int_0^{N_H} Q_i(\nu) \frac{\sigma-1}{\sigma} d\nu \right]^{\frac{\sigma}{\sigma-1}},
\]

where \( \sigma \) is the elasticity of substitution, and \( N_H \) is the measure of intermediate goods available, which is an equilibrium object. Each good \( \nu \) is a CES aggregate over one Home variety and \( K \).
Foreign varieties:

\[ Q_i(\nu) = \left[ q_{Hi,1}(\nu)^{\frac{\alpha-1}{\alpha}} + \sum_{k=1}^{K} q_{Fi,k}(\nu)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{1}{\alpha-1}}, \]  

(10)

where subscript \( ji,k \) refers to the goods produced by firm \( k \) in country \( j \) and sold in country \( i \), \( K \) is a fixed number, and \( \alpha \) is the elasticity of substitution between varieties within each good \( \nu \). Parameters \( \sigma \) and \( \alpha \) govern the price elasticity of demand (i.e., demand elasticity) faced by intermediate producers, to be elaborated below.

The per-period utility of consumer \( n \) at time \( t \) equals

\[ U_{nt} = M_{nt}^\kappa Y_{nt}^{1-\kappa}, \]  

(11)

where \( M_{nt} \) and \( Y_{nt} \) are her consumption of the final composite good and services, respectively, and \( \kappa \) is the share of final good. Consumers maximize the expected present value of their utility stream:

\[ U_n = \sum_{t=1}^{\infty} \beta^{t-1} U_{nt}, \]  

(12)

where \( \beta \) is the discount factor. Workers are risk neutral and do not save.\(^{37}\) For ease of notation, I suppress the time subscript \( t \) in what follows.

The cost minimization problem for final good producers in country \( j \) implies the following demand for the variety produced by country \( i \)’s firm \( k \):

\[ q_{ij,k}(\nu) = A_j P_j(\nu)^{\alpha-\sigma} p_{ij,k}(\nu)^{-\alpha}, \]  

(13)

where \( A_j \) is the aggregate demand shifter in country \( j \), \( P_j(\nu) \) is the price index for good \( \nu \) in country \( j \), and \( p_{ij,k}(\nu) \) is the price of this variety in country \( j \). The price index for good \( \nu \) in country \( j \) is defined as

\[ P_j(\nu) = \left[ p_{Hj,1}(\nu)^{1-\alpha} + \sum_{k=1}^{K} p_{Fj,k}(\nu)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}. \]  

(14)

### 3.3 Intermediate Good Producers

Each intermediate producer receives an idiosyncratic productivity shock (à la Melitz, 2003; Chaney, 2008) and employs labor and the final good to produce its particular variety using the following proprietary technology:

\[ q(\phi, l) = \phi M(\phi, l)^{\eta l^{1-\eta}}, \]  

(15)

\(^{37}\)I assume that \( \beta \) is equal to the inverse of the interest rate and therefore workers do not have any incentive to save.
where $\phi$ is productivity, $l$ is the labor employed at the firm, $M(\phi, l)$ is the composite final good (call it material) demanded by firm $(\phi, l)$, and $\eta$ is the elasticity of output with respect to material (given labor). Intermediate producers are subject to idiosyncratic productivity shocks that follow a stationary Markov process

$$\ln \phi' = \gamma \ln \phi + \varepsilon, \quad \varepsilon \sim N(0, \sigma_\phi^2),$$

where $\gamma$ is the persistence of the productivity process and $\sigma_\phi$ is the variance of shocks. In what follows, $x'$ denotes the variable $x$ in the next period.

### 3.4 Firms’ Static Problem

Final good producers decide only about how much to buy each variety $\nu$ based on the demand equation (13). The rest of this section focuses on the decisions made by Home intermediate producers, which I call “firms” in what follows. Foreign firms’ problem is discussed in the last part of this section.

Firms carry two state variables over time: productivity and labor. Productivity is a state variable since it follows a Markov process, and labor is the other state variable as a result of the presence of labor adjustment costs (see Section 3.5). Moreover, labor market features search and matching frictions à la Mortensen and Pissarides (1994). For an incumbent firm starting this period with the last period productivity and employment $(\phi, l)$ as its state variables, the timing of the events is as follows:

- **exit decision**: pay $f_d$ to draw $\phi'$
- **death shock**: $l'$
- **bargain on $w(\phi', l')$**
- **post vacancies/fire**: $p, l', M(\phi', l')$

Before the new productivity realizes, the firm decides whether to exit the market. If it stays, the firm needs to pay a fixed operation cost $f_d$ to draw the new productivity $\phi'$, and the firm may hit by an exogenous death shock that forces the firm to exit the market. Entrants then replace the firms that exited. Next, the firm decides whether to expand or contract. To expand, the firm posts vacancies and matches with an endogenous number of workers, determined in equilibrium. To contract, the firm simply fires workers. The state of the firm then updates to $(\phi', l')$, after which the labor market closes. Since the labor market is frictional, rents may be generated at the firm. As a result, the firm and workers bargain over the surplus generated at the firm to determine the wage. As the last action within each period, the firm makes decisions about whether to export, how much material to buy, and prices to charge at Home and, if exporting, in the Foreign market.

We solve for firms’ decision variables backward. At the first stage, given the wage and number of workers, firms solve for prices, material usage, and the exporting decision in a static problem.
The second stage solves the wage bargaining problem given the number of workers. At the third stage, firms solve a dynamic vacancy posting/firing problem to determine the number of workers. For exposition purposes, I first describe the static pricing problem, then the dynamic vacancy posting problem, and at the end, the wage bargaining process.

Firms’ pricing decision is a static problem. At this stage, given the number of workers and the negotiated wage (to be discussed in Sections 3.5 and 3.7 below), a Home firm producing good \( \nu \) with productivity \( \phi \) and \( l \) workers makes decisions about prices to charge, whether to export, and the material usage by solving the following static optimization problem:

\[
\Pi(\phi, l) = \max_{p_{HH},p_{HF},M,I} \left[ p_{HH,1}(\phi, l)q_{HH,1}(\phi, l)+p_{HF,1}(\phi, l)I^x(\phi, l) - PM(\phi, l) - f_d - f_x I^x(\phi, l) \right],
\]

subject to

\[
q_{HH,1}(\phi, l) + q_{HF,1}(\phi, l)I^x(\phi, l)d \leq \phi M^n(\phi, l)^{1-\eta},
\]

\[
q_{Hj,1}(\phi, l) = A_j P_j(\nu)^{\alpha-\sigma} p_{Hj,1}(\phi, l)^{-\alpha},
\]

where \( I^x \) is an exporting indicator function that takes a value of 1 if the firm exports and zero otherwise, \( f_d \) is the fixed cost of operation, \( P \) is the price of the homogeneous final good (defined below), \( f_x \) is the fixed exporting cost, and \( d \) is the iceberg cost of exporting—that is, in order to deliver one unit of the good in destination, the producer needs to ship \( d \) units. For future reference, define the firm’s revenue \( (R) \) as sum of the first two terms in (17).

Fixed costs are paid in terms of services. The first constraint above is the feasibility constraint, and the second constraint is the demand schedules derived in equation (13). Notice that since the firm and its matched workers \( l \) have already negotiated and agreed to a particular wage, the wage bill does not enter the optimization problem above. The price of the homogeneous final good at Home is the CES aggregate of the intermediate goods prices:

\[
P = \left[ \int P_i(\nu)^{1-\sigma} d\nu \right]^{\frac{1}{\sigma}},
\]

where the price index \( P_i(\nu) \) is defined in equation (14) above.

The Foreign producer \( k \) of good \( \nu \) solves the following maximization problem:

\[
\Pi_{F,k}(\nu) = \max_{p_{FF,k},p_{FH,k}} \left[ p_{FF,k}(\nu)q_{FF,k}(\nu) + \frac{p_{FH,k}(\nu)}{\tau} q_{FH,k}(\nu) - e \times q(\nu) \right],
\]

subject to

\[
q_{FF,k}(\nu) + q_{FH,k}(\nu)d \leq q(\nu),
\]

\[
q_{Fj,k}(\nu) = A_j P_j(\nu)^{\alpha-\sigma} p_{Fj,k}(\nu)^{-\alpha},
\]
where \((\tau - 1)\) is the import tariff imposed by Home, and \(e\) is the marginal cost of production for all Foreign firms.\(^{38}\) Note that since \(p_{FH,k}\) is the price of the Foreign variety faced by Home consumers, \(p_{FH,k}/\tau\) is the price that the Foreign producer receives.

I assume that Home and Foreign varieties within each good \(\nu\) compete in prices.\(^{39}\) Using demand equations (19) and (23) and following Atkeson and Burstein (2008) to solve the pricing game between Home and Foreign varieties, one shows that the (absolute value of) demand elasticity faced by country \(i\)'s firm \(k\) in market \(j\) can be written as

\[
\varepsilon_{ij,k}(\nu) = s_{ij,k}(\nu)\sigma + (1 - s_{ij,k}(\nu))\alpha,
\]

(24)

where \(s_{ij,k}(\nu)\) is the share of country \(i\)'s firm \(k\) in total sales of good \(\nu\) in market \(j\), which can be written as

\[
s_{ij,k}(\nu) = \left(\frac{p_{ij,k}(\nu)}{P_j(\nu)}\right)^{1-\alpha},
\]

(25)

and this producer charges the markup \(\frac{\varepsilon_{ij,k}(\nu)}{\varepsilon_{ij,k}(\nu) - 1}\) in market \(j\). It is natural to think that the elasticity of substitution between varieties of a good is greater than that between different goods (i.e., \(\alpha > \sigma\)).\(^{40}\) Equation (24) then implies that larger firms face lower demand elasticities and therefore charge higher markups, which is in line with a large body of the literature (e.g., De Loecker and Warzynski, 2012; Atkin et al., 2015; Edmond et al., 2015; De Loecker et al., 2016)

The pricing decision of firms does not have a closed form solution unless \(\alpha = \sigma\)—that is, firms face a constant demand elasticity equal to \(\sigma\) (see equation (24)). In the variable demand elasticity case, however, I solve the pricing game numerically. After solving for prices, the Home firm decides whether to export by comparing the profit from serving only Home with that from serving both Home and Foreign. Finally, total demand can be derived using the demand equations (19), and then I solve for the material usage using the production function:

\[
M(\phi, l) = (\phi l^{1-\eta})^{\frac{\alpha}{\eta}} (q_{HH,1}(\phi, l) + q_{HF,1}(\phi, l)I^x(\phi, l)d)^{\frac{1}{\eta}}.
\]

(26)

The demand structure above allows demand elasticities and markups to respond to trade liberalization. On the one hand, by making the domestic market more accessible to Foreign producers, trade liberalization reduces the market share of domestic producers at Home, which

---

\(^{38}\)To solve the model, what matters is the productivity of Home producers relative to Foreign firms. Hence, I fix the marginal cost of Foreign producers and calibrate the productivity process for Home producers. Notice that while the model can easily be extended to one in which Foreign firms differ in their marginal costs, the assumption that Foreign firms’ marginal costs do not evolve over time is needed to keep the model tractable. This is because in a model with time-varying marginal costs for Foreign firms, the state space would include Foreign firms’ marginal costs as well.

\(^{39}\)Assuming Cournot competition delivers qualitatively similar and quantitatively close results.

\(^{40}\)Calibrating the model shows that this is indeed the case.
puts downward pressure on the markups that domestic producers charge at Home. On the other hand, trade liberalization raises the market share of Home producers in the Foreign market, which raises the markups that these producers charge in the Foreign market. The demand structure employed here therefore allows Foreign producers to charge higher markups in the Home market after trade liberalization, the importance of which is emphasized by, for example, Helpman and Krugman (1989), Edmond et al. (2015), and Arkolakis et al. (2019).

As equation (24) implies, by imposing \( \alpha = \sigma \), all firms would face the same demand elasticity and charge the same markup, which is invariant to trade liberalization. Hence, the demand structure employed here nests the commonly used workhorse model of international trade and macro, that is, monopolistic competition with CES demand. By comparing my model to a nested version with constant markups, I will quantify how variable markups influence labor market outcomes in a frictional labor market.

**Discussion.** The demand structure in this paper is slightly more general than the structure in Bernard et al. (2003), since that paper assumes an infinite elasticity of substitution between Home and Foreign varieties. Moreover, the demand structure here follows Atkeson and Burstein (2008) and Edmond et al. (2015) with one difference: they allow for multiple Home producers to compete at the lower tier. While in principle I could allow for multiple Home producers to compete within each good, doing so would be computationally very expensive. This is because, unlike Atkeson and Burstein (2008) and Edmond et al. (2015), this paper introduces labor market frictions with labor adjustment costs, and therefore allowing for more than one Home producer in (10) would require solving a dynamic vacancy posting game, on top of the static pricing game (see Section 3.5). Moreover, allowing for more than one Home producer would increase the state space since a firm’s decision would depend on competitors’ productivity and employment as well. To explore the robustness of results to the form of the demand structure, Appendix C employs an alternative demand structure similar to Simonovska (2015), which also features variable markups. Simulations of that alternative model deliver qualitatively similar results.\(^{41}\)

### 3.5 Firms’ Dynamic Problem

In the third stage of the firms’ problem, given bargained wages (to be discussed in Section 3.7) and the optimal policies in the first stage, firms choose whether to stay in the market, and if so, decide about vacancy posting/firing. A firm entering the period with state variables \((\phi, l)\) first decides whether to stay active or exit the market:

\[
V(\phi, l) = \max\{(1 - \lambda)E_{\phi|\phi}V^C(\phi', l), 0\},
\]

\(^{41}\)The model in the text and the one in Appendix C have the similar main property as the demand structure in Kimball (1995) (i.e., larger firms face lower demand elasticities); see Klenow and Willis (2016) and Edmond et al. (2018), among others. The main reason that I used a structure similar to Atkeson and Burstein (2008) rather than Kimball (1995) is that it is numerically less expensive to solve for prices in the current structure.
where $V(\phi, l)$ is the continuation value of the firm at state $(\phi, l)$, $\lambda$ is the exogenous exit rate, and $V_C(\cdot)$ is the continuation value of staying active in the market, to be defined below. If the firm exits, its continuation value would be zero; if it stays active in the market, the firm would draw its new productivity and may be hit by an exogenous death shock with probability $\lambda$. Solving the problem above delivers the policy function $I^c(\phi, l)$, which equals 1 if the firm stays in the market and 0 otherwise.

If the firm chooses to stay active, it decides whether to expand or contract. To expand, since the labor market features search and matching frictions, firms need to post vacancies to match with workers. I assume the following matching function (Den Haan et al., 2000; Cosar, 2013; Coşar et al., 2016):

$$M(U, V) = \frac{UV}{(U^\rho + V^\rho)^{\frac{1}{\rho}}} ,$$  \hspace{1cm} (28)

where $U$ is the number of job applicants, $V$ is the number of vacancies, and $\rho$ governs the degree of matching frictions. Higher values of $\rho$ indicate less severe matching frictions since for a given number of vacancies and job applicants, the number of matches rises with $\rho$. Particularly, if $\rho$ goes to infinity, there would be no matching frictions.\footnote{To see this, note that we can write $\lim_{\rho \to \infty} M(U, V) = \min\{U, V\}$. Hence, as $\rho \to \infty$, there would be no matching friction in the labor market.}

The job finding rate, which is the number of matches over the number of applicants, equals

$$m(\theta) = \frac{\theta}{(1 + \theta^\rho)^{\frac{1}{\rho}}} ,$$  \hspace{1cm} (29)

where $\theta := \frac{V}{U}$ is the labor market tightness. The vacancy filling rate, which is the number of matches over the number of vacancies, therefore equals $\frac{m(\theta)}{\theta}$.

The firm decides about vacancy posting/firing by solving the following dynamic optimization problem:

$$V_C(\phi', l) = \max_{l'} [\Pi(\phi', l') - w(\phi', l')l' - C(l, l') + \beta V(\phi', l')] ,$$  \hspace{1cm} (30)

subject to

$$C(l, l') = \begin{cases} (l - l')c_f & l' \leq l \\ c^l v^2 & l' > l \end{cases} ,$$  \hspace{1cm} (31)

$$v = (l' - l) \frac{\theta}{m(\theta)} ,$$  \hspace{1cm} (32)

where $\Pi(\cdot)$ is defined in equation (17), $w(\phi', l')$ is the wage that the firm with productivity $\phi'$ and $l'$ workers pays to all its workers (to be discussed below), and firms discount the future at the same rate $\beta$ as consumers. The term $C(l, l')$ is the cost of adjusting labor from $l$ to $l'$, which consists of a linear firing cost $c_f$ and a convex vacancy posting cost.
expand from $l$ to $l'$, the firm needs to post $(l' - l) \frac{\theta}{m(\theta)}$ vacancies since the vacancy filling rate equals $\frac{m(\theta)}{\theta}$. This is because I assume that the number of matches at each firm is proportional to the number of vacancies that the firm posts. For future reference, note that the job finding rate and the vacancy filling rate are increasing and decreasing functions of labor market tightness, respectively. As a result, as labor market tightness rises, the same expansion in employment requires a higher vacancy posting cost.

Solving the dynamic optimization problem in (30) delivers the employment policy function $l'(\phi', l)$, the vacancy posting policy function $v(\phi', l)$, and the hiring policy function $I^h(\phi', l)$, which takes the value of 1 if the firm expands and 0 otherwise. Note that, in line with the firm dynamics literature, while I assume a linear firing cost, the vacancy posting cost is convex. This formulation implies that while contracting firms adjust their workers at once, expanding firms hire new workers gradually (Coşar et al., 2016; Cooper and Willis, 2009; Cooper et al., 2007; Merz and Yashiv, 2007; Yashiv, 2006). Moreover, as will be discussed in detail in Section 3.7 below, without a convex hiring cost the model would imply no residual wage inequality among expanding firms (see, e.g., Felbermayr et al., 2011), which is not in line with either the literature or the data.

### 3.6 Entry

In equilibrium, an endogenous measure of entrants replace the firms that exit the market either endogenously or exogenously. The timing of the events for entrants is as follows:

<table>
<thead>
<tr>
<th>Event</th>
<th>Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay $f_e$ to draw $\phi$</td>
<td>$l$</td>
</tr>
<tr>
<td>Post vacancies</td>
<td>Bargain on $w(\phi, l)$</td>
</tr>
<tr>
<td>$p, I^r, M(\phi, l)$</td>
<td></td>
</tr>
</tbody>
</table>

Entrants pay the sunk entry cost $f_e$ to start with $l_e$ workers and draw their productivity.\(^{43}\) Entrants draw their productivity from $J^0(\phi)$, which is the ergodic distribution implied by the productivity process (16), and are able to expand right away by posting vacancies. Entrants then bargain with workers and decide about prices, exporting, and material usage. I assume that entrants are not hit by the exogenous death shock upon entry. The value of entry can therefore be expressed as

$$V_e := \int V^C(\phi, l_e) dJ^0(\phi) .$$

(33)

In equilibrium, the value of entry cannot exceed the sunk entry cost:

$$V_e \leq f_e ,$$

(34)

\(^{43}\)Note that workers, who own firms, pay the entry costs. However, since there are no realized profits at the beginning of each period, one might question the financing source of these entry costs. One could assume that workers save a constant amount each period, just enough to finance the entry costs. Note that, as mentioned before, workers have no other reasons to save.
which holds with equality if there is a positive mass of entrants.

3.7 Labor Market and Wage Bargaining

To model the labor market, I build on Coşar et al. (2016) with some modifications. As noted above, the labor market features search and matching frictions à la Mortensen and Pissarides (1994). Figure 3 shows the timing of events in the labor market, which I summarize here. Incumbent firms decide whether to exit or stay active. If a firm exits, either endogenously or exogenously as a result of the death shock, its workers join the unemployment pool. Entrants replace the firms that exit the market and draw their productivities from $J^0(\phi)$. Continuing incumbent firms also draw their new productivities based on the Markov process (16). Based on their realized productivities, firms decide whether to expand (by posting vacancies) or contract. If a firm contracts, fired workers join the unemployment pool. Unemployed individuals decide whether to search for a job in the manufacturing sector or to produce services. If a worker decides to produce services, she can produce one unit of services. Services are sold to consumers as well as intermediate producers in a perfectly competitive market. Fixed costs of operation $f_d$, fixed exporting costs $f_x$, sunk entry costs $f_e$, and labor adjustment costs are paid in terms of services.

Expanding intermediate producers post vacancies, and vacancies are filled at the vacancy filling rate noted above, which is an equilibrium object. If a worker decides to search for a job but does not match with a firm, she produces $b_u < 1$ units of services this period and joins the unemployment pool. After matching has taken place, the labor market closes. As a result, workers and firms cannot search for other alternatives at this point, and firms and matched workers therefore bargain on the wage to split the surplus generated at firms (to be discussed below). If the wage bargaining fails (which is not an equilibrium outcome), the match breaks down, the worker becomes unemployed and produces $b_u$ units of services and joins the unemployment pool.

To split the surplus, since the marginal revenue product of labor varies by employment size, I employ the Stole and Zwiebel (1996) bargaining game, which I summarize here.$^{44}$ The firm bargains with its matched workers one by one. Within each bargaining session, the firm and the worker play the bargaining game of Binmore et al. (1986). Here is what happens at each bargaining session. First, the firm offers a wage to the worker. The worker either accepts or rejects. If the worker accepts, the firm goes on to the next worker and starts bargaining. If the worker rejects, with some probability the match breaks down and the worker has to quit the firm. In this case, the firm starts over bargaining with all remaining workers. If a breakdown does not

$^{44}$ Basically, Stole and Zwiebel (1996) extend the Nash bargaining to the case in which the marginal revenue product of labor varies across workers—for example, because of a decreasing returns to scale production function, as in Elsby and Michaels (2013). Here, although production technology features constant returns to scale and the marginal product of labor is therefore constant, the marginal *revenue* product of labor varies by employment because firms face a downward-sloping demand curve.
happen, the worker makes a counteroffer to the firm. If the firm accepts the offer, this session closes and the firm goes on to bargain with the next worker. Otherwise, with some probability the match breaks down and the worker has to quit the firm. Again, in this case, the firm starts over bargaining with all remaining workers. In each bargaining session, many offers and counteroffers are proposed as either an agreement or a breakdown emerges. Stole and Zwiebel (1996) show that the outcome of this game is such that the firm views each worker as the marginal one, and the firm and each worker therefore split the surplus generated by the marginal worker. As a result, all workers within the same firm are paid the same wage.\footnote{It is worth elaborating more on the bargaining game that I employ in this paper. As Brügemann et al. (2019) emphasize, Stole and Zwiebel (1996)’s bargaining game can be viewed in two different ways. The first view is a perfect information game in which each worker is aware of the bargaining outcome between the firm and other workers. In this case, as Brügemann et al. (2019) show, the Stole and Zwiebel (1996)’s Theorem 2 is wrong and the unique subgame perfect equilibrium of this game is not the one they report in the paper. Second, Stole and Zwiebel (1996) can be interpreted as an imperfect information game in which the outcome of each bargaining session is the private information of the worker. As shown by Fontenay and Gans (2014), if one adds the “passive belief” assumption for the off-equilibrium beliefs, Stole and Zwiebel (1996)’s solution to this modified game is correct. I assume that the underlying bargaining game between firms and matched workers play the Rolodex game, proposed by Brügemann et al. (2019). The Rolodex game results in the the same wage profile as the private information version of Stole and Zwiebel (1996), which is widely used in the empirical macro-labor literature. The Rolodex game is very similar to the Stole and Zwiebel (1996)’s game, with one main difference: in each bargaining session, at most one offer and one counteroffer are involved. If neither agreement nor breakdown happens, the worker becomes the last person in the queue to renegotiate with the firm. In contrast, in Stole and Zwiebel (1996), many offers and counteroffers are proposed as either an agreement or a breakdown emerges.}

I now characterize the bargaining game outcome to determine the wage schedule. At a firm
with productivity $\phi$ and $l$ workers, the firm’s surplus from the marginal worker equals

$$S^F(\phi, l) = \frac{\partial}{\partial l} \left[ \Pi(\phi, l) - w(\phi, l)l + \beta V(\phi, l) \right], \quad (35)$$

where $\Pi(\cdot)$ is defined in equation (17), $w(\phi, l)$ is the wage paid by firm $(\phi, l)$, and $V(\cdot)$ is the continuation value of the firm defined in (27). Note that there is no adjustment cost component in the firm’s surplus function since adjustment costs are assumed to be sunk at this stage and cannot be recovered. Moreover, since firms are not required to pay the firing cost for the workers who voluntarily quit (e.g., after not reaching an agreement with firms), the firms’ marginal surplus above has no firing cost component.

The marginal worker’s surplus from this match equals the wage plus the continuation value of being at this firm minus her outside option:

$$S^W(\phi, l) = w(\phi, l) + \beta W^c(\phi, l) - W^u, \quad (36)$$

where $W^c(\phi, l)$ is the continuation value of being at firm $(\phi, l)$ at the beginning of next period, and the outside option of workers at this stage is joining the unemployment pool, which delivers the value of unemployment $W^u$, both to be defined below.

As discussed above, the outcome of the Stole and Zwiebel (1996)’s bargaining protocol is Nash bargaining on the marginal surplus. As a result, the wage schedule satisfies the following:

$$\xi \max\{S^F(\phi, l), 0\} = (1 - \xi)S^W(\phi, l), \quad (37)$$

where $\xi \in [0, 1)$ is workers’ bargaining power.\(^{46}\)

To elaborate on the wage schedule, I use the solution to the firm’s dynamic optimization problem in (30) to rewrite the right-hand side of (35), which delivers

$$S^F(\phi, l) = \frac{\partial C(l_{-1}, l)}{\partial l}, \quad (38)$$

where the labor adjustment cost $C(\cdot)$ is specified in (31), and $l_{-1}$ is the last period employment at this firm. To explore the marginal surplus generated at firms, I divide firms into two categories: hiring versus non-hiring firms (including contracting firms). For a non-hiring firm, the right-hand side of the equation above is less than or equal to zero. Hence, the marginal worker in such a firm does not generate rents to be split between the firm and its workers. For a hiring firm, however, the right-hand side of the above equation is positive and also varies by the employment level $l$, as a result of the non-linearity of vacancy posting costs. Hence, the marginal worker in an expanding firm generates rents that vary with firm size. This creates a wage dispersion in the economy,

\(^{46}\)Note that workers’ bargaining power has to be less than 1; otherwise, firms would not be able to finance vacancy posting costs, and therefore the labor market would break down.
even though workers are ex ante identical. Intuitively, as a result of the convex hiring cost, it is not optimal for expanding firms to reach their long-run desired size right away. Therefore, there are rents to be split while expanding firms are transiting to their desired size.\footnote{Unlike this paper, which employs a random search framework, Felbermayr et al. (2018) show the importance of non-linear adjustment costs in generating residual wage inequality in a directed search environment.} With a large enough productivity persistence $\rho$, there are enough expanding large firms in the economy (see Bertola and Garibaldi, 2001) so that the model generates a positive size-wage correlation. As Section 4.3 will discuss in detail, this positive size-wage association has crucial implications for the labor market consequences of trade liberalization in the presence of variable markups.

While Appendix E explains in detail the procedure to solve for the wage schedule, it proves useful to characterize the wage schedule at non-hiring firms. The non-hiring firm $(\phi, l)$ generates no surplus, and therefore equations (36) and (37) imply that such a firm pays the workers’ outside option:

$$w(\phi, l) + \beta W^e(\phi, l) = W^u.$$  

(39)

A few points are worth mentioning here. First, the outside option for fired workers is joining the unemployment pool, which delivers $W^u$. Hence, workers are indifferent between working in a non-hiring firm or being laid off from such firms. I assume that a firing firm randomly picks the workers to be laid off, and therefore each worker is equally likely to be fired. Second, non-hiring firms pay different wages because the continuation value $W^e(\phi, l)$ varies across firms (to be discussed below). Finally, note that I assume that firms do not have to pay the firing cost for the workers who voluntarily quit. This assumption implies that workers are not able to threaten to quit at non-hiring firms, and therefore these workers are paid no more than their outside option.

An unemployed worker decides whether to search for a job in the manufacturing sector or to produce services. If she decides to produce services, she would produce one unit of services. Hence, the value of producing services equals

$$W^h = 1 + \beta \max\{W^h, W^a\},$$  

(40)

where $W^a$ is the continuation value of applying for a job:

$$W^a = m(\theta)W^m + (1 - m(\theta))W^u.$$  

(41)

With the job finding probability $m(\theta)$, the worker gets randomly matched with a firm and enjoys the continuation value $W^m$, to be specified below. With probability $1 - m(\theta)$, the worker would be unmatched and joins the unemployment pool. The value of unemployment $W^u$ equals producing $b_u$ units of services this period plus the discounted value of searching for a job or producing services next period:

$$W^u = b_u + \beta \max\{W^h, W^a\}.$$  

(42)
In equilibrium, while some unemployed individuals search for a job in the manufacturing sector, others decide to produce services. Hence, in equilibrium, the value of producing services is equal to the value of applying for a job; that is, \( W^h = W^a = \frac{1}{1-\beta} \), where the last equality uses equation (40).\(^48\)

It remains to specify the expected value of getting matched with a firm, \( W^m \), and the continuation value of being at firm \((x, l)\) at the beginning of a period, \( W^c(x, l) \). Given the continuation policy function \( I^c(x, l) \), the hiring policy function \( I^h(x, l) \), the employment policy function \( l'(x, l) \), and the vacancy posting policy function \( v(x, l) \), all of which were discussed in Section 3.5, the expected value of getting matched \( W^m \) can be written as

\[
W^m = \int_x \int_l W^m(x, l) h(x, l) d\phi dl , \tag{43}
\]

and the distribution of vacancies is

\[
h(x, l) = \frac{v(x, l)G(x, l) + \frac{N_e}{N_H} l_e \theta J_0(\phi)}{\int_x \int_l [v(x, l)G(x, l) + \frac{N_e}{N_H} l_e \theta J_0(\phi)] d\phi dl} , \tag{44}
\]

where \( W^m(x, l) \) is the value of getting matched with firm \((x, l)\), \( G(x, l) \) is the distribution of firms after realization of productivities but before firms decide about their employment, \( N_e \) and \( N_H \) are the measure of entrants and total measure of intermediate producers, respectively, \( l_e \) is entrants’ initial size, and \( 1_x \) is an indicator function equal to one if \( x \) holds and zero otherwise. Note that each entrant posts \( \frac{l_e \theta}{m(\theta)} \) vacancies to start with \( l_e \) number of workers.\(^49\)

The continuation value of getting matched with a firm at state \((x, l)\) consists of the wage \( w(x, l') \) and the expected value of being at this firm at the beginning of next period:

\[
W^m(x, l) = w(x, l') + \beta W^c(x, l') , \tag{45}
\]

where \( l' \) is the outcome of the employment policy function \( l'(x, l) \), defined in Section 3.5. Finally,

\(^{48}\)Recall that services are used by firms to pay fixed costs and also are consumed by consumers. Hence, services must be produced in equilibrium. However, there might exist an equilibrium in which all individuals search for a job, and the number of unmatched workers is just enough to produce the required amount of services. In such an equilibrium, the value of searching for a job does not have to be equal to the value of producing services. However, by matching the model to the data in the next section, it turns out that a fraction of individuals decide to produce services in equilibrium, which means that the continuation value of producing services in equilibrium is equal to that of applying for a job.

\(^{49}\)Entrants are also allowed to expand right after entry. See Section 3.6.
the continuation value of starting the period at firm \((\phi, l)\) consists of three parts:

\[
W^c(\phi, l) = \underbrace{\lambda}_{\text{firm's exogenous exit}} + (1 - \lambda)(1 - I^c(\phi, l))W^u \\
+ (1 - \lambda)I^c(\phi, l)E_{\phi'|\phi}(1 - I^h(\phi', l))W^u \\
+ (1 - \lambda)I^c(\phi, l)E_{\phi'|\phi}I^h(\phi', l)\max\{W^u, W^m(\phi', l)\}.
\]

To elaborate, for an exogenous or endogenous reason, this firm may exit, in which case the worker becomes unemployed and joins the unemployment pool (the first line above). If the firm stays at the market but does not hire, all workers are paid their outside option \(W^u\) (the second line). This is because, as discussed above, no surplus is generated at such firms. If the firm expands, workers may decide whether to stay or quit (the last line). Notice that there would be no voluntary quit in equilibrium since firms pay at least the workers’ outside option, that is, \(W^m(\phi, l) \geq W^u\).

### 3.8 Stationary Equilibrium and Numerical Algorithm

I study the steady-state equilibrium of the model and assume Home is a small open economy that takes the Foreign aggregate demand shifter \(A_F\) as given. I focus on a symmetric equilibrium in which all \(K\) Foreign producers within each good \(\nu\) charge the same price. The aggregate equilibrium objects in the model are the Home aggregate demand shifter \(A_H\), labor market tightness \(\theta\), measure of intermediate producers \(N_H\), the stationary distribution of Home intermediate producers over productivity and labor, and Foreign firms’ marginal cost of production \(e\) (which is relative to the model numeraire).\(^{50}\) The steady-state equilibrium is defined as follows. Given the aggregate equilibrium objects, firms solve problems (17) and (21) to optimally decide about prices to charge, buy the optimal material specified in (26), and decide whether to export; firms optimally decide about the number of vacancies to post by solving the dynamic problem (30); the markets for the final good, intermediate goods, and services clear; trade between Home and Foreign is balanced; free entry condition (34) holds with equality;\(^{51}\) the flow of workers into and out of the unemployment pool are equal; workers are indifferent between applying for a job and producing services; and the distribution of domestic intermediate producers over \((\phi, l)\) evolves through the Markov productivity process, entrants’ productivity draws, and hiring/firing decisions by firms, and reproduces itself. Appendix D provides a formal definition of the stationary equilibrium.

---

\(^{50}\)Note that what matters in equilibrium is the price of services relative to the Foreign producers’ marginal cost. For ease of notation in the model, I take services as the model numeraire and define this relative price as the Foreign producers’ marginal cost relative to services, not the other way around.

\(^{51}\)In general, the equilibrium in this class of models might take one of the following forms: with entry/exit or without. The equilibrium in this paper, however, is the one with entry/exit because of the exogenous exit shock. Therefore, the free entry condition (34) holds with equality.
To solve for the stationary equilibrium, one needs to start with a guess on the aggregate equilibrium objects and iterate over those objects until they converge. The aggregate demand shifter $A_H$ moves around to satisfy the free entry condition \( 34 \) with equality. The relative price $e$ makes trade between Home and Foreign balanced. The measure of intermediate producers $N_H$ makes the demand for services equal to its supply, given that the flows into and out of the unemployment pool are equal in equilibrium. Labor market tightness makes workers indifferent between applying for a job and producing services. Appendix E provides more details on the numerical algorithm to solve for the steady state of the model.

4 Quantitative Analysis

As explained above, the main goal of this paper is to explore pro-competitive consequences of trade in frictional labor markets—that is, to quantify the extent to which accounting for variable markups can affect labor market outcomes and consequently welfare implications of trade in frictional labor markets. To this end, I first fit the model developed in the previous section to the plant-level data from Colombia and then use the calibrated model to perform counterfactual exercises. Colombia is a suitable case to investigate because, by reducing its average import tariffs from 40\% to 7.5\% (Attanasio et al., 2004; Alessandria and Avila, 2020), this country undertook a large trade liberalization in the late 1980s and early 1990s. We can therefore compare the counterfactual outcomes of the model to the data observed after trade liberalization in this country.

4.1 Calibration

I fit the model to the Colombia Annual Manufacturing Survey data in the pre-liberalization episode from 1983 to 1990. These annual data cover all manufacturing plants with at least 10 employees. Five parameters are calibrated outside the model. The discount factor $\beta$ is set to 0.85, consistent with the annual interest rate of 15\% in Colombia (Coşar et al., 2016). The average GDP share of services in Colombia between 1983 and 1990 is around 0.49, so I set $\kappa = 0.51$. Iceberg trade cost $d$ is set to 2.5, following Coşar et al. (2016). The average import tariff rate in manufacturing before trade liberalization in Colombia is 40\% (Alessandria and Avila, 2020), so I set $\tau = 1.4$. Heckman and Pages (2000) document that the average cost of dismissing a worker in Colombia over this period was around six months of wages (see their Graph 1). Given that the average annual wage in the model equilibrium is around 1.8, I put $c_f = 0.9$.$^{52}$ Lastly, to solve

\footnote{Note that the average wage itself depends on $c_f$ and other parameters of the model. Calibrating $c_f$ is therefore based on trial and error to find that, given all calibrated parameters as well as $c_f = 0.9$, the average annual wage in the model equals 1.8.}
Table 2: Data- versus Model-Based Statistics

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\log l_t)$</td>
<td>3.706</td>
<td>3.661</td>
<td>$\text{cov}(I_t^x, \log R_{t+1})$</td>
<td>0.211</td>
<td>0.215</td>
</tr>
<tr>
<td>$E(\log R_t)$</td>
<td>5.655</td>
<td>5.647</td>
<td>$\text{cov}(I_t^x, I_{t+1}^x)$</td>
<td>0.086</td>
<td>0.052</td>
</tr>
<tr>
<td>$E(\log x_t)$</td>
<td>0.117</td>
<td>0.066</td>
<td>$\text{cov} (\log R_t, \log R_{t+1})$</td>
<td>0.211</td>
<td>0.215</td>
</tr>
<tr>
<td>$\text{var}(\log l_t)$</td>
<td>1.002</td>
<td>0.996</td>
<td>$\text{cov} (\log l_t, \log l_{t+1})$</td>
<td>0.086</td>
<td>0.052</td>
</tr>
<tr>
<td>$\text{var}(\log R_t)$</td>
<td>2.589</td>
<td>2.593</td>
<td>$\text{cov} (\log l_t, I_t^x)$</td>
<td>0.086</td>
<td>0.052</td>
</tr>
<tr>
<td>$\text{cov}(\log R_t, I_t^x)$</td>
<td>0.208</td>
<td>0.222</td>
<td>$\text{cov} (\log R_{t+1}, I_t^x)$</td>
<td>0.086</td>
<td>0.052</td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, \log l_{t+1})$</td>
<td>0.979</td>
<td>0.972</td>
<td>$\text{cov} (\log R_{t+1}, \log R_{t+1})$</td>
<td>0.086</td>
<td>0.052</td>
</tr>
</tbody>
</table>

Notes: Data statistics are computed using all plants in the Colombia manufacturing plant-level data set from 1983 to 1990.

the model, I assume the number of Foreign firms within each variety is $K = 5$.\(^{53}\)

This leaves us with 14 parameters to calibrate:

$$\Theta \equiv (\sigma, \alpha, c_\nu, \rho, A_F, \eta, f_d, f_x, f_e, \gamma, \sigma_\phi, b_u, \lambda, \xi) .$$

I calibrate these parameters using the simulated method of moments (SMM) by minimizing the distance between a set of data moments and their model counterparts:

$$\hat{\Theta} \equiv \text{argmin}(D - M(\Theta))^\prime (D - M(\Theta)) ,$$

where $D$ stands for data moments and $M(\Theta)$ denotes model-based simulated moments. I target 28 moments to calibrate the parameters. Table 2 summarizes the data- and model-based statistics. Although all parameters of the model are jointly calibrated, a particular set of moments play a key role in calibrating each particular parameter, which I discuss below.

**Demand parameters $\sigma$ and $\alpha$.** Using the production function (15) as well as demand elasticities $\varepsilon_{H_{i1}}$ faced by Home intermediate producers in (24), the profit maximization problem of these producers implies the following first order condition (see Appendix F for more details):

$$\frac{\varepsilon_{H_{i1}}(\phi, l)}{\varepsilon_{H_{i1}}(\phi, l) - 1} = \eta \frac{p_{H_{i1}}(\phi, l)q_{H_{i1}}(\phi, l)}{PM_{H_{i1}}(\phi, l)} ,$$

\(^{53}\)The model fit and counterfactual results are robust to alternative assumptions, for example, $K = 10$. 29
where \( M_{HI,1}(\phi,l) \) is the material needed to produce \( q_{HI,1} \) for market \( i \). The intuition behind equation (47) follows the work by De Loecker and Warzynski (2012): Since material is a static (i.e., freely adjustable) input, the markup that firm \((\phi,l)\) charges in market \( i \) (i.e., the left-hand-side) \(^{54}\) is equal to the gap between the elasticity of output with respect to material, \( \eta \), and the material share of sale in market \( i \).

To calibrate demand elasticity parameters \( \sigma \) and \( \alpha \), I employ a strategy similar to that in Edmond et al. (2015). In particular, I exploit the relationship between material share of sale and firm size that is implied by equation (47). To elaborate, under \( \alpha > \sigma \ (\alpha < \sigma) \), equation (24) implies that larger firms face lower (higher) demand elasticities and therefore have a lower (higher) material share of sale, according to equation (47). The gap between \( \sigma \) and \( \alpha \) therefore determines to what extent material share of sale varies with firm size. In the data, I regress material share of sale on \( \log(\text{labor}) \) at the firm level, including industry fixed effects to control for cross-industry differences in material share of sale. \(^{55}\) The resulting coefficient in the data is \(-0.006\) and statistically significant; this coefficient implies that firms within the same industry that are 10% larger have on average a 0.06 smaller material share of sale. \(^{56}\) Since this coefficient is negative, we expect that in the model, the lower-tier elasticity \( \alpha \) will be larger than the higher-tier elasticity \( \sigma \), implying that larger firms face lower demand elasticities and charge higher markups. To learn about the level of \( \sigma \) and \( \alpha \), I also target the cross-firm median of material share of sale, guided by equation (47). \(^{57}\) Note that these two demand elasticity parameters influence several other moments as well—for example, the mean of exporters’ export intensity (i.e., share of exports in sales); as \( \alpha \) or \( \sigma \) rises, product markets become more competitive, and therefore export intensity among exporters falls.

**Labor adjustment cost and matching friction.** I use firms’ employment distribution, covariance of employment and revenue \( \text{cov}(\log l_t, \log R_t) \), wage dispersion, job turnover rate, and job turnover rate among continuing firms to learn about labor adjustment cost parameter \( c_\upsilon \) and matching friction \( \rho \). Firms’ revenues are expressed in thousands of 1977 pesos. The job turnover rate is the cross-year average of the annual job turnover rate over the sample period 1983-1990.

\(^{54}\)Note that I define markup as price over marginal cost where marginal cost takes all costs—for example, production costs, labor adjustment costs, and so on—into account.

\(^{55}\)Note that my model does not feature multiple industries.

\(^{56}\)In Edmond et al. (2015), unlike this paper, labor is a freely adjustable input, and they therefore use the labor share of sale to calibrate demand elasticity parameters.

\(^{57}\)As a robustness check, I also employ an alternative calibration strategy: Instead of targeting the median of material share and the relationship between size and material share, I target the median of markups and the relationship between size and markups, where I estimate firm-level markups following De Loecker and Warzynski (2012) methodology. This alternative strategy results in a larger gap between \( \sigma \) and \( \alpha \), and therefore variable markups play a stronger role compared to the benchmark results reported below. Hence, the results below regarding the role of variable markups are conservative.
and the annual job turnover rate is defined as

$$JT_t = \frac{\sum_{i \in c} |l_{it} - l_{it-1}| + \sum_{i \in \text{exit}} l_{it-1} + \sum_{i \in \text{entry}} l_{it} - |L_t - L_{t-1}|}{(L_{t-1} + L_t)/2} \times 100,$$

where $c$ is the set of continuing plants, $l_{it}$ is plant $i$’s employment in year $t$, and $L_t$ is total employment at time $t$. Job turnover among continuing plants is defined in the same way, but it does not include entry and exit in the numerator. The difference between these two measures of job turnover is therefore due to job creation/destruction by entry/exit.

A fall in labor adjustment costs influences the size distribution by shifting employment toward larger plants. In particular, a fall in the hiring cost parameter $c_v$ influences the firm size distribution by inducing firms to expand more after being hit by positive productivity shocks.

Labor adjustment costs influence job turnover as well. On the one hand, since larger firms are less likely to exit, reducing labor adjustment costs tends to reduce job turnover by shifting employment toward larger firms. On the other hand, reducing labor adjustment costs creates more incentive for firms to adjust their workforce in response to transitory productivity shocks, which in turn raises job turnover (Ljungqvist, 2002; Mortensen and Pissarides, 1999). The matching friction parameter $\rho$ affects firms’ responsiveness to their idiosyncratic shocks. This is because, given the labor market tightness $\theta$, a larger $\rho$ implies a higher vacancy filling rate, and therefore firms would have more incentive to respond to their transitory shocks. Hence, the matching friction parameter $\rho$ influences job turnover rate, size distribution, and $\text{cov}(\log l_t, \log R_t)$.

Wage dispersion also rises with the hiring cost parameter $c_v$. As discussed below equation (38), as the (convex) hiring cost rises, the heterogeneity of firms’ surplus across expanding firms rises, and, as a result of wage bargaining, so does the wage dispersion in the economy.

**Other parameters.** The production function parameter $\eta$ influences the employment distribution. In particular, as $\eta$ rises, the size distribution shifts to the left. Moreover, the variance of employment and the covariance between employment and revenue are affected by $\eta$. The means, variances, and covariances of employment, revenue, and export indicator function $I^x$ as well as the firm size distribution help us learn about the productivity process parameters $\gamma$ and $\sigma_\phi$.

The job turnover rate increases with workers’ outside option $b_u$. To elaborate, as $b_u$ rises, firms become more restricted in reducing their wages, since payments to workers have to be above their outside option. As a result, firms will adjust their workforce more frequently, which in turn increases the job turnover rate. Furthermore, wage dispersion helps us learn about workers’ outside option as well as workers’ bargaining power. The intuition is that the range of “feasible” wages falls as $b_u$ rises, and therefore the wage dispersion falls. Moreover, as workers’ bargaining power rises, firms’ heterogeneity translates into a larger wage heterogeneity.

Firms’ exit rate as well as job turnover due to entry and exit help us learn about the exogenous exit rate $\lambda$. The exit rate measures the fraction of plants that exit the market in each year,
averaged over the sample period 1983-1990. Sunk entry cost \( f_e \) and fixed production cost \( f_d \) affect the overall size of the economy because the aggregate demand shifter \( A_H \) is such that the free entry condition (34) holds with equality in equilibrium. The mean of employment and the mean of revenue are therefore influenced by these costs. Moreover, the fixed production cost influences firms’ exit rate. Finally, the fraction of plants that export, the mean of export intensity among exporters, and the covariance of exporting indicator function \( I^x \) and revenue help us learn about fixed exporting cost \( f_x \) and the aggregate demand shifter in the Foreign economy \( A_F \).

4.2 Model Fit, Calibrated Parameters, and Validation

Now that I have explained the intuition behind the set of targeted moments, this section discusses the calibrated parameters and the model fit. Tables 2 and 3 report targeted moments (and their model counterparts) and calibrated parameters, respectively. Overall, the model fits the data reasonably well. In particular, the model matches the negative coefficient of regressing material share of sale on firm size. As will be shown below, the standard constantmarkup model would not be able to generate this relationship.

As explained in Section 2.2, in the data I measure labor in terms of effective units, which means that I control for worker heterogeneity to the extent allowed by the data.\textsuperscript{58} Hence, the notion of wage in the data refers to the wage per effective labor unit. The wage dispersion reported in Table 2 therefore measures the residual wage dispersion, and the model matches it quite well. As discussed above, the fact that the model is able to match the observed residual wage dispersion is due to the convexity of hiring cost. In fact, in a model with a linear vacancy posting cost, all expanding firms pay the same wage (see e.g., Felbermayr et al., 2011), which would substantially reduce the residual wage inequality in the model economy.

To put the calibrated parameters into context, note that the unweighted and size-weighted average markups in the model are equal to 1.10 and 1.12, respectively. These values are broadly in line with the literature (e.g., Edmond et al., 2015; De Loecker and Warzynski, 2012; Coşar et al., 2016; De Loecker et al., 2016; Eslava et al., 2004).\textsuperscript{59} Furthermore, since \( \sigma < \alpha \), larger firms in the model face lower demand elasticities and therefore charge higher markups, which is in line with the literature (De Loecker et al., 2016; Atkin et al., 2015; Edmond et al., 2015; De Loecker and Warzynski, 2012).

The estimated value for the persistence of productivity shocks shows that the productivity process is very persistent, which is in line with the literature (see, e.g., Foster et al. (2008) for the US and Eslava et al. (2010) and Coşar et al. (2016) for Colombia). Moreover, the average

\textsuperscript{58}Within each category of workers, however, there might be some unobservable heterogeneity that the data do not allow me to control for.

\textsuperscript{59}The model by Edmond et al. (2015) implies an average and a median markup of 1.15 and 1.11, respectively. De Loecker and Warzynski (2012) estimate markups for Slovenian firms ranging from 1.13 to 1.28.
Table 3: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>demand elasticity parameter</td>
<td>8.594</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>demand elasticity parameter</td>
<td>13.002</td>
</tr>
<tr>
<td>$c_v$</td>
<td>hiring cost</td>
<td>24.157</td>
</tr>
<tr>
<td>$\rho$</td>
<td>matching friction</td>
<td>2.481</td>
</tr>
<tr>
<td>$\log(A_F)$</td>
<td>aggregate demand shifter in market F</td>
<td>24.604</td>
</tr>
<tr>
<td>$\eta$</td>
<td>material elasticity of output</td>
<td>0.652</td>
</tr>
<tr>
<td>$f_d$</td>
<td>fixed production cost</td>
<td>2.950</td>
</tr>
<tr>
<td>$f_x$</td>
<td>fixed exporting cost</td>
<td>39.094</td>
</tr>
<tr>
<td>$f_e$</td>
<td>sunk entry cost</td>
<td>485.363</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>persistence of productivity shocks</td>
<td>0.936</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>SD of productivity shocks</td>
<td>0.182</td>
</tr>
<tr>
<td>$b_u$</td>
<td>workers’ outside option</td>
<td>0.626</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>death shock</td>
<td>0.099</td>
</tr>
<tr>
<td>$\xi$</td>
<td>workers’ bargaining power</td>
<td>0.266</td>
</tr>
</tbody>
</table>

Notes: Calibrated parameters by minimizing the distance between the set of model moments and their data counterparts, summarized in Table 2.

labor adjustment costs (both hiring and firing) paid by firms are 3.5% of average firms’ sales in the stationary equilibrium of the model. The sunk entry cost $f_e$ is 41% of average firms’ annual sales, while the fixed exporting cost is an order of magnitude smaller.

The Constant-Markup Model. To explore the pro-competitive consequences of trade (i.e., the effects resulting from the responses of markups to trade), I will compare the counterfactual results of my baseline model to those of a nested version that is a standard constant-markup model. In this nested version of the model, I impose constant markups (i.e., $\sigma = \alpha$) and recalibrate all parameters by targeting the same set of moments in Table 2. In what follows, I will refer to the baseline model as the variable-markup model and to the nested version as the constant-markup model.

Appendix Tables B.1 and B.2 report model-based moments and calibrated parameters, respectively, for the constant-markup model. Overall, the constant-markup model also matches the targeted moments reasonably well. As noted before, however, since all firms face the same demand elasticity in the constant-markup model, this model cannot explain the negative relationship between size and material share of sale.

60For the five parameters that I calibrated outside the variable-markup model ($\beta, \kappa, d, \tau, c_f$), I use the same values here. Note that after calibrating this constant markup model with $c_f = 0.9$, the average annual wage is still around 1.8—that is, around six months of wages.

61As mentioned before, I define markup as price over marginal cost where marginal cost takes all costs—for example, production costs, labor adjustment costs, and so on—into account. The constant demand elasticity assumption $\sigma = \alpha$ therefore implies that all firms charge the same markup $\sigma/\sigma - 1$, which is invariant to trade liberalization.
Furthermore, imposing constant markups overestimates the importance of labor market frictions: the share of total labor adjustment costs paid by firms (as a fraction of total income) in the constant-markup model is 11% larger than that in the baseline model.\textsuperscript{62} This is because without markup adjustments, limited hiring and firing responses to idiosyncratic shocks are likely to be attributed entirely to labor market frictions. In a world with variable markups, however, reducing (raising) markups and firing (hiring) workers are substitutes.

**Model Validation and Non-Targeted Moments.** Before employing models to run counterfactual experiments, this section explores how both models perform in generating some non-targeted moments. First, since larger firms in the baseline model face lower demand elasticities, the model generates a correlation of -0.47 between firms’ size and their labor elasticity of revenue, which is in line with the negative size-labor elasticity of revenue association documented in Figure 1b. In the constant-markup model, however, there is no correlation between size and labor elasticity of revenue (since all firms face the same demand elasticity).

Furthermore, as mentioned before, imposing $\sigma = \alpha$ implies that the demand elasticity that firms face would be the same both across firms and across markets. As Appendix G shows, only in such a world would all exporting firms have the same export intensity. For the case of $\sigma \neq \alpha$, however, as equation (24) shows, demand elasticities vary both across firms and across markets, and therefore there would be a dispersion in export intensity across exporters. In the data, export intensity has a standard deviation of 0.25, one-fifth of which is explained by the baseline model. In the constant-markup model, however, all firms face the same demand elasticity in both markets, and the dispersion in export intensity is therefore equal to zero. In terms of total exports, the share of total exports in aggregate manufacturing revenue in Colombia was 9% over the sample period. This ratio equals 13% and 10% in the baseline and constant-markup models, respectively.

Finally, larger firms pay higher wages in the data, and both models replicate this pattern reasonably well: the correlation between wage and size in the data is 0.58 and is 0.49 and 0.45 in the baseline and constant-markup models, respectively. As explained below equation (38), this positive size-wage correlation in both models is due to the non-linear cost of posting vacancies, which implies that there are rents to be split while expanding firms are transiting to their desired size. Since the productivity process is persistent, there are enough expanding firms in the economy, and therefore both models generate this positive size-wage correlation. In a model with a linear vacancy posting cost, however, all expanding firms would pay the same wage, and the size-wage correlation would therefore be very limited.\textsuperscript{63} The counterfactual analysis in

\begin{footnote}{\textsuperscript{62} In the constant-markup model, the share of total labor adjustment costs paid by firms is 3.91% of total income, whereas it is 3.52% in the baseline model.}

\begin{footnote}{\textsuperscript{63} Notice that even if the vacancy posting cost was linear, hiring firms would still pay higher wages than non-hiring firms do. This is because while the marginal surplus in non-hiring firms is zero, it is positive (but the same) for all hiring firms; see the discussion below equation (38). Hence, the degree of the size-wage correlation in a model with linear vacancy posting costs would be limited.}

34
the next section shows that this positive size-wage association has important implications for the pro-competitive consequences of trade in a frictional labor market.

### 4.3 Counterfactual Analysis

This section employs the calibrated models to perform counterfactual analysis. The goal of this section is to explore the pro-competitive consequences of trade in a frictional labor market by comparing the counterfactual outcomes of the baseline model to those of the constant-markup model. This section shows that accounting for variable markups in the presence of labor market frictions increases the unemployment and wage inequality consequences of trade and reduces the gains from trade (i.e., the pro-competitive “gains” from trade are negative).

To do the counterfactual experiments, I simulate in the model the effects of Colombia’s trade liberalization in the early 1990s. In Colombia, import tariffs averaged 40% in the 1980s and reduced to 7.5% on average in the early 1990s (Attanasio et al., 2004; Alessandria and Avila, 2020). In both variable- and constant-markup models, I simulate the effects of this trade liberalization by reducing import tariffs $\tau$ from 1.4 to 1.075. As a result of this trade liberalization, the share of total exports in revenue rises by 221% in both models (in the steady state), which shows that both models have a quite similar (ex post) trade elasticity. In the data, this share rises by 250% from the 1980s to the 2000s (Coşar et al., 2016). The rise in export activities in the data is larger than that in both models, which is partly due to other sources of globalization during this period.

Colombia deregulated the labor market as well in the early 1990s by reducing the cost of dismissing workers by one-half (see Graph 1 and Table 1 in Heckman and Pages, 2000). In a separate counterfactual exercise, I therefore combine the trade liberalization experiment with this labor market deregulation by reducing $c_f$ by 50%. This exercise will help us compare the counterfactual outcomes to what we observe in the data.

Table 4 reports the counterfactual results. The first (last) four columns correspond to the variable- (constant-) markup model. The “Base” columns report the initial steady state of the models. The “Openness,” “Labor,” and “Openness & Labor” columns report the counterfactual results in the new steady state associated with reducing import tariffs $\tau$, reducing the firing cost $c_f$, and reducing both, respectively, as explained above. To make the comparison easier, the “Base” values for aggregate statistics are normalized to one.

As the “Labor” columns in Table 4 show, reducing the firing cost $c_f$ has quite limited effects on the size distribution as well as on most aggregate statistics in both models. The variables that change the most in response to the reduction in $c_f$ are the job turnover rates, to be discussed below. In what follows, I therefore mostly focus on the effects of trade liberalization.

**Firm Size Distribution.** Reducing import tariffs (“Openness” columns) increases import
### Table 4: Counterfactual Results

<table>
<thead>
<tr>
<th>Model-based moments</th>
<th>Variable-Markup Model</th>
<th>Constant-Markup Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
<td>Openness</td>
</tr>
<tr>
<td><strong>firms’ employment distribution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20\textsuperscript{th} percentile</td>
<td>16.15</td>
<td>16.15</td>
</tr>
<tr>
<td>40\textsuperscript{th} percentile</td>
<td>23.44</td>
<td>23.44</td>
</tr>
<tr>
<td>60\textsuperscript{th} percentile</td>
<td>41.01</td>
<td>41.01</td>
</tr>
<tr>
<td>80\textsuperscript{th} percentile</td>
<td>88.75</td>
<td>96.13</td>
</tr>
<tr>
<td>90\textsuperscript{th} percentile</td>
<td>172.68</td>
<td>187.04</td>
</tr>
<tr>
<td>95\textsuperscript{th} percentile</td>
<td>286.37</td>
<td>318.56</td>
</tr>
<tr>
<td><strong>aggregate statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>job turnover rate</td>
<td>1</td>
<td>1.013</td>
</tr>
<tr>
<td>job turnover (continuing firms)</td>
<td>1</td>
<td>1.018</td>
</tr>
<tr>
<td>unemployment rate</td>
<td>1</td>
<td>1.162</td>
</tr>
<tr>
<td>measure of domestic producers</td>
<td>1</td>
<td>0.865</td>
</tr>
<tr>
<td>exit rate</td>
<td>1</td>
<td>1.061</td>
</tr>
<tr>
<td>labor market tightness</td>
<td>1</td>
<td>0.823</td>
</tr>
<tr>
<td>unemployment inflow</td>
<td>1</td>
<td>0.956</td>
</tr>
<tr>
<td>exp. value from searching (W\textsuperscript{m})</td>
<td>1</td>
<td>1.139</td>
</tr>
<tr>
<td>SD of log(wage) across workers</td>
<td>1</td>
<td>1.090</td>
</tr>
<tr>
<td>aggregate price index (P\textkappa)</td>
<td>1</td>
<td>0.979</td>
</tr>
<tr>
<td>real income</td>
<td>1</td>
<td>1.003</td>
</tr>
</tbody>
</table>

Notes: The first (last) four columns correspond to the variable (constant) markup model. The “Base” columns report the initial steady state of models. The “Openness,” “Labor,” and “Openness & Labor” columns report the counterfactual results associated with reducing import tariffs \( \tau \), reducing firing cost \( c_f \), and reducing both, respectively, as explained in the text. To make the comparison easier, the “Base” values for aggregate statistics are normalized to one. Numbers are rounded to the nearest thousandth.
competition and raises the market share of Foreign firms in the Home market, which in turn raises the demand elasticities faced by Home producers in the baseline model (see equation (24)). In the model, in line with a variety of variable-markup studies in the literature, larger firms experience a larger increase in their perceived demand elasticities (and therefore in their labor elasticity of revenue), thereby reducing their markups by a greater amount compared to smaller firms. This result is also consistent with what Figure 2a in Section 2.2 documented in the data (i.e., the labor elasticity of revenue rises by a greater amount for larger firms compared to smaller ones).

The rise in the labor elasticities of revenue, which is operative in the baseline model only, makes firms more responsive to their idiosyncratic shocks and consequently shifts employment toward larger/expanding firms. To elaborate, because of the increase in firms’ responsiveness, less efficient firms exit more frequently, and the exit rate rises. Moreover, firms hit by positive productivity shocks expand more, and therefore large firms become even larger. Since this channel is not operative in the constant-markup model, the rightward shift in the size distribution in this model is limited (to be discussed below).

Since the change in the firm size distribution has crucial implications for various labor market outcomes that will be discussed below, Table 5 explores whether the variable- and constant-markup models are able to capture what happened to the firm size distribution after reforms in Colombia. In the models, pre- and post-reform columns correspond to the initial steady state and the “Openness & Labor” counterfactual experiment, respectively. The pre- and post-reform data statistics are calculated as the average across 1983-1990 and 1992-2012, respectively.

The first two columns in Table 5 report that Colombia has a more dispersed firm size distribution after reforms as a result of a drastic increase in the size of large firms. In particular, the right tail of the size distribution (e.g., the 95th and 97th percentiles) in the data shifts to the right by around 14%-19%. While the post-reform size distribution is obviously not targeted in the calibration procedure, the baseline model explains the expansion of large firms reasonably well. In particular, the right tail of the size distribution in the baseline model rises by around 11%. In the constant-markup model, however, the size distribution does not respond much to the reforms and barely changes. Since the variable- and constant-markup models share the same channels except for the endogenous response in demand elasticities and markups, we conclude

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64 Note that although Home exporters gain market share in the Foreign market, which reduces the demand elasticities that they face there, the rise in the Home demand elasticity is the dominating factor, and overall, Home producers face higher demand elasticities and charge lower markups, as discussed below.

65 In fact, the 5th, 10th, 90th, and 95th percentiles of the markup distribution charged by Home producers fall from [1.085, 1.087, 1.128, 1.130] to [1.084, 1.085, 1.108, 1.111] after trade liberalization, respectively.

66 In addition to this mechanism, Melitz-type forces are operative in both models: openness in both variable- and constant-markup models reallocates resources toward larger, more efficient firms—that is, it induces a rightward shift in the size distribution.

67 Note that some sizes across different models in Tables 4 and 5 are exactly the same because of the discretization of the state space.
Table 5: Firm Size Distribution: Data vs. Model

<table>
<thead>
<tr>
<th>Size Dist.</th>
<th>Data Pre</th>
<th>Post</th>
<th>%∆</th>
<th>Variable-Markup Model Pre</th>
<th>Post</th>
<th>%∆</th>
<th>Constant-Markup Model Pre</th>
<th>Post</th>
<th>%∆</th>
</tr>
</thead>
<tbody>
<tr>
<td>20\textsuperscript{th} percentile</td>
<td>17.51</td>
<td>15.15</td>
<td>-13.5</td>
<td>16.15</td>
<td>15.72</td>
<td>-2.7</td>
<td>16.15</td>
<td>15.72</td>
<td>-2.7</td>
</tr>
<tr>
<td>40\textsuperscript{th} percentile</td>
<td>25.87</td>
<td>26.65</td>
<td>3.0</td>
<td>23.44</td>
<td>22.23</td>
<td>-5.2</td>
<td>24.08</td>
<td>22.83</td>
<td>-5.2</td>
</tr>
<tr>
<td>60\textsuperscript{th} percentile</td>
<td>42.23</td>
<td>48.82</td>
<td>15.6</td>
<td>41.01</td>
<td>38.88</td>
<td>-5.2</td>
<td>41.01</td>
<td>38.88</td>
<td>-5.2</td>
</tr>
<tr>
<td>80\textsuperscript{th} percentile</td>
<td>88.19</td>
<td>113.59</td>
<td>28.8</td>
<td>88.75</td>
<td>88.75</td>
<td>0</td>
<td>91.14</td>
<td>91.14</td>
<td>0</td>
</tr>
<tr>
<td>90\textsuperscript{th} percentile</td>
<td>173.70</td>
<td>224.87</td>
<td>29.5</td>
<td>172.68</td>
<td>182.12</td>
<td>5.5</td>
<td>177.34</td>
<td>177.34</td>
<td>0</td>
</tr>
<tr>
<td>95\textsuperscript{th} percentile</td>
<td>317.68</td>
<td>379.71</td>
<td>19.5</td>
<td>286.37</td>
<td>318.56</td>
<td>11.2</td>
<td>286.37</td>
<td>294.10</td>
<td>2.7</td>
</tr>
<tr>
<td>97\textsuperscript{th} percentile</td>
<td>453.61</td>
<td>517.57</td>
<td>14.1</td>
<td>383.82</td>
<td>426.95</td>
<td>11.2</td>
<td>383.82</td>
<td>394.18</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Notes: “Pre” and “Post” columns refer to the pre- and post-reform episodes, respectively. In the models, pre- and post-reform correspond to the initial steady state and the “Openness & Labor” counterfactual experiment, respectively. The pre- and post-reform data statistics are calculated as the average across 1983-1990 and 1992-2012, respectively.

that the rise in the labor elasticities of revenue plays a major role in explaining the changes in the firm size distribution. As will be discussed below, the same channel crucially influences various labor market outcomes as well.

**Unemployment.** Trade liberalization influences the unemployment rate in both models through two competing channels. On the one hand, as explained above, after trade liberalization jobs get concentrated at larger firms that pay higher wages. Moreover, the rents generated at larger (exporting) firms jump after trade liberalization because larger firms get easier access to the Foreign market, which increases wages at these firms. Therefore, the expected value from searching for jobs $W_m$ rises, which in turn induces more individuals to search for a job and reduces labor market tightness. This reduction in labor market tightness reduces the job finding rate $m(\theta)$, which in turn raises the unemployment rate (see equations (73) and (75) in Appendix D).

On the other hand, however, the rightward shift in the size distribution makes jobs more concentrated in larger firms, which are less likely to exit or downsize. As a result, the mass of individuals that flow into the unemployment pool falls which puts downward pressure on the unemployment rate (see equations (69)-(71) in Appendix D). Overall, the first channel dominates and the unemployment rate rises in both models. Since the shift in the size distribution and therefore the fall in the labor market tightness are more pronounced in the variable-markup model, the rise in the unemployment rate is almost three times larger in the variable-markup model compared to the constant-markup model.

This result highlights that pro-competitive forces of trade liberalization can have adverse unemployment consequences in frictional labor markets, which also has important welfare implications (to be discussed below). The surge in the unemployment rate in the variable-markup model is more in line with the rise in the unemployment rate in Colombia even 20 years after the reforms (see Figure B.4), although one cannot necessarily attribute the entire increase in the
unemployment rate observed in the data to trade liberalization and labor market deregulation.\footnote{As discussed, the rise in unemployment in the model is mainly due to an increase in the number of job seekers in response to higher wages. This presumably takes some time, which might partly explain why unemployment starts to rise a few years after trade liberalization in 1991. Again, I note that other shocks—for example, the financial crisis in 1999—were also going on in Colombia in the 1990s.}

**Wage Inequality.** As Table 4 shows, trade liberalization also increases residual wage inequality in both models. To elaborate, the more dispersed firm size distribution after openness leads to higher wage inequality. This is because larger firms pay higher wages and generated rents at larger (exporting) firms jump after trade liberalization.\footnote{Note that in theory, openness does not necessarily raise wage inequality. As shown by Helpman et al. (2017), since more and more workers get hired by larger firms as openness rises, wage inequality may start to fall at some point.} Since the response of the size distribution to openness is much stronger in the variablemarkup model relative to the constant-markup model, the rise in residual wage inequality is more than two times larger in the former model. This result shows that pro-competitive forces of trade liberalization worsen wage inequality in frictional labor markets. As the “Openness & Labor” columns show, while wage inequality rises by only around 3% in the constant-markup model, the 8.7% increase in wage dispersion in the variable-markup model is more consistent with what is observed in Colombia: Attanasio et al. (2004) document that the standard deviation of log wages within education categories rises by about 15% from 1990 to 1998.\footnote{Note that since Attanasio et al. (2004) divide workers into education groups and look at the wage inequality within each group, their measure is comparable to my measure of residual wage inequality.} \footnote{Since my data set does not report workers’ skill category in the post-1992 data, I cannot measure post-reform residual wage inequality in my data.}

**Job Turnover.** By reducing the firing cost, labor market deregulation creates more incentive for firms to adjust their workforce in response to transitory productivity shocks. This channel, as the “Labor” columns in Table 4 report, raises job turnover in both models (Ljungqvist, 2002; Mortensen and Pissarides, 1999). Job turnover rises by more in the constant-markup model, since in the variable-markup model, firms respond to this change not only on the employment margin but also on the markup margin.

We now focus on the “Openness” columns. Trade liberalization influences job turnover through two channels. The first channel is a general equilibrium effect that works through the size distribution. On the one hand, the rightward shift in the size distribution explained above makes jobs more concentrated in larger firms, which are less likely to exit or downsize, and thereby reduces job turnover. On the other hand, however, the same rightward shift in the size distribution reduces the labor market tightness, as explained above, which tends to increase job turnover. To elaborate, in a less tight labor market, firms are more likely to fill vacancies, which raises firms’ responsiveness to idiosyncratic shocks. Overall, this general equilibrium effect tends to reduce job turnover, as can be seen from the reduction in the mass of individuals that flow into the unemployment pool.
Second, as noted above, a reduction in import tariffs raises firms’ labor elasticity of revenue (in the variable-markup model only), and firms thereby become more responsive to their idiosyncratic shocks. This channel tends to increase job turnover. This channel is dampened by the first force, which results in a moderate increase in job turnover in the variable-markup model. Since the second channel does not exist in the constant-markup model, job turnover barely responds to openness in this model, which is also noted in the literature (e.g., Coşar et al., 2016).

Real Income. Finally, to evaluate the pro-competitive gains/losses from trade, we compare steady-state changes in real income between the variable- and constant-markup models in the “Openness” counterfactual exercise. As Table 4 shows, openness raises real income by only 0.3% in the variable-markup model, a rise that is about 80% smaller than that in the constant-markup model, which is 1.7%. That is, the pro-competitive gains from trade liberalization (in the steady state) are negative. Recall that both models have the same (ex post) trade elasticity and exhibit the same increase in aggregate export share in the steady state in this “Openness” counterfactual exercise.

Real income in both models rises after trade liberalization because of a fall in the aggregate price index. Overall, the aggregate price index in both models tends to fall after openness as a result of cheaper Foreign varieties and also because resources get more concentrated at more efficient firms. As Table 4 shows, real income rises by less in the variable-markup model mainly because the price index in this model falls by less than that in the constant-markup model as a result of the surge in unemployment under variable markups. To elaborate, since unemployed individuals are less productive than those who decide to produce services, the supply of services (demanded by firms for fixed operation cost, fixed exporting cost, and so on) falls as unemployment rises, which in equilibrium reduces the mass of domestic firms/varieties \( N_H \) (see Appendix D for equilibrium conditions). Since unemployment rises by more under variable markups, the fall in the mass of domestic firms/varieties is more pronounced in this model (see Table 4), which puts downward pressure on real income.

In addition to the unemployment effect, variable markups also affect the welfare consequences of trade through two other forces that are standard in the literature. On the one hand, Home producers’ markups fall and become less dispersed.\(^\text{72}\) Moreover, employment shifts toward larger, under-producing firms that charge higher markups. This effect, as in Edmond et al. (2015), tends to reduce misallocation arising from markups and to generate pro-competitive gains from trade liberalization—that is, increasing the welfare consequences of trade relative to the constant-markup model. On the other hand, however, as emphasized by Arkolakis et al. (2019), Foreign firms charge higher markups after trade liberalization, which tends to reduce the pro-competitive gains from trade. To emphasize the role of search frictions in driving the results, it is therefore important to show that the negative pro-competitive gains from trade liberalization that I report

\(^{72}\)Recall that the 5th, 10th, 90th, and 95th percentiles of the markup distribution charged by Home producers fall from [1.085, 1.087, 1.128, 1.130] to [1.084, 1.085, 1.108, 1.111] after trade liberalization, respectively.
in this paper are indeed due to labor market frictions, not to Foreign firms charging higher markups.

To this end, I shut down search frictions in the labor market in both constant- and variable-markup models and perform the “Openness” counterfactual exercise by reducing import tariffs from 40% to 7.5%, as before. Both models generate the same increase in the export share of revenue in the steady state, around 220%, similar to the models with search frictions. This shows that these two models with no labor market frictions have the same (ex post) trade elasticity, which is equal to that in the models with search frictions. Interestingly, in this world without labor market frictions, the pro-competitive gains from trade liberalization are positive: while steady-state real income rises by 2.5% in the variable-markup model, it rises by 2% in the constant-markup model. These positive pro-competitive gains from trade liberalization in a frictionless labor market are consistent with Edmond et al. (2015), as my model structure without search frictions is broadly in line with theirs.

To summarize, as discussed above, the variable- and constant-markup models in the presence of labor market frictions match the same set of moments, have the same (ex post) trade elasticity, and exhibit the same increase in aggregate export (and import) share after trade openness. The welfare implications of trade liberalization in these two models are, however, quite different. Interestingly, while abstracting from labor market frictions in my framework implies positive pro-competitive gains from trade liberalization, taking labor market frictions into account implies that the pro-competitive gains from trade liberalization are negative. We can therefore conclude that accounting for variable markups in a frictional labor market reduces the gains from trade liberalization by around 80%. In the language of Arkolakis et al. (2019), my counterfactual exercises therefore show that the pro-competitive gains from trade can be even more “elusive” in the presence of labor market frictions.

5 Conclusion

This paper develops and estimates a dynamic general equilibrium model of international trade featuring search frictions in the labor market and endogenously variable markups to show that trade-induced changes in markups have crucial impacts on labor market outcomes in a frictional labor market. The link between openness, markups, and labor market outcomes is attributable to trade liberalization increasing competition in product markets, thereby raising demand elasticities and putting downward pressure on markups. This in turn increases the labor elasticities of

\footnote{To do this exercise, I do not recalibrate the models; that is, I use the same parameters as in the models with search frictions. Recalibrating the models would, however, deliver the same conclusion that the pro-competitive gains from trade are positive once we abstract from labor market frictions.}

\footnote{Under a Pareto productivity distribution, frictionless labor market, and some assumptions on preferences (which does not nest my demand structure), Arkolakis et al. (2019) show that the gains from trade liberalization under variable markups are up to 14% smaller than those under constant markups.}
revenue and makes firms’ employment decisions more responsive to their idiosyncratic shocks, which in a frictional labor market increases unemployment and residual wage inequality. I show that once markups are allowed to respond to trade liberalization, unemployment and residual wage inequality rise almost three times more than in a model with constant markups (in the steady state). The presence of labor market frictions makes the pro-competitive gains from trade liberalization negative.

References


Appendices

A Demand Elasticity and Firms’ Responsiveness: Another Perspective

Section 2 of the paper provided a simple model to show that firms with lower labor elasticities of revenue are more responsive to shocks. This section expresses the same idea through a different, but closely related, angle. Here, instead of arguing that the labor elasticity of revenue is affected by changes in output demand elasticity, I take the perspective that labor demand elasticity responds to changes in output demand elasticity. These two perspectives are closely connected since the labor elasticity of revenue and labor demand elasticity are two sides of the same coin. To elaborate, the fundamental law of factor demand, discussed in Hamermesh (1996), shows how derived labor demand elasticity and output demand elasticity are linked:

$$\frac{\partial l}{l} \frac{\partial w}{w} = -(1 - s)\sigma_{l,all} - s|\frac{\partial q}{q} \frac{\partial p}{p}|,$$

(49)

where the left-hand side is labor demand elasticity, s is share of labor in revenue, $\sigma_{l,all}$ is elasticity of substitution between labor and all other factors of production, and $\frac{\partial q}{q} \frac{\partial p}{p}$ is output demand elasticity. This equation shows that as output demand becomes more elastic, so does labor demand. This is intuitive because when output demand is more elastic, changes in wage (by affecting the marginal cost of production and, in turn, output price) have a larger effect on the quantity demanded and therefore on labor demand. Now, the claim is that firms are more responsive (in terms of employment) to their revenue shocks when they face a more elastic labor demand. Figure A.1 shows this idea graphically. Suppose we have two different labor demand curves $D_1$ and $D_2$, before and after trade liberalization, respectively. Suppose a firm faces the same labor supply $S$ and, initially, the firm is at the equilibrium point $A$. Now, consider the same positive revenue shock that hits the firm. Since a revenue shock changes the marginal revenue product of labor, both labor demand curves shift upward with the same magnitude. The new equilibria are $B$ and $C$ for the less and more elastic labor demand, respectively. As seen in the figure, the same revenue shock changes employment by more when labor demand is more elastic.

B Additional Figures and Tables

As noted in the paper, Figure B.1 employs Ackerberg et al. (2015)’s methodology to estimate the labor elasticities of revenue, which shows that the estimated values are robust to the method
as well as to the inclusion of capital. In particular, the pattern and also the estimated values are quite similar to those reported in the paper.

As a robustness check, here I calculate within-employment bin job turnover rate “net of employment change” in year \( t \) for each employment bin \( j \) as

\[
JT_{jt} = \frac{\sum_{i \in c} |l_{ijt} - l_{ijt-1}| + \sum_{i \in \text{exit}} l_{ijt-1} - |L_{jt} - L_{jt-1}|}{(L_{jt-1} + L_{jt})/2} \times 100 ,
\]

where \( i \) denotes plants, \( c \) is the set of continuing plants, and \( L_{jt} \) is total employment for employment bin \( j \) at time \( t \). The first two terms in the numerator measure job turnover by continuing and exiting plants, respectively. The last term in the numerator is the net change in total employment for employment bin \( j \). I take the average job turnover rate for each employment bin over the sample period. Figures B.2 and B.3 confirm the patterns reported as suggestive evidence 1 and 2, respectively, in the paper.
Figure B.1: Job Turnover and Labor Elasticity of Revenue for Each Employment Bin

(a) Labor Elasticity of Revenue- ACF No Capital  (b) Labor Elasticity of Revenue- ACF with Capital

Notes: Plants are divided into 40 employment bins, displayed by the horizontal axes. The shaded areas reflect the linear fitted lines along with their 95% confidence bands.

Figure B.2: Job Turnover across the Firm Size Distribution

Notes: Plants are divided into 40 employment bins, displayed by the horizontal axis. The shaded area reflects the linear fitted line along with its 95% confidence band.
Figure B.3: Changes in Job Turnover versus Changes in Labor Elasticity of Revenue for Each Employment Bin

Notes: Plants are divided into 40 employment bins. The horizontal and vertical axes, respectively, measure the log change in the labor elasticity of revenue and job turnover for each employment bin from pre- to post-trade liberalization episode. The shaded area reflects the linear fitted line along with its 95% confidence band.

Figure B.4: Unemployment rate in Colombia

Notes: Data for unemployment rates are from International Monetary Fund. The red line shows the year 1991, in which Colombia experienced a large reduction in trade barriers.
Table B.1: Data- versus Model-Based Statistics (Constant-Markup Model)

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\log l_t)$</td>
<td>3.706</td>
<td>3.677</td>
<td>$\text{cov}(I^x_t, \log R_{t+1})$</td>
<td>0.211</td>
<td>0.216</td>
</tr>
<tr>
<td>$E(\log R_t)$</td>
<td>5.655</td>
<td>5.611</td>
<td>$\text{cov}(I^x_t, I^x_{t+1})$</td>
<td>0.086</td>
<td>0.049</td>
</tr>
<tr>
<td>$E(I^x_t)$</td>
<td>0.117</td>
<td>0.066</td>
<td>$\text{firms’ employment distribution}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{var}(\log l_t)$</td>
<td>1.002</td>
<td>0.995</td>
<td>20th percentile</td>
<td>17.505</td>
<td>16.149</td>
</tr>
<tr>
<td>$\text{var}(\log R_t)$</td>
<td>2.589</td>
<td>2.612</td>
<td>40th percentile</td>
<td>25.874</td>
<td>24.076</td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, \log R_t)$</td>
<td>1.324</td>
<td>1.282</td>
<td>60th percentile</td>
<td>42.225</td>
<td>41.005</td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, I^x_t)$</td>
<td>0.129</td>
<td>0.124</td>
<td>80th percentile</td>
<td>88.187</td>
<td>91.144</td>
</tr>
<tr>
<td>$\text{cov}(\log R_t, I^x_t)$</td>
<td>0.208</td>
<td>0.227</td>
<td>material share of sale (median)</td>
<td>0.586</td>
<td>0.596</td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, \log l_{t+1})$</td>
<td>0.979</td>
<td>0.969</td>
<td>coefficient of regressing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, \log R_{t+1})$</td>
<td>1.325</td>
<td>1.198</td>
<td>material share of sale on log(labor)</td>
<td>-0.006</td>
<td>0</td>
</tr>
<tr>
<td>$\text{cov}(\log l_t, I^x_{t+1})$</td>
<td>0.133</td>
<td>0.123</td>
<td>job turnover rate (%)</td>
<td>21.458</td>
<td>19.299</td>
</tr>
<tr>
<td>$\text{cov}(\log R_t, \log l_{t+1})$</td>
<td>1.322</td>
<td>1.312</td>
<td>continuing firms’ job turnover (%)</td>
<td>9.371</td>
<td>8.074</td>
</tr>
<tr>
<td>$\text{cov}(\log R_t, \log R_{t+1})$</td>
<td>2.559</td>
<td>2.412</td>
<td>mean of exporters’ export intensity</td>
<td>0.181</td>
<td>0.167</td>
</tr>
<tr>
<td>$\text{cov}(\log R_t, I^x_{t+1})$</td>
<td>0.215</td>
<td>0.229</td>
<td>SD of log wages</td>
<td>0.515</td>
<td>0.514</td>
</tr>
<tr>
<td>$\text{cov}(I^x_t, \log l_{t+1})$</td>
<td>0.129</td>
<td>0.127</td>
<td>exit rate (%)</td>
<td>10.328</td>
<td>9.891</td>
</tr>
</tbody>
</table>

Notes: Data statistics are based on the Colombia manufacturing plant-level data set from 1983 to 1990.

Table B.2: Calibrated Parameters (Constant-Markup Model)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = \alpha$</td>
<td>demand elasticity</td>
<td>8.940</td>
</tr>
<tr>
<td>$c_w$</td>
<td>hiring cost</td>
<td>28.400</td>
</tr>
<tr>
<td>$\rho$</td>
<td>matching friction</td>
<td>2.509</td>
</tr>
<tr>
<td>$\log(A_F)$</td>
<td>aggregate demand shifter in market F</td>
<td>17.109</td>
</tr>
<tr>
<td>$\eta$</td>
<td>material elasticity of output</td>
<td>0.671</td>
</tr>
<tr>
<td>$f_d$</td>
<td>fixed production cost</td>
<td>8.279</td>
</tr>
<tr>
<td>$f_x$</td>
<td>fixed exporting cost</td>
<td>76.643</td>
</tr>
<tr>
<td>$f_e$</td>
<td>sunk entry cost</td>
<td>497.725</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>persistence of productivity shocks</td>
<td>0.913</td>
</tr>
<tr>
<td>$\sigma_{\phi}$</td>
<td>SD of productivity shocks</td>
<td>0.220</td>
</tr>
<tr>
<td>$b_u$</td>
<td>workers’ outside option</td>
<td>0.331</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>death shock</td>
<td>0.088</td>
</tr>
<tr>
<td>$\xi$</td>
<td>workers’ bargaining power</td>
<td>0.225</td>
</tr>
</tbody>
</table>

Notes: Calibrated parameters by minimizing the distance between the set of model moments and their data counterparts, summarized in Table B.1.
C An Alternative Model to Generate Variable Markups

C.1 Environment

The environment is exactly the same as in the model presented in the text, except that intermediate producers sell their goods to the final producers in a monopolistically competitive market.

C.2 Consumers and Final Good Producers

The final good producers make a composite good out of the domestic and Foreign intermediate varieties and sell it to (domestic\(^1\)) consumers as well as intermediate good producers. The final good is produced according to the following commonly available technology:

\[ M = \left[ \int_0^{1+1} (q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} \, d\nu \right]^{\frac{\sigma}{\sigma-1}}, \tag{51} \]

where the intermediate goods are denoted by \( \nu \). I assume the measure of Foreign intermediate producers is one. Moreover, while the measure of potential domestic intermediate producers is one, the measure of active domestic intermediate producers is \( N_H \), an endogenous object determined in equilibrium.\(^2\) Parameters \( \sigma \) and \( \alpha \) govern the price elasticity of demand (i.e., demand elasticity) faced by intermediate goods \( \nu \). This point will be elaborated below. I assume that the final good is produced by a unit measure of producers and is sold at the market-clearing price \( P \).\(^3\) As for consumers, I assume the same utility function as the one in the main text.

Since the main focus of this paper is to show the role that variable demand elasticity plays in the labor market outcomes of an economy, it is worth elaborating more on the demand elasticity implied by equation (51). As I show below, solving the cost minimization problem of country \( i \)'s final good producers implies that these producers demand the following amount of the Home intermediate good \( \nu \) (which is produced by a Home firm with productivity \( \phi \) and \( l \) workers):

\[ q_i(\phi, l) = A_i p_i(\phi, l)^{-\sigma} - \alpha, \tag{52} \]

where I label the intermediate producer by its productivity \( \phi \) and number of workers \( l \). The terms \( p_i(\phi, l) \) and \( q_i(\phi, l) \) are the price of and demand for the Home intermediate producer \( (\phi, l) \) in market \( i \), respectively, and \( A_i \) is aggregate demand shifter. Note that while the aggregate demand shifter in the Home market \( (A_H) \) is an endogenous equilibrium object, the aggregate

\(^1\)Recall that the final good is non-tradable.

\(^2\)Notice that the final good production is not zero even if no intermediate inputs are used; I interpret the parameter \( \alpha \) as managerial ability that could (to some extent) replace the intermediate inputs.

\(^3\)I study the unique symmetric equilibrium in which all producers produce the same amount.
Online Appendix: For Online Publication Only

Demand shifter in the Foreign market is a model parameter that I estimate. Moreover, under the assumption that \( \alpha > 0 \), there is a choke price in the demand curve (52) above which the demand for an intermediate good in market \( i \) is zero:

\[
p_{i}^{\text{choke}} = \left( \frac{A_{i}}{\alpha} \right)^{\frac{1}{\sigma}}.
\] (53)

As I show below, the (absolute value of) demand elasticity that the intermediate producer \((\phi, l)\) faces in market \( i \) is

\[
\varepsilon_{i}(\phi, l) = \sigma \left( 1 + \frac{\alpha}{q_{i}(\phi, l)} \right).
\] (54)

A few points are in order. First, as \( \sigma \) or \( \alpha \) rises, intermediate good producers face more elastic demand curves. Second, more productive intermediate producers are larger and therefore face lower demand elasticities and charge higher markups. Furthermore, any policy that affects the firm size distribution also changes the demand elasticity faced by firms. Finally, in the special case in which \( \alpha \) is set to zero, we are back in the constant demand elasticity world in which all intermediate producers face the same demand elasticity \( \sigma \).

Discussion. While not employing a similar formulation, using a demand curve featuring choke price to generate variable demand elasticity is also used in the literature (Pollak, 1971; Dinopoulous et al., 2011; Simonovska, 2015; Arkolakis et al., 2019). With no income heterogeneity, instead of using a displaced CES production function in (51), one could define consumers’ utility as

\[
U = \left[ \int_{0}^{1+1} \left( q(\nu) + \alpha \right)^{\frac{\sigma - 1}{\sigma}} d\nu \right]^{\sigma_{1}},
\] (55)

and then aggregate the demand for each variety across all workers (as in Simonovska (2015) who uses the above utility function and imposes \( \sigma = 1 \)). However, if individuals earn different incomes, each individual would have her own choke price for each variety \( \nu \). As a result, summing each good’s demand across all individuals is not feasible since this model features wage heterogeneity across a continuum of workers. To solve this aggregation problem, I assume consumers demand a final good that is produced using the displaced CES production function (51). This way, these final good producers collect income from all workers and intermediate firms and demand intermediate goods \( \nu \).

---

4Home is a small open economy.

5This is because, in that case, the demand shifter \( A \) would depend on the consumer’s income. To see this, note that the utility maximization problem would result in first order conditions that are exactly the same as those stated below for the case of the cost minimization problem, except that one needs to replace \( M \) with the utility of the consumer, and also the Lagrange multiplier would be the inverse of \( \Psi \). Hence, one can see that the demand shifter in that case would be a function of the consumer’s income.
C.3 Final Good Producers’ Problem

The cost minimization problem of the final good producers in the Home economy is as follows:

\[
\min_{q(\nu) \geq 0} p(\nu)q(\nu) \quad \text{s.t.} \quad M = \left[ \int_0^{1+1} (q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} \, d\nu \right]^{\frac{1}{\sigma-1}} .
\]  

(56)

The first order conditions yield (\forall q(\nu) > 0)

\[
(q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} \left[ \int_0^{1+1} (q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} \, d\nu \right]^{\frac{1}{\sigma-1}} = \frac{p(\nu)}{\Psi} ,
\]  

(57)

where \(\Psi\) is the Lagrange multiplier. Multiplying both sides by \((q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}}\) and using the definition of \(M\) yields

\[
(q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} M^\frac{1}{\sigma} = \frac{p(\nu)(q(\nu) + \alpha)}{\Psi} .
\]  

(58)

Summing over all varieties with \(q(\nu) > 0\) yields

\[
M^\frac{1}{\sigma} \int_0^{1+1} (q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} 1_{q(\nu) > 0} \, d\nu = \int_0^{1+1} \frac{p(\nu)(q(\nu) + \alpha)1_{q(\nu) > 0}}{\Psi} \, d\nu .
\]  

(59)

Moreover, as in the text, assume that while all the Foreign varieties are imported, only a subset of the domestic varieties with measure \(N_H\) are produced. Hence, using the definition of \(M\), one can write

\[
M^{\frac{\sigma-1}{\sigma}} = \int_0^{1+1} (q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} 1_{q(\nu) > 0} \, d\nu + (1 - N_H)\alpha^{\frac{\sigma-1}{\sigma}} .
\]  

(60)

Combining the last two equations delivers the Lagrange multiplier

\[
\Psi = \frac{\int_0^{1+1} p(\nu)(q(\nu) + \alpha)1_{q(\nu) > 0} \, d\nu}{M(1 - M^{\frac{1-\sigma}{\sigma}} (1 - N_H)\alpha^{\frac{\sigma-1}{\sigma}})} .
\]  

(61)

To find the demand for each variety \(\nu\), rearrange the terms in equation (57) and use the definition of \(M\) to get

\[
(q(\nu) + \alpha)^{\frac{\sigma-1}{\sigma}} = \frac{M^{\frac{1}{\sigma}}}{\Psi} p(\nu) .
\]  

(62)

Rearranging the terms yields

\[
q(\nu) = M^\sigma p(\nu)^{-\sigma} - \alpha .
\]  

(63)

Defining the aggregate demand shifter \(A := M^{\sigma} p(\nu)^{-\sigma}\) delivers the demand for variety \(\nu\) as expressed above

\[
q(\nu) = Ap(\nu)^{-\sigma} - \alpha .
\]  

(64)
Note that the aggregate demand shifter $A$ is an equilibrium object. The (absolute value of) demand elasticity that the variety $\nu$ producer faces is

$$\varepsilon(\nu) = -\frac{\partial q(\nu) p(\nu)}{\partial p(\nu) q(\nu)} = \frac{\sigma A p(\nu)^{\sigma - 1} - \sigma \alpha + \sigma \alpha}{q(\nu)} = \frac{\sigma q(\nu) + \sigma \alpha}{q(\nu)} = \sigma (1 + \frac{\alpha}{q(\nu)}) .$$

The rest of this model is the same as the model presented in the paper. The main implication that markups are variable and rise with firms’ size holds under this structure as well. Simulations of this alternative model show implications similar to those reported in the text.

### D Stationary Equilibrium

The equilibrium notion in this paper is a steady-state equilibrium. Here, I define the steady-state equilibrium of the model. The steady-state equilibrium consists of steady-state distributions $J(\phi, l)$ and $G(\phi, l)$, total production of the composite final good $M$ and its price $P$, measure of intermediate producers $N_H$, measure of entrants $N_e$, aggregate income $I$, measure of employees $L_T$ working for intermediate producers, measure of unemployed individuals $L_U$, measure of producers of services $L_h$, labor market tightness $\theta$, job finding rate $m(\theta)$, relative price $e$, employment policy function $l'(\phi, l)$, continuation policy function $I^c(\phi, l)$, firing policy function $I^f(\phi, l)$, wage schedule $w(\phi, l)$, export policy function $I^x(\phi, l)$, Home intermediate producers’ prices and quantities $p_{HH,1}(\phi, l), q_{HH,1}(\phi, l)$, Foreign intermediate producers’ prices and quantities in the Home market $p_{FH,k}(\phi, l)$ and $q_{FH,k}(\phi, l)$, value functions $V(\phi, l), V^c(\phi, l), W^a, W^a, W^h$, satisfying the following conditions:

I) In equilibrium, the distributions $J(\phi, l)$ and $G(\phi, l)$ reproduce themselves through the Markov productivity process, employment policy function $l'(\phi, l)$, and continuation policy function $I^c(\phi, l)$. Recall that while $J(\phi, l)$ is the end-of-period firms’ distribution, $G(\phi, l)$ is the mid-period distribution—that is, firms’ distribution after realization of productivities (and so after entry and exit) but before firms decide about their employment. Hence, we can write the mid-period distribution $G(\phi, l)$ as a function of the end-of-period distribution $J(\phi, l)$ as follows:

$$G(\phi', l) = \begin{cases} \frac{N_e}{N_H} J^0(\phi') + (1 - \lambda) \int I^c(\phi, l) J(\phi, l) T(\phi'|\phi) d\phi & l = l_e \\ (1 - \lambda) \int I^c(\phi, l) J(\phi, l) T(\phi'|\phi) d\phi & l \neq l_e \end{cases} \quad (65)$$

where $T(\phi'|\phi)$ is the transition density of the productivity Markov process, $J^0(\cdot)$ is the ergodic distribution implied by the productivity Markov process, $l_e$ is the number of workers that entrants

---

$^6$The label $\phi, l$ for the Foreign producer corresponds to the productivity and labor of its Home rival.
start with upon entry, and \( \lambda \) is the exogenous death rate. The first term of the first equation in (65) corresponds to the entrants, who start with \( l_e \) number of workers. In equilibrium, an endogenous measure of firms replace exiters (for endogenous or exogenous reasons) every period so that the total mass of firms remains constant:

\[
N_e = N_H \left[ \lambda \int l(1 - I^c(\phi, l))J(\phi, l)dl d\phi \right].
\]  

(66)

The end-of-period distribution \( J(\phi, l) \), on the other hand, can be written as a function of the mid-period distribution \( G(\phi, l) \):

\[
J(\phi, L) = \frac{\int G(\phi, l)1_{l'=L}dl}{\int \int G(\phi, l)1_{l'=L}dl d\phi},
\]

(67)

where \( 1_x \) is an indicator function equal to one if \( x \) holds, and zero otherwise.

II) In equilibrium, a measure \( L_T \) of workers are employed by the intermediate producers: \(^7\)

\[
L_T = N_H \int \int lJ(\phi, l)dl d\phi.
\]

(68)

At the beginning of each period, some jobs may be destroyed as a result of firms’ endogenous exit or exogenous death. Total inflow to the unemployment pool at the beginning of each period equals

\[
U_{\text{inflow,begin}}^{\text{inflow}} = \lambda L_T + (1 - \lambda)N_H \int \int (1 - I^c(\phi, l))J(\phi, l)dl d\phi.
\]

(69)

Moreover, after realization of the productivity shocks, some firms decide to contract, and the laid-off workers enter the unemployment pool:

\[
U_{\text{inflow,mid}}^{\text{inflow}} = N_H \int \int I^f(\phi, l)(1 - l'(\phi, l))G(\phi, l)dl d\phi.
\]

(70)

Total inflow to the unemployment pool in each period equals the sum of these two:

\[
U_{\text{inflow}}^{\text{inflow}} = U_{\text{inflow,begin}}^{\text{inflow}} + U_{\text{inflow,mid}}^{\text{inflow}}.
\]

(71)

An endogenous fraction (which is the job finding rate) of unemployed individuals \( L_U \) find a job.\(^8\)

\(^7\)Recall that final good producers do not hire labor.

\(^8\)Note that in equilibrium, workers are indifferent between searching for a job and producing services. Hence, no individual who is producing services decides to search for a job.
Hence, outflow from the unemployment pool can be written as
\[ U_{\text{outflow}} = m(\theta)L_U. \] (72)

In equilibrium, flows into and out of the unemployment pool are equal so that the measure of unemployed individuals is constant. Therefore, rearranging equation (72) yields
\[ L_U = \frac{U_{\text{inflow}}}{m(\theta)}. \] (73)

Now that we have \( L_T \) and \( L_U \) from equations (68) and (73), respectively, the measure of individuals who decide to produce services equals
\[ L_h = 1 - L_T - L_U. \] (74)

Then, the unemployment rate equals the fraction of job seekers who cannot find a job:
\[ \text{unemployment rate} = \frac{L_U}{L_U + L_T}. \] (75)

Notice that a rise in labor market tightness, a reduction in the inflow to the unemployment pool, or an increase in the size of the intermediate sector workforce reduces the unemployment rate.

III) Prices \( p_{HH,1}(\phi, l) \) and \( p_{HF,1}(\phi, l) \), quantities \( q_{HH,1}(\phi, l) \) and \( q_{HF,1}(\phi, l) \), material usage \( M(\phi, l) \), and export decision \( I^x(\phi, l) \) solve the domestic intermediate producers’ problem in (17). Moreover, Foreign producers’ price \( p_{FH,k}(\phi, l) \) and quantity \( q_{FH,k}(\phi, l) \) in the Home market solve Foreign firms’ problem in (21).

IV) In equilibrium, international trade balances. Total export (EX) and total import (IM) can, respectively, be expressed as
\[ EX = N_H \int_{\phi} \int_l p_{HF,1}(\phi, l)q_{HF,1}(\phi, l)I^x(\phi, l)J(\phi, l)dld\phi, \] (76)
\[ IM = N_H \sum_{k=1}^{K} \int_{\phi} \int_l p_{FH,k}(\phi, l)q_{FH,k}(\phi, l)J(\phi, l)dld\phi, \] (77)
where \( N_H \) is the measure of intermediate producers.

V) In equilibrium, the market for the non-tradable final good clears. Total supply of the final good in the economy is
\[ M^s = \left[ N_H \int_{\phi} \int_l Q(\phi, l)^{\frac{\sigma-1}{\sigma}} J(\phi, l)dld\phi \right]^{\frac{\sigma}{\sigma-1}}, \] (78)
where

\[ Q(\phi, l) = \left[ q_{HH,1}(\phi, l)^{\frac{\alpha-1}{\alpha}} + \sum_{k=1}^{K} q_{FH,k}(\phi, l)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}}. \]  

(79)

As for the demand, recall that the final good is demanded by both consumers and intermediate producers. Hence, total demand of the final good can be expressed as

\[ M^d = \kappa \frac{I}{P} + N_H \int_{\phi} \int_{l} M(\phi, l) J(\phi, l) dld\phi, \]  

(80)

where the first term is the consumers’ demand and the second term is the demand by intermediate producers. The aggregate income \( I \) can be written as

\[ I = N_H \int_{\phi} \int_{l} \Pi(\phi, l) J(\phi, l) dld\phi - N_H \int_{\phi} \int_{l} C(l, l'(\phi, l)) G(\phi, l) dld\phi - N_e f_e + IM(\tau - 1) + b_u L_U + L_h, \]  

(81)

where \( \Pi(\phi, l) \) is defined in (17). The first term is intermediate producers’ gross profit (i.e., including the wage bill and labor adjustment costs). The second term is the labor adjustment costs paid by intermediate producers. The third and fourth terms are sunk entry cost and tariff revenue rebated to consumers, respectively. The last two terms are total revenue by unemployed individuals as well as producers of services.

VI) Services (i.e., the numeraire) are supplied by those who decide to produce services as well as unemployed individuals. This good is demanded by consumers, and by intermediate producers to pay the sunk entry cost, labor adjustment costs, fixed cost of production, and fixed exporting cost. In equilibrium, the market for services clears:

\[
\begin{aligned}
L_h + b_u L_U &= (1 - \kappa) \frac{I}{P} + N_e f_e + N_H \int_{\phi} \int_{l} C(l, l'(\phi, l)) G(\phi, l) dld\phi + N_H f_d \\
&\quad + N_H f_x \int_{\phi} \int_{l} I^x(\phi, l) J(\phi, l) dld\phi.
\end{aligned}
\]  

(82)

VII) In equilibrium, there is a positive mass of entrants, and therefore the free entry condition holds with equality:

\[ V^e := \int V^C(\phi, l_e) dJ^0(\phi) = f_e. \]  

(83)

VIII) In equilibrium, individuals are indifferent between searching for a job or producing services:

\[ W^a = W^h. \]  

(84)
E Numerical Algorithm to Solve for Stationary Equilibrium

To solve the model, I discretize the state space on a log scale using 20 grid points for productivity and 200 points for labor. The numerical algorithm used to compute the stationary equilibrium consists of the following steps:

**Step 1:** Guess the equilibrium objects $A_H$, $w(\phi, l)$, $\theta$, $e$, and $N_H$.

**Step 2:** Given all the aggregate equilibrium objects, compute the value function $V(\phi, l)$ in (27). Then, compute the value of entry $V^e$ in (33). Reduce (raise) $A_H$ if the value of entry exceeds (falls behind) the sunk entry cost $f_e$. Iterate until $V^e = f_e$.

**Step 3:** Compute the value of unemployment $W^u$ by using in equation (42) the fact that in equilibrium $W^h = W^a = \frac{1}{1-\beta}$. Then, use (45)-(46) to compute the value function $W^e(\phi, l)$ and $W^m(\phi, l)$. To do so, start with a guess on $W^e(\phi, l)$ and solve for $W^m(\phi, l)$ using equation (45). Then compute $W^e(\phi, l)$ using equation (46). Iterate until $W^e(\phi, l)$ converges. Then use equation (43) to solve for $W^m$.

**Step 4:** First, use equation (35) to calculate firms’ surplus. To do so, since I have discretized the state space, compute the marginal surplus generated by the marginal worker as

$$S^F(\phi, l) = [\Pi(\phi, l) - w(\phi, l)l + \beta V(\phi, l)] - [\Pi(\phi, l - \Delta) - w(\phi, l - \Delta)(l - \Delta) + \beta V(\phi, l - \Delta)], \quad (85)$$

where $\Delta$ is the size of the employment grid. To solve this recursive formulation, we start from the marginal surplus generated at a firm with the minimum employment $l_e$:

$$S^F(\phi, l_e) = [\Pi(\phi, l_e) - w(\phi, l_e)l_e + \beta V(\phi, l_e)] - [-f_d]. \quad (86)$$

Notice that, if the marginal worker quits the firm with $l_e$ workers, the firm has to quit the market; however, the firm has already paid the fixed cost $f_d$ to draw its productivity. Now that I have computed the firm’s surplus, use the above two equations as well as equations (36)-(37) to compute the wage schedule $w(\phi, l)$. If the wage schedule does not match the initial guess, go back to step 1 with the new wage schedule. Repeat steps 1-4 until $w(\phi, l)$ converges.

**Step 5:** Given the value of unemployment computed in step 3 above, compute the value of searching for a job $W^a$ using equation (41). If $W^a$ exceeds (falls behind) $W^h = 1/1-\beta$, reduce (raise) labor market tightness and go back to step 1. Iterate steps 1-5 until $W^a$ equals $W^h$.

**Step 6:** Compute total exports and total imports using (76)-(77). If total exports exceed (fall behind) total imports, reduce (raise) the relative price $e$ and go back to step 1. Iterate steps 1-6 until total exports equal total imports.

**Step 7:** Use equation (82) to compute the supply of and demand for services. If the supply
exceeds (falls behind) the demand, raise (reduce) the measure of intermediate producers \( N_H \) and go back to step 1. Iterate steps 1-7 until the supply equals the demand.

## F Markups

To estimate the parameters governing demand elasticities in the model (i.e., \( \sigma \) and \( \alpha \)), I use the distribution of material share of sale. The idea is that, as this section shows, given the material elasticity of output \( \eta \), markup is inversely related to material share of sale. The overview of this methodology is as follows (see Hall et al. (1986) and De Loecker and Warzynski (2012)).

Assume plants produce output \( Q \) using the following production function:

\[
Q = Q(s^1, ..., s^n; d^1, ..., d^m, \phi),
\]  

where \( s^i \) is a static (freely adjustable) input, \( d^i \) a dynamic (costly to adjust) input, and \( \phi \) is productivity. A plant producing output \( Q \) decides about its inputs by minimizing the associated cost function. The Lagrangian for the cost minimization problem is as follows:

\[
L = \sum_{i=1}^{n} p^s_i s^i + \sum_{i=1}^{m} p^d_i d^i + \lambda (Q - Q(.)),
\]  

where \( p^x \) is price of input \( x \), which the plant takes as given. The first order condition implies the following demand for static input \( s^i \):

\[
p^{s^i} = \lambda \frac{\partial Q(.)}{\partial s^i},
\]  

where \( \lambda \) is the Lagrange multiplier. Rearranging terms yields

\[
\frac{P}{\lambda} = \left( \frac{\partial Q(.)}{\partial s^i} \right) \frac{PQ}{Q} \frac{1}{p^{s^i} s^i},
\]  

where \( P \) is the output price. Recalling that the Lagrange multiplier is the marginal cost of production, the left-hand side is the price over marginal cost (i.e., markup). Therefore, markup is equal to the elasticity of output with respect to a static input over the share of that static input in revenue.\(^9\)

\(^9\)This approach doesn’t allow for dynamic pricing, which is consistent with the model presented in the paper.
G  Relation between Export Intensity and Demand Elasticity

Consider a model with two countries, Home and Foreign. Firms are heterogeneous in their productivities. Suppose that a Home exporter $i$ faces demand elasticities $\sigma_{iH}$ and $\sigma_{iF}$ in the Home and Foreign markets, respectively:

$$q_{iJ} = A_{iJ}p_{iJ}^{-\sigma_{iJ}},$$ (91)

where $J = H, F$ denotes the market and $A_{iJ}$ is the aggregate demand shifter in market $J$. The Home exporter $i$ decides the prices to charge in each market by solving the following maximization problem:

$$\max_{p_{iH}, p_{iF}} \left[ p_{iH}q_{iH} + p_{iF}q_{iF} \right],$$ (92)

s.t.

$$\lambda(q_{iH} + q_{iF}d) \leq Q,$$ (93)

where $d$ is the iceberg trade cost, $Q$ is the firm’s total production, and $\lambda$ is the Lagrange multiplier. Substituting the demand equations (91) into (92) yields the following first order conditions:

$$p_{iH} = \frac{\sigma_{iH}}{\sigma_{iH} - 1} \lambda,$$ (94)

$$p_{iF} = \frac{\sigma_{iF}}{\sigma_{iF} - 1} \lambda d.$$ (95)

Using the first order conditions to remove $\lambda$ yields

$$p_{iF} = \psi_i p_{iH}d,$$ (96)

where

$$\psi_i = \frac{\sigma_{iF}}{\sigma_{iF} - 1} \frac{\sigma_{iH} - 1}{\sigma_{iH}}.$$ (97)

Now, I use demand equations (91) to derive the export intensity (EI) for the Home exporter $i$:

$$\text{EI}_i = \frac{p_{iF}q_{iF}}{p_{iH}q_{iH} + p_{iF}q_{iF}} = \frac{A_{F}p_{iF}^{1-\sigma_{iF}}}{A_{H}p_{iH}^{1-\sigma_{iH}} + A_{F}p_{iF}^{1-\sigma_{iF}}}.$$ (98)

Using equation (96) in (98) yields

$$\text{EI}_i = \frac{A_{F}p_{iF}^{1-\sigma_{iF}}(\psi_i d)^{1-\sigma_{iF}}}{A_{H}p_{iH}^{1-\sigma_{iH}} + A_{F}p_{iF}^{1-\sigma_{iF}}(\psi_i d)^{1-\sigma_{iF}}} = \frac{A_{F}p_{iF}^{1-\sigma_{iF}}(\psi_i d)^{1-\sigma_{iF}}}{A_{F}p_{iF}^{1-\sigma_{iF}}(\psi_i d)^{1-\sigma_{iF}} + A_{H}p_{iH}^{1-\sigma_{iH}}(\psi_i d)^{1-\sigma_{iH}}(\psi_i d)^{\sigma_{iF} - 1} + 1};$$ (99)
which simplifies to

$$EI_i = \frac{1}{\frac{A_H}{A_P}^d \sigma_{iF}^{\sigma_{iF} - \sigma_{iH}} (\psi_i d)^{\sigma_{iF} - 1} + 1}. \quad (100)$$

Notice that, in general, export intensity varies by $i$. Moreover, it is easy to verify that if demand elasticities are the same both across markets and across firms (i.e., $\forall i: \sigma_{iF} = \sigma_{iH} = \sigma$), all exporters would have the same export intensity:

$$\forall i: \quad EI_i = \frac{1}{\frac{A_H}{A_P}^d \sigma_{iF}^{\sigma_{iF} - 1} + 1}. \quad (101)$$

However, it can be shown that export share of sale would vary across firms if demand elasticities vary across markets but not across firms (i.e., $\forall i: \sigma_{iH} = \sigma_H$ and $\forall i: \sigma_{iF} = \sigma_F$). In this case, we have $\forall i: \psi_i = \psi$, and so rewriting equation (100) shows that the export intensity varies across firms:

$$EI_i = \frac{1}{\frac{A_H}{A_P}^d \sigma_{iF}^{\sigma_{iF} - \sigma_H} (\psi d)^{\sigma_{iF} - 1} + 1}, \quad (102)$$

where the price $p_{iH}$ varies across firms because firms are heterogeneous. Furthermore, even if demand elasticities are the same across markets but different across firms (i.e., $\sigma_{iH} = \sigma_{iF} = \sigma_i$), the export intensity would still vary across firms:

$$EI_i = \frac{1}{\frac{A_H}{A_P}^d \sigma_i^{\sigma_i - 1} + 1}. \quad (103)$$

Hence, we have proved that exporters would have the same export intensity if and only if demand elasticities are the same, both across markets and across firms.