Trade, Innovation and Optimal Patent Protection

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Abstract

This paper studies optimal patent policy in open economies. By integrating Helpman-Krugman and Eaton-Kortum trade theory, we develop a quantitative model of patenting that captures the trade-off between the innovation and market power effects of patent policy. We calibrate the model to estimate patent protection by destination and the geography of innovation. Counterfactual analysis shows that stronger patent protection has global benefits, but local costs, resulting in inefficiently weak protection. There are large potential gains from global cooperation, but realizing these gains requires larger, more innovative economies to offer stronger protection. By pushing towards policy harmonization, the TRIPS agreement reduces global welfare with developing countries bearing the cost.

1   Introduction

How should intellectual property be protected in the global economy? This is perhaps the most contentious question in modern trade policy, leading to recurring frictions between the Global North and the Global South. Rich countries argue that strong intellectual property rights are needed to stimulate innovation and are therefore willing to grant innovators substantial monopoly rights. Poor countries counter that these monopoly rights inflate consumer prices and argue that strong intellectual property rights amount to a transfer from poor-country households to rich-country firms.

The tensions surrounding the Trade-Related Aspects of Intellectual Property Rights (TRIPS) agreement are an important case in point. TRIPS was the most controversial part of the Uruguay Round negotiations that led to the creation of the World Trade Organization (WTO). It sought to strengthen intellectual property rights in developing economies by requiring countries to adopt intellectual property policies similar to those already implemented in rich countries. As Saggi (2016) reports, TRIPS was pushed through by the United States (US), Europe, and Japan against strong opposition from Brazil, India, and China. A common assessment is that these tensions drove a lasting wedge between WTO members, thereby playing a crucial role in the failure of the Doha Round and ultimately the stalemate at the WTO.

In this paper, we develop a quantitative model of trade, growth, and patenting and use it to study optimal patent policy in open economies. By so doing we are able to undertake the first comprehensive quantitative analysis of the TRIPS agreement. We also characterize cooperative and noncooperative patent policy equilibria, as well as analyzing unilateral policy changes. Our results shed new light on how welfare depends upon both domestic and foreign patent protection and on the distributional conflicts at the heart of the policy debate.

The classic trade-off patent policy faces is that stronger protection brings dynamic benefits from faster innovation, but also generates static costs through higher prices (Nordhaus 1969). We formalize this trade-off using a trade and growth model with endogenous patenting. Building on Grossman and Lai (2004), the model implies that in open economies stronger patent protection has global benefits, but local costs. While all countries reap the dynamic benefits of the increased product variety brought about by innovation, only households in the country issuing the patent pay the static costs. This is because a patent gives a firm the exclusive right to sell in a particular market, thus establishing local monopoly power and raising prices. Since patent policies have cross-border spillovers, there is scope for international policy coordination.

A key property of the model is that the global growth rate is more sensitive to the level of patent protection in markets that are more profitable for innovators. Intuitively, such markets account for a higher fraction of the value of innovation and, therefore, have a disproportionate effect on R&D

investment. It follows that not only larger countries, but also (because of home bias in trade) more innovative countries generate greater dynamic benefits by increasing patent rights. Consequently, optimal patent protection differs across economies and tends to be weaker in smaller, less innovative countries.

The paper's theoretical contribution is to develop a patenting model that is quantitatively tractable in an open economy setting. We achieve this by integrating Helpman-Krugman trade theory with Eaton-Kortum trade theory. Our model inherits the tractability of these frameworks, but endogenizes their relative importance in production and trade. Innovators start out as monopolists who produce and sell differentiated products as in Helpman and Krugman (1987). But once technology diffuses, products that are not under patent protection are produced and sold competitively as in Eaton and Kortum (2002). In equilibrium, the split between Helpman-Krugman trade and Eaton-Kortum trade depends upon patent policy in all countries.

We calibrate the model to a world economy divided into the US, Europe, Japan, China, Brazil, India, and a residual rest of the world. We choose the year 2005 as our baseline so that we can relate our quantitative analysis to TRIPS, which came into force in 1995 but was phased in over a 10 year implementation period. The key policy parameter in our model is the Poisson rate at which a patent expires, which can be interpreted as capturing a combination of the statutory patent length and the probability of patent enforcement. Our estimates imply that a patent in the US has an expected duration of 14 years. Since the legal term of US patents is 20 years, this value is consistent with relatively high levels of patent enforcement in the US. We find that Europe and Japan have similar patent protection to the US, while protection is weaker elsewhere. Expected patent duration is around 10 years in China and Brazil and around 5 years in India.

In the calibrated economy, innovation is highly concentrated in the richer economies. The US alone accounts for over 60 percent of global innovation and the US, Europe and Japan together account for more than 95 percent of innovation. Since trade is home biased, this pattern of specialization in innovation implies that innovation and growth are much more sensitive to patent policy in Europe, Japan and especially the US, than to patent policy in the developing world.

We begin our counterfactual analysis by studying countries' incentives to unilaterally deviate from the 2005 status quo. We find that all countries apart from the US have an incentive to weaken their patent protection, since the local static costs of protection exceed the dynamic benefits. For the US, the costs and benefits are roughly equal, but there is a weak incentive to strengthen protection. We also find that when any country increases its patent protection, all other countries benefit due to higher growth. However, the magnitude of these international spillovers varies greatly because patent protection in the United States, Japan, and Europe has a much stronger effect on growth than patent protection in China, Brazil, and India.

Next, we turn to the noncooperative scenario and simulate Nash patent policies. We find that

only the United States offers patent protection in the Nash equilibrium and that all other countries free ride on the innovation incentives US patents provide. Strikingly, the Nash equilibrium is nevertheless similar to the 2005 baseline in welfare terms, because the dynamic costs of reducing patent protection are offset by the static benefits. Welfare falls by 1.2 percent for the United States, but rises by 0.8 percent in China. For the world on aggregate, welfare declines by 0.8 percent when all individuals are weighted equally.

Then, we consider the cooperative scenario that maximizes world welfare. In the baseline case with equal welfare weights for all individuals, efficiency requires that the US, Europe, and Japan provide complete patent protection, while other countries do not offer protection. Policy divergence is optimal because growth is more sensitive to patent protection in larger and more innovative economies. World welfare is 7.4 percent higher, but these large gains mask substantial distributional effects. China, Brazil and India are the big winners, whereas gains are lower for the US and Japan, and Europe experiences a small decline in welfare.

Finally, we study the effects of TRIPS. Starting from the calibrated equilibrium in 2005, we consider two counterfactuals. A pre-TRIPS counterfactual in which we return patent protection in China, Brazil, India and the rest of the world to levels calibrated on data from before the implementation of TRIPS. And a harmonization counterfactual in which we set patent protection in all countries equal to US protection. Both counterfactuals lead to the same conclusion. Increasing patent protection in developing countries, as TRIPS sought to do, reduces welfare in those countries and for the world as a whole, while delivering little benefit to developed economies.

Taken together, our results suggest that there is significant scope for TRIPS reform. The main reason is that it pushes policy towards common patent rights across countries, which is highly suboptimal according to our analysis. This finding substantiates the theoretical point of Grossman and Lai (2004) that the harmonization of patent policies is neither necessary nor sufficient for maximizing world social welfare.

Our analysis contributes to a small quantitative literature on TRIPS and intellectual property policy in open economies. Eaton and Kortum (1996, 1999) use a quantitative model of innovation and patenting to shed light on which countries drive growth and the extent of international technology diffusion. However, in their model mark-ups are unaffected by patent protection. Building on Eaton and Kortum's theory, McCalman (2001) quantifies how patent policy harmonization under TRIPS reallocates producer surplus across countries, while McCalman (2005) shows that TRIPS benefits all countries through higher innovation. However, McCalman's analysis does not allow TRIPS to effect market power. Chaudhuri et al. (2006) use estimated price and expenditure elasticities to quantify the welfare effects of product patent enforcement under TRIPS in the fluoroquinolones subsegment of the Indian anti-bacterials market, but do not study changes in innovation. Jakobsson and Segerstrom (2016) find that TRIPS raises welfare in a two-region product

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cycle model of foreign direct investment, but their model does not include patenting. In contrast to existing research, we quantify how optimal patent policy depends upon the trade-off between dynamic benefits and static costs of stronger protection.

The theory we develop builds upon the extensive trade and growth literature.[1] It is most closely related to the theories of innovation and patenting in Eaton and Kortum (1999) and Grossman and Lai (2004). However, we go beyond the existing literature by introducing a quantitative model of trade and patenting that incorporates trade in both monopolistic and competitive products. The model also has points of contact with Lind and Ramondo (2022) and Hsieh et al. (2022), but neither of these papers endogenizes innovation or considers the effects of patent policy on growth and welfare. Milicevic et al. (2023) analyze optimal innovation policy in a two country world, but focus on subsidies rather than intellectual property rights. Akcigit et al. (2021) study the interplay between trade policy and innovation policy and use patent data to calibrate their model. But, like other quantitative dynamic trade models that use patent data for calibration (e.g. Cai et al. 2022, Sampson 2023), they do not model patenting decisions or institutions, nor do they analyze strategic interactions between countries.

Consistent with our model, recent evidence establishes that innovation responds to patent protection (Williams 2017). Moscona (2021) finds that the introduction of patent protection for plants in the US in 1985 led to the development of new varieties for the affected crops. TRIPS itself generated exogenous variation in the duration of patent protection: in the US, patent protection changed from 17 years after grant to 20 years after application. Bertolotti (2022) uses this source of variation to show that both patenting and R&D increase with patent length. Kyle and McGahan (2012) exploit cross-country variation in disease prevalence and patent laws during the implementation of TRIPS to assess whether stronger patent protection induces more innovation in pharmaceuticals. They find a positive effect for protection in developed economies, but no effect for developing countries. This result is in line with our model, since we find that the impact of patent rights on innovation depends upon market size and levels of innovation. Looking at the price effects of patents, Duggan et al. (2016) study the effect of India's TRIPS-induced patent reform on pharmaceutical prices. They find moderate price increases for molecules that receive a patent. Also in line with our model, De Rassenfosse et al. (2022) show that exports collapse when firms lose patent protection in a market.

In the tradition of Nordhaus (1969) and Grossman and Lai (2004), we quantify the trade-off between innovation and market power. Although this trade-off is at the heart of policy debates over intellectual property rights, there are other channels through which patent policy may affect eco-

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[1]Helpman (1993) provides the first general equilibrium analysis of the welfare effects of intellectual property rights in an endogenous growth model. Saggi (2016) reviews the literature on trade and intellectual property rights. Akcigit and Melitz (2021) and Melitz and Redding (2021) review the broader trade and growth literature.

nomic outcomes. Future research could extend our analysis by allowing patent protection to impact export market entry (Ivus 2015, Cockburn et al. 2016), foreign investment choices (Branstetter et al. 2011, Bilir 2014), knowledge spillovers (Moser and Voena 2012), technology transfer within multinational firms (Branstetter et al. 2006) and/or investment in imitation and technology adoption. It would also be interesting to shift the focus from multilateral to bilateral policies and study the 'TRIPS-plus' provisions in many recent free trade agreements that further strengthen intellectual property rights (Mercurio 2006).

The remainder of the paper is organized as follows. Section 2 introduces the model and characterizes equilibrium behavior. Section 3 explains how we calibrate the model and discusses model fit and the calibrated parameters. Section 4 presents the counterfactual analysis. Finally, Section 5 concludes.

2 A Theory of Trade and Patents

We develop a dynamic model of trade and patenting with endogenous innovation. In the model, patent protection determines the expected duration of innovators' monopoly control over their ideas. Consequently, as in Grossman and Lai (2004), stronger patent rights incentivize innovation, but generate a static distortion due to monopoly pricing. In our model, greater patent protection also generates a sourcing distortion because monopoly control over production restricts buyers' ability to source from the lowest cost supplier. The theory embeds the trade-off between static costs and dynamic benefits into a quantitative open economy model that is suitable for counterfactual analysis of changes in patent protection.

2.1 Economic Environment

We consider an economy with N countries and $S + 1$ sectors. Sectors $s \neq 0$ feature endogenous innovation and patenting, while in sector zero innovation is exogenous and there is no patenting. Each country n has a fixed labor endowment L_n and labor is the only factor of production. Time t is continuous.

Demand. In each country and sector, non-tradable sectoral output is produced competitively as a constant elasticity of substitution aggregate of tradable intermediate product varieties indexed by ω. Products differ in their quality $\psi(\omega)$, except in sector zero where $\psi(\omega) = 1$ for all varieties. Let M_t^s denote the mass of products available in sector s at time t. Since the model does not feature an extensive margin of trade, M_t^s is the same in all countries. Sectoral output Y_{nt}^s then satisfies:

$$Y_{nt}^s = \left(\int_0^{M_t^s} \psi(\omega)^{\frac{1}{\sigma^s}} c_{nt}^s(\omega)^{\frac{\sigma^s-1}{\sigma^s}} d\omega \right)^{\frac{\sigma^s}{\sigma^s-1}}, \tag{1}$$

where $c_{nt}^s(\omega)$ denotes demand for product ω in country n at time t and σ^s is the elasticity of substitution between varieties. Producer optimization yields a constant elasticity demand function given by:

$$c_{nt}^s(\omega) = \psi(\omega) \left(\frac{p_{nt}^s(\omega)}{P_{nt}^s} \right)^{-\sigma_s} Y_{nt}^s,$$

where $p_{nt}^s(\omega)$ is the price of variety ω in country n and P_{nt}^s is the sectoral price index.

Output from each sector is combined using a Cobb-Douglas aggregator to produce a non-tradable final good according to:

$$Y_{nt} = \prod_{s=0}^{S} \left(\frac{Y_{nt}^s}{\beta^s} \right)^{\beta^s}. \tag{2}$$

The final good is used for consumption and as an intermediate input in variety production.

Final consumption demand comes from each country's representative agent whose intertemporal preferences are given by:

$$U_{nt} = \int_t^\infty e^{-\rho(\tilde{t}-t)} \frac{C_{n\tilde{t}}^{1-1/\gamma}}{1-1/\gamma} d\tilde{t}. \tag{3}$$

In this equation ρ is the discount rate, γ is the elasticity of intertemporal substitution and C_{nt} denotes aggregate final good consumption in country n. Agents earn income from wages w_{nt} and by investing in a risk free asset with interest rate r_{nt}. Consequently, the representative agent's intertemporal budget constraint is:

$$\dot{W}_{nt} = w_{nt}L_n + r_{nt}W_{nt} - P_{nt}C_{nt},$$

where W_{nt} denotes total assets owned by the representative agent and P_{nt} is the price index of the final good in country n. We assume there is no international borrowing or lending, implying that asset markets clear at the national level.

Variety Production. Variety production combines labor with intermediate inputs that are produced one-to-one from the final good. Producers of variety ω in country i have productivity $z_i^s(\omega)$ and produce output $y_i^s(\omega)$ given by:

$$y_i^s(\omega) = \frac{z_i^s(\omega)}{(\alpha^s)^{\alpha^s}(1-\alpha^s)^{1-\alpha^s}} l_i^s(\omega)^{\alpha^s} q_i^s(\omega)^{1-\alpha^s}, \tag{4}$$

where $l_i^s(\omega)$ and $q_i^s(\omega)$ denote the quantities of labor and intermediate inputs, respectively, used to produce variety ω in country i. Labor and intermediate inputs are purchased in competitive markets and the parameter $\alpha_s \in (0, 1)$ equals labor's share of production costs.

Following Eaton and Kortum (2002), productivity levels are drawn from a country-sector specific Fréchet distribution $F_i^s(z) = \exp\left(T_i^s z^{-\theta^s}\right)$. The scale parameter T_i^s captures variables, such as institutions and infrastructure, that affect productivity conditional on innovation and diffusion outcomes. The shape parameter $\theta^s > \sigma^s - 1$ is an inverse measure of productivity dispersion across varieties. Productivity draws are independent across varieties and also across countries within a variety.

Product varieties are tradable subject to iceberg trade costs. In order to sell one unit of output in country n, a producer in country i must ship $\tau_{ni}^s \geq 1$ units. We assume $\tau_{ii}^s = 1$ for all countries i.

Innovation and Patenting. In all sectors $s \neq 0$, new products are created by risk neutral innovators. Innovation uses labor and each worker employed in innovation in country i and sector s successfully innovates at Poisson rate $\eta_i^s \left(L_{Rit}^s\right)^{-\kappa}$. The parameter η_i^s determines the efficiency of R&D, which is country-sector specific, while L_{Rit}^s denotes total employment of R&D workers in innovation in section s and country i. We assume $\kappa \in (0, 1)$ implying that innovation is subject to a stepping-on-the-toes externality whereby the marginal productivity of R&D labor declines as the innovation sector expands (Jones 1995). Imposing $\kappa > 0$ also ensures that all countries innovate in all sectors in equilibrium.

Let Ψ_t^s be the aggregate quality of all varieties produced in sector s, defined by:

$$\Psi_t^s = \int_0^{M_t^s} \psi(\omega)\, d\omega.$$

Assume that, when innovation occurs, each invention creates Ψ_t^s new product varieties. This assumption introduces knowledge spillovers into the innovation technology and is sufficient to ensure there is balanced growth in the steady state equilibrium.

There is free entry into innovation. Let V_{it}^s be the expected value of inventing a new variety in country i and sector s at time t. The free entry condition requires that the wage rate equals the product of the probability of innovation, the number of products an invention creates and the expected value of each product. That is:

$$w_{it} = \eta_i^s \left(L_{Rit}^s\right)^{-\kappa} \Psi_t^s V_{it}^s. \tag{5}$$

Prior to invention, both product quality $\psi(\omega)$ and the productivity $z_n^s(\omega)$ with which varieties can be produced in each country are unknown. When innovation occurs, the innovator immediately learns the quality of their invention, which is drawn from a Pareto distribution $H(\psi) = 1 - \psi^{-k}$

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with shape parameter $k > 1$ and scale parameter 1. All Ψ_t^s products that compose an invention have the same quality. Before commencing production, inventors in each country i also learn the domestic productivity $z_i^s(\omega)$ with which their products can be produced.

Initially, only the innovator knows how to produce its new varieties giving them a technological monopoly over their invention. However, as technologies are non-rival and imperfectly excludable we assume that technology diffusion occurs at Poisson rate $\nu^s > 0$. Before an invention diffuses, only domestic production in the innovator's home country is possible. After diffusion, any firm in any country can produce the diffused products. Moreover, all firms produce products with the same quality and all firms in a given country have the same country-specific productivity $z_n^s(\omega)$. Consequently, diffusion strips the innovator of its technological monopoly. Innovators may also lose their monopoly due to product obsolescence. We assume each variety become obsolete at Poission rate ζ^s.

Anticipating the possibility of technology diffusion, the innovator may also secure a legal monopoly over their invention by purchasing a patent. Patents are country-specific and cover all Ψ_t^s product varieties created by an invention. We assume that an inventor who holds a country n patent has the monopoly right to sell varieties covered by the patent to country n. Once purchased, a patent expires at Poisson rate δ_n^s, where δ_n^s is an inverse measure of the strength of patent protection in country n and sector s. An increase in patent protection reduces δ_n^s.

A successful innovator must immediately decide whether to patent their invention in each country. Imposing this restriction captures the fact that innovators have a strong incentive to file a patent application as soon as possible in order to assert priority over an invention (Dechezleprêtre et al. 2017). We also assume that innovators choose whether to patent after learning the quality of their invention, but before knowing their productivity. This timing convention captures the idea that innovators are well-informed about the quality of their inventions, but initially know less about whether an invention is commercially viable and how much it will cost to produce.

The benefit of patenting for an innovator is that it extends the expected duration of their monopoly over an invention. An innovator that purchases a country n patent loses its monopoly in country n only when both the technology has diffused and the patent has expired. When choosing whether to patent, the innovator compares this benefit to the costs of patenting. In order to patent, an innovator from country i must first hire $f_i^{s,o}$ units of domestic labor to prepare their patent application by codifying the invention in terms comprehensible to patent offices. We will refer to $f_i^{s,o}$ as the patent preparation cost. Let $L_{it}^{s,o}$ denote total labor employed in the preparation of patent applications.

In addition, to purchase a patent in country n, the innovator must also hire $f_n^{s,e}$ units of country n labor. This country-specific patenting cost captures the payments needed to submit a patent application, for example application fees, agent payments, translation fees and maintenance fees.

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We will refer to $f_n^{s,e}$ as the patent application cost. Let $L_{int}^{s,e}$ denote total labor employed in country n by innovators from country i to cover patent application costs. Payments to these workers $w_{nt}L_{int}^{s,e}$ represent an export of patenting services from n to i.

To complete the specification of the model we return to sector zero. Sector zero is an Eaton and Kortum (2002) sector with no endogenous innovation or patenting. Instead, all varieties are produced competitively and the aggregate quality Ψ_t^0, which equals the mass of varieties produced M_t^0, grows exogenously at rate g^0.

2.2 Equilibrium

The model has two types of products sold in each destination: Helpman-Krugman products and Eaton-Kortum products. Varieties for which either the technology has not diffused or the inventor holds a non-expired patent are Helpman-Krugman products. These varieties are sold by a monopolist inventor who faces constant elasticity demand under monopolistic competition (Helpman and Krumgan 1987). Varieties that are not under patent protection and for which technology diffusion has occurred are Eaton-Kortum products. These varieties are produced and sold competitively as in Eaton and Kortum (2002). Because patents are country-specific, whether a variety is a Helpman-Krugman product or an Eaton-Kortum product may differ across destinations.

Before solving the model, it is useful to decompose aggregate quality Ψ_t^s by product type. Let Ψ_{Mnit}^s denote the aggregate quality of all Helpman-Krugman products sold monopolistically from country i to destination n at time t and let Ψ_{Cnt}^s be the aggregate quality of all Eaton-Kortum products sold competitively in country n at time t. Then we have:

$$\Psi_t^s = \Psi_{Cnt}^s + \sum_{i=1}^{N} \Psi_{Mnit}^s. \tag{6}$$

Note that since all products are sold to all countries, this equation holds for any destination n.

2.2.1 Static Equilibrium

A convenient feature of the model is that the equilibrium conditions can be split into a static equilibrium and a dynamic equilibrium. The static equilibrium solves for wages, output levels, prices and trade flows conditional on knowing for all i, n and s the aggregate quality of products sold competitively Ψ_{Cnt}^s and monopolistically Ψ_{Mnit}^s, total labor employed in output production L_{Yit} and total labor employed to purchase patents $L_{int}^{s,e}$. The dynamic equilibrium solves for optimal innovation and patenting decisions.

In this section we sketch the main features of the static equilibrium. A formal definition together with the full set of static equilibrium conditions can be found in Appendix A.1. Although all

variables are time dependent, to simplify notation we henceforth drop the time subscript t, except where needed to avoid confusion.

Solving the static equilibrium requires decomposing production and trade into Helpman-Krugman products sold monopolistically and Eaton-Kortum products sold competitively. Start by considering Helpman-Krugman varieties. Monopoly producers face constant elasticity demand with demand elasticity σ^s. Consequently, they charge a mark-up $\sigma^s / (\sigma^s - 1)$ above their marginal cost of serving a market. Using the production function (4) to solve for marginal cost and recalling that exports are subject to iceberg trade costs τ_{ni}^s, it follows that the price $p_{ni}^s (\omega)$ of a Helpman-Krugman variety ω produced in i and sold in n satisfies:

$$p_{ni}^s (\omega) = \frac{\sigma^s}{\sigma^s - 1} \frac{\tau_{ni}^s w_i^{\alpha^s} P_i^{1-\alpha^s}}{z_i^s (\omega)}.$$

Therefore, the monopolists profits per variety $\pi_{ni}^s (\omega)$ are given by:

$$\pi_{ni}^s (\omega) = \psi (\omega) \frac{(\sigma^s - 1)^{\sigma^s - 1}}{(\sigma^s)^{\sigma^s}} \left(\frac{\tau_{ni}^s w_i^{\alpha^s} P_i^{1-\alpha^s}}{z_i^s (\omega)} \right)^{1-\sigma^s} (P_n^s)^{\sigma^s} Y_n^s. \tag{7}$$

Since $z_i^s (\omega)$ is drawn after the patenting decision, the distribution of productivity z (but not quality ψ) is independent of whether varieties are patented. This allows us to aggregate prices across source countries and varieties to derive a subprice index P_{Mn}^s for Helpman-Krugman products sold in country n:

$$P_{Mn}^s = \left(\frac{\sigma^s}{\sigma^s - 1} \right) \left[\Gamma \left(\frac{\theta^s + 1 - \sigma^s}{\theta^s} \right) \right]^{\frac{1}{1-\sigma^s}} \left(\sum_{j=1}^{N} \Psi_{Mnj}^s \left(\Phi_{nj}^s \right)^{\frac{\sigma^s - 1}{\theta^s}} \right)^{\frac{1}{1-\sigma^s}}, \tag{8}$$

where $\Gamma (\cdot)$ is the Gamma function and:

$$\Phi_{ni}^s \equiv T_i \left(\tau_{ni}^s w_i^{\alpha^s} P_i^{1-\alpha^s} \right)^{-\theta^s}, \tag{9}$$

gives the supply potential of country i in country n, which is an inverse measure of the average cost of producing for country n in country i.

Aggregation also yields that the value of exports X_{Mni}^s of Helpman-Krugman products from i to n is given by:

$$X_{Mni}^s = \frac{\Psi_{Mni}^s \left(\Phi_{ni}^s \right)^{\frac{\sigma^s - 1}{\theta^s}}}{\sum_{j=1}^{N} \Psi_{Mnj}^s \left(\Phi_{nj}^s \right)^{\frac{\sigma^s - 1}{\theta^s}}} \left(\frac{P_{Mn}^s}{P_n^s} \right)^{1-\sigma^s} P_n^s Y_n^s. \tag{10}$$

We see that Helpman-Krugman trade is increasing in both the aggregate quality Ψ_{Mni}^s of products sold monopolistically from i to n and the supply potential Φ_{ni}^s of i in n. Substituting equation

(9) into equation (10) also implies that the elasticity of Helpman-Krugman trade to trade costs τ_{ni}^s equals $1 - \sigma^s$.

Now, consider Eaton-Kortum varieties. As in Eaton and Kortum (2002), each variety is sourced from the lowest cost supplier. Consequently, the subprice index P_{Cn}^s for Eaton-Kortum products sold in country n is:

$$ P_{Cn}^s = \left[\Gamma \left(\frac{\theta^s + 1 - \sigma^s}{\theta^s} \right) \Psi_{Cn}^s \right]^{\frac{1}{1-\sigma^s}} \left(\sum_{j=1}^{N} \Phi_{nj}^s \right)^{-\frac{1}{\theta^s}}, \tag{11} $$

and exports X_{Cni}^s of Eaton-Kortum products from i to n satisfy:

$$ X_{Cni}^s = \frac{\Phi_{ni}^s}{\sum\limits_{j=1}^{N} \Phi_{nj}^s} \left(\frac{P_{Cn}^s}{P_n^s} \right)^{1-\sigma^s} P_n^s Y_n^s. \tag{12} $$

It follows that the elasticity of Eaton-Kortum trade to τ_{ni}^s is given by the Fréchet dispersion parameter $-\theta^s$.

Using equations (8)–(12), the remaining static equilibrium conditions can be obtained by aggregating Helpman-Krugman with Eaton-Kortum products in each sector to obtain price indices and trade flows and then imposing output market clearing and trade balance conditions (see Appendix A.1 for details).

2.2.2 Dynamic Equilibrium

The dynamic equilibrium solves for R&D investment levels, patenting decisions and how the aggregate quality of each product type changes over time.

Value of firms and patenting decisions. Consider a firm in country i and sector s that creates an invention with quality ψ at time t_0. Let $V_{nit_0}^{s,NP}(\psi)$ denote the expected present discounted value of profits per variety that the firm makes from sales in destination n if it chooses not to patent in n. A firm that does not patent loses its monopoly when its technology either diffuses (at rate ν^s) or becomes obsolete (at rate ζ^s). Recalling that firms make patenting decisions before learning their productivity, we have:

$$ V_{nit_0}^{s,NP}(\psi) = \int_{t_0}^{\infty} \mathbb{E}_z \pi_{ni}^s(\psi, z) \exp\left(-\int_{t_0}^{t} (r_i + \zeta^s + \nu^s) \, d\widetilde{t} \right) dt, \tag{13} $$

where $\mathbb{E}_z \pi_{ni}^s(\psi, z)$ denotes expected profits computed over the distribution of productivity z.

By contrast, a firm that patents in n loses its monopoly only when its technology becomes obsolete, or both its patent has expired and its technology has diffused. For a product invented at t_0, the probability that both diffusion and patent expiration occur before t is $\left[1 - e^{-\nu^s(t-t_0)} \right] \left[1 - e^{-\delta_n^s(t-t_0)} \right]$.

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Therefore, the expected present discounted value of profits per variety in destination n conditional on patenting $V_{nit_0}^{s,P}(\psi)$ satisfies:

$$V_{nit_0}^{s,P}(\psi) = \int_{t_0}^{\infty} \mathbb{E}_z \pi_{ni}^s(\psi, z) \exp\left(-\int_{t_0}^{t}(r_i + \zeta^s + \nu^s)\, d\tilde{t}\right)$$
$$\times \left\{1 - \exp\left[-\delta_n^s(t - t_0)\right] + \exp\left[(\nu^s - \delta_n^s)(t - t_0)\right]\right\} dt. \tag{14}$$

We can now solve for expected profits by noting from equation (7) that profits per variety are proportional to the monopolist's quality ψ and equal to a fraction $1/\sigma^s$ of revenue. Therefore, aggregate profits made by monopolists in i from sales to n are given by X_{Mni}^s/σ^s and expected profits are:

$$\mathbb{E}_z \pi_{ni}^s(\psi, z) = \psi \mathbb{E}_z \pi_{ni}^s(1, z) = \psi \frac{X_{Mni}^s}{\sigma^s \Psi_{Mni}^s}. \tag{15}$$

Substituting this expression into equations (13) and (14) implies that the value functions are proportional to quality ψ, i.e. $V_{nit_0}^{s,J}(\psi) = \psi V_{nit_0}^{s,J}(1)$ for $J = NP, P$.

After paying the patent preparation cost $w_i f_i^{s,o}$, a firm patents in country n if the difference between $V_{nit_0}^{s,P}(\psi)$ and $V_{nit_0}^{s,NP}(\psi)$ exceeds the patent application cost per variety. Since each invention comprises Ψ^s varieties, it follows that the firm patents in n if at the time of application t_0:

$$\Psi^s \left[V_{nit_0}^{s,P}(\psi) - V_{nit_0}^{s,NP}(\psi)\right] \geq w_n f_n^{s,e}.$$

Because the value functions are proportional to ψ, this inequality defines a quality threshold $\psi_{ni}^{s,e*}$ such that only firms with quality ψ above the threshold opt to patent in n (conditional on having paid the application preparation cost). Rearranging the expression above and remembering that ψ is drawn from a distribution with lower bound one, we have:

$$\psi_{ni}^{s,e*} = \max\left(\frac{w_n f_n^{s,e}}{\Psi^s \left[V_{nit_0}^{s,P}(1) - V_{nit_0}^{s,NP}(1)\right]}, 1\right). \tag{16}$$

Next, we need to determine which firms pay the patent preparation cost $w_i f_i^{s,o}$. Appendix A.2 shows that there exists a second quality threshold $\psi_i^{s,o*}$ such that only firms with quality above this threshold pay the preparation cost. It follows that firms from country i with quality below $\psi_i^{s,o*}$ do not patent anywhere and that firms patent in country n if and only if $\psi \geq \psi_{ni}^{s,*}$ where the patenting threshold $\psi_{ni}^{s,*}$ is defined by:

$$\psi_{ni}^{s*} = \max\left(\psi_{ni}^{s,e*}, \psi_i^{s,o*}\right). \tag{17}$$

Patenting increases the expected duration of an innovator's monopoly over their varieties. A longer monopoly is more valuable to higher quality firms since expected profits are proportional to quality by equation (15). Consequently, the benefits exceed the fixed costs of patenting only for firms with quality above the patenting thresholds defined in equation (17).

The expected value V_{it}^s of inventing a new variety at t equals the expected present discounted value of profits in all markets less expected patenting costs. Using the optimal patenting thresholds, summing across destinations and taking expectations over the quality distribution, Appendix A.2 shows that V_{it}^s is given by:

$$V_{it}^s = \sum_{n=1}^{N} \left\{ \frac{k}{k-1} \left[V_{nit}^{s,NP}(1)\left(1 - (\psi_{ni}^{s*})^{-k+1}\right) + V_{nit}^{s,P}(1)(\psi_{ni}^{s*})^{-k+1} \right] - (\psi_{ni}^{s*})^{-k} \frac{w_n f_n^{s,e}}{\Psi^s} \right\}$$
$$- (\psi_i^{s,o*})^{-k} \frac{w_i f_i^{s,o}}{\Psi^s}. \tag{18}$$

Laws of motion for aggregate qualities. We can decompose the aggregate quality Ψ_{Mni}^s of Helpman-Krugman products sold from i to n into the aggregate quality of products that are not patented $\Psi_{Mni}^{s,NP}$, the aggregate quality of products that are patented but whose technology has not diffused $\Psi_{Mni}^{s,P,ND}$ and the aggregate quality of products that are patented and whose technology has diffused $\Psi_{Mni}^{s,P,D}$. We have:

$$\Psi_{Mni}^s = \Psi_{Mni}^{s,NP} + \Psi_{Mni}^{s,P,ND} + \Psi_{Mni}^{s,P,D}. \tag{19}$$

Together with the aggregate quality of Eaton-Kortum products Ψ_{Cn}^s, these aggregate qualities compose the state variables of the economy. To solve for the dynamic equilibrium, we need to characterize how they evolve over time.

The law of motion for the aggregate quality of Helpman-Krugman products that are not patented $\Psi_{Mni}^{s,P,ND}$ is given by:

$$\dot{\Psi}_{Mni}^{s,NP} = \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \Psi^s \frac{k}{k-1} \left[1 - (\psi_{ni}^{s*})^{1-k}\right] + \delta_n^s \Psi_{Mni}^{s,P,ND} - (\nu^s + \zeta^s)\Psi_{Mni}^{s,NP}. \tag{20}$$

The first term on the right hand side of this expression gives the aggregate quality of new goods invented in country i and sector s that are not patented in country n. There are L_{Ri}^s R&D workers each of whom innovates at rate $\eta_i^s \left(L_{Ri}^s\right)^{-\kappa}$ and each innovation produces Ψ^s new varieties. Innovations with quality below ψ_{ni}^{s*} are not patented in country n, implying that a unit mass of innovations contributes aggregate quality $\int_1^{\psi_{ni}^{s*}} \psi \, dH(\psi) = \frac{k}{k-1}\left[1 - (\psi_{ni}^{s*})^{1-k}\right]$ to $\Psi_{Mni}^{s,P,ND}$. Combining these observations yields the first term. The second term gives the increase in $\Psi_{Mni}^{s,P,ND}$ due to patent expiration among Helpman-Krugman varieties whose technology has not diffused. And the third

term captures the decline in $\Psi_{Mni}^{s,P,ND}$ due to technology diffusion and product obsolescence.

Analogous reasoning gives the laws of motion for the other state variables. Helpman-Krugman products that are patented, but whose technology has not diffused are generated by patenting and lost due to by patent expiration, technology diffusion and product obsolescence, which yields:

$$\dot{\Psi}_{Mni}^{s,P,ND} = \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \left(\Psi^s\right) \frac{k}{k-1} \left(\psi_{ni}^{s*}\right)^{1-k} - \left(\delta_n^s + \nu^s + \zeta^s\right) \Psi_{Mni}^{s,P,ND}. \tag{21}$$

Helpman-Krugman products that are patented and whose technology has diffused are generated by technology diffusion and lost due to by patent expiration and product obsolescence, implying:

$$\dot{\Psi}_{Mni}^{s,P,D} = \nu^s \Psi_{Mni}^{s,P,ND} - \left(\delta_n^s + \zeta^s\right) \Psi_{Mni}^{s,P,D}. \tag{22}$$

Finally, Eaton-Kortum products are generated by either technology diffusion among not patented products or patent expiration among products whose technology has already diffused. And Eaton-Kortum products are destroyed by product obsolescence. Therefore:

$$\dot{\Psi}_{Cn}^s = \sum_{i=1}^N \left(\nu^s \Psi_{Mni}^{s,NP} + \delta_n^s \Psi_{Mni}^{s,P,D}\right) - \zeta^s \Psi_{Cn}^s. \tag{23}$$

Combining these laws of motion with the patenting thresholds and firm value functions derived above and imposing labor market clearing gives the dynamic equilibrium, which is formally defined in Appendix A.2.

Let g^s denote the growth rate of aggregate quality Ψ^s in sector s. Using equations (6) and (19) to decompose the growth rate of Ψ^s in terms of the growth rates of Eaton-Kortum and Helpman-Krugman products and then combining equations (20)–(23) implies that in any dynamic equilibrium:

$$g^s = \sum_{i=1}^N \frac{k}{k-1} \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} - \zeta^s, \qquad \text{for } s \neq 0. \tag{24}$$

This expression shows how sector-level growth in aggregate quality depends upon R&D employment in all N countries. The first term on the right hand side captures the contribution of innovation to growth. Innovations occur at rate $\eta_i^s \left(L_{Ri}^s\right)^{1-\kappa}$ in country i and $k/\left(k-1\right)$ is the average quality of an innovation. The second term captures the decline in aggregate quality due to product obsolescence.

2.3 Steady State

We define a steady state of the global economy as a balanced growth path equilibrium in which all aggregate and industry-level variables grow at constant rates. This section describes the main features of a steady state, while Appendix A.3 provides further details.

Knowledge spillovers in the innovation and patenting technologies are global in scope. Consequently, steady state growth rates do not vary by country. Steady state also requires that the aggregate qualities of Eaton-Kortum products and of each type of Helpman-Krugman products in sector s grow at rate g^s. Let g be the growth rate of final consumption C_i in any country i. Wages w_i, final good output Y_i and trade flows X_{ni}^s also grow at rate g, which is given by:

$$g = \frac{1}{\sum_{s=0}^{S} \beta^s \alpha^s} \sum_{s=0}^{S} \frac{\beta^s}{\sigma^s - 1} g^s, \tag{25}$$

Thus, in steady state, growth results from increases in the aggregate quality of varieties produced in each sector. In turn, growth in aggregate quality results from innovation as shown by equation (24).

Computing the integrals in equation (13) gives that the steady state value of a variety with quality one that is not patented satisfies:

$$V_{nit}^{s,NP}(1) = R^{s,NP} \mathbb{E}_z \pi_{nit}^s(1, z) \text{ where } R^{s,NP} \equiv \frac{1}{r + \zeta^s + \nu^s - g + g^s}. \tag{26}$$

The term $R^{s,NP}$ in this expression captures the expected value that a firm that does not patent obtains from future profit flows. It is the inverse of the firm's effective discount rate and is decreasing in the interest rate r, the product obsolescence rate ζ^s and the technology diffusion rate ν^s, but increasing in the growth rate of profits $g - g^s$.

Similarly, equation (14) implies that the value of a patented variety is:

$$V_{nit}^{s,P}(1) = R_n^{s,P} \mathbb{E}_z \pi_{nit}^s(1, z), \tag{27}$$

where $R_n^{s,P} = R^{s,NP} + \Delta R_n^s$ and:

$$\Delta R_n^s \equiv \frac{1}{r + \zeta^s + \delta_n^s - g + g^s} - \frac{1}{r + \zeta^s + \nu^s + \delta_n^s - g + g^s} > 0.$$

Patenting reduces the firm's effective discount rate by extending the expected duration of its monopoly. Consequently, its valuation of future profit flows increases by ΔR_n^s. We will refer to ΔR_n^s as the benefit of patenting in n. Stronger patent protection increases ΔR_n^s by reducing δ_n^s. The benefit of patenting is also increasing in the rate of technology diffusion ν^s, implying that patenting is complementary to technology diffusion. The complementarity arises because the

probability that patent protection is needed to maintain the firm's monopoly is greater when technology diffusion is faster. Indeed, if $\nu^s = 0$, meaning that there is no technology diffusion, then $\Delta R_n^s = 0$ and firms have no incentive to patent. Intuitively, ΔR_n^s is also decreasing in r and ζ^s, but increasing in the growth rate of profits $g - g^s$.

To characterize the steady state equilibrium it is convenient to detrend all variables. Detrending yields normalized variables that are constant in steady state, which we denote using tildes. We normalize variables that grow at rate g^s by writing them relative to Ψ^s and normalize variables that grow at rate g by writing them relative to Ψ defined by:

$$\Psi \equiv \left[\prod_{s=0}^{S} (\Psi^s)^{\frac{\beta^s}{\sigma^s - 1}} \right]^{\frac{1}{\sum_{s=0}^{S} \beta^s \alpha^s}}.$$

Thus, $\widetilde{\Psi}_{Cn}^s = \Psi_{Cn}^s / \Psi^s$ and $\widetilde{w}_i = w_i / \Psi$, for example. Likewise, we normalize variables that grow at rate $g - g^s$, such as profits and value functions, by writing them relative to Ψ / Ψ^s. In particular, we define normalized expected profits as:

$$\widetilde{\pi}_{ni}^s \equiv \Psi^s \frac{\mathbb{E}_z \pi_{ni}^s (1, z)}{\Psi} = \frac{\widetilde{X}_{Mni}^s}{\sigma^s \widetilde{\Psi}_{Mni}^s}, \tag{28}$$

where the equality uses equation (15). Using equations (26) and (27) to substitute for $V_{nit}^{s,NP}(1)$ and $V_{nit}^{s,P}(1)$ in equation (16), we now obtain that the patenting threshold $\psi_{ni}^{s,e*}$ satisfies:

$$\psi_{ni}^{s,e*} = \max \left(\frac{\widetilde{w}_n f_n^{s,e}}{\Delta R_n^s \widetilde{\pi}_{ni}^s}, 1 \right). \tag{29}$$

The (interior) patenting threshold depends upon the cost of patenting in n, the benefit of patenting in n and the profitability of market n. A higher patenting cost $\widetilde{w}_n f_n^{s,e}$ increases the patenting threshold. By contrast, an increase in either the benefit of patenting ΔR_n^s or normalized profits $\widetilde{\pi}_{ni}^s$ reduces the patenting threshold. Appendix A.3 shows that the patent preparation threshold $\psi_i^{s,o*}$ satisfies a similar expression, but accounting for the option value of patenting in all destinations.

Using equation (18), we can also write the normalized expected value of inventing a new variety $\widetilde{V}_i^s$ as:

$$\widetilde{V}_i^s = \sum_{n=1}^{N} \left[\frac{k}{k-1} \widetilde{\pi}_{ni}^s \left(R^{s,NP} + \Delta R_n^s (\psi_{ni}^{s*})^{1-k} \right) - \widetilde{w}_n f_n^{s,e} (\psi_{ni}^{s*})^{-k} \right] - \widetilde{w}_i f_i^{s,o} (\psi_i^{s,o*})^{-k}. \tag{30}$$

The expected value of invention comprises four terms. The first term gives the expected value if there is no patenting. The second term captures the additional value that arises because firms have the opportunity to patent their inventions. The value that patenting creates is increasing in

profitability $\widetilde{\pi}_{ni}^s$ and in the benefit of patenting ΔR_n^s, but decreasing in the patenting threshold ψ_{ni}^{s*}. The final two terms in equation (30) give the expected patenting costs a firm pays.

Free entry into innovation (5) implies:

$$(L_{Ri}^s)^\kappa = \eta_i^s \frac{\widetilde{V}_i^s}{\widetilde{w}_i}. \tag{31}$$

Together with equation (30), this expression determines the allocation of labor to R&D and, therefore, the sectoral growth rate g^s by equation (24).

Finally, equation (3) implies that steady state welfare is given by:

$$U_{nt} = \frac{\Psi_t^{1-1/\gamma}}{1-1/\gamma} \frac{\widetilde{C}_n^{1-1/\gamma}}{\rho - g\left(1 - \frac{1}{\gamma}\right)}. \tag{32}$$

Conditional on the initial value Ψ_t, an increase in either the normalized consumption level $\widetilde{C}_n$ or the growth rate g raises steady state welfare in country n. The trade-off between static costs and dynamic benefits of patent protection arises when stronger protection raises growth g, but reduces consumption $\widetilde{C}_n$.

2.4 Understanding the Model

Before calibrating the model, it is useful to develop more intuition about how patent protection affects the steady state equilibrium. Therefore, in this section we characterize the direct effects of changes in the strength of patent protection δ_n^s in country n on steady state outcomes in all countries, without allowing for general equilibrium adjustments.

Suppose country n increases the strength of its patent protection by reducing δ_n^s. Equation (32) shows that changes in steady state welfare can be decomposed into a static effect on normalized consumption and a dynamic effect on growth. We start by analyzing the static effect. The direct impact of stronger patent protection on consumption levels operates through changes in the shares of aggregate quality accounted for by competitive Eaton-Kortum versus monopolistic Helpman-Krugman products. Using the laws of motion for aggregate qualities we obtain:

$$\widetilde{\Psi}_{Mni}^s = \frac{k}{k-1} \eta_i^s (L_{Ri}^s)^{1-\kappa} \left[\frac{1}{g^s + \nu^s + \zeta^s} + \frac{(\psi_{ni}^{s*})^{1-k}}{g^s + \delta_n^s + \zeta^s} \frac{\nu^s}{g^s + \delta_n^s + \nu^s + \zeta^s} \right], \tag{33}$$

implying that stronger patent protection directly increases the share of aggregate quality sold in country n and sector s that is supplied monopolistically by country i, $\widetilde{\Psi}_{Mni}^s$. And since the equation above holds for any i, stronger patent protection directly reduces the share of aggregate quality sold competitively in country n, $\widetilde{\Psi}_{Cn}^s = 1 - \sum_{i=1}^N \widetilde{\Psi}_{Mni}^s$. Thus, stronger patent protection increases the

average monopoly power of suppliers to country n.

Monopoly power creates two distortions that raise prices in country n: a mark-up distortion and a sourcing distortion. The mark-up distortion arises because monopolists set a mark-up $\sigma^s/(\sigma^s - 1)$ above their marginal costs. The sourcing distortion arises because country n sources Eaton-Kortum products from its lowest cost supplier, whereas Helpman-Krugman products can only be sourced from the monopolist's home country. Differentiating the sectoral price index with respect to $\widetilde{\Psi}^s_{Mni}$ and accounting for the decline in $\widetilde{\Psi}^s_{Cn}$ yields:

$$
\frac{\partial \widetilde{P}^s_n}{\partial \widetilde{\Psi}^s_{Mni}} \propto 1 - \underbrace{\left(\frac{\sigma^s}{\sigma^s - 1}\right)^{1-\sigma^s}}_{\text{Mark-up distortion}} \underbrace{\frac{\left(\widetilde{\Phi}^s_{ni}\right)^{\frac{\sigma^s-1}{\theta^s}}}{\left(\sum_{j=1}^{N} \widetilde{\Phi}^s_{nj}\right)^{\frac{\sigma^s-1}{\theta^s}}}}_{\text{Sourcing distortion}} . \tag{34}
$$

It follows that stronger patent protection in country n has a direct positive effect on P^s_n because it raises $\widetilde{\Psi}^s_{Mni}$. In addition, we see that the price increase is greater when the mark-up is higher (i.e. σ^s is lower) and when the supply potential of country i in country n given by $\widetilde{\Phi}^s_{ni} = T_i \left(\tau^s_{ni} \widetilde{w}^{\alpha^s}_i P^{1-\alpha^s}_i\right)^{-\theta^s}$ is low relative to other countries.

Monopoly power also generates profits for innovators. Aggregate profits made by innovators from i in country n in sector s satisfy:

$$
\frac{\widetilde{X}^s_{Mni}}{\sigma^s} = \Gamma\left(\frac{\theta^s + 1 - \sigma^s}{\theta^s}\right) \frac{(\sigma^s - 1)^{\sigma^s-1}}{(\sigma^s)^{\sigma^s}} \beta^s \widetilde{\Psi}^s_{Mni} \widetilde{\Phi}^s_{ni} \left(\widetilde{P}^s_n\right)^{\sigma^s-1} P_n \widetilde{Y}_n.
$$

This expression implies that the direct effect of stronger patent protection in n is to increase the profits that all countries make in n by raising both $\widetilde{\Psi}^s_{Mni}$ and $\widetilde{P}^s_n$. The increase in profits is greater when country n has higher final good expenditure $P_n \widetilde{Y}_n$.

The level of normalized consumption $\widetilde{C}_i$ in each country is affected by both price distortions and profit levels. Manipulating the static equilibrium trade balance and market clearing conditions yields:

$$
\widetilde{C}_i = \frac{\widetilde{w}_i}{P_i}\left(L_{Yi} + \sum_{s=1}^{S}\sum_{n=1}^{N} L^{s,e}_{ni} - \sum_{s=1}^{S}\sum_{n=1}^{N} \frac{\widetilde{w}_n}{\widetilde{w}_i} L^{s,e}_{in}\right) + \frac{\Pi_i}{P_i} - TB_i \sum_{n=1}^{N} \frac{P_n}{P_i} \widetilde{Y}_n,
$$

where $\Pi_i \equiv \sum_{s=1}^{S}\sum_{n=1}^{N} \widetilde{X}^s_{Mni}/\sigma^s$ denotes aggregate profits made by country i. This expression gives consumption as a function of the real wage $\widetilde{w}_i/P_i$, real profits Π_i/P_i and trade imbalances. The real wage can in turn be written as:

$$\frac{\widetilde{w}_i}{P_i} = \left\{ \prod_{s=0}^{S} \left[\Gamma \left(\frac{\theta^s + 1 - \sigma^s}{\theta^s} \right) \right]^{\frac{\beta^s}{\sigma^s - 1}} \left(\frac{T_i}{\lambda_{Cii}^s} \right)^{\frac{\beta^s}{\theta^s}} \left(\frac{\widetilde{\Psi}_{Ci}^s}{\mu_{Ci}^s} \right)^{\frac{\beta^s}{\sigma^s - 1}} \right\}^{\frac{1}{\sum_{s=0}^{S} \alpha^s \beta^s}}. \tag{35}$$

In this equation $\lambda_{Cii}^s \equiv \widetilde{X}_{Cii}^s / \sum_{j=1}^{N} \widetilde{X}_{Cij}^s = \widetilde{\Phi}_{Cii}^s / \sum_{j=1}^{N} \widetilde{\Phi}_{Cij}^s$ denotes the domestic share of expenditure on Eaton-Kortum products in sector s and country i, while μ_{Ci}^s denotes the expenditure share of Eaton-Kortum products in sector s and country i. When all products are sold competitively $\mu_{Ci}^s = 1$ and equation (35) reduces to a multi-sector version of the Arkolakis et al. (2012) formulation of the gains from trade.

But in our model, real wages depend not only upon the domestic trade share for competitive products λ_{Cii}^s, but also upon the ratio of the share of aggregate quality supplied competitively $\widetilde{\Psi}_{Ci}^s$ to the expenditure share of competitive products μ_{Ci}^s. Because of the pricing distortions for monopolistic products this ratio is less than one, meaning that expenditure on Eaton-Kortum products exceeds their share of aggregate quality and that real wages are lower than when all products are supplied competitively. In fact, we have:

$$\frac{\widetilde{\Psi}_{Ci}^s}{\mu_{Ci}^s} = 1 - \sum_{j=1}^{N} \widetilde{\Psi}_{Mij}^s \left[1 - \left(\frac{\sigma^s}{\sigma^s - 1} \right)^{1 - \sigma^s} \frac{\left(\widetilde{\Phi}_{ij}^s \right)^{\frac{\sigma^s - 1}{\theta^s}}}{\left(\sum_{k=1}^{N} \widetilde{\Phi}_{ik}^s \right)^{\frac{\sigma^s - 1}{\theta^s}}} \right], \tag{36}$$

implying that real wages are decreasing in the share of aggregate quality supplied monopolistically by each exporter j, $\widetilde{\Psi}_{Mij}^s$. Moreover, comparing this expression with equation (34) shows that the impact of an increase in $\widetilde{\Psi}_{Mij}^s$ on real wages is greater when the pricing distortions are larger. Equation (36) formalizes how expanding the share of quality supplied monopolistically increases the static inefficiencies in this economy.

Putting everything above together, we can now characterize the direct effect of a reduction in δ_n^s on steady state consumption levels $\widetilde{C}_i$ in each country i. For countries $i \neq n$, the only direct effect is a rise in profits that occurs because of an increase in the share of aggregate quality supplied monopolistically by country i, $\widetilde{\Psi}_{Mni}^s$. This means that stronger domestic patent protection generates positive direct spillovers to foreign countries by giving their suppliers greater market power. However, in country n itself higher profits are offset by a decline in real wages caused by an increase in the share of aggregate quality supplied monopolistically. Thus, the direct cost of the static pricing distortions generated by increased monopoly power is borne by country n.

Next, we turn to the dynamic effect of stronger patent protection on growth. To understand this effect, we use a version of the model where $f_i^{s,0} = 0$, meaning that there are no patent preparation costs, which implies $\psi_{ni}^{s*} = \psi_{ni}^{s,e*}$. With this simplification, we can write the value of inventing a variety as $\widetilde{V}_i^s = \sum_{n=1}^{N} \widetilde{V}_{ni}^s$ where $\widetilde{V}_{ni}^s$ denotes the value that comes from supplying destination

n. Assuming that all patenting thresholds are interior and substituting equation (29) into equation (30) yields:

$$\widetilde{V}_{ni}^s = \frac{k}{k-1}\widetilde{\pi}_{ni}^s \left(R^{s,NP} + \frac{\Delta R_n^s \left(\psi_{ni}^{s*}\right)^{1-k}}{k} \right),$$

showing that destination n is more valuable when it generates higher expected profits $\widetilde{\pi}_{ni}^s$, when the benefits of patenting ΔR_n^s in n are greater and when the threshold ψ_{ni}^{s*} for patenting in n is lower.

Taking the partial derivative of equations (24), (29), (30) and (31) with respect to δ_n^s while holding $\widetilde{w}_i$, $\widetilde{\pi}_{ni}^s$, $R^{s,NP}$ constant gives that the direct effect of stronger patent protection on the sectoral growth rate g^s satisfies:

$$\frac{\partial \ln g^s}{\partial \ln \delta_n^s} = \frac{\partial \ln \Delta R_n^s}{\partial \ln \delta_n^s} k \frac{1-\kappa}{\kappa} \sum_{i=1}^N \underbrace{\frac{\frac{k}{k-1}\eta_i^s \left(L_{Ri}^s\right)^{1-\kappa}}{g^s}}_{\substack{\text{Contribution of} \\ i \text{ to global} \\ \text{innovation}}} \underbrace{\frac{\widetilde{V}_{ni}^s}{\widetilde{V}_i^s}}_{\substack{\text{Contribution} \\ \text{of } n \text{ to value} \\ \text{of innovation in } i}} \underbrace{\frac{\Delta R_n^s \left(\psi_{ni}^{s*}\right)^{1-k}}{kR^{s,NP} + \Delta R_n^s \left(\psi_{ni}^{s*}\right)^{1-k}}}_{\substack{\text{Contribution of} \\ \text{patent protection} \\ \text{to value of supplying } n}}.$$

(37)

The growth elasticity is negative because ΔR_n^s is decreasing in δ_n^s as discussed above. Therefore, stronger patent protection has a positive direct effect on growth. Intuitively, this occurs because stronger protection raises the returns to innovation by extending the expected duration of an innovator's monopoly.

The effect of patent protection in country n on growth is greater when destination n accounts for a larger share of the value of innovating $\widetilde{V}_i^s$ in more innovative countries, i.e. when $\widetilde{V}_{ni}^s/\widetilde{V}_i^s$ is higher for countries where $\eta_i^s \left(L_{Ri}^s\right)^{1-\kappa}$ is large. It follows that growth is more sensitive to the strength of patent protection in larger, more profitable markets. In addition, because of home bias in trade we expect domestic sales to make the biggest contribution to the value of innovating in each country. Therefore, all else equal, growth is more sensitive to δ_n^s when country n contributes a greater share of global innovation. Finally, we note that whereas the static costs of patent protection are domestic in scope, the dynamic benefits are global because all countries have the same steady growth rate. This contrast generates an incentive for countries to choose weaker patent protection than is globally optimal when acting unilaterally. These implications of equation (37) will play an important role in our quantitative results.

3 Model Calibration

We calibrate the model's steady state to fit the world economy in 2005. The calibrated model has two sectors, i.e. S = 1, one sector with patenting and one without. We map the Manufacturing, Information and Professional, scientific and technical services industries to the patenting sector. These industries accounted for 93 percent of US patent applications in 2008 (NSF 2013) while producing 31 percent of gross output (BEA 2022). All other industries are mapped to the no patenting sector.

Countries are aggregated into $N = 7$ economies: US, Europe, Japan, China, Brazil, India and the rest of the world. Europe comprises the 32 countries that are members of the European Patent Office and are also included in the OECD's Input-Output Tables (OECD 2021). The six economies not included in the rest of the world account for 78 percent of global GDP and 84 percent of global R&D expenditure (World Bank 2023).

3.1 Data

We obtain data on patent applications from PATSTAT (2022) and WIPO (2023). We use PATSTAT to group applications into patent families that cover the same invention. We also obtain the origin country for each patent family and the destination of each application. This allows us to measure the flow of applications at the patent family level between each pair of countries in our sample. Appendix B.1 provides further details about how we measure patent flows.

Our data shows that international patent flows in 2005 are highly concentrated among richer economies. US, Europe and Japan together are the origin of 77 percent of cross-border flows and the destination for 64 percent of flows. Developing economies account for a small share of international applications by origin, but play a larger role as destinations. China, Brazil and India together are the origin of only 2 percent of cross-border flows, but the destination for 33 percent of flows.

Data on trade, output, expenditure and intermediate input costs are from the OECD's Input-Output Tables 2021 (OECD 2021). Country-level GDP, working age population, R&D expenditure, price level data and GDP deflator data are from the World Development Indicators (World Bank 2023). Innovation statistics for US firms and sectors are from the National Science Foundation (NSF 2005, 2013). And we obtain sectoral price index and gross output data for the US from the Bureau of Economic Analysis (BEA 2022).

3.2 Calibration

To calibrate the model we set some parameters equal to values from the prior literature, choose others to exactly match selected data moments and then jointly calibrate the remaining parameters using simulated method of moments estimation. This section describes the moments that we use and how we implement the calibration. Appendix B.2 provides further details on how we measure moments in the data. Appendix B.3 explains how we simulate moments in the model.

The parameters ρ, γ, κ, σ^1 and θ^0 are chosen based on previous work. Drawing on Acemoglu et al. (2018), we let the discount rate $\rho = 0.02$, the intertemporal elasticity of substitution $\gamma = 0.5$ and the concavity of the innovation technology $\kappa = 0.5$. This value for κ is consistent with evidence that the elasticity of R&D expenditure to R&D costs, which in our model equates to $(1-\kappa)/\kappa$, is around one (Bloom et al. 2019). Using industry-level mark-up estimates from Hall (2018), we set $\sigma^1 = 2.7$, which implies that the mark-up ratio in the patenting sector $\sigma^1/(\sigma^1 - 1) = 1.59$. This value is similar to De Loecker et al.'s (2020) estimate of the median mark-up ratio in US manufacturing in 2005. We also set $\theta^0 = 5$, which implies that the trade elasticity equals five in the no patenting sector (Head and Mayer 2014).

We use exact moment matching to infer TB_i, β^s, α^s, τ^s_{ni} and L_i. Trade imbalances TB_i are measured relative to world output. Expenditure shares are set to each sector's share of world output, which gives $\beta^0 = 0.63$. We calibrate labor's share of production costs to equal the ratio of value-added to output by sector, which implies $\alpha^0 = 0.64$ and $\alpha^1 = 0.41$. Trade costs τ^s_{ni} are chosen such that the equilibrium trade flows exactly match observed trade shares X^s_{ni}/X^s_{nn} in the input-output tables. Finally, the population of each country L_i is set equal to its working age population.

This leaves $4NS + N(S+1) + 3S + 2 = 47$ parameters: δ^1_n, $f^{1,e}_n$, $f^{1,o}_i$, η^1_i, T^s_i, ν^1, ζ^1, θ^1, k and g^0. We calibrate these parameters using moments that capture information on patenting choices, the value and costs of patenting, R&D expenditure, growth, the trade elasticity, prices and production. To simplify notation, we henceforth drop the one superscript from parameters that only apply to the patenting sector, e.g. δ^1_n becomes δ_n.

From the patent data we observe the patent flow PAT_{ni} from i to n defined as the number of applications belonging to distinct patent families in destination n by applicants from country i in 2005. For each pair of economies with $n \neq i$, we target international patent shares defined as the ratio of PAT_{ni} to total international patents $\sum_{n=1,n\neq i}^{N} \sum_{i=1}^{N} PAT_{ni}$. International patent shares are informative about relative innovation levels in each origin country and the relative strength of patent protection δ_n in each destination.

Firm-level surveys find that patent applications are made for around 40 percent of US innovations (Cohen, Nelson and Walsh 2000) and around 35 percent of European innovations (Arundel and Kabla 1998). We match these moments to the share of innovations that are patented domes-

tically in the US and Europe, respectively. In addition to depending upon the costs and benefits of patenting in the two economies, these moments are important for calibrating dispersion in the quality of innovations, k. For the US and Europe, we also target the share of domestic patents in total inward patents, which is given by the ratio of PAT_{nn} to $\sum_{i=1}^{N} PAT_{ni}$. This ratio depends upon the size of the patent preparation cost f_n^o relative to the patent application cost f_n^e.[2]

We target two moments that are important for inferring the level of patent protection in the US δ_{US} and the technology diffusion rate ν. Kogan et al. (2017) use stock market responses to news about patents to estimate the private value of holding a patent. Averaging their estimates for 1995-2007, we target an aggregate value of patents relative to R&D expenditure of 9.3 percent for the US. We also use trade data from Schott (2008) to compute a measure of turnover in US imports. As in Hsieh et al. (2022), turnover depends upon the rate at which technology diffusion leads to changes in where products are produced.

In the model, patenting costs are denominated in labor units, which implies large cost differences between high and low wage countries. However, measures of patent application costs are not strongly correlated with income levels (Park 2010, De Rassenfosse and Van Pottelsberghe 2013). This may be because patenting costs reflect wages for skilled workers, which vary less across countries than average wages (Hjort et al. 2022). To capture this feature of the data, we parameterize patenting costs as:[3]

$$f_n^o = f^o h_n, \qquad f_n^e = f^e h_n, \qquad \text{where } h_n \equiv \left(\frac{\text{Real GDP per capita in US}}{\text{Real GDP per capita in n}} \right)^{1-\chi},$$

where f^o and f^e are common across countries. Imposing this parameterization reduces the number of parameters to calibrate to 35. The cost adjustment h_n shrinks cross-country variation in patenting costs. We compute h_n using observed data on real GDP per capita and setting $\chi = 0.16$ based on the estimated elasticity of real middle management costs to real GDP per capita in Hjort et al. (2022). We also adjust f_{Europe}^e upwards to account for the fact that applicants must pay patent fees in multiple countries to obtain patent protection in Europe (see Appendix B.2 for details). To pin down the level of patent application costs f^e, we target total US expenditure on domestic patent applications, which we compute by multiplying observed PAT_{nn} for the US by Park's (2010) estimate that a US patent application costs $17,078$ dollars.

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[2]In order to reduce measurement error resulting from differences across patent offices in the average scope of a patent, we do not use any information on domestic patenting in other countries. Dechezleprêtre et al. (2017) show that international patents are more comparable across patent offices than domestic patents.

[3]Given the challenges of obtaining comprehensive measures of patenting costs, and the fact that available measures differ considerably by source, we choose not to use international variation in estimates of patent application costs to calibrate f_n^e. In practice, this means that our calibration will load unmodeled cross-country differences in patenting costs onto the patent protection parameters δ_n.

Data on R&D expenditure and the market share of innovative firms allows us to discipline the allocation of resources to innovation. We target the ratio of R&D expenditure to GDP in the US, Europe and Japan. We do not use R&D data for the other four economies because, as with patenting costs, R&D workers in these economies are likely to be relatively more expensive, compared to the average wage, than in high income countries. The model can absorb differences in the relative cost of R&D workers in η_i, but when such differences are significant it does not make sense to target measured R&D expenditure in all countries.[4] We measure the market share of innovative firms as the revenue share of firms that invest in R&D. Firms that perform R&D account for 65 percent of sales in the patenting sector in the US (NSF 2005, BEA 2022). As discussed further in Section 3.3, the calibrated model reveals a tension between this moment and the US R&D expenditure to GDP ratio because it implies investment in innovation exceeds measured R&D expenditure. We use both moments in the calibration.

We target two growth rates: aggregate growth g and the difference between price growth in the patenting and no patenting sectors. We measure g as trend growth in US real GDP per capita from 1980-2019 and sector-level price growth using US gross output price indices from 1997-2019. Conditional on innovation and patenting, these moments pin down the product obsolescence rate in the patenting sector ζ and the exogenous growth rate in sector zero g^0.

The trade elasticity in the patenting sector is a weighted average of $\sigma^1 - 1$ and θ^1, where the weights depend upon the share of trade in Helpman-Krugman versus Eaton-Kortum products, which varies by country pair. To calibrate θ^1, we target the average trade elasticity across all international trade flows, which we set equal to five (Head and Mayer 2014). Finally, we target world gross output, each economy's share of world real GDP and each economy's price index relative to the US. These moments are informative about the Fréchet scale parameters T_i^s that capture any productivity variation across countries that does not stem from differences in innovative capacity.

Let Ω denote the set of parameters calibrated using the simulated method of moments and K the set of targeted moments. Using m^k to denote moment k with elements m_i^k that have target values $m_i^{k,target}$, the objective function that the calibration seeks to minimize is:

$$F\left(\Omega\right) = \sum_{k \in K} \sum_{i=1}^{\dim\left(m^k\right)} \left[\frac{v_k}{\dim\left(m^k\right) \sum_{j \in K} v_j} \mathcal{L}^k\left(m_i^k\left(\Omega\right), m_i^{k,target}\right)\right]^2,$$

where v_k is the weight given to moment k, $m_i^k\left(\Omega\right)$ denotes the simulated value of element i of moment k and $\mathcal{L}^k\left(\cdot\right)$ is a loss function. Appendix B.4 describes the algorithm we use to solve for the model's steady state conditional on knowing Ω. Appendix B.5 provides further details on the calibration procedure we use to obtain Ω.

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[4]Consistent with this observation, the calibrated model under-predicts R&D expenditure relative to GDP in China, Brazil, India and the rest of the world.

3.3 Model Fit and Calibrated Parameters

Figure 1 and Table 1 report how the calibrated model matches the targeted moments. Figure 1 plots international patent shares implied by the model against their observed values. The model performs well in matching both cross-country and within-country variation in patenting shares. Regressing the log of the observed shares against their model-based counterparts yields an elasticity of 0.97 with a standard error of 0.03 and an R-squared of 0.96. Note that the calibrated model successfully matches variation in bilateral patenting flows without assuming any country-pair specific differences in patenting costs or patent protection. Table 1 shows that other moments related to patenting are also closely matched with the exception of the domestic patent share in the US, which the model over-predicts.
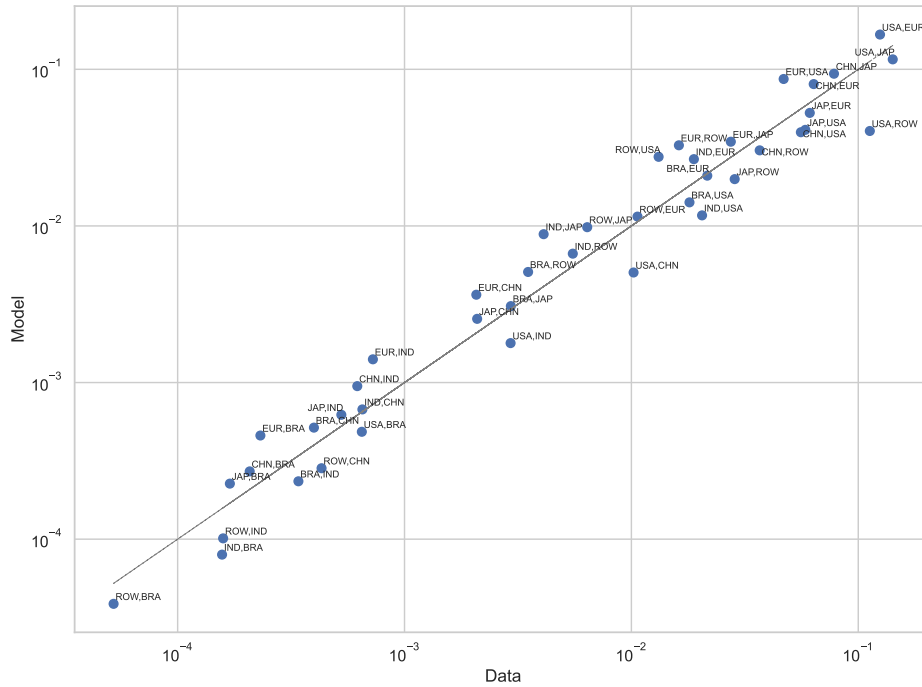
Notes: Model-implied versus observed international patent shares. Points labelled (destination, origin). Solid line is 45 degree line.

Figure 1: International patent shares

The most notable discrepancy between the targeted and model-implied moments is that the model cannot simultaneously match both the market share of innovative firms and the R&D expenditure to GDP ratio in the US. In the calibrated model the market share of innovative firms is lower than its observed value, while the R&D to GDP ratio is higher. In practice, the observed market share moment is likely to overstate the market share of Helpman-Krugman products to the extent that firms that perform R&D sell Eaton-Kortum products as well as Helpman-Krugman products. By contrast, the R&D expenditure moment likely understates innovation because not all

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innovative investment is captured in R&D data. Consequently, we choose to target both moments and allow the patenting moments, which themselves depend upon innovation levels, to resolve the tension between them.

The model closely matches almost all the remaining moments including turnover in US imports, growth rates and the trade elasticity in the patenting sector. It slightly over-predicts China's price level and share of world real GDP, while under-predicting these moments for the rest of the world. However, price levels and real GDP shares are well matched for all other countries.

The calibrated parameters are reported in Table 2. We calibrate $\delta_{US} = 0.070$, which implies an expected patent duration of $1/\delta_{US} = 14.3$ years. Since the legal term of US patents is 20 years, this value implies relatively high levels of patent enforcement in the US. Patent protection in Europe and Japan is similar to US levels, while China, Brazil and especially India offer somewhat weaker protection.[5] It follows that, even after the implementation of TRIPS, there exists substantial cross-country variation in patent rights. This finding is consistent with evidence from the Ginarte-Park patent rights index, which shows that TRIPS narrowed, but did not close, the gap in patent rights between developed and developing economies (Park 2008).

The calibration implies technology diffusion is moderately slow. We estimate $\nu = 0.036$, implying it takes 27.5 years on average for an innovation to diffuse.[6] We also find that $f^o/f^e = 1.4$ implying that the patent preparation cost is greater than the patent application cost. A relatively large patent preparation cost is required to match the observed prevalence of domestic patents. Setting $f^o = 0$ leads to an equilibrium with too much domestic, relative to international, patenting.

We estimate that R&D efficiency η_i is highest in the US, followed by Japan and then Europe. Moreover, it is an order of magnitude greater in these economies than in China, Brazil and India. Combining these differences in R&D efficiency with variation in the allocation of labor to R&D implies that innovation is highly concentrated in developed economies. In the calibrated steady state, the US accounts for 62 percent of world innovations, Japan 20 percent and Europe 14 percent. China, Brazil and India each account for fewer than 1 percent of innovations. Calibrated productivity T_i^s is higher in the US, Europe and Japan than in the other economies in both the patenting and no patenting sectors. However, the implied productivity gaps are notably smaller than the variation in R&D efficiency.

Although we calibrate trade costs to exactly match trade shares in both the patenting and no patenting sectors, the division of trade between monopolistic Helpman-Krugman and competitive

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[5]Calibrated patent protection is substantially weaker in the rest of the world than the other six economies. This gap largely reflects the need to patent in multiple jurisdictions to obtain a "rest of world" patent. As we do not adjust patenting costs to reflect this need, the model infers that the rest of world has weaker patent protection.

[6]Though note that, because firms discount future profit flows, the "certainty equivalent" technology diffusion lag is shorter than the expected duration of the Poisson process. The value of $R^{s,NP}$ in the calibrated equilibrium is the same as if technology diffusion occurred with certainty after 15.3 years.

Eaton-Kortum products within the patenting sector is endogenous. In the calibrated economy monopolistic products account for 15 percent of world output of the patenting sector. However, since monopolistic products are more traded than competitive products, they account for 44 percent of international trade in the patenting sector. Nevertheless, there is still substantial home bias in sales of monopolistic products. Domestic sales account for 65 percent of world output of monopolistic products.

Unsurprisingly, monopolistic products account for larger shares of output and trade in more innovative countries. Helpman-Krugman products account for 96 percent of US exports in the patenting sector, 90 percent of Japanese exports and 52 percent of European exports. By contrast, Eaton-Kortum products account for more than 90 percent of patenting sector exports in each of China, Brazil and India. Indeed, the biggest exporters of Eaton-Kortum products in the patenting sector are the rest of the world and China, while US and Japan have the lowest exports. These differences illustrate how international variation in innovation levels leads to stark within-sector specialization in exports across product types.

4 Patent Policy

We use the calibrated model to study optimal patent policy by analyzing counterfactual changes in the strength of patent protection δ_n. We start by analyzing the effect of unilateral changes in patent protection in a single country. Next, we solve for the non-cooperative Nash equilibrium in patent protection levels. Then, we solve for the cooperative equilibrium where countries jointly choose patent protection to maximize global welfare. Finally, we characterize the welfare effects of TRIPS by simulating a return to pre-TRIPS levels of patent protection. In each case, we analyze the impact of unanticipated, one-off, permanent changes in patent policies at time zero and assume that the global economy is in steady state initially. We also assume that, from time zero onwards, the new protection levels apply to all patents that had not expired prior to the change in policy.

We measure welfare changes using the equivalent variation in consumption. The equivalent variation EV_i for country i is defined as the percentage increase in consumption in the initial steady state that delivers the same welfare as the new equilibrium. Let EV_i^{SS} denote the equivalent variation in steady state welfare. Then the difference $EV_i - EV_i^{SS}$ captures how the transition dynamics between the initial and new steady states affect welfare.

Obtaining steady state welfare from equation (32) and using SS, O to denote the initial steady state and SS, N to denote the new steady state gives a simple expression for the change in steady state welfare:

$$EV_i^{SS} = \frac{\widetilde{C}_i^{SS,N}}{\widetilde{C}_i^{SS,O}} \left[\frac{\rho - g^{SS,O}\left(1 - \frac{1}{\gamma}\right)}{\rho - g^{SS,N}\left(1 - \frac{1}{\gamma}\right)} \right]^{\frac{\gamma}{\gamma-1}}.$$

By contrast, to calculate EV_i we need to know the entire dynamic equilibrium starting from the initial steady state. Appendix B.6 explains how we solve for the transition dynamics between steady states in order to compute EV_i.

When calculating the equivalent variation for the world as a whole, we consider two alternative measures of global welfare. First, an equal weights measure that sums the welfare of each individual: $U_{W,Equal} = \sum_{i=1}^{N} L_i u_i = \sum_{i=1}^{N} L_i^{\frac{1}{\gamma}} U_i$, where u_i denotes the welfare of an individual with normalized consumption per capita $\widetilde{C}_i/L_i$ and U_i denotes aggregate welfare in country i as defined by equation (3). Second, a measure that uses Negishi (1960) weights based on individuals' inverse marginal utility of consumption in the initial steady state $U_{W,Negishi} = \sum_{i=1}^{N} \left(\widetilde{C}_i^{SS,O}/L_i\right)^{\frac{1}{\gamma}} L_i^{\frac{1}{\gamma}} U_i = \sum_{i=1}^{N} \left(\widetilde{C}_i^{SS,O}\right)^{\frac{1}{\gamma}} U_i$. The Negishi measure puts greater weight on the welfare of richer economies than the equal weights measure since it implies a social planner has no incentive to redistribute income across countries.

4.1 Unilateral Patent Policy

To explore the effect of unilateral changes in patent policy we vary δ_n in one country at a time, while holding patent protection in all other countries constant. Figure 2 plots the steady state world growth rate g as a function of the proportional change in δ_n for each country n. It also shows how the growth rate changes when all countries change δ by the same proportion simultaneously.

The figure shows that stronger patent protection (i.e. lower δ) increases growth, but that the magnitude of the effect varies greatly across countries. The effect is largest for the US followed by Europe and then Japan. By contrast, patent protection in China or the rest of the world has only a small effect on growth and the growth rate is effectively inelastic to changes in protection in Brazil and India. These results are consistent with our observation in Section 2.4 that growth is more sensitive to the level of patent protection in larger and more innovative countries. We also see that strengthening protection in all countries simultaneously has a bigger effect than unilateral changes. However, varying δ_{US} alone is responsible for more than half of the total effect.

Turning to welfare, Figure 3 plots welfare effects EV_i by country, and for the world as a whole, against proportional changes in δ_n. In each panel we change patent protection in a different country. Panel (a) shows the impact of variation in US patent protection δ_{US}. Since stronger protection raises growth, it increases welfare in all countries other than the US and also for the world as a whole. However, in the US, the dynamic benefits of stronger protection are offset by the static
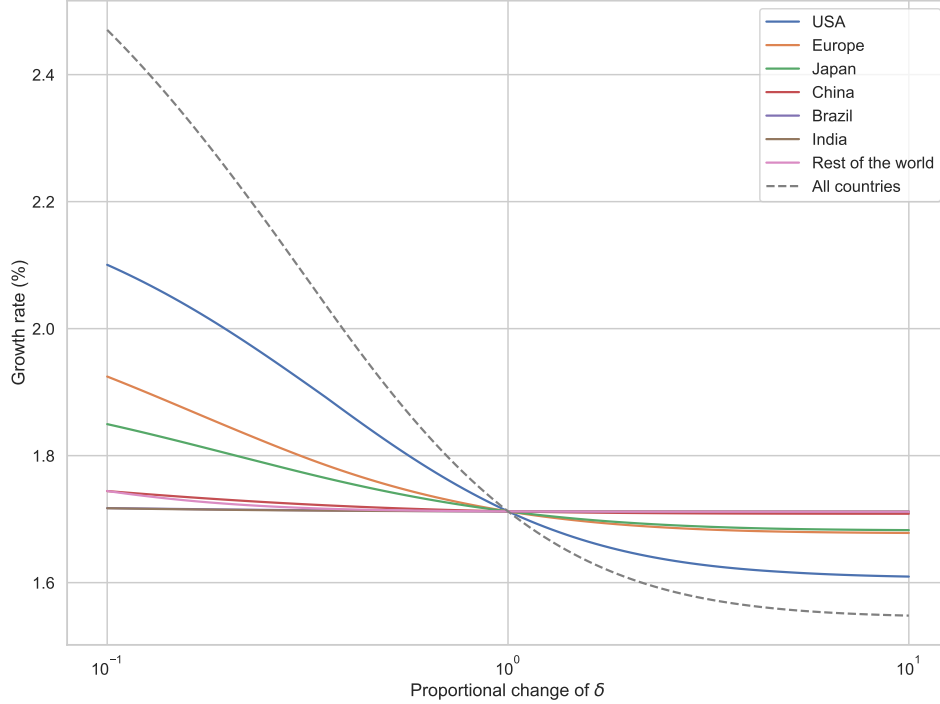
Notes: Effect of proportional change in calibrated patent protection on steady state growth rate.
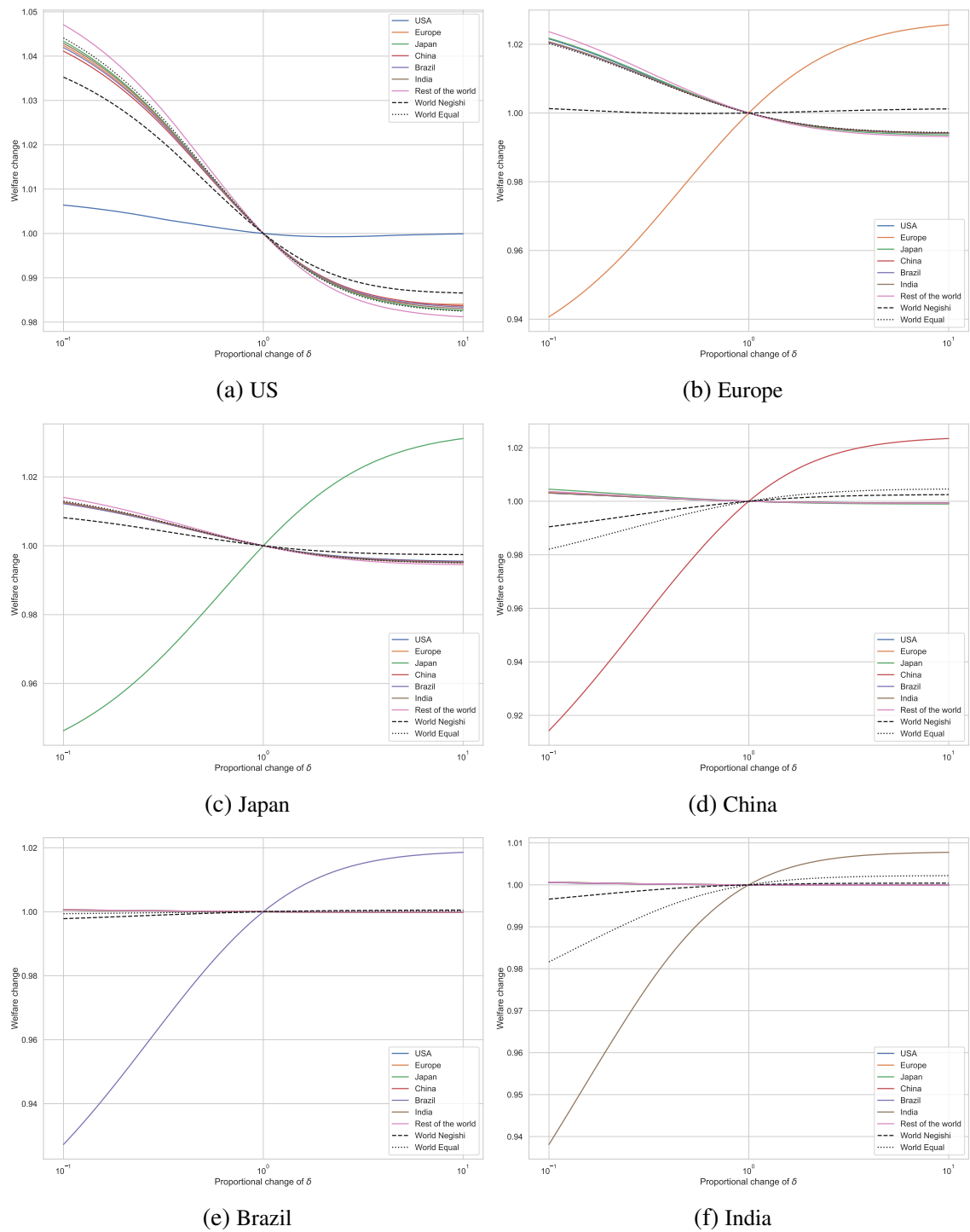
Figure 2: Unilateral patent policy changes and growth

costs of higher prices caused by increased market power in the patenting sector. We find that these effects have similar magnitudes. Consequently, US welfare varies little with δ_{US}, although starting from the calibrated equilibrium the US has a weak unilateral incentive to reduce δ_{US}.

Panels (b)-(f) show welfare effects from varying δ_n in Europe, Japan, China, India and Brazil, respectively. Each figure shows similar patterns, but with different magnitudes (note that the scale of the y-axis differs across panels). Strengthening patent protection in country n raises welfare in countries $i \neq n$, but reduces welfare in country n itself. Thus, all countries other than the US have a unilateral incentive to weaken patent protection. Compared to the US case in panel (a), stronger protection generates smaller spillover benefits for foreign countries because it has a weaker effect on growth (as shown in Figure 2). For Europe and Japan, the benefits of stronger protection to foreign countries exceed the domestic costs and world welfare rises. However, for China, Brazil and India, domestic costs exceed foreign benefits and stronger protection reduces world welfare.

4.2 Nash Equilibrium

How does the calibrated steady state compare to a Nash equilibrium where each country chooses patent protection δ_n to maximize its own welfare taking the response functions of other countries as given? To address this question, we solve for a Nash equilibrium in which each country makes

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(a) US

(b) Europe

(c) Japan

(d) China

(e) Brazil

(f) India

Notes: Effect of proportional change in calibrated patent protection on welfare relative to calibrated equilibrium.

Figure 3: Unilateral patent policy changes and welfare

a one-off, permanent change in its patent protection in order to maximize its steady state welfare.[7] We bound the expected duration of each country's patent protection between one month and 100 years. This range corresponds to values of δ_n between 12 and 0.01, which we refer to as no protection and complete protection, respectively. Numerically, we find that the Nash equilibrium of this game is unique.

Table 3 reports patent protection levels in the Nash equilibrium, together with changes in growth and welfare relative to the calibrated steady state. From the analysis above, we know that all countries other than the US have a unilateral incentive to weaken patent protection. Therefore, it is not surprising that in the Nash equilibrium there is no patent protection in Europe, Japan, China, Brazil, India or the rest of the world. The US also increases its δ, though only slightly, by setting $\delta_{US} = 0.085$, which corresponds to an expected patent duration of 11.7 years.

With all countries offering weaker patent protection, there is less innovation and the steady state growth rate g is 0.09 percentage points lower than in the calibrated economy. However, the decline in growth is offset by a reduction in market power. The share of monopolistic products in world output of the patenting sector falls from 15.5 percent to 13.0 percent. In welfare terms, these two effects roughly cancel each other out at the global level. World welfare is 0.8 percent lower in the Nash equilibrium using equal weights and 0.1 percent lower using Negishi weights. For individual countries, welfare changes are small, though heterogeneous. Welfare declines in US, India and rest of the world, but increases in Europe, Japan, China and Brazil. These results imply that, although the calibrated economy has stronger patent protection and faster growth than the Nash equilibrium, it does not generate significantly higher welfare.

Table 3 also decomposes the welfare effects into changes in steady state welfare and changes due to transition dynamics between steady states. The decomposition shows that the steady state component dominates overall welfare effects. Incorporating transition dynamics reduces welfare gains in all countries, but does not make a major difference to the quantitative results. The welfare effect of including transition dynamics is negative because the growth rate g adjusts to its new lower steady state level more quickly than normalized consumption $\widetilde{C}_i$ increases to its higher steady state value.

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[7]We also assume countries seek to maximize steady state welfare when solving for the cooperative equilibrium in Section 4.3. Alternatively, we could allow countries to maximize welfare including transition dynamics. However, in this case policy choices generally depend upon the initial steady state, which creates time inconsistencies. Implementing the alternative approach makes no difference to the cooperative equilibrium. But in the Nash equilibrium the US chooses the strongest patent protection available, which increases welfare in all other countries by raising the growth rate. However, US welfare is still 0.5 percent lower in this Nash equilibrium than in the calibrated model.

4.3 Cooperative Equilibrium

We assess the potential gains from global cooperation over patent policy by solving for the cooperative equilibrium where countries choose all δ_n jointly to maximize world welfare. Again we allow δ_n to vary between 0.01 and 12 for each country. The results are shown in Table 4. Panel (a) reports the case where the objective function uses equal weights and panel (b) the case with Negishi weights.

Starting with equal weights, we see that the cooperative equilibrium is for the US, Europe and Japan to provide complete patent protection, while all other countries offer no protection. This pattern occurs because growth is more sensitive to patent protection in more innovative economies, as discussed in Section 2.4. Consequently, it is efficient for the world to delegate the job of incentivizing innovation to these economies. And with stronger incentives to innovate, growth increases by 0.60 percentage points compared to the calibrated equilibrium.

World equal weights welfare is 7.4 percent higher in the cooperative equilibrium, showing that there are large gains to cooperation. But the overall gains mask strong distributional effects. While welfare in China, Brazil, India and the rest of the world rises by nearly 10 percent, the US and Japan experience much smaller gains and European welfare declines. This cross-country variation occurs because the static costs of stronger protection are borne by the US, Europe and Japan, while developing economies are able to free ride on the protection offered by the developed world. In steady state, the share of monopolistic products in patenting sector expenditure increases more than 20 percentage points in US, Europe and Japan compared to the calibrated equilibrium, but increases less than 2 percentage points in China, Brazil and India.

The cooperative equilibrium with Negishi weights is similar to the equal weights equilibrium except that Europe switches from complete protection to no protection. This benefits Europe, but dampens the increase in growth and leads to lower welfare gains for all other countries. On a Negishi-weighted basis, world welfare increases by 4.4 percent. At the country level, welfare falls in Japan and increases by only 0.7 percent in the US, while the beneficiaries of cooperation are the countries that free ride upon US and Japanese patent policy.

In both the equal weights and Negishi weights equilibria, including transition dynamics reduces welfare gains in countries that weaken patent protection and increases welfare gains in countries that strengthen protection. In the former case, the transition dynamics component is always small relative to the steady state component. However, when protection gets stronger the two effects are of comparable magnitudes. For example, in the cooperative equilibria European welfare declines 9.6 percent in steady state, but only 1.3 percent with transition dynamics. This example illustrates the importance of accounting for transition dynamics in welfare calculations.

4.4   TRIPS

The TRIPS Agreement came into effect at the start of 1995, but allowed developing countries to phase-in implementation over ten years, while giving least developed countries even longer to adjust. TRIPS sought to narrow the gap between the strength of intellectual property rights in developed and developing economies by: introducing minimum standards of protection and enforcement for all WTO members; applying the principles of national treatment and most-favoured nation treatment to intellectual property, and; placing intellectual property rights under the remit of the WTO's dispute settlement mechanism. For patents, TRIPS mandated that countries make patents available for inventions in all fields of technology and that patents should be enforceable for at least 20 years. In practice, implementation of TRIPS required developing countries to strengthen intellectual property rights, but had little or no impact on policies in developed countries (Saggi 2016).

Figure 4 shows the number of international patent family applications by destination in our data starting in 1990. Between 1990 and 2005 there was a rapid increase in applications filed in China, India and, to a much lesser extent, Brazil. This observation is consistent with TRIPS leading to stronger patent rights in these countries, but it could also result from rapid growth raising market size. We use the model to disentangle these alternatives and quantify changes in patent rights by calibrating patent protection prior to TRIPS.

Since countries may have initiated patent reforms in anticipation of TRIPS we use 1992 data for the pre-TRIPS calibration. We recalibrate δ_i, η_i and T_i^s in 1992, while holding the remaining parameters from the simulated method of moments estimation fixed (see Appendix B.7 for details). Table 5 reports the estimated pre-TRIPS protection levels. We find that China and India had considerably weaker protection in 1992 than 2005, but that protection in Brazil and the developed economies is similar in both periods.

Using these estimates, we study the welfare effects of TRIPS by simulating a counterfactual return to pre-TRIPS patent protection starting from the calibrated steady state in 2005. The assumption that TRIPS was the main cause of changes in patent protection between 1992 and 2005 is more plausible for developing than developed countries. Therefore, in column (a) of Table 5 we consider the case where patent protection in China, Brazil, India and the rest of the world reverts to pre-TRIPS levels, while protection in the US, Europe and Japan remains unchanged. Then in column (b) we set protection to pre-TRIPS levels in all countries.

The counterfactual results show that returning to pre-TRIPS patent policy benefits China, India and the world as a whole, while having little effect on other countries. Welfare increases by 1.0 percent in China, 0.5 percent in India and 0.3 percent for the world (using equal weights). Whether or not patent protection is held constant in US, Europe and Japan does not affect these numbers. It follows that TRIPS reduced welfare in countries that strengthened patent protection. These coun-
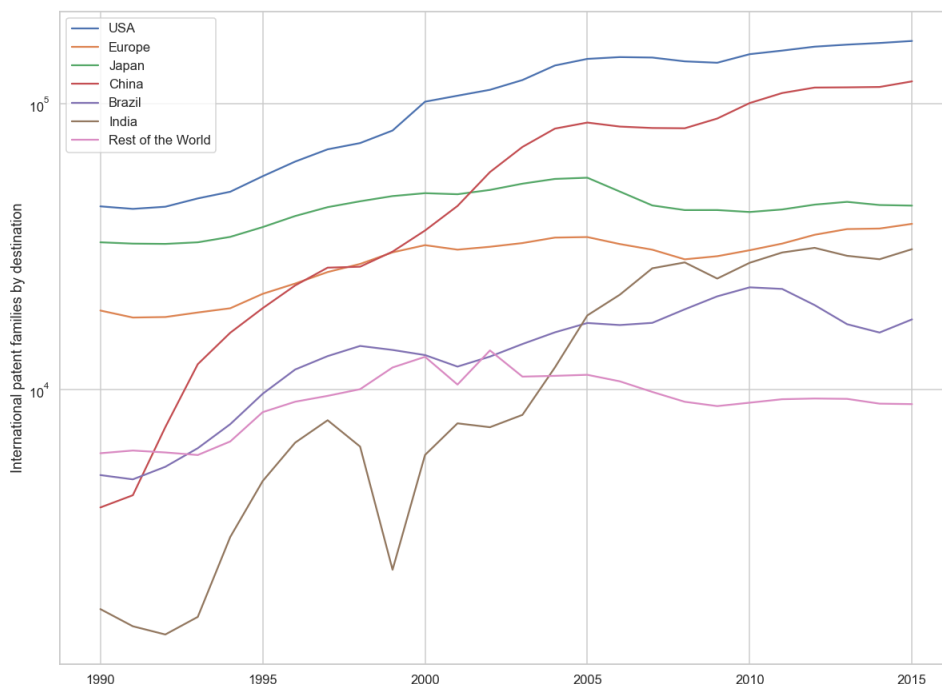
Notes: Count of cross-border applications by destination at the family level.

Figure 4: International patent flows by destination

tries faced local static costs from increased market power, but did not realize offsetting dynamic benefits because growth is inelastic to patent policy in developing countries. For the same reason, TRIPS had negligible spillover benefits for countries that did not change patent policy.

Our calibration implies that TRIPS did not lead to harmonization of patent protection across countries. Nevertheless, we can still use the model to study what would happen if policies were harmonized. Starting from the 2005 calibration, Figure 5 plots welfare changes from setting patent protection in all countries to the same δ. The vertical line on the figure marks the calibrated level of patent protection in the US $\delta_{US} = 0.07$.

Figure 5 shows that harmonizing protection to the calibrated value of δ_{US} would benefit US, Japan and Europe, but reduce welfare in all other countries and for the world as a whole. World equal weights welfare falls 1.5 percent in this counterfactual. Thus, harmonizing policy to US levels is globally inefficient and benefits richer countries at the expense of poorer nations. This finding supports the argument that it would be a mistake for developing countries to adopt US-style patent rights.

We also see that welfare is generally non-monotonic in the harmonized level of protection and that, conditional on harmonization, all countries other than China would prefer δ to be lower than δ_{US}. World equal weights welfare is maximized when $\delta = 0.03$. Countries prefer stronger protection when policies are harmonized than when policy is set unilaterally because international

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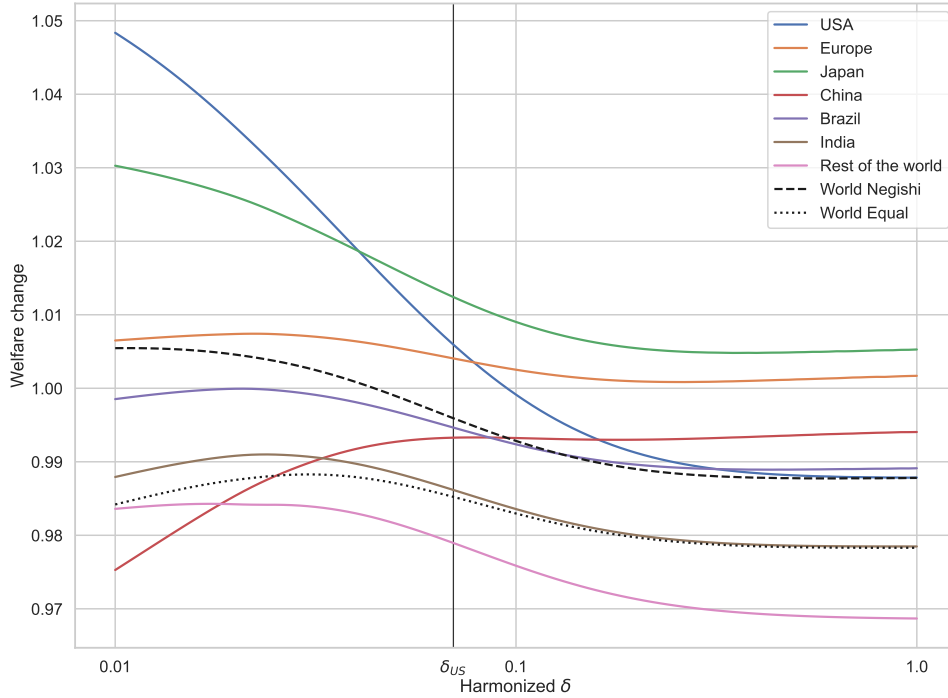
Notes: Welfare changes relative to the calibrated steady state in 2005 of setting patent protection in all countries equal to δ.

Figure 5: Welfare effects of harmonizing patent protection

spillovers imply that the dynamic benefits of stronger protection are greater when all countries reduce δ together.

5 Conclusions

Whether and how patent rights should vary across countries has long been controversial. But debate has suffered from a lack of evidence on the general equilibrium effects of patent policy in open economies. To address this gap, we develop a new quantitative model of trade and patenting. By allowing innovation, patenting and market power to respond endogenously to domestic and foreign patent policy, the model captures the trade-off between static costs and dynamic benefits of stronger patent protection.

We calibrate the steady state of the model to match the world economy in 2005 and use the calibrated economy to analyze the impact of changes in patent policy. We study unilateral changes in patent protection, cooperative and non-cooperative equilibria, and the TRIPS agreement. The counterfactual results imply that there are large potential gains from global cooperation over patent policy. However, realizing these gains requires not that policies are harmonized across countries, but that larger and more innovative economies offer stronger protection. Moreover, the gains and

loses from patent policy are not equally shared.

Two mechanisms are most important in driving these findings. First, international spillovers. The dynamic benefits from higher growth caused by stronger patent rights are global in scope, whereas the static costs due to higher prices are primarily borne domestically. Consequently, non-cooperative patent policies tend to be weaker than is globally efficient because countries do not fully internalize the dynamic benefits. Starting from the calibrated economy, all countries except the US have an incentive to weaken their patent protection and in the Nash equilibrium only the US offers any protection.

Second, heterogeneity in growth effects. The global growth rate is more sensitive to patent policy in countries that account for higher shares of innovators' profits. This means that stronger protection has greater benefits if provided by a large, innovative country such as the US, than if provided by a smaller, less innovative economy such as India. Consequently, optimal patent policy varies greatly across countries. We find that in the cooperative equilibrium, US and Japan offer complete patent protection, whereas China, Brazil, and India provide no protection. This pattern of protection increases growth and raises global welfare, but not all economies benefit equally. Countries that do not provide protection gain more than those that shoulder the burden of encouraging innovation.

The TRIPS agreement required developing countries to increase patent protection towards levels provided in developed economies. Our results imply that TRIPS reforms reduced welfare in developing countries and that further policy harmonization would exacerbate these effects. This finding suggests that some of the opposition to TRIPS may be well-founded, although we caution that our analysis represents only a first attempt at quantifying the effects of patent policy in open economies. Nevertheless, the theory and calibration method that we have developed provide a framework that future research can build upon to better understand this important topic.

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Moment	Target	Model
International patent shares	See Figure 1	
Share of innovations patented in US	0.40	0.42
Share of innovations patented in Europe	0.35	0.33
Share of domestic patents in inward patents in US	0.57	0.74
Share of domestic patents in inward patents in Europe	0.53	0.51
Value of patents relative to R&D expenditure in US	0.093	0.093
Turnover in US imports	0.016	0.015
Expenditure on domestic patent applications in US (trillion $)	0.0032	0.0032
Market share of innovative firms in US	0.65	0.34
Aggregate growth rate	0.017	0.017
Price growth difference (non-patenting minus patenting)	0.0088	0.0088
Trade elasticity in patenting sector	5.0	4.9
World output (trillion $)	82	82
R&D expenditure relative to GDP in US	0.025	0.047
R&D expenditure relative to GDP in Europe	0.017	0.017
R&D expenditure relative to GDP in Japan	0.031	0.032

Moment	World real GDP shares		Price indices relative to US	
Country	Target	Model	Target	Model
US	0.22	0.22		
Europe	0.23	0.24	1.09	1.07
Japan	0.07	0.07	1.18	1.17
China	0.11	0.14	0.35	0.43
Brazil	0.03	0.03	0.44	0.45
India	0.06	0.06	0.24	0.24
Rest of world	0.29	0.25	0.61	0.51

Notes: Targets and model-implied values for moments used in simulated method of moments.

<center>Table 2: Calibrated parameter values</center>

Parameter	Value
Technology diffusion rate, ν	0.036
Product obsolescence rate, ζ	0.012
Shape parameter of Pareto quality distribution, k	1.15
Shape parameter of Fréchet productivity distribution in patenting sector, θ^1	7.26
Growth rate of no patenting sector, g^0	0.013
Patent preparation cost, f^o	0.062
Patent application cost, f^e	0.046

	Patent protection		R&D efficiency, $\eta_i \times 100$	Patenting sector productivity, $(T_i^1)^{\wedge}(1/\theta^1)$	No patenting sector productivity, $(T_i^0)^{\wedge}(1/\theta^0)$
	δ_i	$1/\delta_i$			
US	0.070	14.3	2.67	6.4	16.0
Europe	0.072	13.9	0.74	4.9	13.0
Japan	0.056	17.7	1.61	6.2	13.9
China	0.100	10.0	0.04	3.1	4.8
Brazil	0.104	9.7	0.04	3.6	7.8
India	0.183	5.5	0.02	1.2	4.8
Rest of world	0.754	1.3	0.12	2.0	5.6

Notes: Table reports parameters calibrated using simulated method of moments.

Table 3: Nash equilibrium

	Patent protection, δ_i	Welfare change (percent)	
		Total, EV_i	Steady state, EV_i^{SS}
US	0.085	-1.2	-0.9
Europe	None	1.7	1.9
Japan	None	2.1	2.2
China	None	0.8	1.2
Brazil	None	0.3	0.7
India	None	-0.8	-0.5
Rest of world	None	-1.7	-1.5
World Equal		-0.8	-0.5
World Negishi		-0.1	0.2
Growth rate change (percentange points)			-0.09

Notes: No protection corresponds to δ_i = 12. Changes relative to calibrated steady state. Welfare change expressed as equivalent variation in consumption.

Table 4: Cooperative equilibrium

	(a) Equal weights			(b) Negishi weights		
		Welfare change (percent)			Welfare change (percent)	
	Patent protection, δ_i	Total, EV_i	Steady state, EV_i^{SS}	Patent protection, δ_i	Total, EV_i	Steady state, EV_i^{SS}
US	Complete	2.7	1.8	Complete	0.7	-0.7
Europe	Complete	-1.3	-9.6	None	8.0	9.3
Japan	Complete	0.7	-4.3	Complete	-1.6	-7.4
China	None	9.3	11.1	None	6.9	7.8
Brazil	None	8.6	10.3	None	6.3	7.2
India	None	7.4	9.2	None	5.1	6.1
Rest of world	None	7.1	10.1	None	4.6	6.1
World Equal		7.4	9.4		5.3	6.5
World Negishi		3.9	2.0		4.4	4.5
Growth rate change (percentange points)		0.60				0.40

Notes: Complete protection corresponds to δ_i = 0.01. No protection corresponds to δ_i = 12. Changes relative to calibrated steady state.
Welfare change expressed as equivalent variation in consumption.

Table 5: Pre-TRIPS counterfactuals

	Patent protection, δ_i		(a) Developing countries		(b) All countries	
			Welfare change (percent)		Welfare change (percent)	
	Baseline, 2005	Pre-TRIPS, 1992	Total, EV_i	Steady state, EV_i^{SS}	Total, EV_i	Steady state, EV_i^{SS}
US	0.070	0.076	0.0	0.0	0.1	0.2
Europe	0.072	0.055	0.0	0.0	-0.9	-1.4
Japan	0.056	0.059	0.0	-0.1	0.1	0.2
China	0.100	0.157	1.0	1.3	1.0	1.4
Brazil	0.104	0.107	0.0	0.1	0.0	0.1
India	0.183	0.370	0.5	0.6	0.5	0.6
Rest of world	0.754	0.630	-0.1	-0.1	0.0	0.0
World Equal			0.3	0.4	0.3	0.4
World Negishi			0.1	0.2	0.0	-0.1
Growth rate change (percentange points)			0.00		0.00	

Notes: Column (a): China, Brazil, India and rest of world revert to pre-TRIPS patent protection. Column (b): all countries revert to pre-TRIPS patent protection. Changes relative to calibrated steady state in 2005. Welfare change expressed as equivalent variation in consumption.

Appendices

A   Theory

A.1   Static Equilibrium

Cost minimization using the sectoral production function (1) implies that the sectoral price index satisfies:

$$
\begin{aligned}
P_n^s &= \left(\int_0^{M^s} \psi\left(\omega\right) p_n^s\left(\omega\right)^{1-\sigma^s} d\omega \right)^{\frac{1}{1-\sigma^s}}, \\
&= \left[\left(P_{Cn}^s\right)^{1-\sigma^s} + \left(P_{Mn}^s\right)^{1-\sigma^s} \right]^{\frac{1}{1-\sigma^s}}, \\
&= \left[\Gamma\left(\frac{\theta^s + 1 - \sigma^s}{\theta^s} \right) \right]^{\frac{1}{1-\sigma^s}} \left[\Psi_{Cn}^s \left(\sum_{j=1}^N \Phi_{nj}^s \right)^{\frac{\sigma^s-1}{\theta^s}} + \left(\frac{\sigma^s}{\sigma^s-1} \right)^{1-\sigma^s} \sum_{j=1}^N \Psi_{Mnj}^s \left(\Phi_{nj}^s\right)^{\frac{\sigma^s-1}{\theta^s}} \right]^{\frac{1}{1-\sigma^s}},
\end{aligned}
\tag{38}
$$

where the final equality uses equations (8) and (11) to substitute for P_{Mn}^s and P_{Cn}^s, respectively.

Likewise, cost minimization using the final good production function (2) implies that sectoral output satisfies:

$$
P_n^s Y_n^s = \beta^s P_n Y_n,
\tag{39}
$$

where the final good price index is given by:

$$
P_n = \prod_{s=0}^S \left(P_n^s\right)^{\beta^s}.
\tag{40}
$$

We now impose market clearing conditions. Income from producing sector s output in country i is divided between wages paid to production workers, expenditure on intermediate inputs and profits. Let L_{Yi}^s denote production labor and Q_i^s denote total intermediate input usage in country i and sector s. Each monopolist's profits equal a fraction $1/\sigma^s$ of revenue. Consequently, aggregate profits made by monopolists in i from sales to n are given by X_{Mni}^s/σ^s. Setting production income equal to total sales in each sector then yields:

$$
w_i L_{Yi}^s + P_i Q_i^s + \frac{1}{\sigma} \sum_{n=1}^N X_{Mni}^s = \sum_{n=1}^N X_{ni}^s,
\tag{41}
$$

where total exports X_{ni}^s from i to n equals the sum of Eaton-Kortum trade and Helpman-Krugman trade:

$$X_{ni}^s = X_{Cni}^s + X_{Mni}^s. \tag{42}$$

The variety production technology (4) implies that intermediate input expenditure equals a fraction $1 - \alpha^s$ of production costs. Therefore, at the sectoral level we have:

$$P_i Q_i^s = \frac{1 - \alpha^s}{\alpha^s} w_i L_{Yi}^s. \tag{43}$$

Substituting this expression into equation (41) and rearranging yields:

$$L_{Yi}^s = \frac{\alpha^s}{w_i} \left(\sum_{n=1}^{N} X_{ni}^s - \frac{1}{\sigma} \sum_{n=1}^{N} X_{Mni}^s \right). \tag{44}$$

Note that since $X_{Mni}^0 = 0$ for all n, i, the equation above also holds in sector zero. Summing over sectors then gives:

$$w_i L_{Yi} = \sum_{s=0}^{S} \alpha^s \sum_{n=1}^{N} X_{ni}^s - \frac{1}{\sigma^s} \sum_{s=1}^{S} \alpha^s \sum_{n=1}^{N} X_{Mni}^s, \tag{45}$$

where L_{Yi} is total labor employed in production in country i, which we take as given in the static equilibrium.

Final good market clearing in each country requires that:

$$Y_i = C_i + \sum_{s=0}^{S} Q_i^s,$$

and substituting for Q_i^s using equation (43) yields:

$$Y_i = C_i + \sum_{s=0}^{S} \frac{1 - \alpha^s}{\alpha^s} \frac{w_i}{P_i} L_{Yi}^s. \tag{46}$$

We allow for the possibility of exogenous trade imbalances. Let TB_i be the trade surplus of country i relative to the value of global final output. Accounting for trade in both varieties and patenting services and setting the trade balance plus imports equal to exports gives:

$$TB_i \sum_{n=1}^{N} P_n Y_n + \sum_{s=0}^{S} \sum_{n=1}^{N} X_{in}^s + \sum_{s=1}^{S} \sum_{n=1}^{N} w_n L_{in}^{s,e} = \sum_{s=0}^{S} \sum_{n=1}^{N} X_{ni}^s + \sum_{s=1}^{S} \sum_{n=1}^{N} w_i L_{ni}^{s,e}, \tag{47}$$

where we note that $\sum_{n=1}^{N} X_{in}^s = P_i^s Y_i^s$, which implies $\sum_{s=0}^{S} \sum_{n=1}^{N} X_{in}^s = P_i Y_i$.

47

Letting the final good in country one be the numeraire, meaning $P_1 = 1$, we can now define the static equilibrium as follows.

Definition 1. *Static equilibrium. Assume that the aggregate quality of products sold competitively Ψ_{Cn}^s and monopolistically Ψ_{Mni}^s, labor allocated to output production L_{Yi} and labor allocated to patent purchases $L_{in}^{s,e}$ are known for all countries i and n and sectors s. Then a static equilibrium is defined as a set of N wage rates w_n, N final good output levels Y_n and N final good price indices P_n that solve:*

N final good price index equations (40) subject to the normalization $P_1 = 1$;

N income equals sales equations (45), and;

N trade balance equations (47), where:

P_n^s are defined in (38); Φ_{ni}^s are defined in (9); X_{ni}^s are defined in (42); X_{Mni}^s are defined in (10); P_{Mn}^s are defined in (8); X_{Cni}^s are defined in (12); P_{Cn}^s are defined in (11), and; Y_n^s are defined in (39). The allocation of production labor across sectors L_{Yi}^s is then given by (44) and aggregate consumption C_i by (46).

A.2 Dynamic Equilibrium

Intertemporal demand. Solving the representative agent's intertemporal optimization problem yields the Euler equation:

$$r_{nt} = \rho + \frac{1}{\gamma} \left(\frac{\dot{C}_{nt}}{C_{nt}} + \frac{\dot{P}_{nt}}{P_{nt}} \right), \tag{48}$$

and the transversality condition:

$$\lim_{t \to \infty} \exp\left(-\int_{t_0}^{t} r_{n\tilde{t}} d\tilde{t} \right) W_{nt} = 0. \tag{49}$$

Patenting thresholds. Equation (16) gives the quality threshold above which firms in country i choose to patent in country n if they have paid the patent preparation cost $w_i f_i^{s,o}$. Paying this cost gives firms the option of applying for a patent in each destination. Therefore, a firm that creates an invention with quality ψ at time t_0 opts to pay the preparation cost if and only if:

$$\sum_{n | \psi \geq \psi_{ni}^{s,e*}} \left[\Psi^s \left(V_{nit_0}^{s,P}(\psi) - V_{nit_0}^{s,NP}(\psi) \right) - w_n f_n^{s,e} \right] \geq w_i f_i^{s,o}.$$

The left hand side of this expression gives the value of patenting net of application costs, while the right hand side is the patent preparation cost. We can rewrite the inequality as:

$$\sum_n \max\left[\psi\left(V_{nit_0}^{s,P}(1) - V_{nit_0}^{s,NP}(1)\right) - \frac{w_n f_n^{s,e}}{\Psi^s}, 0\right] \geq \frac{w_i f_i^{s,o}}{\Psi^s}.$$

Let $n_i^* \equiv \arg\min_n \psi_{ni}^{s,e*}$ denote the country n with the lowest threshold in equation (16). The left hand side of the expression above is strictly increasing in ψ whenever $\psi \geq \psi_{n_i^* i}^{s,e*}$. Consequently, there exists a unique threshold $\psi_i^{s,o*}$ defined by:

$$\psi_i^{s,o*} = \psi_{n_i^* i}^{s,e*} \text{ if } \sum_n \max\left[\psi_{n_i^* i}^{s,e*}\left(V_{nit_0}^{s,P}(1) - V_{nit_0}^{s,NP}(1)\right) - \frac{w_n f_n^{s,e}}{\Psi^s}, 0\right] \geq \frac{w_i f_i^{s,o}}{\Psi^s},$$
$$\sum_n \max\left[\psi_i^{s,o*}\left(V_{nit_0}^{s,P}(1) - V_{nit_0}^{s,NP}(1)\right) - \frac{w_n f_n^{s,e}}{\Psi^s}, 0\right] = \frac{w_i f_i^{s,o}}{\Psi^s} \text{ otherwise,}$$

(50)

such that only firms with quality $\psi \geq \psi_i^{s,o*}$ pay the patent preparation cost.

Value of invention. Quality is drawn from a Pareto distribution with scale parameter one and shape parameter k. An innovator from country i and sector s pays the patent preparation cost if quality exceeds $\psi_i^{s,o*}$ and patents in country n if quality exceeds ψ_{ni}^{s*}. Each innovation creates Ψ^s varieties. Therefore, an innovator's expected total patenting costs per variety equal:

$$\sum_{n=1}^{N} (\psi_{ni}^{s*})^{-k} \frac{w_n f_n^{s,e}}{\Psi^s} + (\psi_i^{s,o*})^{-k} \frac{w_i f_i^{s,o}}{\Psi^s}.$$

The expected present discounted value of profits per variety that a time t innovator makes from sales to destination n is:

$$\int_1^{\psi_{ni}^{s*}} V_{nit}^{s,NP}(\psi) k\psi^{-k-1} d\psi + \int_{\psi_{ni}^{s*}}^{\infty} V_{nit}^{s,P}(\psi) k\psi^{-k-1} d\psi = \frac{k}{k-1}\left[V_{nit}^{s,NP}(1)\left(1 - (\psi_{ni}^{s*})^{-k+1}\right) \right.$$
$$\left. + V_{nit}^{s,P}(1)(\psi_{ni}^{s*})^{-k+1}\right].$$

Summing this expression over n and subtracting expected patenting costs per variety yields that the expected value V_{it}^s of inventing a new variety at time t satisfies equation (18).

Labor market clearing. Innovation in country i and sector s occurs at rate $\eta_i^s (L_{Ri}^s)^{1-\kappa}$ and a fraction $(\psi_{ni}^{s*})^{-k}$ of innovations are patented in country n. Therefore, total labor employed to purchase patents in country n satisfies:

$$L_{in}^{s,e} = \eta_i^s (L_{Ri}^s)^{1-\kappa} (\psi_{ni}^{s*})^{-k} f_n^{s,e}. \tag{51}$$

Likewise total labor employed in country i for the preparation of patent applications is:

$$L_i^{s,o} = \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \left(\psi_i^{s,o*}\right)^{-k} f_i^{s,o}. \tag{52}$$

The labor market clearing condition is then given by:

$$L_i = L_{Yi} + \sum_{s=1}^{S} L_{Ri}^s + \sum_{s=1}^{S} L_i^{s,o} + \sum_{s=1}^{S} \sum_{n=1}^{N} L_{ni}^{s,e}. \tag{53}$$

We can now define a dynamic equilibrium.

Definition 2. *Dynamic equilibrium. A dynamic equilibrium is defined as a set of labor allocations to R&D, patenting and production L_{Ri}^s, $L_{in}^{s,e}$, $L_i^{s,o}$ and L_{Yi}; aggregate qualities of Helpman-Krugman and Eaton-Kortum products Ψ_{Mni}^s, $\Psi_{Mni}^{s,NP}$, $\Psi_{Mni}^{s,P,ND}$, $\Psi_{Mni}^{s,P,D}$ and Ψ_{Cn}^s; patenting thresholds ψ_{ni}^{s*} and $\psi_i^{s,o*}$; value functions $V_{nit}^{s,NP}(1)$, $V_{nit}^{s,P}(1)$ and V_{it}^s; interest rates r_i; wage rates w_i; final good output levels Y_i, and; final good price indices P_i, such that in all time periods:*

w_i, Y_i and P_i obey a static equilibrium according to Definition 1;

labor market clearing (53) holds with employment in patenting given by (51) and (52);

$\Psi_{Mni}^{s,NP}$, $\Psi_{Mni}^{s,P,ND}$, $\Psi_{Mni}^{s,P,D}$ and Ψ_{Cn}^s satisfy the laws of motion in (20) – (23) and Ψ_{Mni}^s is given by (19);

$V_{nit}^{s,NP}(1)$ and $V_{nit}^{s,P}(1)$ are defined by (13) and (14) with $\psi = 1$ and expected profits obeying (15);

V_{it}^s is given by (18);

ψ_{ni}^{s*} and $\psi_i^{s,o*}$ are defined by (16), (17) and (50);

L_{Ri}^s satisfies the innovation free entry condition (5), and;

r_i satisfies the Euler equation (48) and the transversality condition (49) holds.

A.3 Steady State

Growth rates. We solve for a steady state equilibrium. Labor market clearing (53) implies that the allocation of labor to R&D, patenting and production in each country is constant in steady state. Equations (51) and (52) then imply that the patenting thresholds ψ_{ni}^{s*} and $\psi_i^{s,o*}$ are constant.

Let g^s denote the growth rate of Ψ^s. Equations (6) and (19) imply that the aggregate qualities of Helpman-Krugman and Eaton-Kortum products in sector s denoted by Ψ_{Mni}^s, $\Psi_{Mni}^{s,NP}$, $\Psi_{Mni}^{s,P,ND}$, $\Psi_{Mni}^{s,P,D}$ and Ψ_{Cn}^s all grow at rate g^s in steady state. Note that the growth rate of aggregate quality is the same in all countries. Using equations (8), (9), (46) and (47) it then follows that the growth rates of wages w_i, final good output Y_i, consumption C_i, final good price indices P_i and trade flows X_{ni}^s are all constant across countries. Since the final good in country one is the numeraire, we must have that steady state final good prices are constant in all countries.

Let g denote the growth rate of consumption C_i. The final good clearing condition (46) implies that wages w_i and final good output Y_i also grow at rate g, while the trade balance condition (47)

implies that trade flows X_{ni}^s grow at rate g. Equation (9) then gives that Φ_{ni}^s grows at rate $-\alpha^s \theta^s g$ and combining this result with equation (38) yields that the sectoral price index P_n^s grows at rate:

$$g_{P^s} = \alpha^s g - \frac{g^s}{\sigma^s - 1}. \tag{54}$$

From equation (39) we have that sectoral output Y_n^s grows at rate:

$$g_{Y^s} = g - g_{P^s},$$

and the production technology (2) implies:

$$g = \sum_{s=0}^{S} \beta^s g_{Y^s}.$$

Combining the three equations above implies that g satisfies equation (25) in the main text.

The Euler equation (48) then implies that the steady state interest rate is constant across countries and given by:

$$r = \rho + \frac{g}{\gamma}, \tag{55}$$

and since total assets W_n grow at rate g the transversality condition is satisfied if and only if $r > g$, which requires $\rho > g\left(1 - 1/\gamma\right)$. We also note from (13), (14) and (18) that the value functions $V_{nit}^{s,NP}(1)$, $V_{nit}^{s,P}(1)$ and V_{it}^s grow at the same rate as expected profits $\mathbb{E}_z \pi_{ni}^s(1, z)$, which equals $g - g^s$ by (15).

Laws of motion for normalized aggregate qualities. Normalizing each of the aggregate quality variables by Ψ^s, the laws of motion in equations (20)–(23) can be rewritten as:

$$
\begin{aligned}
(g^s + \nu^s + \zeta^s) \, \widetilde{\Psi}_{Mni}^{s,NP} &= \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \frac{k}{k-1} \left[1 - (\psi_{ni}^{s*})^{1-k}\right] + \delta_n^s \widetilde{\Psi}_{Mni}^{s,P,ND}, \\
(g^s + \delta_n^s + \nu^s + \zeta^s) \, \widetilde{\Psi}_{Mni}^{s,P,ND} &= \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \frac{k}{k-1} (\psi_{ni}^{s*})^{1-k}, \\
(g^s + \delta_n^s + \zeta^s) \, \widetilde{\Psi}_{Mni}^{s,P,D} &= \nu^s \widetilde{\Psi}_{Mni}^{s,P,ND}, \\
(g^s + \zeta^s) \, \widetilde{\Psi}_{Cn}^s &= \sum_{i=1}^{N} \left(\nu^s \widetilde{\Psi}_{Mni}^{s,NP} + \delta_n^s \widetilde{\Psi}_{Mni}^{s,P,D}\right).
\end{aligned}
\tag{56}
$$

Patenting thresholds. Substituting equations (26) and (27) into (50) and using the definition of normalized profits in (28), the threshold for paying the application preparation cost satisfies:

$$\psi_i^{s,o*} = \psi_{n_i^*i}^{s,e*} \text{ if } \sum_n \max\left(\psi_{n_i^*i}^{s,e*}\Delta R_n^s \widetilde{\pi}_{ni}^s - \widetilde{w}_n f_n^{s,e}, 0\right) \geq \widetilde{w}_i f_i^{s,o},$$

$$\sum_n \max\left(\psi_i^{s,o*}\Delta R_n^s \widetilde{\pi}_{ni}^s - \widetilde{w}_n f_n^{s,e}, 0\right] = \widetilde{w}_i f_i^{s,o} \text{ otherwise.} \tag{57}$$

We can now define a steady state equilibrium.

Definition 3. *Steady state.* *A steady state equilibrium is defined as a set of labor allocations to R&D, patenting and production L_{Ri}^s, $L_{in}^{s,e}$, $L_i^{s,o}$ and L_{Yi}; normalized aggregate qualities of Helpman-Krugman and Eaton-Kortum products $\widetilde{\Psi}_{Mni}^s$, $\widetilde{\Psi}_{Mni}^{s,NP}$, $\widetilde{\Psi}_{Mni}^{s,P,ND}$, $\widetilde{\Psi}_{Mni}^{s,P,D}$ and $\widetilde{\Psi}_{Cn}^s$; patenting thresholds ψ_{ni}^{s*} and $\psi_i^{s,o*}$; normalized value functions $\widetilde{V}_{ni}^{s,NP}(1)$, $\widetilde{V}_{ni}^{s,P}(1)$ and $\widetilde{V}_i^s$; normalized wage rates $\widetilde{w}_i$; normalized final good output levels $\widetilde{Y}_i$; final good price indices P_i; growth rates g^s and g, and; interest rate r such that:*

$\widetilde{w}_i$, $\widetilde{Y}_i$ and P_i obey a static equilibrium according to Definition 1 (with all variables normalized);

labor market clearing (53) holds with employment in patenting given by (51) and (52);

$\widetilde{\Psi}_{Mni}^{s,NP}$, $\widetilde{\Psi}_{Mni}^{s,P,ND}$, $\widetilde{\Psi}_{Mni}^{s,P,D}$ and $\widetilde{\Psi}_{Cn}^s$ satisfy the laws of motion in (56) and $\widetilde{\Psi}_{Mni}^s$ is given by the normalized version of (19);

$\widetilde{V}_{ni}^{s,NP}(1)$ and $\widetilde{V}_{ni}^{s,P}(1)$ are given by the normalized versions of (26) and (27) with normalized profits obeying (28);

$\widetilde{V}_{it}^s$ satisfies (30);

ψ_{ni}^{s*} and $\psi_i^{s,o*}$ are defined by (17), (29) and (57);

L_{Ri}^s is given by (31);

g^s and g are given by (24) and (25), and;

r satisfies the Euler equation (55) and the transversality condition $r > g$ holds;

B   Calibration

B.1   Patent Flows

We use PATSTAT (2022) to obtain data on applications for "Patent of Inventions" filed at patent offices around the world. Patent applications covering the same invention are grouped into families. Since we are interested in unique innovations, we aggregate patent applications to the level of DOCDB simple patent families. A DOCDB family is a collection of patent documents that are considered to cover a single invention and have the same priorities. Each application belongs to exactly one DOCDB family. We date each patent family to the year of the earliest filing date of the root priority application. We then use the steps below to compute bilateral patent flows by year at

the family level from 1990 onwards.

Using the probability mappings from Lybbert and Zolas (2012) we map the CPC/IPC technology classes associated with each patent family to ISIC sectors. We then drop patent families for which all CPC/IPC codes map to our patenting sector with probability less than one half. This leads to us dropping around 5 percent of patent families. We keep patent families for which CPC/IPC codes are not recorded.

We determine the origin country for each patent family based on the location of applicants. When different applicants within a patent family have different origins, we assign the patent fractionally across origins based on the share of applicants from each origin that are listed on any application belonging to the family. When applicant information is not available, we use the location of inventors. When data on both applicants and inventors is missing, but all applications in the family are filed at the same patent office, we assign the origin of the patent family using the location of the patent office. Otherwise, we drop the patent family. This leads to us dropping around 1 percent of patent families.

We assign a patent family to a destination country if any of the applications belonging to the family are filed in the destination (including national phase entries for applications filed under the Patent Cooperation Treaty). For patents granted by the European Patent Office (EPO), we use data on PGFP (Post Grant Fees Paid) events to determine which EPO countries the application is transferred to. For non-granted EPO applications, which account for around two-thirds of EPO applications, we use a machine learning algorithm to predict which countries each application would have been transferred to if granted. We train and test a multi-label classifier on granted EPO patents using the following family-level features: year, number of applicants, number of inventors, number of other patent offices applied to, number of citations, number of applications in the family, and share of other offices that have granted applications in the family. Because Europe and the rest of the world comprise many individual countries, we weight counts by GDP shares when aggregating patent flows into Europe and into the rest of the world.

PATSTAT has poor coverage of applications filed at the Indian Patent Office. Consequently, to compute patents flows into India we use data from WIPO (2023) on patent applications (direct and PCT national phase entries) filed in India by applicant's origin. The WIPO data is at the application (not family) level, includes patents in all sectors and assigns origin using the first named applicant on the root priority application. We adjust for these differences by using PATSTAT to construct origin-year specific deflators based on applications filed in other large developing countries (China, Brazil, Russia and Mexico). In 2005 the cross-origin averages of the deflators are: 1.02 applications per family; 1.04 ratio of all patent families to families that map to our patenting sector, and; 0.95 adjustment for assigning origin using first named applicant.

B.2 Calibration Moments: Data

Mark-ups. We use Hall's (2018, Table 2) estimates of mark-ups by US industry, which are computed from 1987-2015 data. The value-added weighted average mark-up ratio across industries in the patenting sector (i.e. the Manufacturing, Information and Professional, scientific and technical services industries) in Hall's estimates is 1.38. In the model the mark-up ratio in the patenting sector is one for Eaton-Kortum products and $\sigma^1/(\sigma^1 - 1)$ for Helpman-Krugman products. We calibrate σ^1 such that the revenue weighted average of these ratios equals 1.38, which requires $\sigma^1 = 1 + InnovativeShare_{US}^1/0.38$ where $InnovativeShare_{US}^1$ denotes the revenue share of innovative firms in the patenting sector in the US. As explained below, we measure $InnovativeShare_{US}^1 = 0.65$, which implies $\sigma^1 = 2.7$.

Market share of innovative firms. To measure the market share of innovative firms, we use the revenue share of firms that invest in R&D. We compute this share as the ratio of total domestic net sales by firms that perform industrial R&D in 2005 in the Manufacturing, Information and Professional, scientific, and technical services industries from NSF (2005) to total gross output of the same industries in BEA (2022).

Share of innovations patented. Cohen, Nelson and Walsh (2000) survey R&D labs in US manufacturing in 1994. Weighting responses by R&D expenditure, they find that respondents apply for patents on 49 percent of their product innovations and 31 percent of their process innovations. Arundel and Kabla (1998) survey large European industrial firms in 1993. On an R&D weighted basis, they estimate that patent applications are made for 44 percent of product innovations and 26 percent of process innovations. To obtain our targets we take the simple average across product and process innovations, which is consistent with data from Bena and Simintzi (2022) on the share of patents that correspond to process innovations.

Turnover in US imports. The turnover measure captures the rate at which the origin of US imports switches across countries. We use US trade data at the HS 8-digit level from Schott (2008) and, for any base year t, restrict the sample to 8-digit products for which the US was a net importer in both t and $t+5$. We then aggregate countries to the regions used in our calibration and define the turnover rate as the import-weighted share of products for which there is a significant change in the origin of US imports between t and $t+5$. We classify a product as experiencing a significant change in origin if three conditions are met: (i) the leading country (in terms of US imports) changes; (ii) the initial leader has an import share at least 25 percentage points higher than any other country in year t, and; (iii) the new leader has an import share at least 25 percentage points higher than any other country in year $t+5$. These conditions are chosen to identify products that experience a clear switch in the origin of imports. The average turnover rate calculated over all base years from 1996-2016 is 1.56 percent.

European patenting cost. Prior to the introduction of the unitary European patent in June 2023,

patents granted by the European Patent Office (EPO) were only protected in countries where the patent was validated, which required the payment of national fees. Inventors could also seek protection in individual European countries by filing applications with national patent offices directly. The absence of a unitary European patent increased the cost of patenting in Europe. Berger (2004) estimates that a typical EPO patent in 2003 cost $30,530$ euros and was validated in six countries. Adjusting for the share of European GDP covered by the six largest European economies and converting from 2003 euros to 2005 dollars, Berger's estimate implies that a European patent is 1.88 times more expensive than a US patent (using Park's (2010) estimate that a US patent application cost 17,078 dollars in 2005 for comparison). Based on this result we set $f_{Europe}^e = 1.88 f^e h_{Europe}$.

Other moments. The target growth rate is computed by regressing the ratio of US real GDP to working age population on a time trend using data for 1980-2019 from the World Development Indicators (World Bank 2023). We construct sectoral price indices for the patenting and no patenting sectors using Bureau of Economic Analysis gross output price indices (BEA 2022). Industries are weighted using gross output shares in 2000 and we compute the trend growth in each sector from 1997-2019.

Country characteristics in 2005 from the World Development Indicators (World Bank 2023) are defined as follows. GDP and R&D expenditure are measured in current US dollars. Population is the working age population aged 15-64. The price level is the ratio of the PPP conversion factor of GDP to market exchange rates. For Europe and the rest of the world we take the GDP weighted average of the price level in all countries with available data. We compute real GDP as the ratio of GDP in current US dollars to the price level.

B.3 Calibration Moments: Model

This section derives expressions for the moments used in the simulated method of moments calibration. All moments are computed in the model's steady state equilibrium.

The patent flow PAT_{ni}^s from origin i to destination n is:

$$PAT_{ni}^s = \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \left(\psi_{ni}^{s*}\right)^{-k}.$$

International patent shares are then given by $PAT_{ni}^s / \sum_{n=1,n\neq i}^N \sum_{i=1}^N PAT_{ni}^s$, while the share of domestic patents in inward patents for country n equals $PAT_{nn}^s / \sum_{i=1}^N PAT_{ni}^s$. In addition, the share of innovations patented in the US and Europe is given by $(\psi_{ii}^{s*})^{-k}$ for $i = US$ and $i = Europe$, respectively. And we calculate total expenditure on domestic patent applications in the US as $\sum_{s=1}^S PAT_{ii}^s \widetilde{w}_i f_i^{s,e}$ for $i = US$.

The private value of holding a patent in destination n for an invention of quality ψ invented at time t in origin i equals $\Psi^s \psi \left[V_{nit}^{s,P}(1) - V_{nit}^{s,NP}(1)\right]$. Therefore, the aggregate value of patents

purchased by US innovators in the US at time t is:

$$\sum_{s=1}^{S} \eta_i^s \left(L_{Ri}^s\right)^{1-\kappa} \Psi^s \left[V_{nit}^{s,P}(1) - V_{nit}^{s,NP}(1)\right] \int_{\psi_{ni}^{s*}}^{\infty} \psi dH(\psi),$$

where $i = n = US$. We compute the value of patents relative to R&D expenditure in the US by taking the ratio of this expression to R&D expenditure RD_i given by:

$$RD_i = \sum_{s=1}^{S} \left(w_i L_{Ri}^s + w_i L_i^{s,o} + \sum_{n=1}^{N} w_n L_{in}^{s,e}\right).$$

for $i = US$. Taking the ratio allows us to write all variables in their normalized forms. The market share of innovative firms in the US equals $\sum_{n=1}^{N} \widetilde{X}_{Mni}^s / \sum_{n=1}^{N} \widetilde{X}_{ni}^s$ for $i = US$.

We match the turnover moment to the model-implied ratio of the value of US imports that switch origin between t and $t + 5$ to total US imports of products imported in both t and $t + 5$, which we denote TO_n^s with $n = US$. In the model, the US sources each variety from a single country and the origin of imports only changes when varieties switch from Helpman-Krugman to Eaton-Kortum products (due to either technology diffusion or patent expiration) and the previous monopolist's country is not the lowest cost Eaton-Kortum supplier. Let $\epsilon(x) \equiv 1 - e^{-x}$. Then a little calculation yields:

$$TO_n^s = \frac{\left(\sum_k \widetilde{\Phi}_{nk}^s\right)^{\frac{\sigma^s-1}{\theta^s}-1} \sum_i \sum_{j \neq i,n} \left[\widetilde{\Psi}_{Mni}^{s,NP} \epsilon(\nu\Delta t) + \widetilde{\Psi}_{Mni}^{s,P,ND} \epsilon(\nu\Delta t)\epsilon(\delta_n^s\Delta t) + \widetilde{\Psi}_{Mni}^{s,P,D} \epsilon(\delta_n^s\Delta t)\right] \widetilde{\Phi}_{nj}^s}{\sum_i \left[\begin{array}{c} \left(\sum_k \widetilde{\Phi}_{nk}^s\right)^{\frac{\sigma^s-1}{\theta^s}-1} \widetilde{\Psi}_{Cn}^s \widetilde{\Phi}_{ni}^s \\ + \left(\frac{\sigma^s}{\sigma^s-1}\right)^{1-\sigma^s} \left[\widetilde{\Psi}_{Mni}^{s,NP} e^{-\nu\Delta t} + \widetilde{\Psi}_{Mni}^{s,P,ND}\left(e^{-\nu\Delta t} + \epsilon(\nu\Delta t)e^{-\delta_n^s\Delta t}\right) + \widetilde{\Psi}_{Mni}^{s,P,D} e^{-\delta_n^s\Delta t}\right]\left(\widetilde{\Phi}_{ni}^s\right)^{\frac{\sigma^s-1}{\theta^s}} \\ + \left(\sum_k \widetilde{\Phi}_{nk}^s\right)^{\frac{\sigma^s-1}{\theta^s}-1} \sum_{j \neq n}\left[\widetilde{\Psi}_{Mni}^{s,NP} \epsilon(\nu\Delta t) + \widetilde{\Psi}_{Mni}^{s,P,ND} \epsilon(\nu\Delta t)\epsilon(\delta_n^s\Delta t) + \widetilde{\Psi}_{Mni}^{s,P,D} \epsilon(\delta_n^s\Delta t)\right] \widetilde{\Phi}_{nj}^s \end{array}\right]},$$

where $\Delta t = 5$.

The trade elasticity TE_{ni}^s for exports from country i to country n is defined as the negative of the elasticity of trade value X_{ni}^s to trade costs τ_{ni}^s. In our model, the trade elasticity is the trade-share weighted average of the trade elasticity for Helpman-Krugman products $\sigma^s - 1$ and the trade elasticity for Eaton-Kortum products θ^s, which gives:

$$TE_{ni}^s = \frac{\widetilde{X}_{Mni}^s}{\widetilde{X}_{ni}^s}\left(\sigma^s - 1\right) + \frac{\widetilde{X}_{Cni}^s}{\widetilde{X}_{ni}^s}\theta^s.$$

We target the average trade elasticity across all country pairs defined as:

$$\frac{1}{N(N-1)} \sum_{n=1, n \neq i}^{N} \sum_{i=1}^{N} TE_{ni}^s.$$

The aggregate growth rate equals g. Using equation (54), the difference between price growth in the non-patenting and patenting sectors is:

$$\left(\alpha^0 - \alpha^1\right) g + \frac{g^1}{\sigma^1 - 1} - \frac{g^0}{\sigma^0 - 1}.$$

We calculate world gross output as $\sum_{i=1}^{N} P_i \widetilde{Y}_i$. Price levels relative to the US are given by P_i/P_{US}. We define the nominal GDP of country i as:

$$GDP_i = P_i C_i + TB_i \sum_{n=1}^{N} P_n Y_n + w_i \left(L_i - L_{Yi}\right).$$

This allows us to compute R&D expenditure relative to GDP in US, Europe and Japan as RD_i/GDP_i and world real GDP shares as GDP_i/P_i divided by $\sum_{n=1}^{N} GDP_n/P_n$.

B.4 Steady State Solution Algorithm

Let $\widetilde{Z}_i \equiv P_i \widetilde{Y}_i$, $\widetilde{B}_{ni}^s \equiv \widetilde{\pi}_{ni}^s / \widetilde{w}_i$ and $\widetilde{\phi}_{ni}^s \equiv \left(\widetilde{\Phi}_{ni}^s\right)^{\frac{1}{\theta^s}}$. We solve for the steady state equilibrium using a fixed point approach in the vector of fundamental variables $VF = \left(\widetilde{w}_i, \widetilde{Z}_i, L_{Ri}^s, \widetilde{B}_{ni}^s, \widetilde{\phi}_{ni}^s\right)$.

Given an initial guess for VF, we compute the auxiliary variables as follows. Profits: $\widetilde{\pi}_{ni}^s = \widetilde{w}_i \widetilde{B}_{ni}^s$. Growth rates: equation (24) gives g^s, equation (25) gives g and equation (55) gives r. Patenting thresholds: equation (29) gives $\psi_{ni}^{s,e*}$, equation (57) gives $\psi_i^{s,o*}$ and equation (17) gives ψ_{ni}^{s*}. Aggregate qualities: equation (33) gives $\widetilde{\Psi}_{Mni}^s$ for $s \neq 0$, $\widetilde{\Psi}_{Mni}^0 = 0$ and $\widetilde{\Psi}_{Cn}^s = 1 - \sum_{i=1}^{N} \widetilde{\Psi}_{Mni}^s$. Sectoral relative prices:

$$\frac{P_{Mn}^s}{P_n^s} = \left[\frac{\left(\frac{\sigma^s}{\sigma^s-1}\right)^{1-\sigma^s} \sum_{j=1}^{N} \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s\right)^{\sigma^s-1}}{\left(\frac{\sigma^s}{\sigma^s-1}\right)^{1-\sigma^s} \sum_{j=1}^{N} \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s\right)^{\sigma^s-1} + \widetilde{\Psi}_{Cn}^s \left(\sum_{j=1}^{N} \left(\widetilde{\phi}_{nj}^s\right)^{\theta^s}\right)^{\frac{\sigma^s-1}{\theta^s}}}\right]^{\frac{1}{1-\sigma^s}}, \quad (58)$$

$$\frac{P_{Cn}^s}{P_n^s} = \left[\frac{\widetilde{\Psi}_{Cn}^s \left(\sum_{j=1}^N \left(\widetilde{\phi}_{nj}^s\right)^{\theta^s}\right)^{\frac{\sigma^s-1}{\theta^s}}}{\left(\frac{\sigma^s}{\sigma^s-1}\right)^{1-\sigma^s} \sum_{j=1}^N \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s\right)^{\sigma^s-1} + \widetilde{\Psi}_{Cn}^s \left(\sum_{j=1}^N \left(\widetilde{\phi}_{nj}^s\right)^{\theta^s}\right)^{\frac{\sigma^s-1}{\theta^s}}}\right]^{\frac{1}{1-\sigma^s}}. \tag{59}$$

Labour allocations: equation (51) gives $L_{in}^{s,e}$, equation (52) gives $L_i^{s,o}$ and equation (53) gives L_{Yi}. Trade flows:

$$\widetilde{X}_{Mni}^s = \frac{\widetilde{\Psi}_{Mni}^s \left(\widetilde{\phi}_{ni}^s\right)^{\sigma^s-1}}{\sum\limits_{j=1}^N \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s\right)^{\sigma^s-1}} \left(\frac{P_{Mn}^s}{P_n^s}\right)^{1-\sigma^s} \beta^s \widetilde{Z}_n, \tag{60}$$

$$\widetilde{X}_{Cni}^s = \frac{\left(\widetilde{\phi}_{ni}^s\right)^{\theta^s}}{\sum\limits_{j=1}^N \left(\widetilde{\phi}_{nj}^s\right)^{\theta^s}} \left(\frac{P_{Cn}^s}{P_n^s}\right)^{1-\sigma^s} \beta^s \widetilde{Z}_n, \tag{61}$$

$$\widetilde{X}_{ni}^s = \widetilde{X}_{Cni}^s + \widetilde{X}_{Mni}^s. \tag{62}$$

Final good price indices:

$$P_n = \prod_{s=0}^S \left\{ \Gamma\left(\frac{\theta^s + 1 - \sigma^s}{\sigma^s}\right) \left[\left(\frac{\sigma^s}{\sigma^s - 1}\right)^{1-\sigma^s} \sum_{j=1}^N \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s\right)^{\sigma^s-1} + \widetilde{\Psi}_{Cn}^s \left(\sum_{j=1}^N \left(\widetilde{\phi}_{nj}^s\right)^{\theta^s}\right)^{\frac{\sigma^s-1}{\theta^s}}\right]\right\}^{\frac{\beta^s}{1-\sigma^s}}. \tag{63}$$

We then update the fundamental variables using:

$$\frac{(L_{Ri}^s)^\kappa}{\eta_i^s} = \sum_{n=1}^N \left[\frac{k}{k-1}\widetilde{B}_{ni}^s R^{s,NP} + (\psi_{ni}^{s*})^{-k} \frac{\widetilde{w}_n h_n f^{s,e}}{\widetilde{w}_i} \left(\frac{k}{k-1} \frac{\psi_{ni}^{s*}\widetilde{w}_i \widetilde{B}_{ni}^s \Delta R_n^s}{\widetilde{w}_n h_n f^{s,e}} - 1\right)\right] - (\psi_i^{s,o*})^{-k} h_i f^{s,o},$$

$$\widetilde{B}_{ni}^s = \frac{\widetilde{X}_{Mni}^s}{\sigma^s \widetilde{\Psi}_{Mni}^s \widetilde{w}_i},$$

$$\widetilde{\phi}_{ni}^s = \frac{(T_n^s)^{\frac{1}{\theta^s}}}{\widetilde{w}_n^{\alpha^s} P_n^{1-\alpha^s}} \left[\left(\frac{\widetilde{X}_{ni}^s}{\widetilde{X}_{nn}^s} \right)^{\text{Data}} \frac{\frac{\widetilde{\Psi}_{Mnn}^s \left(\widetilde{\phi}_{nn}^s \right)^{\sigma^s-1-\theta^s}}{\sum_{j=1}^N \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s \right)^{\sigma^s-1}} \left(\frac{P_{Mn}^s}{P_n^s} \right)^{1-\sigma^s} + \frac{1}{\sum_{j=1}^N \left(\widetilde{\phi}_{nj}^s \right)^{\theta^s}} \left(\frac{P_{Cn}^s}{P_n^s} \right)^{1-\sigma^s}}{\frac{\widetilde{\Psi}_{Mni}^s \left(\widetilde{\phi}_{ni}^s \right)^{\sigma^s-1-\theta^s}}{\sum_{j=1}^N \widetilde{\Psi}_{Mnj}^s \left(\widetilde{\phi}_{nj}^s \right)^{\sigma^s-1}} \left(\frac{P_{Mn}^s}{P_n^s} \right)^{1-\sigma^s} + \frac{1}{\sum_{j=1}^N \left(\widetilde{\phi}_{nj}^s \right)^{\theta^s}} \left(\frac{P_{Cn}^s}{P_n^s} \right)^{1-\sigma^s}} \right]^{\frac{1}{\theta^s}},$$

$$\widetilde{w}_i = \frac{1}{L_{Yi}} \sum_{s=0}^S \alpha^s \sum_{n=1}^N \left(\widetilde{X}_{ni}^s - \frac{1}{\sigma^s} \widetilde{X}_{Mni}^s \right), \tag{64}$$

$$\widetilde{Z}_i = \sum_{s=0}^S \sum_{n=1}^N \widetilde{X}_{ni}^s + \sum_{s=1}^S \sum_{n=1}^N \widetilde{w}_i L_{ni}^{s,e} - \left(TB_i \sum_{n=1}^N \widetilde{Z}_n + \sum_{s=1}^S \sum_{n=1}^N \widetilde{w}_n L_{in}^{s,e} \right), \tag{65}$$

and iterate to stabilize VF using a fixed-point iterative algorithm. Note that the algorithm takes the trade shares $\left(\widetilde{X}_{ni}^s / \widetilde{X}_{nn}^s \right)^{\text{Data}}$ directly from the data since the trade costs τ_{ni}^s are chosen to match these shares exactly. We use a type-I stationary Anderson accelerated process (Anderson 1965), following the implementation of Zhang et al. (2020). Convergence is ensured by enforcing a damping hyper-parameter (Evans et al. 2020). The damping hyper-parameter is weakly optimized, following a modified implementation of Chen and Vuik (2022). The key advantage of this approach is that we do not need to compute any Jacobian or Hessian matrices, either analytically or numerically. We measure a time complexity of $O(N^2 S \ln S)$ for the solver up to 100 countries and 100 sectors.

After the iteration stabilizes, we apply the transformation $\left(\widetilde{w}_i, \widetilde{Z}_i, L_{Ri}^s, \widetilde{B}_{ni}^s, \widetilde{\phi}_{ni}^s \right) \rightarrow$ $\left(\widetilde{w}_i/P_1, \widetilde{Z}_i/P_1, L_{Ri}^s, \widetilde{B}_{ni}^s, \widetilde{\phi}_{ni}^s P_1 \right)$, which yields a solution that respects our numeraire condition $P_1 = 1$.

To solve for the steady state when undertaking counterfactual analysis, we follow the same procedure described above except that when updating $\widetilde{\phi}_{ni}^s$ we use:

$$\widetilde{\phi}_{ni}^s = \frac{(T_i^s)^{\frac{1}{\theta^s}}}{\tau_{ni}^s \widetilde{w}_i^{\alpha^s} P_i^{1-\alpha^s}}. \tag{66}$$

B.5 Calibration Algorithm

The loss function $\mathcal{L}^k(\cdot)$ is either the log difference of the simulated and targeted moments, or the absolute value of the difference of the ratio of the moments from one. Table A1 reports the dimension, weight and loss function for each of the calibration moments. The weights are chosen to optimize the model's match to the target moments using an informal application of the epsilon constraint method of multi-objective optimization.

The calibration uses a trust-region algorithm, which we find to be more robust and less prone to finding a local minimum than variations of Newton or gradient-descent methods. We use the formulation of trust-region sub-problems of Branch et al. (1999), and the solving of the sub-problems in the trust regions follows an implementation of the Levenberg-Marquardt algorithm from Moré (2006). We use the reflective characterization of the trust-region algorithm in Coleman and Li (1996) to avoid stepping directly into bounds.

B.6 Transition Dynamics

Suppose there is an unanticipated change in one or more parameters at time zero and that the economy was in steady state before time zero. To characterize the transition dynamics between steady states we need to derive expressions for the time derivatives of the value functions $\widetilde{V}_{nit}^{s,NP}(1)$, $\widetilde{V}_{nit}^{s,P}(1)$ and $\widetilde{V}_{nit}^{s,P,D}(1)$, where $\widetilde{V}_{nit}^{s,P,D}(1)$ denotes the expected present discounted value of profits per variety that a firm from country i makes in destination n if at time t it owns a non-expired patent over an invention with quality 1 for which the technology has already diffused. We have:

$$\widetilde{V}_{nit}^{s,P,D}(1) = \frac{\Psi_t^s}{\Psi_t} \int_t^\infty \mathbb{E}_z \pi_{ni\hat{t}}^s(1,z) \exp\left(-\int_t^{\hat{t}} (r_{i\widetilde{t}} + \zeta^s + \delta_n^s) d\widetilde{t}\right) d\hat{t},$$

and differentiating this expression with respect to t yields:

$$\dot{\widetilde{V}}_{nit}^{s,P,D}(1) = (r_{it} + \zeta^s + \delta_n^s + g_t^s - g_t) \widetilde{V}_{nit}^{s,P,D}(1) - \widetilde{\pi}_{nit}^s. \tag{67}$$

Likewise, differentiating (13) implies:

$$\dot{\widetilde{V}}_{nit}^{s,NP}(1) = (r_{it} + \zeta^s + \nu^s + g_t^s - g_t) \widetilde{V}_{nit}^{s,NP}(1) - \widetilde{\pi}_{nit}^s, \tag{68}$$

while differentiating (14) gives:

$$\dot{\widetilde{V}}_{nit}^{s,P}(1) = (r_{it} + \zeta^s + \nu^s + \delta_n^s + g_t^s - g_t) \widetilde{V}_{nit}^{s,P}(1) - \nu^s \widetilde{V}_{nit}^{s,P,D}(1) - \delta_n^s \widetilde{V}_{nit}^{s,NP}(1) - \widetilde{\pi}_{nit}^s. \tag{69}$$

To solve for the transition dynamics, we use a fixed point algorithm with fundamental variables $\widetilde{\Psi}_{Cnt}^s$, $\widetilde{\Psi}_{Mnit}^{s,NP}$, $\widetilde{\Psi}_{Mnit}^{s,P,ND}$, $\widetilde{\Psi}_{Mnit}^{s,P,D}$, $\widetilde{V}_{nit}^{s,NP}(1)$, $\widetilde{V}_{nit}^{s,P}(1)$, $\widetilde{V}_{nit}^{s,P,D}(1)$, P_{it}, $\widetilde{w}_{it}$ and $\widetilde{Z}_{it} \equiv P_{it}\widetilde{Y}_{it}$. We start by guessing time paths for the fundamental variables on the time interval $[0, T]$ under the assumption that the economy is in the new steady state from time $T/2$ onwards and that at time zero the state variables $\widetilde{\Psi}_{Cnt}^s$, $\widetilde{\Psi}_{Mnit}^{s,NP}$, $\widetilde{\Psi}_{Mnit}^{s,P,ND}$, $\widetilde{\Psi}_{Mnit}^{s,P,D}$ equal their values in the old steady state. In practice, we set $T = 500$ and our results shows that the economy is always extremely close to the new steady state after 100 years.

Given our initial guess for the fundamental variables, we compute the auxiliary variables as follows. Equation (66) gives $\widetilde{\phi}^s_{nit}$. Normalized version of equation (19) gives $\widetilde{\Psi}^s_{Mnit}$. Equation (58) gives P^s_{Mnt}/P^s_{nt} and equation (59) gives P^s_{Cnt}/P^s_{nt}. Equation (16) gives $\psi^{s,e*}_{nit}$, equation (50) gives $\psi^{s,o*}_{it}$ and equation (17) gives ψ^{s*}_{nit}. Normalized version of equation (18) gives $\widetilde{V}^s_{it}$. Equation (31) gives L^s_{Rit}. Equation (24) gives g^s_t and equation (25) gives g_t. Equation (51) gives $L^{s,e}_{int}$, equation (52) gives $L^{s,o}_{it}$ and equation (53) gives L_{Yit}. Equation (60) gives $\widetilde{X}^s_{Mnit}$, equation (61) gives $\widetilde{X}^s_{Cnit}$ and equation (62) gives $\widetilde{X}^s_{nit}$. Equation (28) gives $\widetilde{\pi}^s_{nit}$. Normalized version of equation (46) gives $P_{it}\widetilde{C}_{it}$ and equation (48) gives:

$$r_{it} = \rho + \frac{1}{\gamma}\left[\frac{\frac{\partial}{\partial t}\left(P_{it}\widetilde{C}_{it}\right)}{P_{it}\widetilde{C}_{it}} + g_t\right].$$

Computing numerical derivatives as necessary, we then update the fundamental variables. Using equations (20)-(23) we set:

$$\widetilde{\Psi}^s_{Cnt} = \frac{1}{\zeta^s + g^s_t}\left[\sum_{i=1}^N\left(\nu^s\widetilde{\Psi}^{s,NP}_{Mnit} + \delta^s_n\widetilde{\Psi}^{s,P,D}_{Mnit}\right) - \dot{\widetilde{\psi}}^s_{Cnt}\right],$$

$$\widetilde{\Psi}^{s,NP}_{Mnit} = \frac{1}{\zeta^s + \nu^s + g^s_t}\left(\eta^s_i\,(L^s_{Rit})^{1-\kappa}\,\frac{k}{k-1}\left[1-(\psi^{s*}_{nit})^{1-k}\right] + \delta^s_n\Psi^{s,P,ND}_{Mnit} - \dot{\widetilde{\psi}}^{s,NP}_{Mnit}\right),$$

$$\widetilde{\Psi}^{s,P,ND}_{Mnit} = \frac{1}{\zeta^s + \nu^s + \delta^s_n + g^s_t}\left(\eta^s_i\,(L^s_{Rit})^{1-\kappa}\,\frac{k}{k-1}\left[1-(\psi^{s*}_{nit})^{1-k}\right] - \dot{\widetilde{\psi}}^{s,P,ND}_{Mnit}\right),$$

$$\widetilde{\Psi}^{s,P,D}_{Mnit} = \frac{1}{\zeta^s + \delta^s_n + g^s_t}\left(\nu^s\widetilde{\Psi}^{s,P,ND}_{Mnit} - \dot{\widetilde{\psi}}^{s,P,D}_{Mnit}\right).$$

From equations (67)-(69) we set:

$$\widetilde{V}^{s,P,D}_{nit}\,(1) = \frac{1}{r_{it} + \zeta^s + \delta^s_n + g^s_t - g_t}\left[\widetilde{\pi}^s_{nit} + \dot{\widetilde{V}}^{s,P,D}_{nit}\,(1)\right],$$

$$\widetilde{V}^{s,NP}_{nit}\,(1) = \frac{1}{r_{it} + \zeta^s + \nu^s + g^s_t - g_t}\left[\widetilde{\pi}^s_{nit} + \dot{\widetilde{V}}^{s,NP}_{nit}\,(1)\right],$$

$$\widetilde{V}^{s,P}_{nit}\,(1) = \frac{1}{r_{it} + \zeta^s + \nu^s + \delta^s_n + g^s_t - g_t}\left[\widetilde{\pi}^s_{nit} + \nu^s\widetilde{V}^{s,P,D}_{nit} + \delta^s_n\widetilde{V}^{s,NP}_{nit}\,(1) + \dot{\widetilde{V}}^{s,P}_{nit}\,(1)\right].$$

Finally, we update P_{it} using equation (63), $\widetilde{w}_{it}$ using equation (64) and $\widetilde{Z}_{it}$ using equation (65). We iterate this procedure until the fundamental variables stabilize.

B.7   Pre-TRIPS Calibration

For the pre-TRIPS calibration, we divide parameters into time-varying parameters that we calibrate in 1992 and other parameters that we hold fixed at their values from the baseline calibration. The time-varying parameters are: TB_i, β^s, α^s, τ_{ni}^s, L_i, h_i, δ_i, η_i and T_i^s. As before, we use exact moment matching to infer TB_i, β^s, α^s, τ_{ni}^s and L_i and differences in real GDP per capita to determine the patenting cost adjustment h_i.

We calibrate δ_i, η_i and T_i^s using simulated method of moments estimation. We match those moments used for the baseline calibration that are informative about patent protection, innovation and productivity. We use: international patent shares; share of innovations patented in US and Europe; share of domestic patents in inward patents in US and Europe; expenditure on domestic patent applications in US; market share of innovative firms in US; world output; R&D expenditure relative to GDP in US, Europe and Japan; world real GDP shares, and; price indices relative to US.

The data moments for the 1992 calibration are calculated following the same procedures used for the 2005 calibration with the following exceptions. Country of origin is missing from the WIPO (2023) data on patent applications filed in India in 1992 for around two-thirds of applications. We impute origin countries for these applications using the origin of applications filed in India in 1994. To compute expenditure on domestic patent applications in US in 1992, we deflate Park's (2010) estimate of the cost of a US patent application from 2005 dollars to 1992 dollars using the US GDP deflator. The OECD data (OECD 2021) used to compute trade shares, output, expenditure and intermediate inputs is from 1995, the earliest year available. Likewise, we measure R&D expenditure relative to GDP in US, Europe and Japan in 1996, the earliest year for which it is available in the World Development Indicators (World Bank 2023).

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Table A1: Calibration moments

Moment	Dimension	Weight	Loss function
International patent shares	N(N-1)	1	Log
Share of innovations patented in US	1	1	Log
Share of innovations patented in Europe	1	1	Log
Share of domestic patents in inward patents in US	1	1	Log
Share of domestic patents in inward patents in Europe	1	1	Log
Value of patents relative to R&D expenditure in US	1	5	Log
Turnover in US imports	1	3	Log
Expenditure on domestic patent applications in US	1	1	Log
R&D expenditure relative to GDP in US, Europe and Japan	3	3	Log
Market share of innovative firms in US	1	1	Log
Aggregate growth rate	1	3	Lin
Price growth difference (non-patenting minus patenting)	1	1	Lin
Trade elasticity in patenting sector	1	3	Lin
World output	1	4	Lin
World real GDP shares	N	3	Log
Price indices relative to US	N-1	3	Log

Notes: moments used in simulated method of moments. In the calibration N=7. Log loss function is log difference of simulated and target moments. Lin loss function is absolute value of difference of ratio of simulated and target moments from one.