Innovations in Entrepreneurial Finance and Top Wealth Inequality

Magnus Irie*

First version: October 2, 2023
This version: November 8, 2023

Latest version

Abstract

How does improved entrepreneurial equity financing affect top wealth inequality? On the one hand, better equity financing enables entrepreneurs to scale up, which tends to raise top wealth inequality. On the other hand, better risk sharing allows entrepreneurs to reduce the idiosyncratic volatility in their wealth portfolios. This risk reduction lowers wealth inequality by making extreme wealth trajectories less likely and by weakening entrepreneurs' precautionary savings motive. The novel insight in this paper is that which of these two effects dominates depends crucially on how much economic activity is reallocated to entrepreneurial firms from elsewhere in the economy when entrepreneurs try to scale up. When this reallocation is large, wealth inequality rises rapidly when equity financing improves, and the model makes sense of several empirical trends, most notably the dramatic rise of firms with a history of venture capital-backing. Numerical simulations suggest that improved risk sharing through better equity financing has been a quantitatively important contributor to rising wealth concentration.

*I would like to thank Andrew Atkeson, Ian Martin, Benjamin Moll, and Ricardo Reis for their guidance throughout the project. I thank Ulf Axelsson, Cynthia Balloch, Emil Bustos, Nils Lager, Jonas Lundstedt, Walker Ray, Anna Rogantini Picco, Dimitri Vayanos, Viking Waldén, Karl Walentin, Roine Vestman, and numerous seminar participants, especially at Sveriges Riksbank and the Federal Reserve Bank of Minneapolis, for helpful comments and questions. I gratefully acknowledge financial support from ESRC.
Department of Finance, London School of Economics, e-mail: `magnus.irie@gmail.com`

1 Introduction

A cursory glance at the names appearing on lists of wealthy Americans uncovers a striking fact: many of the richest individuals became wealthy through a risky investment in a single entrepreneurial firm. Prior work has emphasized the role of entrepreneurs with high exposures to idiosyncratic risk in explaining both the thick right tail of the wealth distribution and the prevalence of newly minted fortunes at the top. While improved equity financing allows entrepreneurs to scale up, it also allows them to offload idiosyncratic risk. With lower levels of idiosyncratic risk, the extreme upward wealth trajectories that help account for the thick right tail of the wealth distribution become less likely. Moreover, with less idiosyncratic investment risk, entrepreneurs' precautionary savings motives are weaker, which slows their wealth accumulation. In addition, better equity financing means that returns to successful firms are spread over a larger set of investors. Therefore, it is not immediately clear how wealth inequality is affected by better risk sharing. The question studied in this paper is therefore: how is top wealth inequality affected by improvements in equity financing for entrepreneurs?

I develop a tractable general equilibrium framework to answer this question. The framework concisely summarizes the impact of improved equity financing in three economic forces. Consider a hypothetical entrepreneur, Jeff. Suppose Jeff's equity financing constraints have just been relaxed. Specifically, he can now finance a larger fraction of his online bookstore startup by issuing equity to outsiders. Jeff could use the risk-sharing properties of improved equity financing to reduce his own idiosyncratic risk exposures. This *risk-reduction effect* would lower top wealth inequality in the long run by making extreme upward wealth trajectories for entrepreneurs less likely. However, Jeff could also use the improved financing to scale up and, with some luck, turn his online bookstore into a retail giant. If this *scaling-up effect* is strong enough, top wealth inequality rises.

The tractability of the framework allows me to highlight a novel theoretical mechanism. Whether the risk-reduction or scaling-up effect dominates depends on a third force: the *general equilibrium reallocation effect*. This measures the extent to which productive resources are reallocated to cutting-edge entrepreneurial firms from other firms in the economy when entrepreneurial financing improves. When every entrepreneur tries to scale up, competition among them reduces their equilibrium profitability. This reduces the attractiveness of scaling up. Why does entrepreneurs' equilibrium profitability fall when they all want to scale up? First, their profitability is reduced because their equilibrium cost of capital rises as they compete for financing. Second, their profitability is diminished because their labor costs rise, and the equilibrium prices of the goods they sell fall as they compete for labor and customers.

This downward pressure on entrepreneurial profitability is ameliorated if entrepreneurs as a group can poach customers, attract labor, and raise capital at the expense of other firms in the economy. A crucial feature of the model is that entrepreneurial firms compete not only with one another but also with traditional firms. These traditional firms produce goods that are imperfectly substitutable with the goods the entrepreneurial firms produce. The higher the elasticity of substitution between these goods, the stronger the reallocation effect. This is because with a high elasticity of substitution, entrepreneurs can expand by poaching demand and productive resources from traditional firms rather than competing down their equilibrium profitability. In this case, entrepreneurs' excess return remains relatively stable, while improvements in equity financing allow them to carry less risk per dollar invested, thus improving the risk-reward trade-off they face. Then, they choose to scale up so much that their *total* risk exposure increases even if improvements in equity financing allow them to carry a smaller *fraction* of the risk in their firm. In this case, wealth inequality rises. In contrast, when the elasticity of substitution between the goods is low, there is limited room for entrepreneurs to expand in equilibrium at the expense of the traditional firms. Hence, the downward pressure on their excess return is high. If the excess return falls enough for the risk-reward trade-off associated with entrepreneurial activity to deteriorate, entrepreneurs choose to reduce their idiosyncratic risk exposures, lowering wealth inequality in the long run.

As a second contribution, the framework makes sense of several other empirical trends documented in U.S. data, if the elasticity of substitution between entrepreneurial firms' goods and traditional firms' goods is high. Most notable among these trends is the rapid growth in the share of economic activity associated with venture capital-backed firms. Other consistent trends include the stability of the accounting return to the aggregate capital stock despite these falling safe rates, and the fall in the aggregate labor share despite stable firm-level labor shares and a falling safe rate. The model exhibits these patterns precisely when the reallocation effect is strong.

Model overview. The extent to which entrepreneurs choose to bear idiosyncratic risk is an equilibrium outcome, so a comprehensive understanding of how improvements in entrepreneurial financing affect top wealth inequality requires an equilibrium model. To this end, I build a stylized but complete general equilibrium model where risks and expected returns associated with entrepreneurship are endogenously determined.

The immediate precursors to the model are the modified neoclassical growth models of Angeletos (2007), Brunnermeier and Sannikov (2017), and Di Tella and Hall (2022). The model features two sectors of production: an innovative entrepreneurial sector and a traditional sector. The firms in the innovative entrepreneurial sector are more productive than the traditional firms. However, a portion of each firm's idiosyncratic risk must be borne by the associated entrepreneur for incentive alignment purposes. Equity issuance is possible but limited. The traditional sector is less productive but has no idiosyncratic risk costs. The uninsurable risk of entrepreneurial production implies that entrepreneurs earn a positive idiosyncratic excess return. Entrepreneurs choose how much idiosyncratic risk to bear by weighing the excess return against the risk.

The allocation of capital to each type of firm is determined by the trade-off between the higher productivity of the entrepreneurial sector, the lower risk costs of the traditional sector, and the substitutability of the goods they produce. I model improvements in entrepreneurial financing as an increase in the fraction of the firm's risk that entrepreneurs can offload to financial markets. This greater offloading lowers the risk cost associated with entrepreneurial production, which, in turn, triggers a reallocation of economic activity from the traditional firms to the entrepreneurial firms.

The model makes stark predictions regarding the effect of improvements in equity financing on top wealth inequality, and the effect depends on the strength of the reallocation. When the degree of substitutability between the goods the two types of firms produce is high, even minor improvements in entrepreneurial equity financing cause a considerable reallocation of capital, labor, and sales to entrepreneurial firms. The large reallocation means the competitive pressure among entrepreneurial firms for financing, workers, and customers is less severe. Entrepreneurs can then expand more aggressively without their expected excess returns declining much. Moreover, better risk sharing reduces the risk per unit invested. If the risk-reward trade-off improves despite the slightly lower expected excess return, then entrepreneurs scale up not only their firms but their total idiosyncratic risk exposures. This raises top wealth inequality by making extreme wealth trajectories more likely.

The setup with two types of firms is essential to deliver these results. The presence of traditional firms from which entrepreneurial firms can draw productive resources and customers enables entrepreneurs, in the aggregate, to scale up without adversely affecting their returns. To the best of my knowledge, this aspect is new to the literature. Finally, I derive a closed-form solution for the model's steady state level of Pareto inequality. The formula reveals an intimate link between entrepreneurs' risk exposures, the share of wealth they hold in aggregate, and the thickness of the right tail of the overall cross-sectional distributional wealth.

Empirical overview. The framework can make sense of four key empirical trends under the assumption of a high elasticity of substitution between the goods that entrepreneurial firms and traditional firms produce:

1. The dramatic growth in the fraction of firms with a history of venture capital-backing among the largest publicly traded firms in the U.S.[1]
2. The fall in the aggregate labor share, despite relatively stable firm-level labor shares.[2]
3. The stable or slightly rising accounting return to the aggregate capital stock despite the falling real safe interest rate.[3]
4. The fall in real safe interest rates.[4]

The first trend is directly related to the mechanism at the heart of this paper. The three others are auxiliary because their connection to the main mechanism is more subtle than that of the first. Their significance arises from the fact that they are implied by the model precisely when the general equilibrium reallocation effect is so strong that the scaling-up effect dominates the risk-reduction effect. The model exhibits these trends under the same conditions under which improvements in entrepreneurial financing lead to higher top wealth inequality.

The growth of venture capital-backed firms. For improvements in entrepreneurial financing to be associated with rising top wealth inequality, the model requires that it causes a substantial reallocation of capital from traditional firms to cutting-edge entrepreneurial firms. Gornall and Strebulaev (2021) document precisely such a reallocation. For instance, they document that firms with a history of VC-backing constituted less than 5% of the market capitalization of publicly traded firms before 1980 but that this share has risen to around 45% in 2020. Since venture capital is explicitly aimed at providing financing for cutting-edge entrepreneurial firms, this suggests that there has been a significant reallocation to such firms over the past half-century.

Labor share. Improvements in entrepreneurial financing have two offsetting effects on the labor share in the model. First, the reallocation of production toward the low-labor-share entrepreneurial firms reduces the aggregate labor share via a composition effect. Conversely, it increases the labor share at the firm level for entrepreneurial firms, as they must raise wages to attract workers. The model displays the empirically observed pattern of a declining aggregate labor share alongside stable or weakly rising firm-level labor shares precisely when the reallocation effect is strong.

Rates of return to capital. The same reasoning applies to the accounting return to the overall capital stock. Reallocating capital to high-return entrepreneurial firms raises the aggregate return to capital. On the other hand, diminishing returns within the entrepreneurial sector

[1]See Gornall and Strebulaev (2021) and Greenwood, Han, and Sanchez (2022).

[2]See Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and Hartman-Glaser, Lustig, and Xiaolan (2019).

[3]See Reis (2022), Moll, Rachel, and Restrepo (2019) and Farhi and Gourio (2018).

[4]Holston, Laubach, and Williams (2017), Auclert, Malmberg, Martenet, and Rognlie (2021), Rachel and Summers (2019).

exerts a counteracting downward pressure. Improvements in entrepreneurial financing only increase the return to the aggregate capital stock if the reallocation effect is strong.

Safe real interest rate. Improvements in entrepreneurial equity financing, combined with a strong general equilibrium reallocation force, lead entrepreneurs to take on more idiosyncratic risk. Higher idiosyncratic risk exposure increases entrepreneurs' precautionary savings motive, which depresses the equilibrium real safe interest rate.

Numerical assessment. To gauge the quantitative role played by improved equity financing, I study the model through stripped-down numerical experiments. The tractability of the framework allows me to compute the model's transition dynamics straightforwardly. In this experiment, I feed in a decline in equity financing frictions that reproduces the rise in the average rate of equity issuance by firms associated with entrepreneurs at the top of the Forbes 400, as documented by Gomez and Gouin-Bonenfant (2023). The model can account for the transition dynamics of Pareto inequality in response to improved equity financing for entrepreneurs, provided that the general equilibrium reallocation effect is large enough. In particular, when the general equilibrium reallocation effect is strong enough to account for the rise in the market capitalization share of firms with a history of venture capital-backing, the model can account for almost all of the rise in Pareto inequality.

Literature. This paper contributes to the literature on the consequences of idiosyncratic investment risk and entrepreneurship for wealth inequality. This literature was pioneered by Quadrini (2000) and further developed by Meh and Quadrini (2006), Cagetti and De Nardi (2006), with recent contributions by Benhabib, Bisin, and Zhu (2014), Gabaix, Lasry, Lions, and Moll (2016), Jones and Kim (2018), Peter (2021), Atkeson and Irie (2022), Hui (2023) and Gomez and Gouin-Bonenfant (2023). This literature aims to account for three stylized facts: the thick right tail of the wealth distribution, the rapid dynamics of the wealth distribution over time, and the prevalence of new fortunes at the top.

Because the literature has emphasized the role played by entrepreneurs with high idiosyncratic risk exposures to account for these facts, studies in this literature have concluded that less restrictive debt financing could raise top wealth inequality while better risk sharing would reduce wealth inequality. For instance, in contrast with the results presented in this paper, recent studies by Peter (2021) and Hui (2023) conclude that improved risk sharing for entrepreneurs lowers wealth inequality.[5] This is because the risk-sharing rather than the scaling-up force dominates in their settings. I discuss this in detail in Section 3.3.

Compared to models in this literature based on the framework of Aiyagari (1994), the model in this paper is closer to Angeletos (2007) in that the economy aggregates tractably

[5]In Atkeson and Irie (2022), entrepreneurs' return and idiosyncratic risk are exogenous. In this paper, they are endogenized and shown to be tightly linked.

despite the presence of idiosyncratic risk. The specification of the risk-sharing environment as one where equity issuance is possible but limited due to agency frictions is based on Brunnermeier and Sannikov (2017). Relative to their model, I introduce labor and imperfect substitutability between the goods produced by the two types of firms. I also consider a demographic setup where the between-type distribution of wealth and the cross-sectional distribution of wealth are stable in the long run despite the presence of idiosyncratic risk. These modifications are essential for considering the issues at the heart of this paper: top wealth inequality, the factor income distribution, and returns to wealth in the long run.

An ongoing discussion in the literature is the extent to which the rise in wealth inequality can be accounted for by changes in relative valuations of broad asset classes, perhaps driven by falling interest rates.[6] Irie (2023) points out that the increase in top wealth inequality is associated with more unequal distributions of the income flows that wealth generates, suggesting that valuation effects do not entirely drive the rise in top wealth inequality. When studying declining interest rates, Gomez and Gouin-Bonenfant (2023) find that lower interest rates primarily raise Pareto inequality by lowering costs of capital for entrepreneurs, not through valuation effects.[7] Other such explanations include Aoki and Nirei (2017), Hubmer, Krusell, and Smith (2021) and Kaymak and Poschke (2016), focusing on taxes; Moll et al. (2019), focusing on automation; and Jones and Kim (2018) and Atkeson and Irie (2022), who focus on entrepreneurs and business owners. Aoki and Nirei (2017) attribute rising Pareto inequality to lower taxes, making entrepreneurs want to increase their exposure to their firms. In the present study, it is instead reduced equity financing frictions that make entrepreneurs want to scale up.

2 Scaling Up and Risk Reduction in a Simple Framework

In this section, I present a simplified partial equilibrium framework where improved risk sharing for entrepreneurs unambiguously leads to increases in risk-taking. In other words, the *scaling-up effect* dominates when risk sharing improves. I also show how moving from partial equilibrium to general equilibrium *can* turn this result on its head. This section thus serves as motivation for the full model presented in Section 3. In that full model, the extent to which entrepreneurial firms can poach customers, raise capital, and attract labor at the expense of other firms in the economy is what determines precisely how strong the

[6] See Kuhn, Schularick, and Steins (2020), Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2021), Cioffi (2021), Gomez (2017), and others.

[7] Interestingly, the long-run response of the model presented in this paper includes a fall in the cost of capital as well. Nevertheless, in contrast to Gomez and Gouin-Bonenfant (2023), this is an outcome rather than a driving force.

scaling-up effect ends up being in equilibrium.

2.1 Partial equilibrium: The scaling-up force dominates

Consider a continuum $i \in [0,1]$ of entrepreneurs operating firms. Each firm produces an output flow using capital, labor, and a Cobb-Douglas production technology. Entrepreneur i accumulates capital subject to idiosyncratic risk:

$$\begin{aligned} y_{it}dt &= \bar{A}k_{it}^{\alpha}l_{it}^{1-\alpha}dt \\ dk_{it} &= (\iota_{it} - \delta)\,k_{it}dt + k_{it}\tilde{\sigma}dZ_{it} \end{aligned} \tag{1}$$

where ι_{it} is the investment rate, δ is the depreciation rate, and $k_{it}\tilde{\sigma}dZ_{it}$ is the idiosyncratically risky part of capital accumulation. In particular, Z_{it} is an individual specific Brownian motion. The entrepreneur finances the capital stock by investing their own wealth, by issuing risk-free securities d_{it}, and by issuing risky equity v_{it}^{out}. The capital structure of the firm is therefore

$$k_{it} = n_{it} + v_{it}^{\text{out}} + d_{it}.$$

The equity issued to outsiders carries the same risk as the risk in the firm's capital. Risk sharing through equity issuance is limited. In particular, the entrepreneur is subject to a skin-in-the-game constraint:

$$\frac{k_{it} - v_{it}^{\text{out}}}{k_{it}} \geq \chi \tag{2}$$

where χ is the fraction of the firm's risk that the entrepreneur must bear. The interest rate on risk-free debt is r_t. The required return on equity issued to outsiders is r_t^{out}. Because outsiders hold this equity as part of diversified portfolios, and since all risk is idiosyncratic and therefore washes away in such a portfolio, no-arbitrage implies that $r_t = r_t^{\text{out}}$. The wage rate is w_t. The entrepreneur consumes at rate c_{it} and has logarithmic utility. The entrepreneur's problem is therefore

$$\max_{\{c_{it},k_{it},l_{it},v_{it}^{out},d_{it}\}} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\log(c_{it})\,dt\right]$$

$$dn_{it} = \left(y_{it} - w_t l_{it} - \delta k_{it} - (d_{it} + v_{it}^{\text{out}})r_t - c_{it}\right)dt + (k_{it} - v_{it}^{\text{out}})\tilde{\sigma}dZ_{it}$$

$$\text{subject to} \quad \frac{k_{it} - v_{it}^{\text{out}}}{k_{it}} \geq \chi.$$

To solve this, we first define the instantaneous return on the firm's capital as

$$dR_t^k \equiv \underbrace{\left(\frac{y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}\right)}_{\text{expected return: } r_{it}^k} dt + \tilde{\sigma} dZ_{it}.$$

Next, we note that the firm's labor demand decision is static. In particular, the associated first-order condition pins down the labor-to-capital ratio as $(1-\alpha)\left(\frac{k_{it}}{l_{it}}\right)^{\alpha} = w_t \Rightarrow \frac{l_{it}}{k_{it}} = \left(\frac{1-\alpha}{w_t}\right)^{1/\alpha}$. Because the optimal labor-to-capital ratio does not depend on i, the expected return to capital r_{it}^k does not depend on i. Let r_t^k denote this common expected return (which will depend on the prevailing wage rate). We also note that the equity issuance constraint is always binding in optimum; if it were not, then the entrepreneur could issue more outside equity and invest the proceeds in the risk-free asset with the same expected return but no risk. This would reduce risk without affecting expected returns and, therefore, make the entrepreneur better off. Hence, $v_{it}^{\text{out}} = (1-\chi)k_{it}$. We can then redefine the entrepreneur's problem as a Merton portfolio choice problem instead:

$$\max_{\left\{\frac{c_{it}}{n_{it}}, \frac{k_{it}}{n_{it}}\right\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_{it})\, dt\right]$$

$$\frac{dn_{it}}{n_{it}} = \left(r_t + \frac{k_{it}}{n_{it}}(r_t^k - r_t) - \frac{c_{it}}{n_{it}}\right) dt + \frac{k_{it}}{n_{it}} \chi \tilde{\sigma} dZ_{it}.$$

This problem has the following well-known solution for the optimal choice of firm size relative to wealth:[8]

$$\frac{k_{it}}{n_{it}} = \frac{r_t^k - r_t}{(\chi\tilde{\sigma})^2}. \tag{3}$$

The entrepreneur's risk exposure, defined as the volatility of net worth, implied by this solution is

$$\tilde{\sigma}_{it}^E \equiv \frac{k_{it}}{n_{it}} \chi \tilde{\sigma} = \underbrace{\frac{r_t^k - r_t}{\chi\tilde{\sigma}}}_{\text{Sharpe ratio}} \tag{4}$$

where $\tilde{\sigma}_{it}^E$ is the resulting volatility of the entrepreneurs' net worth. In other words, entrepreneurs choose an exposure to the idiosyncratic risk equal to the Sharpe ratio associated with investing in entrepreneurial capital, taking into account that they only carry a fraction

[8]The fact that entrepreneurs can freely issue risk-free debt d_{it} means that they can attain this firm size even though equity financing is limited. However, entrepreneurs are risk averse and the issuance of risk-free debt raises the risk entrepreneurs carry.

χ of the risk. Taking the wage rate w_t and the risk-free rate r_t as given, it is clear that improved risk sharing would induce entrepreneurs to increase their risk exposures: a fall in χ improves the risk-reward trade-off as measured by the Sharpe ratio. A higher Sharpe ratio means a higher optimal risk exposure. This is the *scaling-up force* in action. When risk sharing improves so that entrepreneurs can carry a smaller fraction of the risk in their firm, they scale up so much that their total risk exposure rises. The following lemma summarizes this discussion.

Lemma 1. *Keeping fixed expected returns, a fall in χ raises the Sharpe ratio $\frac{r_t^k - r_t}{\chi\tilde{\sigma}}$ and, therefore, entrepreneurs' optimal risk exposure.*

However, the expected excess return $r_t^k - r_t$ is an equilibrium object, and it is easy to see that the effect of improved risk sharing on entrepreneurs' risk exposure can easily go the other way around in equilibrium. For instance, consider a framework where, in equilibrium, the aggregate capital stock of the economy K_t is equal to the aggregate net worth of the entrepreneurs $N_t^E \equiv \int_i n_{it} di$. An example of such a framework would be one where all owners of capital in the economy were entrepreneurs, and capital was the only asset in positive net supply. In such a setting, the optimal portfolio choice of entrepreneurs (3) combined with the condition $K_t = N_t^E$ would imply

$$\frac{r_t^k - r_t}{(\chi\tilde{\sigma})^2} = \frac{k_{it}}{n_{it}} = \frac{K_t}{N_t^E} = 1 \Rightarrow r_t^k - r_t = (\chi\tilde{\sigma})^2$$

so that the *equilibrium* excess return is in fact proportional to χ^2. In this economy, entrepreneurs' equilibrium risk exposure would then be

$$\tilde{\sigma}_{it}^E = \frac{r_t^k - r_t}{\chi\tilde{\sigma}} = \chi\tilde{\sigma}. \tag{5}$$

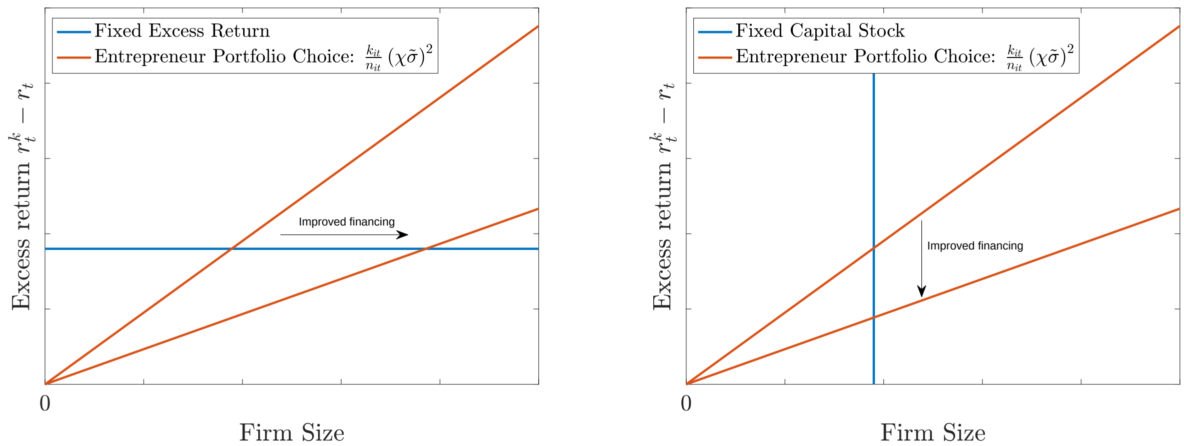
In this case, a looser inside equity constraint (a fall in χ) instead leads to a fall in the risk exposure. In other words, the partial equilibrium result in Lemma 1 is completely reversed. The intuition behind this result is straightforward. When risk sharing improves so that entrepreneurs can carry a smaller fraction of the risk in their firm, they attempt to scale up. However, in equilibrium, they cannot all scale up in the aggregate. To ensure that entrepreneurs are content with operating the existing capital stock, the excess return has to fall. Because firm sizes (relative to the entrepreneurs' net worth) are the same as before, but entrepreneurs now hold a smaller fraction of the risk, their total risk exposure is lower.

The intuition behind the contrasting results in the partial equilibrium case, where excess returns are fixed, and this very particular general equilibrium example where firm sizes relative to entrepreneurs' net worth are fixed, can be understood through Figure 1. The left panel, Figure 1a, represents the partial equilibrium framework. We see an upward-sloping relationship between the excess return and entrepreneurs' choice of firm size. This upward-sloping curve represents the entrepreneurs' portfolio choice. When the excess return is high, entrepreneurs supply their firms with a lot of capital. The slope is determined by, among other things, the inside equity fraction χ. When χ falls, this supply schedule rotates outwards. In the left panel, where excess returns are fixed, the improvements in risk sharing lead to a substantial increase in optimal firm sizes. In contrast, in the right panel, Figure 1b, firm size relative to entrepreneurs' net worthis fixed at $\frac{K_t}{N_t^E} = \frac{k_{it}}{n_{it}} = 1$. Any improvement in risk sharing is, therefore, immediately accompanied by a reduction in the excess return.

In this paper, I will argue that both the partial equilibrium framework represented by Figure 1a, and the particular general equilibrium framework represented by Figure 1b are too extreme. Specifically, I will develop a general equilibrium model where neither the excess return nor the amount of capital relative to entrepreneurs' net worth, is fixed. The equilibrium response of entrepreneurs' choice of risk exposure will then depend on exactly how sensitive excess returns are when entrepreneurs try to scale up. When entrepreneurial firms are the only firms in the economy, they can not scale up at all in the aggregate. One way of avoiding this stark implication is, therefore, to introduce other types of firms into the economy.[9] The entrepreneurial firms will then be able to scale up in the aggregate at the expense of these other firms. The easier it is for the entrepreneurial firms to poach demand, raise capital, and attract labor from these other firms, the more closely this general equilibrium model will resemble the partial equilibrium framework represented by Figure 1a.

In Section 3, I present precisely such a model. In that model, the ease with which entrepreneurial firms can attract economic activity from the other firms in the economy is governed by the elasticity of substitution between the goods that the entrepreneurial firms produce and the goods that these other firms produce. When the elasticity is high, entrepreneurial firms will be able to attract a lot of economic activity from the other firms in the economy when entrepreneurial financing improves, and the resulting equilibrium will resemble the partial equilibrium framework in Figure 1a, where the scaling-up effect dominates. When the elasticity is low, entrepreneurs will have a hard time attracting economic activity from these other firms, and the equilibrium will more closely resemble the one represented by Figure 1b, where the risk-reduction effect dominates.

[9]Another way would be to allow capital inflow from abroad.



(a) Partial equilibrium response of better risk sharing (fall in χ). Excess return is stable, optimal firm size increases substantially.

(b) One-sector general equilibrium response of better risk sharing (fall in χ). Excess return falls substantially, no change in firm size.

Figure 1: Partial equilibrium versus one-sector general equilibrium response of excess return and optimal firm size k_{it} when risk sharing improves.

3 Full Model

Relative to the simplified framework in the previous section, I now consider a model with three types of agents and two types of firms. In addition to entrepreneurs and hand-to-mouth workers, the model will also feature diversified investors. The model will now include a standard neoclassical firm as well as those operated by the entrepreneurs. The entrepreneur-operated firms will be more productive but will be constrained in their equity issuance, as in the previous section. The neoclassical firm, referred to as the traditional firm, will be less productive but will not face any financing constraints. The substitutability of the goods produced by the entrepreneurial firms and those produced by the traditional firm will be governed by a constant elasticity of substitution (CES) parameter ε.

Demographics. The demographics in the model are set up to allow the distribution of wealth to be stable in the long run. Specifically, the economy is populated by a continuum of hand-to-mouth workers endowed with L units of labor and a continuum $i \in [0,1]$ of capitalists. The group of capitalists consists of two types: entrepreneurs and diversified capitalists, denoted by E and D, respectively. Entrepreneurs own a project and can choose to run a firm based on this project. Diversified capitalists do not have a viable project and instead passively invest their wealth. Entrepreneurs lose their ability to operate a firm at rate ϕ^l and then become diversified capitalists. Capitalists die at rate $\tilde{\delta}_d$. When this happens, the capitalist is replaced with offspring who either inherit the wealth and type of their parent, leaving the dynasty intact, or the dynasty breaks, and the new agent is reborn with the average wealth level of capitalists. The probability that the dynasty is broken conditional on death is π_0. We denote by $\delta_d = \tilde{\delta}_d \pi_0$, the rate at which dynasties are broken. When dynasties are broken, the newborn agent becomes an entrepreneur with probability ψ^0. Setting the initial fraction of entrepreneurs in the economy to $\bar{\psi} = \frac{\delta_d \psi^0}{\delta_d + \phi^l}$ ensures that the population structure remains intact over time.

Firms and technology. There are two types of intermediate goods-producing firms, namely (i) a representative traditional firm and (ii) a continuum of entrepreneurial firms. The representative traditional firm is entirely standard and owns and operates a capital stock K_t^T that evolves according to

$$\frac{dK_t^T}{K_t^T} = \left(\iota_t^T - \delta\right) dt + \sigma dZ_t \tag{6}$$

where ι_t^T and δ are the investment and depreciation rates respectively, and Z_t is an aggregate shock.[10] The firm finances this capital stock externally by issuing equity to the capitalists in the economy. The capital structure of the traditional firm is therefore $K_t = V_t^{T,\text{out}}$, where $V_t^{T,\text{out}}$ is the total amount of equity issued. The cost of this equity capital, its required return, is determined by competitive capital markets. In particular, this equity pays an expected return of r_t^T, to be determined in equilibrium, and has the same risk as the risk in the capital, so the return for investing in the equity of the traditional firm is

$$dR_t^T = r_t^T dt + \sigma dZ_t. \tag{7}$$

The firm hires labor from the workers at the wage rate w_t. The traditional firm uses a standard Cobb-Douglas technology to produce an output flow $Y_t^T dt = \underline{A}(K_t^T)^\alpha (L_t^T)^{1-\alpha} dt$. This is sold to at price p_t^T. The traditional firm maximizes expected profit flows, $\pi_t^T = \max_{L_t^T, K_t^T} p_t^T Y_t^T - w_t L_t^T - \left(\delta + r_t^T\right) V_t^{T,\text{out}}$ subject to $K_T = V_t^{T,\text{out}}$.[11] This implies that wages and rates of returns are equated to the value of marginal products of the respective factors of production:

$$w_t = p_t^T (1-\alpha) \frac{Y_t^T}{L_t^T}, \quad \text{and} \quad r_t^T + \delta = p_t^T \alpha \frac{Y_t^T}{K_t^T}. \tag{8}$$

Entrepreneurial firms produce the second type of intermediate goods. They also employ a Cobb-Douglas technology to produce an output flow $y_{it} dt = \bar{A} k_{it}^\alpha l_{it}^{1-\alpha} dt$, but where $\bar{A} > \underline{A}$ so that entrepreneurial firms have higher total factor productivity than does the traditional firm. Entrepreneurial firms hire labor at the wage rate w_t in the same competitive labor

[10] In Appendix B.2, I define the continuum of traditional firms that the representative traditional firm represents.

[11] Investment drops out of the optimization problem because investing one unit of capital decreases cash flows by one unit but instantaneously increases the value of the capital stock by one unit.

market as the traditional firm. The intermediate good entrepreneurial firms produce is sold to the final goods producer at a price p_t^E. The total quantity of this intermediate good is $Y_t^E = \int_{i \in E} y_{it}\, di$.

Each entrepreneurial capitalist manages the stock of capital used by their firm. The capital is subject to idiosyncratic risk. The stock of capital evolves according to

$$dk_{it} = (\iota_{it} - \delta)\, k_{it} dt + y_{it} \tilde{\sigma} dZ_{it} + k_{it} \sigma dZ_t + d\Delta_{it}^k \tag{9}$$

where ι_{it} and δ are the investment and depreciation rates, respectively, $d\Delta_{it}^k$ is net purchases of capital, Z_{it} is an idiosyncratic Brownian motion, Z_t is an aggregate Brownian motion, $\tilde{\sigma}$ and σ are scalars governing the loadings on these Brownian risks.

Note that the idiosyncratic shocks are proportional to output. This specification of the idiosyncratic risk is directly related to the risk specification in Di Tella and Hall (2022). One interpretation is that the idiosyncratic shocks become larger the more intensely the capital is used in production. This assumption has two implications. First, it makes the model *more* tractable because it will imply that the entrepreneurial firms and the traditional firms will choose the same labor-to-capital ratio. If the shocks were proportional to capital instead of output, the entrepreneurial firms would be less capital intensive than the traditional firms. The intuition for this is the following: for traditional firms, expanding production is associated with some marginal cost determined by the wage rate and required return on capital. For entrepreneurial firms, expanding production is also associated with higher risk. If the idiosyncratic shocks depended on capital alone, expanding by increasing capital would be risky on the margin, whereas expanding by hiring more labor would not. Hence, compared to the traditional firm, capital would be a relatively more costly factor of production when taking into account this risk cost. By having the idiosyncratic risk proportional to output, expanding by hiring more labor also becomes risky on the margin. This re-establishes the symmetry between capital and labor and ensures that the trade-off is not distorted by risk. Secondly, this assumption also implies that the entrepreneurs' share of income will come at the expense of both the pure labor share and the pure capital share. This will imply that the entrepreneurial firms have lower labor shares and lower pure capital shares, providing the model with non-trivial testable implications for factor income shares.

The return on capital for an entrepreneurial firm is

$$dR_{it}^k = \left(\frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}} \right) dt + \tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t \tag{10}$$

where $\tilde{\sigma}^k_{it} = \frac{y_{it}}{k_{it}}\tilde{\sigma}$ is the loading on the idiosyncratic Brownian.[12] Final output Y_t is produced by a representative firm using a CES-technology and the two types of intermediate goods,

$$Y_t dt = \left[\nu \left(Y^E_t\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(Y^T_t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} dt$$

where ε is the elasticity of substitution between the intermediate goods. This parameter governs the strength of the competition between the sectors.[13] The final goods producer's first-order conditions are

$$p^E_t = \nu \left(\frac{Y^E_t}{Y_t}\right)^{-\frac{1}{\varepsilon}}, \quad p^T_t = (1-\nu)\left(\frac{Y^T_t}{Y_t}\right)^{-\frac{1}{\varepsilon}}. \tag{11}$$

Financial markets. Any capitalist can issue or invest in zero-net supply riskless debt at the riskless rate r_t. Entrepreneurial capitalists can also issue equity. However, this outside financing is constrained. In particular, the entrepreneur faces a skin-in-the-game constraint so that at least a fraction χ of the risk in the firm must be retained. Letting v^{out}_{it} denote the total value of the liabilities issued to outsiders by entrepreneur i, the constraint is

$$\frac{k_{it} - v^{\text{out}}_{it}}{k_{it}} \geq \chi. \tag{12}$$

The risk in the liabilities issued to outsiders is determined by the riskiness of the productive assets of their firm, but the price of those liabilities, and hence their expected return, is determined in a competitive financial market. Outsiders hold the liabilities of firm i as part of a diversified portfolio of the liabilities of all firms and, therefore, do not require a risk premium for the idiosyncratic risk associated with firm i. Pricing by arbitrage then implies that the equilibrium expected return on the liabilities of firm i is $r^{\text{out}}_t = r_t + \varsigma_t \sigma = r^T_t$, where ς_t is the price of aggregate risk in the economy and r_t is the risk-free rate. Note in particular that the expected return on equity issued to outsiders is identical to the expected return to equity issued by traditional firms, r^T_t. This is because both carry the same amount of aggregate risk, and outsiders do not require compensation for idiosyncratic risk as they can diversify it away. The total return is therefore

$$dR^{\text{out}}_{it} = r^T_t dt + \tilde{\sigma}^k_{it} dZ_{it} + \sigma dZ_t. \tag{13}$$

[12] This is the instantaneous return to the existing capital stock and therefore does not include any references to capital purchases $d\Delta^k_{it}$.

[13] Relative to sector-specific capital adjustment costs, this imperfect substitutability assumption is more tractable. This is because sector-specific capital adjustment costs are both an intratemporal and intertemporal friction. Imperfect substitutability is solely intratemporal.

From the perspective of households investing in the firms, it is without loss of generality to assume that they invest in a mutual fund consisting of the liabilities of all firms in the economy, traditional and entrepreneurial, with return

$$dR_t^{\text{fund}} = r_t^T dt + \sigma dZ_t. \tag{14}$$

I purposely model improvements in entrepreneurial financing in a stylized fashion rather than modeling it after the particularities of today's venture capital industry. Specifically, I model innovation in the financing of entrepreneurial firms as a fall in χ, the minimum inside equity financing fraction. This is motivated by two considerations. First, this paper focuses on the consequences improvements in entrepreneurial financing have for top wealth inequality rather than on the sources of those improvements. Second, although this study focuses on a particular historical episode, the framework is applicable more generally. Other contexts in which improvements in entrepreneurial financing have impacted top wealth inequality differ in the details while sharing the operational mechanism studied in this paper.

The valuation of entrepreneurial firms. Note that the formulation of how entrepreneurial firms are financed in the model does not reference the number of shares the entrepreneurs issue or the prices of these shares. Instead, the financing of the entrepreneurial firms is expressed in terms of the amount of capital raised from outsiders and the expected return these outsiders receive. There is, of course, a link between the two ways of formulating the financing of these firms. Making this link explicit clarifies two things. First, it clarifies that the entrepreneurs' insider equity financing fraction χ should not be confused with their insider ownership fraction. Second, it demonstrates that the model produces deviations from Tobin's $q = 1$ using neither capital adjustment costs nor market power, which are the more common modeling devices that accomplish this.

An entrepreneur who has decided on operating a firm with total capital stock k_{it} must provide at least χk_{it} of the financing themself and can raise at most $(1-\chi)k_{it}$ from outsiders. Let N_0 be the initial number of shares, all owned by the entrepreneur. The number of shares the entrepreneur has to issue to the outsider, Δ_{Nt}, is then defined by

$$v_{it}^{\text{out}} \equiv \Delta_{Nt} p_{it} = (1-\chi)k_{it} \tag{15}$$

where p_{it} is the price per share issued. The equilibrium price per share issued, on the other hand is pinned down by the condition that the equilibrium expected return on equity to outsiders is $r_t^T dt$. In other words,

$$\frac{\left(\frac{\Delta_{Nt}}{N_0+\Delta_{Nt}}\right) k_{it}(1+r_{it}^k dt)}{p_{it}\Delta_{Nt}} = 1 + r_t^T dt \tag{16}$$

where the numerator is the payoff to the outsider and the denominator is the amount invested by the outsider. Equations (15) and (16) jointly pin down the price and the number of shares issued in terms of the expected returns and the outside financing fraction $1-\chi$:

$$\Delta_{Nt} = \frac{(1+r_t^T dt)(1-\chi)}{(r_{it}^k - r_t^T)dt + \chi(1+r_t^T dt)} N_0$$

$$p_{it} = \left(\frac{(r_{it}^k - r_t^T)dt + \chi(1+r_t^T dt)}{1+r_t^T dt}\right)\frac{k_{it}}{N_0}.$$

Note that measuring outsiders' stake in the firm as $p_{it}\Delta_{Nt}$, the price per share times the number of shares they hold, coincides with the model notion of the value of their stake in the firm, since by construction $p_{it}\Delta_{Nt} = (1-\chi)k_{it}$. That is, however, not true for the entrepreneur if there is a risk premium associated with entrepreneurship so that $(r_t^k - r_t^T) > 0$. In particular, the post-money valuation of the entrepreneur's shares is

$$p_{it}N_0 = \left(\frac{(r_{it}^k - r_t^T)dt + \chi(1+r_t^T dt)}{1+r_t^T dt}\right) k_{it} > \chi k_{it} \tag{17}$$

where the inequality follows from the fact that $(r_t^k - r_t^T) > 0$. This also illustrates that χ should not be confused with the entrepreneur's ownership share measured as the fraction of the outstanding shares the entrepreneur holds. Rather, χ is the insider financing share, the share of the financing that the entrepreneur provides.

The discrepancy stems from the fact that p_{it} is the price an investor with no exposure to the idiosyncratic risk in firm i is willing to pay for a share. This is more than what the entrepreneur associated with that firm is willing to pay for a share because the entrepreneur has to maintain a non-negligible exposure to the idiosyncratic risk in the firm and requires a risk premium for that.

This has important implications for the measurement of the value of an entrepreneurial firm, both in the context of this model and in reality. First, there is a gap between the market cap of the firm, as measured as the price per share times the number of shares outstanding, and the value of the capital stock invested in the firm. In this sense, the entrepreneurial firms in the model have Tobin's Q's that differ from 1. Specifically, the deviation from $q = 1$

is

$$q_{it} - 1 = \frac{p_{it}\left(N_0 + \Delta_{Nt}\right)}{k_{it}} = \frac{1 + r_{it}^k dt}{1 + r_t^T dt} - 1, \tag{18}$$

which is the geometric excess return to entrepreneurship. In other words, the model produces deviations from $q = 1$ without adjustment costs to capital and without market power. This is but one of the dimensions along which idiosyncratic risk and the payoff entrepreneurs earn from carrying it have similar implications as the presence of market power. Another such instance will be discussed when we examine the model's implications for the labor share of income.

Aggregates. The financial wealth in the economy is $N_t = N_t^E + N_t^D$, where $N_t^j = \int_{i \in j} n_{it}\, di$ is the financial wealth of capitalists of group $j \in \{E, D\}$. The share of financial wealth held by entrepreneurial capitalists is denoted by $\eta_t = \frac{N_t^E}{N_t}$. The financial wealth consists of claims on the productive assets of the economy, in other words, the real capital of the economy K_t. Since the financial wealth of the economy constitutes claims on the capital stock of the economy, we have $K_t = N_t^E + N_t^D$. The use of the capital stock is split between the traditional firm and the entrepreneurial firms. The share of the capital stock used by entrepreneurial firms is denoted $\kappa_t = \frac{K_t^E}{K_t}$. The labor-to-capital ratio is equalized across firms in equilibrium because the trade-off between labor and capital in production is the same for all firms. Therefore, the aggregate output can be written as

$$Y_t = A(\kappa_t) K_t^{\alpha} L^{1-\alpha} \tag{19}$$

where the aggregate total factor productivity (TFP) is

$$A(\kappa_t) = \left[\nu \left(\bar{A}\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(\underline{A}(1-\kappa_t)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{20}$$

which depends on the capital allocation. Aggregate investment in the economy is output less consumption. Therefore, the aggregate capital stock evolves according to

$$dK_t = \left(Y_t - C_t^E - C_t^D - C_t^W - \delta K_t\right) dt + \sigma K_t dZ_t \tag{21}$$

where C_t^E, C_t^D, and C_t^W are the consumption of entrepreneurial capitalists, diversified capitalists, and workers, respectively.

Entrepreneurs' problem. In this section, I solve for the entrepreneurs' consumption and portfolio choices. In particular, I will solve for the entrepreneurs' choice of how much idiosyncratic risk to bear, which will be key for this paper's result on top wealth inequality because these choices determine the dynamics of the entrepreneurs' wealth accumulation process. The net worth of an individual entrepreneur can be written as

$$n_{it} = \underbrace{k_{it}}_{\text{capital}} - \underbrace{v_{it}^{\text{out}}}_{\text{outsiders' equity}} - \underbrace{d_{it}}_{\text{debt}} + \underbrace{v_{it}^{\text{fund}}}_{\text{diversified holdings}}. \tag{22}$$

Each of the components of an entrepreneur's net worth is associated with some expected excess return and some risk. Table 1 summarizes the returns and risk associated with each component.

	Expected return	Risk
k_{it}:	$\frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}$	$\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t$
v_{it}^{out}:	$r_t + \varsigma_t \sigma$	$\tilde{\sigma}_{it}^k dZ_{it} + \sigma dZ_t$
v_{it}^{fund}:	$r_t + \varsigma_t \sigma$	σdZ_t
b_{it}:	r_t	0

Table 1: Risk-return profiles. $\tilde{\sigma}_{it}^k \equiv \frac{y_{it}}{k_{it}} \tilde{\sigma}$

As in the simplified framework of Section 2, we can express the entrepreneur's problem as a combination of a portfolio choice problem and a problem of choosing the optimal factor input mix. In particular, expressing each component of the firms capital structure relative to the entrepreneur's financial wealth by letting $\theta_{it}^k = \frac{k_{it}}{n_{it}}$, $\theta_{it}^{\text{out}} = \frac{v_{it}^{\text{out}}}{n_{it}}$, $\theta_{it}^{\text{fund}} = \frac{v_{it}^{\text{fund}}}{n_{it}}$, $-\theta_{it}^d = 1 - \theta_{it}^k + \theta_{it}^{\text{out}} - \theta_{it}^{\text{fund}}$, and by letting $x_{it} = \frac{y_{it}}{k_{it}}$ denote the ratio of output to firm capital, we can write the entrepreneurs' problem as follows:[14]

$$\max_{\{c_{it}, x_{it}, \theta_{it}^k, \theta_{it}^{\text{out}}, \theta_{it}^{\text{fund}}\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_{it})\, dt\right]$$
$$\frac{dn_{it}}{n_{it}} = \left(r_t + \theta_{it}^k \left(r_{it}^k - r_t\right) - \theta_{it}^{\text{out}} \varsigma_t \sigma + \theta_{it}^{\text{fund}} \varsigma_t \sigma - \frac{c_{it}}{n_{it}}\right) dt + \left(\theta_{it}^k - \theta_{it}^{\text{out}}\right) x_{it} \tilde{\sigma} dZ_{it}$$
$$+ \left(\theta_{it}^k - \theta_{it}^{\text{out}} + \theta_{it}^{\text{fund}}\right) \sigma dZ_t, \quad \text{where} \quad r_{it} = p_t^E x_{it} - w_t \left(\frac{x_{it}}{\bar{A}}\right)^{\frac{1}{1-\alpha}} - \delta \quad \text{and} \quad \frac{\theta_{it}^k - \theta_{it}^{\text{out}}}{\theta_{it}^k} \geq \chi. \tag{23}$$

As shown in Section B.4 of the Appendix, solving this problem and expressing the solution

[14]One implication of writing the entrepreneurs' problem as a Merton optimal portfolio choice problem is that we view the entrepreneur as choosing how much capital to hold and supply to their firm k_{it}, instead of how much capital to purchase $d\Delta_{it}^k$. Hence, as in Brunnermeier and Sannikov (2017), we make no explicit reference to the capital purchase decision.

in the unscaled variables implies

$$
\begin{aligned}
c_{it} &= \rho n_{it}, \quad y_{it} = \bar{A}\left(\frac{1-\alpha}{\alpha}\frac{r_t^T+\delta}{w_t}\right)^{1-\alpha} k_{it}, \quad k_{it} = \frac{r_{it}^k - r_t^T}{(\chi\tilde{\sigma}_t^k)^2} n_{it} \\
v_{it}^{\text{fund}} &= \frac{r_t^T - r_t}{\sigma^2} n_{it} - \chi k_{it}, \quad v_{it}^{\text{out}} = (1-\chi)k_{it}.
\end{aligned}
\tag{24}
$$

Note three important things. Firstly, all the decision rules are proportional to the entrepreneur's wealth, with the same proportionality for all entrepreneurs. This implies that the distribution of wealth within the group of entrepreneurs does not matter for aggregate quantities and prices. In particular, because $x_{it} \equiv y_{it}/k_{it}$ is identical for all entrepreneurs, the expected return to entrepreneurial capital is identical for all entrepreneurial firms so that we can write $r_{it}^k = r_t^k$. The same goes for the idiosyncratic risk exposure, $\tilde{\sigma}_{it}^k = \frac{y_{it}}{k_{it}}\tilde{\sigma} = \tilde{\sigma}_t^k$. Secondly, note that the skin-in-the-game constraint is always binding. This is because entrepreneurs have access to both issuing outside equity and buying shares of the mutual fund. The mutual fund has the same expected return as issuing outside equity does, but it has lower risk. Hence, entrepreneurs will want to short (issue) as much outside equity as possible. Finally, the labor-to-capital ratio in each entrepreneurial firm is $\frac{l_{it}}{k_{it}} = \frac{1-\alpha}{\alpha}\frac{r_t^T+\delta}{w_t}$, which is the same as in the traditional sector. This means that κ_t is not only the fraction of capital employed by the entrepreneurial sector but also the fraction of labor employed by the entrepreneurial sector, so aggregate supply of the intermediate good produced by entrepreneurial firms is $Y_t^E = \bar{A}\kappa_t K_t^\alpha L_t^{1-\alpha}$.

Diversified capitalists and workers. Diversified capitalists have wealth N_t^D in the aggregate. They invest this wealth in the mutual fund and riskless bonds. Diversified capitalists have log utility. Their consumption as a group is $C_t^D = \rho N_t^D$, and the fraction of their wealth invested in the mutual fund is $\theta_t^D = \frac{r_t^T - r_t}{\sigma^2}$. Workers supply labor inelastically and consume their labor income so that $C_t^W = w_t L$.

3.1 Characterizing the equilibrium

In this section, I begin by characterizing the equilibrium of the model at a given point in time by considering the interactions between supply and demand for capital to entrepreneurial firms and traditional firms, respectively. I then characterize the dynamic equilibrium by describing how the economy's aggregate state variables evolve over time.

The equilibrium at a given point in time can be characterized in terms of the capital stock K_t and the share of wealth owned by entrepreneurs $\eta_t \equiv \frac{\int_{i\in E} n_{it}\, di}{N_t} = \frac{\int_{i\in E} n_{it}\, di}{K_t}$. Given values of these state variables, the equilibrium fraction of the capital stock operated by entrepreneurial firms $\kappa_t = \frac{K_t^E}{K_t}$ and the equilibrium excess return to entrepreneurial capital $r_t^k - r_t^T$ are jointly pinned down by the following system of equations:

$$\begin{aligned} \frac{\kappa_t}{\eta_t}\left(\chi\tilde{\sigma}_t^k\right)^2 &= r_t^k - r_t^T \\ r_t^k - r_t^T &= \left(\bar{A}p^E(\kappa_t) - \underline{A}p^T(\kappa_t)\right)\left(\frac{L}{K_t}\right)^{1-\alpha} \end{aligned} \tag{25}$$

where the prices are expressed as functions of κ_t as

$$p_t^E = \nu\left(\frac{\bar{A}\kappa_t}{A(\kappa_t)}\right)^{-1/\varepsilon}, \quad p_t^T = (1-\nu)\left(\frac{\underline{A}(1-\kappa_t)}{A(\kappa_t)}\right)^{-1/\varepsilon},$$

and aggregate TFP $A(\kappa_t)$ is defined in equation (20). The first of these equations is the relative supply of capital to entrepreneurial firms. It is relative because the quantity variable is κ_t, the *fraction* of the aggregate capital stock operated by the entrepreneurial firms, and the price variable is the *excess return* $r_t^k - r_t^T$. This comes directly from the solution to the entrepreneurs' problem in (24), noting that the linearity of entrepreneurs' decision rules implies $\frac{k_{it}}{n_{it}} = \frac{K_t^E}{W_t^E} = \frac{\kappa_t}{\eta_t}$. The supply is upward sloping in the excess return to capital in the entrepreneurial sector as entrepreneurs are willing to invest larger amounts of capital in their firms when the excess return is high. From an asset pricing and portfolio choice perspective, this is commonly referred to as the entrepreneurs' risky asset *demand*, productive capital being the risky asset. However, of course, an entrepreneur's demand for capital as an investment vehicle constitutes the supply of capital to that entrepreneur's firm.

The second equation is instead the entrepreneurial firms' relative demand schedule, which can be derived by combining market clearing for capital, $K_t^E = K_t - K_t^T$, with the fact that the traditional sector's demand for capital is $K_t^T = \alpha\left(\frac{Y_t^T}{r_t^T+\delta}\right)$, according to (8).

In Section B.6 of the Appendix, I provide the definition of equilibrium. In Section B.7 of the Appendix, I show that there is a unique resource allocation κ_t that solves this system. Given this equilibrium allocation of productive resources across the two sectors, all other prices and quantities are pinned down as well at the given point in time. These time-t prices and quantities determine the evolution of the state variables K_t and η_t going forward. These results are summarized in the following proposition.

Proposition 1. *Given values of K_t and η_t, there is a unique solution $\kappa_t \in [0,1]$ to the system of equations in (25). Given this solution, the relative prices of the intermediate goods are given by (11), while the other prices are given by*

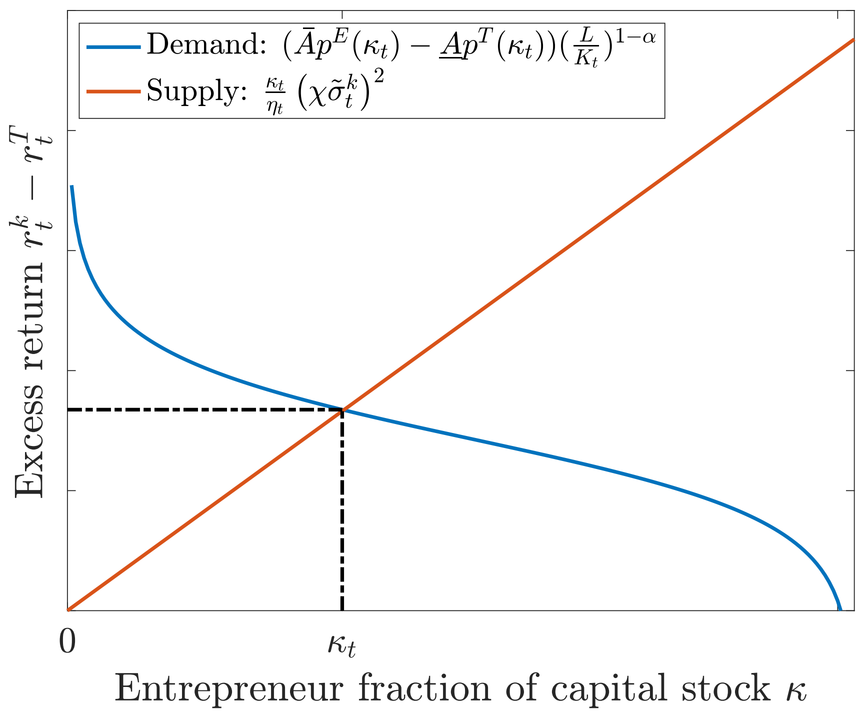


Figure 2: The equilibrium allocation of capital to entrepreneurs and excess return to entrepreneurial capital.

$$r_t^T + \delta = p_t^T \alpha \underline{A}\left(\frac{L}{K_t}\right)^{1-\alpha}, \quad r_t^k = r_t^T + \frac{\kappa_t}{\eta_t}\left(\chi\tilde{\sigma}_t^k\right)^2$$
$$r_t = r_t^T - \sigma^2, \quad w_t = (1-\alpha)\underline{A}\left(\frac{L}{K_t}\right)^{1-\alpha}. \tag{26}$$

The evolution of an individual entrepreneur's wealth is

$$\frac{dn_{it}}{n_{it}} = \left(r_t^E - \rho\right)dt + \tilde{\sigma}_t^E dZ_{it} + \sigma dZ_t \tag{27}$$

where $\tilde{\sigma}_t^E = \frac{\kappa_t}{\eta_t}\chi\tilde{\sigma}_t^k$ *is the entrepreneurs' idiosyncratic risk exposure and* $r_t^E = r_t + \left(\tilde{\sigma}_t^E\right)^2 + \sigma^2$ *is the expected return to the entrepreneurs' invested wealth. Finally, the system of stochastic differential equations that govern the evolution of the aggregate capital stock and entrepreneurs' wealth share is*

$$\frac{dK_t}{K_t} = \left(\kappa_t r_t^k + (1-\kappa_t)r_t^T - \rho\right)dt + \sigma dZ_t$$
$$\frac{d\eta_t}{\eta_t} = \left((1-\eta)\left(\tilde{\sigma}_t^E\right)^2 + \frac{(\bar{\psi} - \eta_t)}{\eta_t}(\delta_d + \phi^l)\right)dt. \tag{28}$$

The entrepreneurial appraisal ratio. A critical determinant of both the evolution of the wealth share of entrepreneurs as a group and the wealth accumulation process of individual entrepreneurs is their idiosyncratic risk exposure $\tilde{\sigma}_t^E$. This idiosyncratic risk exposure is key for understanding the level of top wealth inequality and the prevalence of "self-made" fortunes because it determines the likelihood of extreme upward wealth trajectories.

$\tilde{\sigma}_t^E$ appears directly as entrepreneurs' risk loading on their idiosyncratic risk process. However, because the equilibrium risk premium is determined by entrepreneurs' risk bearing, it also appears in the drift term of entrepreneurs' wealth growth process through its effect on the return on their invested wealth $r_t^E = r_t + \left(\tilde{\sigma}_t^E\right) + \sigma^2$.

Closer inspection reveals that this idiosyncratic wealth exposure is, in fact, equal to the so-called appraisal ratio associated with investments in entrepreneurial capital. The appraisal ratio, sometimes called the information ratio, is a close cousin of the more well-known Sharpe ratio but measures instead the risk-reward trade-off associated with investing in an asset with idiosyncratic risk relative to an asset with the same systematic risk but no idiosyncratic risk. The fact that entrepreneurs choose an idiosyncratic risk exposure equal to the appraisal ratio is a special case of the solution to the standard optimal portfolio choice problem of Merton (1969). The fact that entrepreneurs have logarithmic utility greatly simplifies the analysis of the model, as the optimal risk exposure is unaffected by changes in the investment opportunity set.

In this model, the appraisal ratio is defined relative to the mutual fund:

$$\text{appraisal ratio} = \frac{r_t^k - r_t^T}{\chi \tilde{\sigma}_t^k} = \frac{\kappa_t}{\eta_t} \chi \tilde{\sigma}_t^k = \tilde{\sigma}_t^E. \tag{29}$$

In other words, entrepreneurs choose an idiosyncratic risk exposure equal to the appraisal ratio associated with entrepreneurial capital. When the idiosyncratic risk-reward trade-off is more attractive, they choose a larger exposure, and their wealth grows faster on average at the individual level, as does the wealth share of entrepreneurs as a group. As shown below, this appraisal ratio will also determine top wealth inequality, and the effect of improved entrepreneurial financing on top wealth inequality will work through its effect on this appraisal ratio.

3.2 Steady state

In this section, I derive a closed-form formula for Pareto tail inequality in steady state as a function of the steady state wealth share of entrepreneurs. I thereby show a direct analytical link between the share of wealth entrepreneurs hold and the level of tail inequality. In particular, tail inequality will increase when entrepreneurs hold a larger fraction of wealth. Then, I describe how the steady state risk-reward trade-off that entrepreneurs face pins down the amount of risk they bear and how that, in turn, determines Pareto inequality.

A steady state of the economy is characterized by a pair of values for the capital stock and entrepreneurs' wealth share, K_{ss} and η_{ss}, such that $\frac{dK_t}{K_t} = 0$ and $\frac{d\eta_t}{\eta_t} = 0$. The presence of aggregate shocks to capital will, in general, prevent the economy from reaching, let alone staying in, any steady state. For this section, I study the economy's behavior along a path of zero realized aggregate shocks. In other words, I assume $dZ_t = 0$ for an indefinite time, which corresponds to studying the median path of the economy. This differs from shutting down aggregate shocks by setting $\sigma = 0$. In particular, we study the *realized* behavior of the economy in a setting where shocks are still possible but happen not to materialize.

Entrepreneurs' wealth share and Pareto inequality. In a steady state, the drift and volatility governing the wealth accumulation process of each entrepreneur is described by a geometric Brownian motion. In particular,

$$\frac{dn_{it}}{n_{it}} = \left(r_{ss} + \left(\tilde{\sigma}^E_{ss}\right)^2 + \sigma^2 - \rho\right) dt + \tilde{\sigma}^E_{ss} dZ_{it}. \tag{30}$$

The combination of individual wealth growing according to a geometric Brownian motion with entrepreneurial dynasties interrupted by death or type-switching implies that the steady state distribution of entrepreneurs' wealth follows a double Pareto distribution. The so-called Pareto tail coefficient describes the thickness of the right tail of this distribution. This tail coefficient is determined by the drift and volatility of the wealth accumulation process and the death and switching rates. Specifically, in Appendix C.3, I show that the stationary Kolmogorov forward equation that pins down the Pareto tail coefficient ζ_{ss} is of the well-known form:

$$0 = \zeta_{ss}\mu^E_{ss} + \frac{(\tilde{\sigma}^E_{ss})^2}{2}\zeta_{ss}\left(\zeta_{ss} - 1\right) - \left(\delta_d + \phi^l\right) \tag{31}$$

where $\mu^E_{ss} = r_{ss} + \left(\tilde{\sigma}^E_{ss}\right)^2 - \rho$ is the drift of the entrepreneurs' wealth accumulation process. The key to the results in this paper is that the model implies a direct relationship between drift and the volatility of entrepreneurs' wealth accumulation process in a steady state equilibrium. In particular, in equation (28), $\frac{dK_t}{K_t} = 0$ implies that $\mu^E_{ss} = (1 - \eta_{ss})\left(\tilde{\sigma}^E_{ss}\right)^2$, and $\frac{d\eta_t}{\eta_t} = 0$ implies that $(\tilde{\sigma}^E_{ss})^2 = \frac{(1-\frac{\bar{\psi}}{\eta_{ss}})(\delta_d+\phi^l)}{1-\eta_{ss}}$. In other words, the drift and the volatility are both directly related to the wealth share of entrepreneurs in a steady state equilibrium. This allows us to characterize the Pareto tail coefficient in terms of the steady state wealth share of entrepreneurs. The following proposition is proved in Appendix C.3.

Lemma 2. *The steady state right Pareto tail coefficient of entrepreneurs' wealth is*

$$\zeta = \eta_{ss} - \frac{1}{2} + \sqrt{\left(\eta_{ss} - \frac{1}{2}\right)^2 + \frac{2\eta_{ss}(1-\eta_{ss})}{\eta_{ss} - \bar{\psi}}} \tag{32}$$

where η_{ss} is the steady state share of wealth entrepreneurs own. Moreover $\frac{\partial \zeta}{\partial \eta_{ss}} < 0$. Hence, keeping fixed the population fraction $\bar{\psi}$, tail inequality $1/\zeta$ will be higher when entrepreneurial capitalists hold a larger fraction of wealth in the economy.

Equation 32 is strictly decreasing in η_{ss} so that the tail is thicker the higher the share of wealth owned by entrepreneurs. This expression for the tail coefficient provides a direct analytical link between the cross-sectional distribution of wealth and the share of wealth held by entrepreneurs. Understanding how structural changes in the economy affect steady state top wealth inequality thus boils down to understanding how those structural changes affect the steady wealth share of entrepreneurs.

What determines the steady state wealth share of entrepreneurs? Looking at equation (28), $\frac{d\eta_t}{\eta_t} = 0$, we see that the steady state value η_{ss} is pinned down by the exogenous demographic parameters, δ_d, ϕ_l, and $\bar{\psi}$, as well as the idiosyncratic volatility of entrepreneurs' wealth $\tilde{\sigma}^E_{ss}$, which is endogenous:

$$(\tilde{\sigma}^E_{ss})^2 = \frac{(1 - \frac{\bar{\psi}}{\eta_{ss}})(\delta_d + \phi^l)}{1 - \eta_{ss}}. \tag{33}$$

From this equation, we see a strictly positive relationship between the steady state wealth share of entrepreneurs and their steady state idiosyncratic risk exposure. In other words, given the values of the demographic parameters, a steady state associated with a higher level of idiosyncratic risk exposure will be associated with a higher wealth share for entrepreneurs. This is because a higher idiosyncratic risk exposure will be associated with a larger idiosyncratic risk premium or, equivalently, a larger precautionary savings motive for entrepreneurs. That implies that their expected wealth growth rate will be higher than that of the diversified capitalists, which in turn implies a larger steady state wealth share.

No other endogenous objects appear in the steady state equation (33) for the entrepreneurs' wealth share and, consequently, in the steady state Pareto tail coefficient. In particular, apart from the entrepreneurs' idiosyncratic risk exposure $\tilde{\sigma}^E_{ss}$, which is endogenous, only exogenous demographic parameters appear in equation (33). Therefore, the response of the steady state Pareto tail coefficient to any non-demographic change in the economy must go via changes in the idiosyncratic risk exposure of entrepreneurs. Specifically, any change in the economy that increases the idiosyncratic risk exposure of entrepreneurs will increase entrepreneurs' share of wealth, lower the Pareto tail coefficient, and thereby increase top wealth inequality.

Finally, recall that entrepreneurs choose an idiosyncratic risk exposure equal to the appraisal ratio associated with entrepreneurial investment. These insights will allow us to thoroughly summarize the effect of improved entrepreneurial financing on top wealth inequality since we only need to determine how improved entrepreneurial financing affects the appraisal ratio associated with entrepreneurial investment.

Improved entrepreneurial financing and steady state Pareto inequality. The model features only one friction, the constraint on equity issuance that entrepreneurs face. Recall that the severity of this constraint is captured by the parameter χ, the fraction of the risk in the firm that must be borne by the entrepreneur themself. Improved entrepreneurial financing in this context thus refers to a fall in the parameter χ. Using the fact that any non-demographic change in the economy that affects the Pareto tail coefficient must operate via its effect on the amount of idiosyncratic risk that entrepreneurs choose to bear, we therefore have the following proposition:

Lemma 3. *Improvements in entrepreneurial financing, understood as a relaxation of the equity issuance constraint (a fall in χ), leads to a fall in the Pareto tail coefficient ζ (and, therefore a rise in Pareto inequality $1/\zeta$) if and only if it raises the idiosyncratic risk exposure of entrepreneurs, which is, in turn, equal to the appraisal ratio associated with entrepreneurship:*

$$\tilde{\sigma}^E_{ss} = \frac{r^k_{ss} - r^T_{ss}}{\chi \tilde{\sigma}^k_{ss}} \equiv \textit{appraisal ratio}. \tag{34}$$

In other words, improved financing for entrepreneurs leads to a rise in top wealth inequality if it makes the trade-off related to idiosyncratic risk bearing more attractive. Figure 3 depicts the relationship between the Pareto inequality $1/\zeta$ and the appraisal ratio associated with entrepreneurship, capturing the essence of the above proposition.

Examining the mechanism: the role of the elasticity of substitution ε. To understand the mechanism behind the effect of improved entrepreneurial financing, we consider the effect of a fall in χ, the inside equity constraint. This fall in χ induces a reallocation of capital from the traditional sector towards the entrepreneurial sector. This can be seen from (25), which we recall can be written as

$$\frac{\kappa_t}{\eta_t}(\chi \tilde{\sigma}^k_t)^2 = \left(\bar{A} p^E(\kappa_t) - \underline{A} p^T(\kappa_t)\right) \left(\frac{L}{K_t}\right)^{1-\alpha} \tag{35}$$

where the prices are expressed in terms of κ_t as

$$p^E_t = \nu \left(\frac{\bar{A}\kappa_t}{A(\kappa_t)}\right)^{-1/\varepsilon}, \quad p^T_t = (1-\nu)\left(\frac{\underline{A}(1-\kappa_t)}{A(\kappa_t)}\right)^{-1/\varepsilon},$$

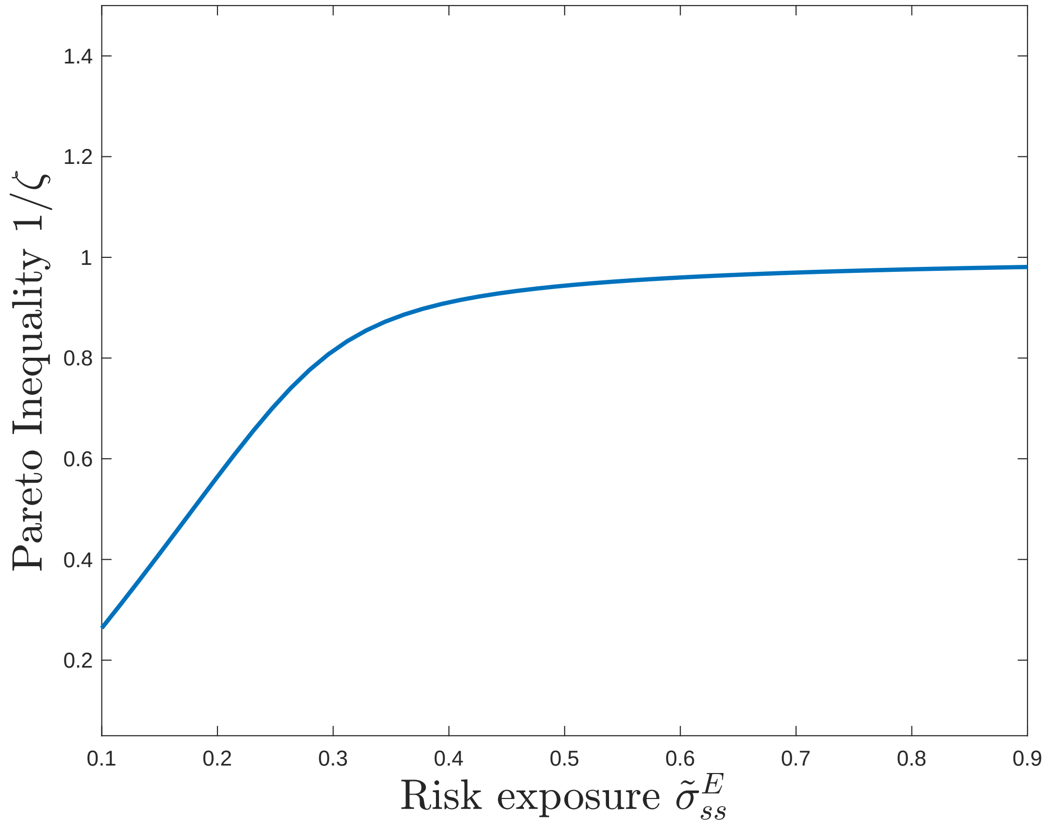


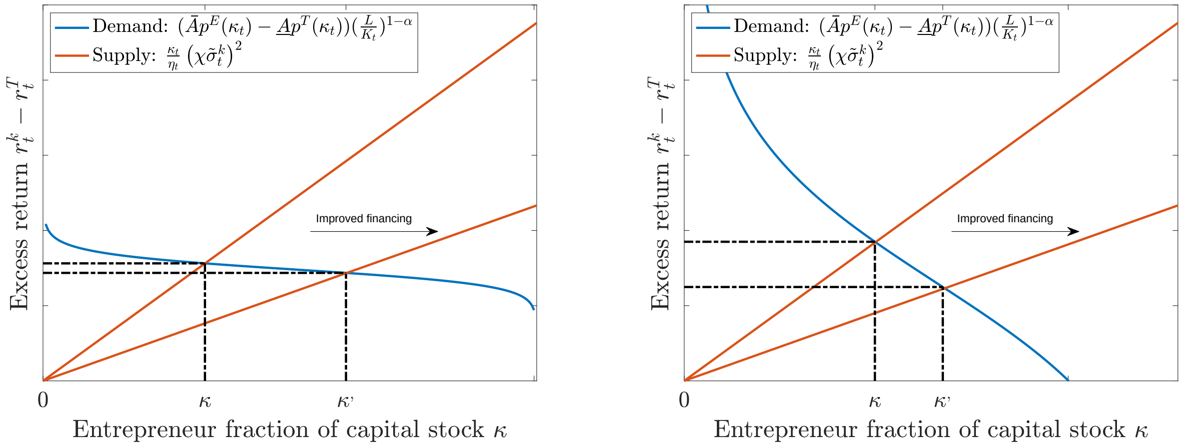
Figure 3: Pareto tail inequality $1/\zeta$ and entrepreneurs' idiosyncratic risk exposure $\tilde{\sigma}^E_{ss}$ across steady states.

and $A(\kappa_t)$ is defined in equation (20). On impact, η_t and K_t, and therefore also $\tilde{\sigma}^k_t = \frac{Y^E_t}{K^E_t}\tilde{\sigma} = \bar{A}\tilde{\sigma}\left(\frac{L}{K_t}\right)^{1-\alpha}$, are fixed, so that the left-hand side is simply an increasing linear function of κ_t. In contrast, the right-hand side is a strictly decreasing function of κ_t. The fall in χ shifts the supply of capital to entrepreneurial firms, on the left-hand side, outward, leaving the demand schedule unaffected. The new equilibrium features a higher κ_t and a lower excess return than in the initial steady state. Figure 4 is a graphical representation of this for two different values of the elasticity ε.

What happens to the appraisal ratio, which we know determines entrepreneurs' risk exposure as well as Pareto inequality? Rewriting the above equation in terms of the appraisal ratio, we see that

$$\tilde{\sigma}^E_t = \text{appraisal ratio} = \frac{\bar{A}p^E_t(\kappa_t) - \underline{A}p^T_t(\kappa_t)}{\chi\tilde{\sigma}\bar{A}} \tag{36}$$

so that the appraisal ratio may move up or down depending on how much of the fall in χ in the denominator is offset by a fall in the excess return in the numerator. The fall in



(a) High elasticity

(b) Low elasticity

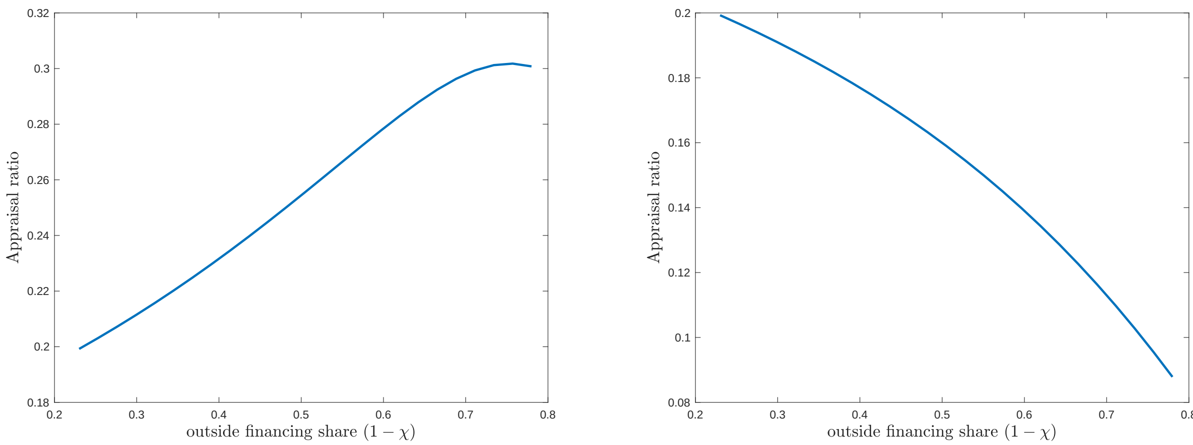
Figure 4: The effect of reduced financing frictions χ on capital allocation to entrepreneurs.

the numerator is determined by how much the intermediate goods prices $p_t^E(\kappa_t)$ and $p_t^T(\kappa_t)$ change. The sensitivity of these prices is, because of the CES setup, determined by the constant elasticity of substitution parameter ε.

If the elasticity of substitution between the two sectors is high, the market adjusts primarily via quantities and not prices; that is, the excess return in the numerator is relatively stable. In this case, the appraisal ratio rises. If, on the other hand, ε is low, prices react strongly in response to any reallocation of capital. The fall in the excess return in the numerator will then be larger than the fall in the denominator, and the appraisal ratio will fall. Figure 5 displays the relationship between the steady state appraisal ratio and the outside financing fraction $1 - \chi$. Figure 5a depicts the relationship when the elasticity of substitution is high, and 5b when this elasticity is low.

An interesting observation is that as the outside financing fraction becomes very large, the steady state appraisal ratio starts to decline. This happens because as the risk costs associated with production in the entrepreneurial sector decline, the entrepreneurial sector starts taking over all production in the economy. When this happens, the competitive pressure within the entrepreneurial sector becomes more severe since there is not much capital that can be squeezed out of the traditional sector anymore. The increased competitive pressure between entrepreneurs for the existing capital stock then drives down the excess return to entrepreneurship so that the appraisal ratio falls.

The dynamic response of the economy after impact will also depend on whether the appraisal ratio rises or falls. Recalling the equations for the evolution of the state variables in (28):



(a) High elasticity

(b) Low elasticity

Figure 5: Relationship between the steady state appraisal ratio and outside financing share $1-\chi$.

$$
\begin{aligned}
\frac{dK_t}{K_t} &= \left(\kappa_t r_t^k + (1-\kappa_t) r_t^T - \rho\right) dt + \sigma dZ_t \\
\frac{d\eta_t}{\eta_t} &= \left((1-\eta)\left(\tilde{\sigma}_t^E\right)^2 + \frac{(\bar{\psi}-\eta_t)}{\eta_t}(\delta_d + \phi^l)\right) dt,
\end{aligned}
\tag{37}
$$

we see that a rise in the appraisal ratio will lead to an increase in the appraisal, which will raise the drift of η_t, which consequently starts to grow. The behavior of the capital stock also depends on the strength of the reallocation of capital relative to the reaction of prices to this reallocation. The expected accounting return to the capital stock is the weighted average expected return in the two sectors $\kappa_t r_t^k + (1-\kappa_t) r_t^T = r_t^T + \kappa_t(r_t^k - r_t^T)$. When capital is reallocated in response to the fall in χ, the capital stock will grow to the extent that the reallocation of capital to the higher-return entrepreneurial sector constitutes a stronger force than the excess return in that sector.

In this case, the resulting growth in the entrepreneurs' wealth share and the capital stock will induce entrepreneurs to scale up further and, therefore, increase κ_t over time. This increase in κ_t will lower the excess return and the appraisal ratio over time relative to the level reached on impact. What matters for the behavior of top inequality, in the long run, is whether or not the economy settles on an appraisal ratio that is higher or lower than in the initial steady state. Suppose the elasticity of substitution between the sectors is high enough. Then, the new steady state appraisal ratio will be higher than before, implying a larger share of wealth owned by entrepreneurs, faster wealth dynamics for entrepreneurs, and higher Pareto inequality. I summarize this discussion in the following proposition that says that inequality increases when χ falls, provided that the elasticity of substitution is high enough:

Proposition 2. *Suppose the economy is in an initial steady state $s_0 = (\eta_0, K_0, \kappa_0)$, where $\kappa_0 \in (0, 1)$ and the initial inside equity constraint parameter is χ_0. Let $\zeta(\chi)$ denote the steady state Pareto tail coefficient as a function of χ. Then, there exists a $\varepsilon^*_{s_0}$ such that if $\varepsilon > \varepsilon^*_{s_0}$, then $\frac{d\zeta}{d\chi}(\chi_0) < 0$.*

Proof. By Lemma 3, we need to show that the steady state appraisal ratio is increasing in χ for high values of ε. Let $\kappa(\chi)$ denote the steady state κ as a function of χ. Then, we have by equation (36) evaluated in steady state:

$$\frac{d \log \tilde{\sigma}^E_{ss}}{d \log \chi} = \frac{d \log \left(\bar{A} p^E(\kappa(\chi)) - \underline{A} p^T(\kappa(\chi))\right)}{d \log \chi} - 1.$$

The prices can be made arbitrarily insensitive to changes in $\kappa(\chi)$ by picking a high enough value for ε. That is, there exists some $\varepsilon^*_{s_0}$ such that if $\varepsilon > \varepsilon^*_{s_0}$, we have $\frac{d \log\left(\bar{A} p^E(\kappa(\chi)) - \underline{A} p^T(\kappa(\chi))\right)}{d \log \chi} < 1$. This proves the result. □

Note that this proposition is local because the threshold value $\varepsilon^*_{s_0}$ depends on the steady state the economy starts in. This means that the relationship between χ and Pareto inequality can be non-monotone. As we saw in Figure 5a, the relationship turns around when the outside financing fraction $1 - \chi$ becomes large. This holds even in the case where the goods are perfect substitutes. In particular, we have the following proposition, which I prove in Appendix C.5:

Proposition 3. *Even with perfect substitutes, $\varepsilon = \infty$, there is a value χ^* such that if $\chi < \chi^*$, a further fall in χ reduces Pareto inequality.*

The intuition is that with perfect substitutes when χ becomes low enough, the entrepreneurial firms take over the entire economy so that $\kappa_{ss} = 1$. When κ_{ss} reaches this maximum value, entrepreneurs can no longer scale up at the expense of traditional firms. In this case, the model reduces to a one-sector model, and inequality falls when risk sharing improves because the scaling-up effect is mute. As I discuss in the next section, the absence of a sector from which entrepreneurs can attract resources is one of the reasons earlier work has found that improvements in risk sharing reduce inequality.

3.3 Why the two-sector setup is important

One key prediction of the model is that wealth inequality increases in response to improved entrepreneurial financing, provided that excess returns associated with entrepreneurship do not fall too much when entrepreneurs scale up. The degree to which the equilibrium excess return to entrepreneurial activity falls when entrepreneurs want to scale up depends on how fiercely they compete with each other for the existing capital stock. The presence of the traditional sector is critical for this model prediction. The traditional sector constitutes a source from which the entrepreneurial sector can attract capital, relieving the competitive pressure. Instead of drawing capital from other entrepreneurial firms, which puts downward pressure on the excess return to entrepreneurship, they can draw capital from the traditional sector.

Consider the extreme case where there is no sectoral reallocation possible. This would be the case in a one-sector model where all firms are entrepreneurial or where the output produced by entrepreneurial firms and traditional firms are perfect complements so that they do not compete in the output market. In response to improved risk sharing, entrepreneurs want to scale up. However, because the capital stock is fixed in the short run, they cannot scale up in the aggregate.[15] Hence, the equilibrium expected return has to fall in order to render the entrepreneurs content with operating the existing capital stock. Without the ability to scale up, entrepreneurs' risk exposure cannot rise. Because they operate a capital stock of the same size as before but carry a smaller fraction of the associated risk, their risk exposure unambiguously falls, as does wealth inequality. This mechanism is at the heart of why Hui (2023) finds that improved risk sharing for entrepreneurs lowers wealth inequality in a one-sector model. Peter (2021) also studies a model wherein all production is in the hands of entrepreneurs. The model is rich and closer in spirit to Quadrini (2000) and Cagetti and De Nardi (2006), which in turn are based on the Aiyagari (1994) framework. Peter (2021) also finds that improved risk sharing for entrepreneurs reduces steady state wealth inequality. However, because of the richer model framework, it is slightly harder to evaluate analytically precisely which of the model features produces this result. The results of the present paper suggest that it may be that the general equilibrium reallocation effect is not strong enough, perhaps because of the one-sector setup.

4 An Empirical Connection Between Improved Equity Financing and Top Wealth Inequality?

In the previous section, I established conditions under which improvements in risk sharing for entrepreneurs lead to higher top wealth inequality. In the next section, I will conduct a simple numerical exercise to understand whether improved risk sharing has any quantitative bite. In this section, I briefly describe the empirical motivation behind the numerical

[15] This remains roughly true in the long run as well *despite* the fact that the long-run elasticity of capital supply is perfectly elastic in this model.

experiment in the next section. That experiment is motivated by two sets of observations. The first set relates to the characteristics of the wealthiest Americans today, how they became wealthy, and the rise of venture capital and venture capital-backed firms. Specifically, the wealthiest Americans of today are, to a larger extent than in decades past, founders or early investors in entrepreneurial firms. These individuals were propelled to the top of the wealth distribution by raising substantial amounts of capital from outside investors, often venture capital funds. This allowed them to operate much larger firms than their wealth would have admitted.[16] Relatedly, venture capital has grown tremendously and has undergone what has been referred to as a revolution.[17] The second set of facts relates to the evolution of measured top wealth inequality. Top wealth shares have risen substantially over the past half-century.[18] Especially noteworthy is that the observed rise in top wealth shares has been fractal, meaning that wealth inequality has risen *within* the top as well: not only has the top 1% wealth share risen, the top 0.01% share of the top 1% has risen as well. In other words, Pareto inequality has increased.

The central proposition advanced by this paper is that these two sets of facts may be intimately related: improvements in the ability of entrepreneurs to raise outside equity capital and offload risk to financial markets, as exemplified by, but not limited to, the growth of the venture capital industry, may have contributed to the observed pattern of increased top wealth inequality. I now briefly discuss these motivating facts in more detail before presenting the parameterized model in Section 5.

The rise of venture capital-backed firms. The mechanism at the heart of this paper connects changes in the ability of innovative entrepreneurs to raise equity capital and offload risk to financial markets to the reallocation of economic activity to cutting-edge entrepreneurial firms and rising top inequality. Regarding the reallocation of economic activity, Gornall and Strebulaev (2021) and Greenwood et al. (2022) document that firms with a history of VC-backing constituted around 0–5% of the total market capitalization before and up to 1980, rising to around 45–50% in 2020. Moreover, among firms founded after 1968, Gornall and Strebulaev (2021) document that firms with a history of VC-backing constituted around 50% of market cap in 1980, rising to 75% in 2020. Figure 6 from Gornall and Strebulaev (2021) summarizes the evolution of venture capital-backed firms. They argue that regulatory changes implemented through the 1974 Employee Retirement Income Security Act (ERISA) and its subsequent reinterpretation in 1979 created a substantial divergence in the creation rate of large successful companies between the U.S. and comparable

[16]Kaplan and Rauh (2013)

[17]See Gompers and Lerner (2001).

[18]Although there is some disagreement regarding the precise magnitudes of the increases (see Saez and Zucman (2016) and Smith, Zidar, and Zwick (2021)).

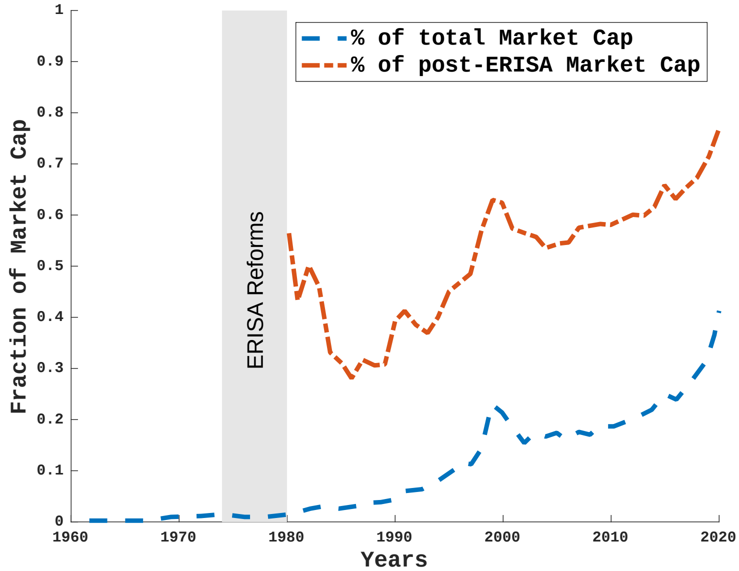


Figure 6: Evolution of venture capital-backed firms. From Gornall and Strebulaev (2021).

countries. These reforms allowed a broader set of investors to invest in venture capital, previously regarded as too risky.[19] Many successful venture capital-backed firms and their associated founders are household names by now: Tesla, Amazon, Google, Uber, and Apple, to name but a few. It is important to note that the venture capital is but a fraction of the outside financing that these firms receive. The purpose of highlighting the growth of firms with a history of venture capital-backing is not to argue that venture capital was responsible for this growth. Rather, because venture capital is explicitly aimed at providing financing for cutting-edge entrepreneurial firms, the growth of venture capital is evidence of a reallocation of economic activity to these types of firms.

To highlight the role of firms with a history of venture capital-backing for top wealth, Table 2 compares the ten wealthiest individuals on the Forbes 400 list in 2022 with the ten wealthiest individuals on the first edition of that list in 1982. As pointed out by Kaplan and Rauh (2013) and more recently by Gomez (2023), we see that the table reflects the observation that the number of "self-made" entrepreneurs within the top 10 is markedly higher now. We also see that many of the wealthiest individuals in 2022 are associated with venture capital-backed firms.[20]

Pareto inequality. Saez and Zucman (2016) document a 13-percentage-point increase in

[19] See also Greenwood et al. (2022) for additional evidence.

[20] This is not to say that the VC connection is necessarily causally responsible for the rise of these firms. It could have been that they had just brief encounters with venture capitalists at some early stage. Instead, the point is that the fact that they have a VC connection suggests that their firms are the types of firms that correspond to the entrepreneurial firms in the model of this paper.

Table 2: Comparison of Forbes Top 10: 2022 and 1982

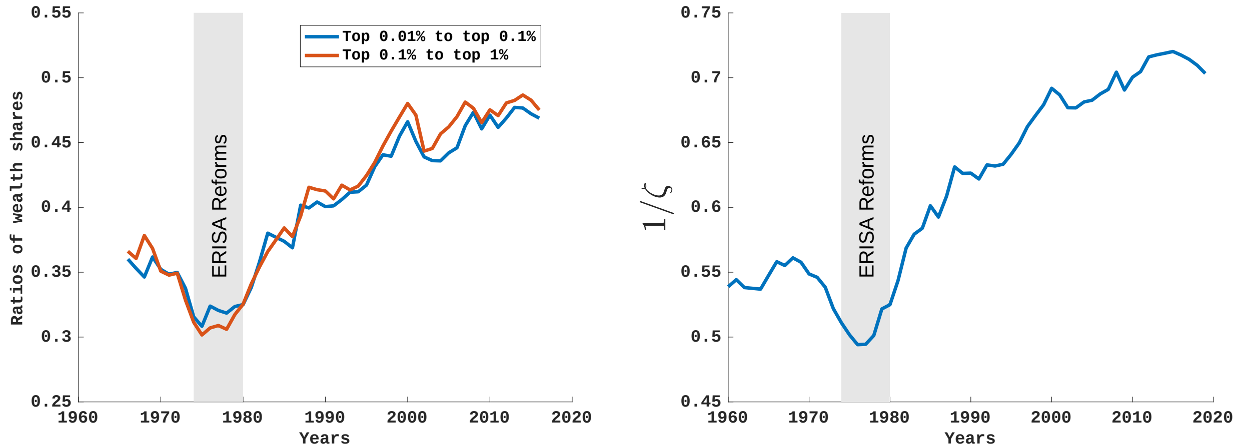
	2022				1982				
	Name	Firm	Self-made	VC connected		Name	Firm	Self-made	VC connected
1	Elon Musk	Tesla	✓	✓	1	Daniel K. Ludwig	Exportadora de Sal	✓	✗
2	Jeff Bezos	Amazon	✓	✓	2	Gordon Getty	Getty Oil	✗	✗
3	Bill Gates	Microsoft	✓	✓	3	Margaret Hunt Hill	-	✗	✗
4	Larry Ellison	Oracle	✓	✓	4	William H. Hunt	Halcon	✗	✗
5	Warren Buffet	Berkshire Hathaway	✓	✗	5	Marvin H. Davis	Davis Oil	✓	✗
6	Larry Page	Alphabet Inc.	✓	✓	6	David Packard	Hewlett-Packard	✓	✗
7	Sergey Brin	Alphabet Inc.	✓	✓	7	Lamar Hunt	-	✗	✗
8	Steve Ballmer	Microsoft	✓	✓	8	David Rockefeller Sr.	-	✗	✗
9	Michael Bloomberg	Bloomberg LP	✓	✗	9	Caroline R. Hunt	-	✗	✗
10	Jim Walton	Walmart	✗	✗	10	Nelson B. Hunt	Halcon	✗	✗

the wealth share of the top 1%, from a low of 22% in 1978 to 35% in 2016. Similarly, Smith et al. (2021) find an increase of 10 percentage points, to 33%, over the same period.

Interestingly, they also document substantial changes in the distribution of wealth *within* the top 1%. It is precisely these changes within the top 1% that are the subject of this paper. Figure (7a) depicts the evolution of the ratio of the top 0.1% to the top 1% and the top 0.01% to the top 0.1%. As in Figure 6, the grey area marks the period of the ERISA regulatory changes that Gornall and Strebulaev (2021) argue gave rise to the expansion of the venture capital industry. The similar level and evolution of these ratios indicate that the top of the wealth distribution roughly follows a Pareto distribution and that Pareto inequality, the inverse of the Pareto tail coefficient, has increased. Figure (7b) depicts an estimate of Pareto inequality based on these ratios of top wealth shares, using a formula provided by Jones and Kim (2018).

These figures capture the essence of the stylized facts on which the literature on top wealth inequality has centered, accounting for the rise in the level of Pareto inequality as well as the speed with which this rise has occurred. Gabaix et al. (2016) point out that the speed of transition to higher Pareto inequality is not captured well by basic random growth models of wealth accumulation. Atkeson and Irie (2022) point out a direct relationship between the ability of random growth models to match the speed of transition of top wealth inequality and the existence of rapidly amassed "self-made" fortunes. In particular, there is a direct relationship between the existence of a subset of extremely fast upwardly mobile agents and the speed of transition of the Pareto tail coefficient over time.

The present paper incorporates one of the critical insights of Gabaix et al. (2016) and



(a) Ratios of top wealth shares.

(b) Pareto inequality $1/\zeta$.

Figure 7: Ratios of top wealth shares and the Pareto tail coefficient. Data from Saez and Zucman (2016)

Atkeson and Irie (2022) in order to address the shortcomings of the basic random growth model exhibit. Namely, it includes a small minority of entrepreneurial capitalists with very high idiosyncratic risk exposures and higher expected returns to wealth than the other agents in the model. Importantly, and in contrast to Gabaix et al. (2016) and Atkeson and Irie (2022), entrepreneurs' high idiosyncratic risk exposures are endogenous outcomes of their optimal portfolio choice problems rather than exogenous parameters. I explore this model further in the next section.

5 The Quantitative Impact of the Reallocation Effect: A Numerical Approach

Section 3 described that the crucial determinant of how improved entrepreneurial financing affects top wealth inequality is how much economic activity is reallocated to entrepreneurs in equilibrium. When the reallocation is substantial, top wealth inequality rises, and when it is not, top wealth inequality falls. The size of this reallocation is, in turn, determined by the elasticity of substitution between the goods that entrepreneurial firms produce and those that traditional firms produce. When the substitutability is high, the economy reallocates much capital to the entrepreneurial firms in response to the reduced risk cost associated with production in that sector.

In this section, I examine the role played by the strength of the general equilibrium reallocation effect numerically. Specifically, I parameterize the model to be roughly consistent with key aspects of the data. Then, I investigate how the strength of the general equilibrium reallocation effect, as determined by the elasticity of substitution ε, impacts how reductions in equity financing constraints affect top wealth inequality. The tractability of the framework allows me to compute the transition dynamics of the model straightforwardly. This is important because we are interested in understanding how the strength of the equilibrium reallocation mechanism affects the speed of the dynamics of Pareto inequality. In particular, recall that the remarkable speed with which Pareto inequality has increased is one of the key stylized facts that Gabaix et al. (2016) argued that models of top wealth inequality should ideally be able to account for.

The takeaway from this exercise is that the elasticity of substitution between the goods produced by the two sectors of the economy needs to be very high for the model to feature a general equilibrium reallocation effect that is large enough to produce a transition of top wealth inequality that is roughly in line with the data. One way of interpreting this is that the model requires the entrepreneurial and traditional firms to operate together, producing similar goods across a wide range of industries, rather than being isolated from one another in separate industries. However, it does not imply that entrepreneurial and traditional firms use the same production technologies. The entrepreneurial firms may use cutting-edge high-tech production technology but produce output that is highly substitutable with traditional firms' goods. Despite using various cutting-edge technologies in their production processes, Uber is in the taxi business, Amazon is in the retail business, and Google is in the advertising business.

With this in mind, a natural follow-up question is whether there is empirical evidence that justifies this large reallocation effect. I answer this in the affirmative by pointing to four well-documented empirical trends, showing that the model captures these trends precisely when the reallocation effect is large. These trends are (i) the dramatically growing fraction of venture capital-backed innovative entrepreneurial firms among the largest publicly traded firms in the U.S.; (ii) the fall in the aggregate labor share, despite relatively stable firm-level labor shares; (iii) falling safe real interest rates: and (iv) the stable or slightly rising accounting return to the aggregate capital stock, despite a falling real safe interest rate. I go through each of these trends in turn and explain how they are impacted by improvements in entrepreneurial financing qualitatively. The impact will depend on whether the general equilibrium reallocation force is strong or weak. I conclude with a numerical examination of the model-implied transition dynamics for each trend.

The takeaway from this exercise is that when the elasticity of substitution ε is set to a value such that the increase in the size of the entrepreneurial sector is roughly in line with the growth of venture capital-backed firms, improvements in entrepreneurial financing accounts for a large fraction of the rise in top wealth inequality. It also produces a smaller but still meaningful fraction of the fall in the aggregate labor share, a temporarily elevated but long-run stable rate of return to the aggregate capital stock, and a sizeable fraction of the fall in the risk-free rate.

5.1 Parameterization

We want to numerically examine the effect of improved entrepreneurial financing, modeled as a reduction in equity frictions captured by the parameter χ, on top of wealth inequality. More specifically, we want to study how this effect is influenced by the strength of the general equilibrium reallocation of capital towards entrepreneurial firms, governed by ε. The focal parameters for this exercise are, therefore, χ and ε. I discuss how I parameterize the fall in χ below. Since we want to see how the transition dynamics are affected by the size of ε, I will consider a range of values for this parameter.

The remaining parameters are set to roughly match relevant moments of the data on top wealth inequality, factor income shares, rates of return to business capital, the risk-free rate, the capital-output ratio, the average volatility of wealth growth at the top of the wealth distribution, and various facts regarding the share of economic activity accounted for by venture capital-backed firms.

Parameterizing the fall in χ. I parameterize the fall in χ by selecting an initial value χ_0 and a final value χ_1. I then let χ fall from χ_0 to χ_1 smoothly over time according to the sigmoid curve depicted in Figure 8. To emphasize the connection between the fall in χ and improvements in entrepreneurial financing, I let the lion's share of the fall occur in the period 1974–1979, which is the period of the ERISA regulatory reforms that Gompers and Lerner (2001), Greenwood et al. (2022), and Gornall and Strebulaev (2021) argue triggered the venture capital revolution.

I pick χ_0 and χ_1 by matching the average rate at which firms associated with entrepreneurs at the top of the Forbes 400 list issued equity. Specifically, Gomez and Gouin-Bonenfant (2023) document that the average lifetime growth rate of shares outstanding associated with entrepreneurs at the top of the Forbes 400 list has increased from 0.5% in 1985 to 2.9% in 2015. In Section D of the Appendix, I show that the average lifetime rate of equity issuance of the entrepreneurial firms in the model is

$$\text{Lifetime equity issuance rate} = \left(1 + \frac{(1+r^T)(1-\chi)}{(r^k - r^T) + \chi(1+r^T)}\right)^{1/T_l} - 1 \tag{38}$$

where T_l is the number of years that the firm is considered to be associated with the entrepreneur. A few comments regarding this choice of calibrating χ are in order: The

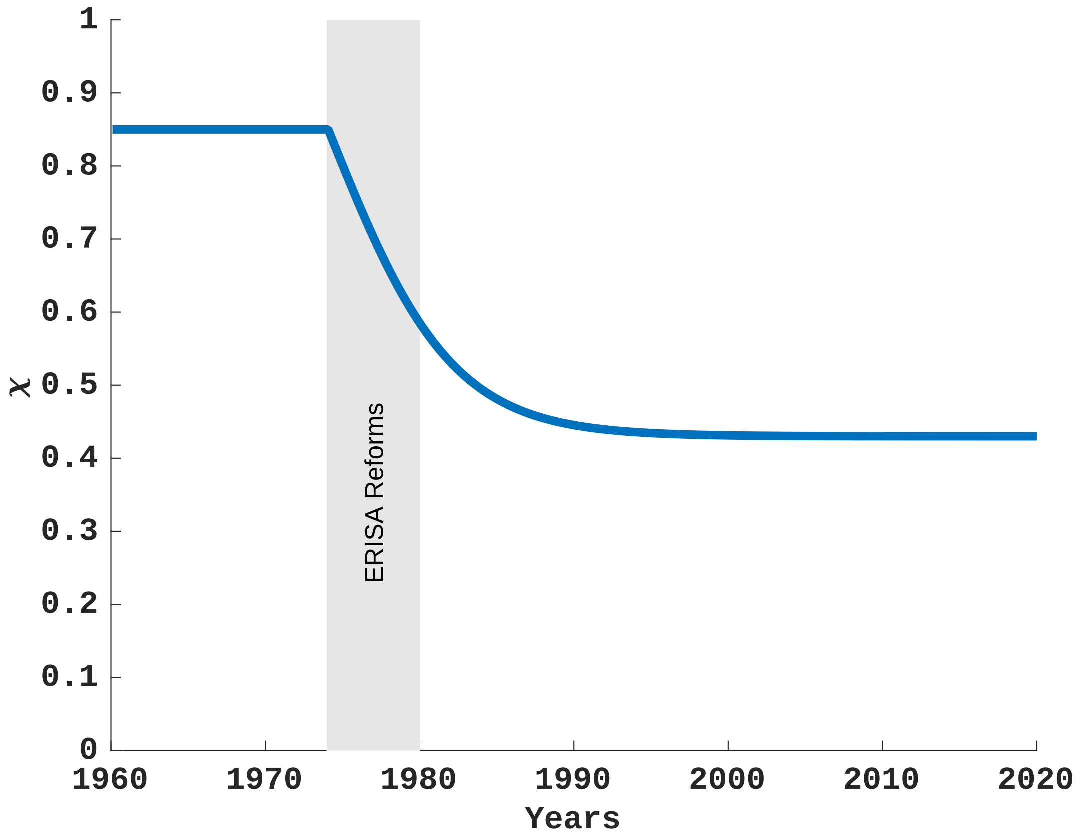


Figure 8: Baseline fall in χ

insider financing fraction χ should not be confused with the insider ownership fraction. As noted in Section 3, these are not the same. The constraint determines the financing fraction, while the ownership fraction is determined by competition in capital markets. Moreover, Brunnermeier, Sannikov, and Merkel (2022) pursue a different way of calibrating χ. They look at the share of privately held business wealth in the economy and argue that this is the share of business capital that insiders hold. This approach would, however, be a problem in the present setting because we are specifically interested in firms that are not necessarily privately held. Finally, Gomez and Gouin-Bonenfant (2023) measure the equity issuance rate in 1985, while the regulatory changes that motivate the fall in χ are prior to that. However, it seems reasonable to assume that many of the entrepreneurs at the top of the Forbes 400 list in 1985 had companies that were at the very least a decade old and so had operated mainly in the pre-ERISA era. This would mean that their estimate of the average *lifetime* equity issuance rate from 1985 reflects entrepreneurial financing conditions in the pre-ERISA era.

Baseline parameterization. I consider two values of the elasticity of substitution between the goods produced by the entrepreneurial firms and the traditional firms: $\varepsilon = 10$ and $\varepsilon = 100$. The remaining parameters are $\{\alpha, \rho, \delta, \sigma, \nu, \bar{A}, \tilde{\sigma}, \delta_d, \phi_l, \bar{\psi}, T_l\}$. In this general equilibrium framework, most target moments are affected to some extent by most parameters. However, some moments are more sensitive to some parameters and less sensitive to others. I use this to parameterize the model by altering each parameter to match a moment especially sensitive to that parameter. I chose α, ρ, and σ to produce a steady state that matches the labor share, the rate of return to business capital, and the risk-free rate in 1960. These are important quantities for the trends we want to study. In addition, I use δ to target a (business) capital-output ratio of 2. This results in a value of $\delta = 0.1$, which is larger than in most standard calibrations. I choose $\bar{A}$, ν and $\tilde{\sigma}$ to match an initial Pareto tail coefficient of $\zeta_0 = 1.85$, an initial fraction of the capital stock operated by innovative entrepreneurial firms of $\kappa_0 \approx 5\%$, and idiosyncratic volatility of stock returns of 30%.[21] The demographic parameters, the rate at which dynasties are broken δ_d, the rate at which innovative entrepreneurs become diversified capitalists ϕ_l, and the fraction of innovative entrepreneurs among capitalists $\bar{\psi}$, strongly influence the fraction of entrepreneurs found at various points in the wealth distribution. Kaplan and Rauh (2013) document that 69% of the Forbes 400 list in the 2011 edition were the first in their family to run their business, up from 40% in the first 1982 edition. I therefore target a share of entrepreneurs in the initial steady state of 40%. Moreover, Gomez (2023) estimates that the average level of idiosyncratic volatility within the top 0.01% of the wealth distribution for the period 1960–1980 was 10%, slightly lower in 1960 than in 1980, so I target a level of 8% in the initial steady state. In the present model, this will be the weighted average of the volatility of entrepreneurs and diversified capitalists within this top quantile. I use $\bar{\psi}$ and ϕ_l to match these moments. I set the dynasty breaking rate to once a generation, $\delta_d = 1/30$, to reflect the risk of generational handover. I set the parameter T_l, the lifetime over which the model-implied average lifetime equity issuance rate is computed, to 30 years.

Finally, changing the value of ε while keeping all other parameters constant will, of course, alter most of the moments that the model produces in the initial steady state. In the extreme case, this could imply that each value of ε would need to be paired with a different parameterization of all the other variables. However, it turns out that changing the value of ε requires only a parsimonious re-parameterization of the other variables. In particular, different parameterizations of ε need to be coupled with different parameterizations of ν, but other than that, the model produces roughly the same moments across the two specifications.

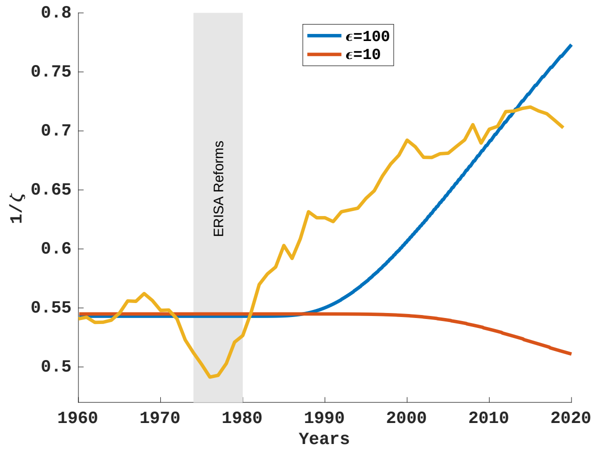
[21] According to Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), this was the average idiosyncratic volatility of stock returns in 1960.

Table 3: Baseline parameterization and model fit. All rates are annualized.

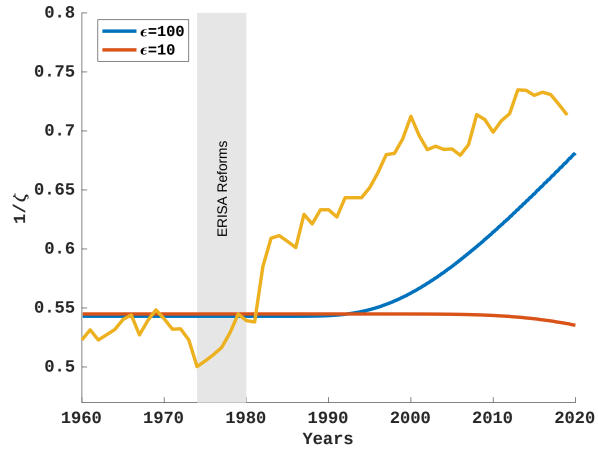
Parameter	Value	Description
Macro parameters		
α	0.34	Output elas. of capital
ρ	0.071	Discount rate
δ	0.095	Depreciation
σ	0.15	Aggregate volatility
Distribution and allocation parameters		
ε	100, 10	Elas. of substitution
ν	0.5041, 0.43905	CES share parameter
$\bar{A}$	1.06	TFP of ent. firms
$\tilde{\sigma}$	0.3	Idiosyn. vol. scalar
δ_d	1/30	Dissipation rate
ϕ_l	1/15	Ent. switching rate
$\bar{\psi}$	1/25	Ent. capitalist frac.
χ_0	0.85	Ent. financing fraction
T_l	30	Top ent. firm lifetime
Moment	**Target**	**Model**
Pareto tail coefficient	1.85	1.85
Labor share	65%	65%
Average return to capital	7.41%	7.41%
Risk-free rate	4.51%	4.48%
Capital-output ratio	2	2.03
Equity issuance rate	0.5%	0.5%
Ent. share of capital	<5%	4.83%, 5.22%
Ent. firms idios. vol	30%	31%
Ent. share of Forbes 400	40%	40%
Average vol. wealth for top 0.01%	8%	8%

5.2 Transition dynamics of wealth inequality

In this section, I study how the value of ε affects the model-implied transition of Pareto inequality in response to ameliorated equity issuance frictions captured by the fall in χ depicted in Figure 8. That this high elasticity is indeed key is illustrated in Figure 9, where we examine the transition dynamics of Pareto inequality, the inverse of the Pareto tail coefficient, for the two different values of ε in Table 3. In Figure **??**, we examine the transition of tail inequality measured at the top 0.1%, and in Figure **??**, we examine the transition of tail inequality measured at the top 0.01%. The reason to look at tail inequality at two different points in the wealth distribution is that although tail inequality is the same throughout the wealth distribution in steady state, this is not the case in the transition.



(a) Tail inequality measured at top 0.1%



(b) Tail inequality measured at top 0.01%

Figure 9: Transition of Pareto inequality for $\varepsilon = 100$ and $\varepsilon = 10$. Pareto tails based on ratio of top 0.01% to 0.1% and top 0.1% to 1% wealth shares, respectively. Data (yellow) from Piketty et al. (2018).

As pointed out by Gabaix et al. (2016), the transition speed is slower higher up in the wealth distribution. A comprehensive understanding of how well the model does with respect to the transition speed, therefore, requires us to look at various points along the wealth distribution. When $\varepsilon = 100$, wealth is measured, and Pareto inequality rises at a rate roughly consistent with the data. In contrast, when $\varepsilon = 10$, the downward pressure on entrepreneurial expected excess returns in response to the capital reallocation is so heavy that the risk-reward trade-off deteriorates: the risk falls as improved entrepreneurial financing enables more risk sharing, but the expected excess return declines even more so that the appraisal ratio falls. In this case, top wealth inequality declines slightly as entrepreneurs reduce their idiosyncratic risk exposure.

This exercise demonstrates that the model can account for a meaningful fraction of the rise in Pareto inequality, provided the elasticity of substitution is very high. In the following sections, I examine how other model predictions are affected by the size of the general equilibrium capital allocation effect, as captured by the value of ε. In particular, I focus on the model's predictions along three dimensions: the growing fraction of various measures of economic activity accounted for by innovative entrepreneurial firms (§5.3), factor income shares (§5.4), and rates of return to savings and investment (§5.5). The aim is to let empirical evidence in those dimensions guide our understanding of what reasonable values of ε might be to obtain a quantitative sense of how much of the rise in top wealth inequality can be accounted for by the central mechanism explored in this study.

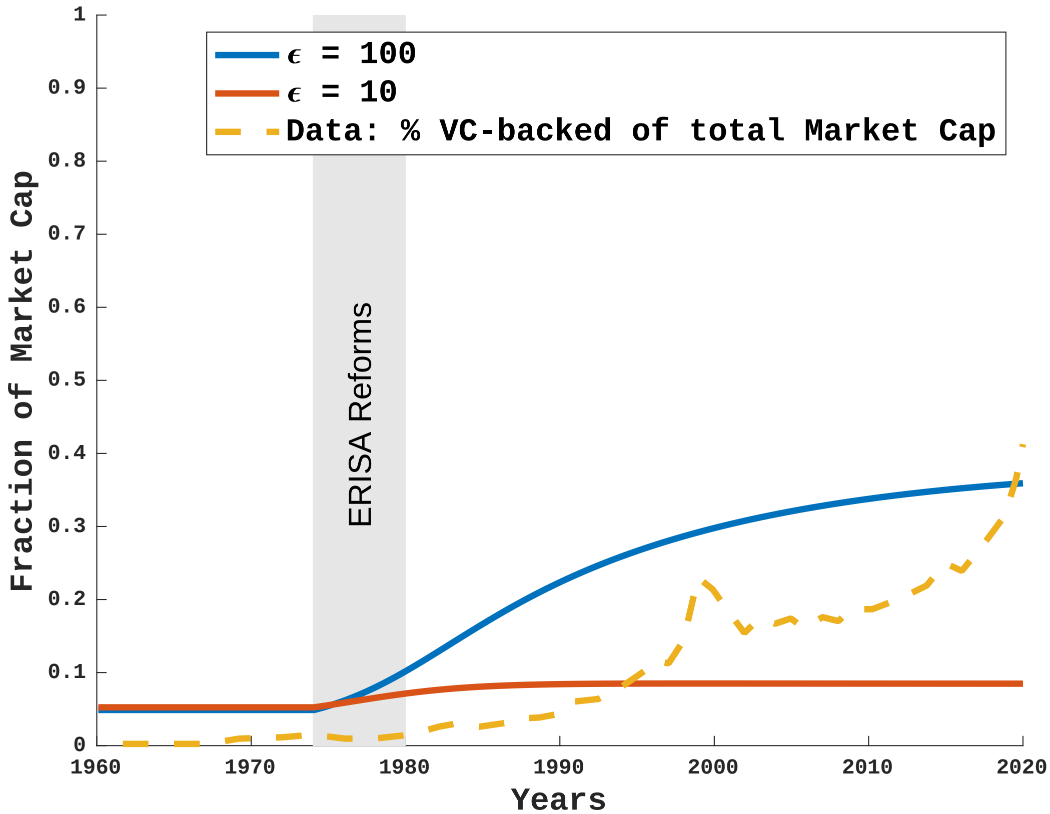


Figure 10: Transition of κ_t for $\varepsilon = 100$ and $\varepsilon = 10$. Data from Gornall and Strebulaev (2021).

5.3 Reallocation to cutting-edge entrepreneurial firms

In the model, a reduction in equity-issuance-related agency frictions increases the fraction of the capital stock operated by the entrepreneurial sector relative to the traditional sector. In other words, $\kappa_t = \frac{K_t^E}{K_t}$ rises. Exactly how much it rises depends on the elasticity of substitution between the goods that the two sectors produce. When the elasticity is high, the falling risk costs associated with entrepreneurial production motivate a substantial reallocation to that sector, and vice versa when the elasticity is low. This was illustrated in Figure 4. In this section, I study this question numerically. In particular, taking as a starting point the initial steady state associated with the baseline calibration in Table 3, I examine the transition dynamics of κ_t. Figure 10 illustrates the result of this exercise. We see that $\varepsilon = 100$ is associated with a rise in the relative size of the entrepreneurial sector. In contrast, we hardly see a budge with $\varepsilon = 10$.

The reallocation in the data. If we interpret the entrepreneurial sector in the model as consisting of innovative entrepreneurial firms similar to venture capital-backed U.S. firms, we can compare the model-implied transitions with some relevant data. Specifically, in

Section 4, I discussed the so-called "venture capital revolution." In Figure 6 from Gornall and Strebulaev (2021), we see that venture capital-backed firms constituted around 0–5% of the total market capitalization before and up to 1980, rising to around 45–50% of market cap in 2020.

5.4 Factor income shares

In the model, improved entrepreneurial financing leads to a fall in the aggregate labor share despite stable or increasing labor shares at the firm level when the elasticity of substitution between the sectors is high. To see why, note first that the labor share in the traditional sector is $1-\alpha$. This is unaffected by changes in entrepreneurial financing.

Both the labor share and the pure capital share in the entrepreneurial sector are lower than in the traditional sector. This is because, following Di Tella and Hall (2022), the idiosyncratic risk in the firm renders the marginal product of each factor of production locally uncertain. This takes seriously the Knightian view (Knight, 2013) that entrepreneurs engage in risk-taking when renting capital and hiring labor because the marginal products of each are uncertain at the time that the cost of capital and wages are determined. Rental rates and wages are, therefore, equal to their respective expected marginal products, less a risk premium. This risk premium constitutes the foundation for the entrepreneurial share of income.[22] Algebraically, the labor share in the entrepreneurial sector is

$$\frac{w_t L_t^E}{p^E(\kappa_t) Y_t^E} = (1-\alpha)\frac{p^T(\kappa_t)}{p^E(\kappa_t)}\frac{\underline{\underline{A}}}{\bar{A}} = (1-\alpha)\left(1 - \underbrace{\frac{(r_t^k - r^T)K_t^E}{p^E(\kappa_t)Y_t^E}}_{\text{"entrepreneurial" share}}\right) \tag{39}$$

and the pure capital share is analogously $\frac{r^T K_t^E}{p^E(\kappa_t)Y_t^E} = \alpha\left(1 - \frac{(r_t^k - r^T)K_t^E}{p^E(\kappa_t)Y_t^E}\right)$.

The overall factor shares in the economy are the sales-weighted averages of the shares in each sector. There are two channels along which improved entrepreneurial financing affects these factor shares. Let us focus on the aggregate labor share, although the reasoning is identical for the pure capital share. Firstly, there is a composition effect coming from the reallocation of capital to low-labor-share entrepreneurial firms. This puts downward pressure on the aggregate labor share. The pressure is stronger when ε is higher because reallocation is more substantial. Secondly, the reallocation causes a rise in the labor share

[22] I refer to it as the "entrepreneurial share" rather than the "entrepreneur's share" because the entrepreneur also gets some pure capital income. The entrepreneur's share is, therefore, the entrepreneurial share *plus* the entrepreneur's pure capital income share.

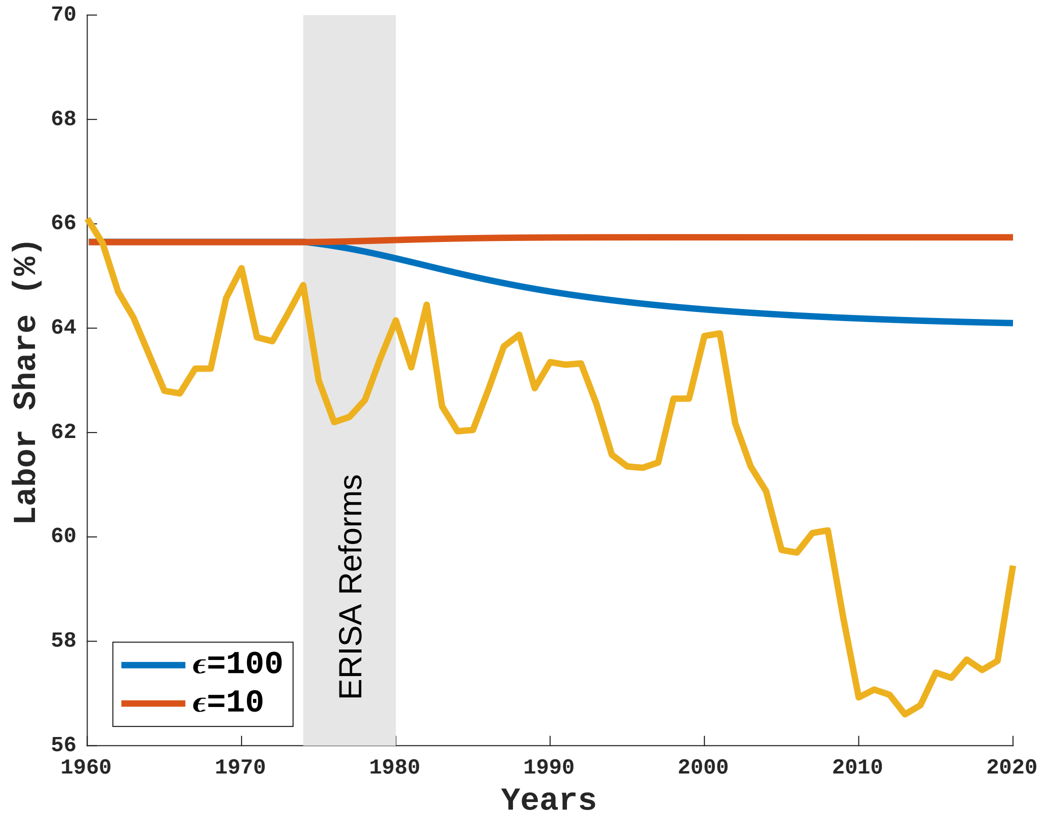


Figure 11: Transition dynamics of the aggregate labor share for $\varepsilon = 100$, and $\varepsilon = 10$

within the entrepreneurial sector. This is because entrepreneurs need to raise wages to attract labor in response to the reallocation of capital. Specifically, the price of the intermediate goods produced by the traditional sector $p^T(\kappa_t)$ rises as resources are allocated away from that sector. This raises the value of the marginal product of labor in that sector, which puts upward pressure on wages. Hence, the labor share within the entrepreneurial sector rises. This upward pressure on wages is higher if the elasticity of substitution ε is small because then the rise in $p^T(\kappa_t)$, and consequently the marginal product of labor in the traditional sector, is higher. The aggregate labor share only falls if the composition effect is stronger than the within-sector effect. Because the composition effect is larger than the within-sector effect when ε is large, this is, again, the key parameter for this prediction. Figure 11 depicts the evolution of the labor share in response to improved entrepreneurial financing for different values of ε in the baseline calibration.

Evolution of factor income shares in the data. The debate on the magnitude of the fall in the labor share of income since 1970 is ongoing. On the very high end of estimates of this fall, we find Barkai (2020), showing a 13 percentage point drop. On the lower end, we find Smith, Yagan, Zidar, and Zwick (2022) estimating a fall of around 5–6 percentage points. Most estimates fall in the 6–7 percentage point range. Key to understanding the discussion around the fall in the labor share is the observation that this fall has been driven by a reallocation of economic activity towards firms with low labor shares rather than by a general fall in the labor share at the firm level, which has remained stable (Autor et al., 2020) or even increased (Hartman-Glaser et al., 2019). Qualitatively, the model presented in this study is very much in line with that observation.

Moreover, it has also been pointed out that the fall in the labor share has not been accompanied by a rise in the pure capital share of income. Instead, both the labor share and the capital share have fallen relative to what has been referred to as factorless income (Karabarbounis and Neiman, 2019). The nature and causes of this rise in factorless income have yet to be fully understood, and many studies have pointed out potential sources. Barkai (2020) emphasizes the role of pure profits, market power, and declining competition. Eisfeldt, Falato, and Xiaolan (2021) and Smith et al. (2022) instead focus on the role of human capital income of key employees and business owners. In the model presented in this paper, the rise in the factorless income share comes from the rise in innovative entrepreneurs' share of income. In this sense, the explanation is closer in spirit to Eisfeldt et al. (2021) and Smith et al. (2022), focusing on idiosyncratic risk bearing as the source of this entrepreneurial share.

How much of such a fall can be accounted for quantitatively by the mechanism presented in this study depends on the value of ε. Looking at Figure 11, with $\varepsilon = 100$, around 20% of the fall is accounted for.

5.5 The return to the aggregate capital stock, and the risk-free rate

A similar reasoning as for the labor share applies to the accounting return to the aggregate capital stock as well. In particular, the accounting return to the aggregate capital stock in the economy is given by

$$r_t^K \equiv \frac{Y_t - w_t L_t - \delta K_t}{K_t} = \kappa_t r_t^k + (1 - \kappa_t) r^T . \tag{40}$$

In other words, the aggregate return is the capital allocation weighted average of the return in each sector. As with the labor share, a reallocation of resources towards the entrepreneurial firm creates upward pressure on the aggregate return through a composition effect and downward pressure by lowering excess returns within the entrepreneurial sector. The composition effect is stronger than the within effect when ε is high. The aggregate return rises if ε is high enough.

So far, the mechanism is analogous to that for the labor share. However, there are additional implications for returns to wealth in the long run. Recall that the basis for the model in this study is a version of the neoclassical growth model. This means that in the long-run steady state, the return to wealth settles down to the consumption rate out of wealth.[23] In other words, $r^K_{ss} = \rho$. This means that any movements in the return to the aggregate capital stock are temporary, and hence, the reallocation effect has no bite in the long run. However, the long-run stability of the return to aggregate capital is what makes the model's implication for the risk-free rate interesting.

Since the total wealth of the economy is the total capital stock, the return to capital has to be the wealth-weighted average return to wealth for entrepreneurs and diversified agents:

$$r^K_{ss} = \eta_{ss} r^E_{ss} + (1 - \eta_{ss}) r^T_{ss}. \tag{41}$$

Noting that the difference in return between entrepreneurs and diversified capitalists is $r^E_{ss} - r^T_{ss} = \left(\tilde{\sigma}^E_{ss}\right)^2$, one obtains the following expression for the return to aggregate capital:

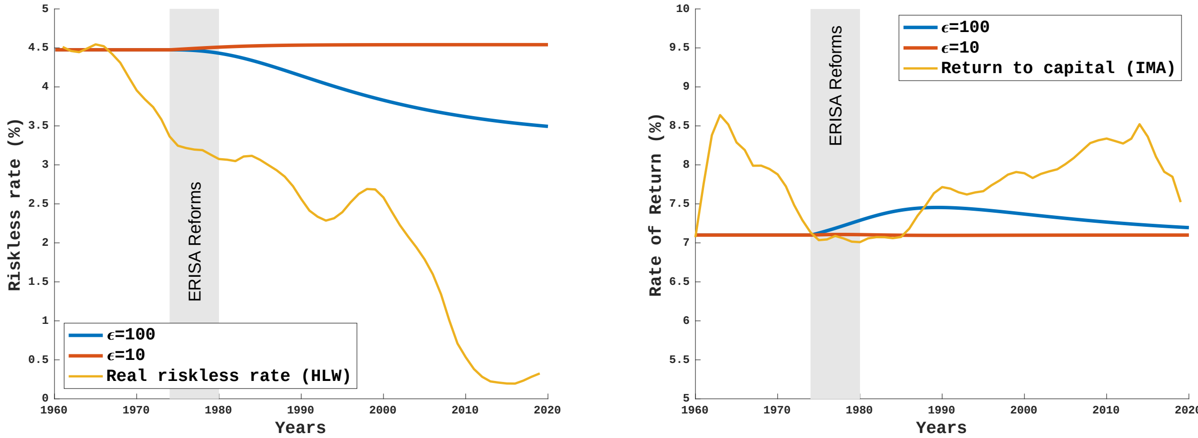
$$r^K_{ss} = r^T_{ss} + \eta_{ss} \left(\tilde{\sigma}^E_{ss}\right)^2, \tag{42}$$

which states that the return to the aggregate capital stock is the return in the traditional sector plus the risk premium received from investment in the entrepreneurial firms. More precisely, the risk premium that entrepreneurs receive per unit of wealth invested in their firms is $\left(\tilde{\sigma}^E_{ss}\right)^2$, and their fraction of all wealth is η_{ss} so that the risk premium for the economy as a whole is $\eta_{ss}\left(\tilde{\sigma}^E_{ss}\right)^2$. We know that when the general equilibrium capital reallocation effect is strong enough, entrepreneurs' risk exposure and share of wealth both increase, so that $\eta_{ss}\left(\tilde{\sigma}^E_{ss}\right)^2$ rises. However, the fact that $r^K_{ss} = \rho$ is fixed in the long run means that the rise in the risk premium $\eta_{ss}\left(\tilde{\sigma}^E_{ss}\right)^2$ must be associated with a fall in r^T_{ss}. Since the risk-free rate is $r_{ss} = r^T_{ss} - \sigma^2$, it also falls.[24] Figure 12 depicts the model-implied evolution of the rate of return to the aggregate capital stock and the risk-free rate for the two values of ε.

The rate of return to business capital and the risk-free rate in the data. Several recent studies document a relatively stable or slightly rising return to business capital in the U.S. (Farhi and Gourio, 2018; Gomme, Ravikumar, and Rupert, 2011; Moll et al., 2019; Reis, 2022). With the different estimates these studies provide, one finds a return that hoovers around 7–10%. In contrast, estimates of the return on safe assets show a downward trend for the last half-century (Rachel and Summers, 2019). Holston et al. (2017) estimates a

[23] Again focusing on the median path of the economy where aggregate shocks dZ_t happen to be 0 for a long time.

[24] An alternative way of interpreting the increase in $\eta_{ss}\left(\tilde{\sigma}^E_{ss}\right)^2$ and the resulting fall in the risk-free rate is as a more pronounced precautionary savings motive of entrepreneurs. As they take on more idiosyncratic risk, their precautionary savings motive rises, which puts downward pressure on the risk-free rate.



(a) Riskless rate

(b) Return to aggregate capital stock

Figure 12: Transition dynamics of the riskless rate and the rate of return to capital for $\varepsilon = 100$ and $\varepsilon = 10$. Data on riskless rate from Holston et al. (2017). Data on return to capital from Moll et al. (2019) (smoothed).

decline of 3–4 percentage points in the long-run return on safe assets between 1960 and 2020. Powerful forces, like demographic changes and the so-called international "savings glut," can account for most of the fall in the risk-free rate (see Auclert et al. (2021) and Rachel and Summers (2019)). Looking at Figure 12, we see, however, that for the larger values of ε, the mechanism discussed in this study also puts meaningful downward pressure on the risk-free rate, accounting for between around 30% of the drop when $\varepsilon = 100$.

6 Conclusion

This paper studies the effects of improvements in entrepreneurial equity financing on the level and dynamics of top wealth inequality. By developing a tractable general equilibrium model, I show that this impact is summarized by three key effects: the risk-reduction effect, the scaling-up effect, and the general equilibrium reallocation effect. First, improved financing enables entrepreneurs to offload more of their firms' risk to financial markets. This gives them the opportunity to reduce their idiosyncratic risk exposure, which would lower top wealth inequality by making extreme wealth trajectories less likely and by reducing entrepreneurs' precautionary savings motive. This is the risk-reduction effect. In contrast, improved financing also allows entrepreneurs to raise more capital and scale up, which raises top wealth inequality. This is the scaling-up effect.

The central theoretical contribution of the paper is the insight that a third general equilibrium effect determines the relative strengths of the risk-reduction and scaling-up effects: the reallocation effect. If entrepreneurs can attract substantial amounts of economic activity from other sectors of the economy without putting too much downward pressure on their equilibrium expected excess returns, the scaling-up effect dominates the risk-reduction effect, and wealth inequality rises. More generally, it illustrates that the relationships between top wealth inequality, entrepreneurial finance, and idiosyncratic risks and returns may be quite subtle.

The second contribution of the paper is to show that several well-documented trends in U.S. data point to the strength of the general equilibrium reallocation effect in practice. In particular, the dramatically growing fraction of venture capital-backed innovative entrepreneurial firms among the largest publicly traded firms in the U.S., the fall in the aggregate labor share despite relatively stable firm-level labor shares, and the stable or slightly rising accounting return to the aggregate capital stock despite falling safe rates, are reflected by the model precisely when the general equilibrium capital reallocation effect is strong enough for the scaling-up effect to dominate the risk-reduction effect.

References

S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994. doi: 10.2307/2118417.

George-Marios Angeletos. Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics*, 10(1):1–30, 2007. doi: https://doi.org/10.1016/j.red.2006.11.001.

Shuhei Aoki and Makoto Nirei. Zipf's law, Pareto's law, and the evolution of top incomes in the United States. *American Economic Journal: Macroeconomics*, 9(3):36–71, 2017. doi: 10.1257/mac.20150051.

Andrew G. Atkeson and Magnus Irie. Rapid dynamics of top wealth shares and self-made fortunes: What is the role of family firms? *American Economic Review: Insights*, 4(4):409–24, 2022. doi: 10.1257/aeri.20210560.

Adrien Auclert, Hannes Malmberg, Frederic Martenet, and Matthew Rognlie. Demographics, wealth, and global imbalances in the twenty-first century. Working Paper 29161, National Bureau of Economic Research, August 2021. URL https://www.nber.org/papers/w29161.

David Autor, David Dorn, Lawrence F Katz, Christina Patterson, and John Van Reenen. The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135 (2):645–709, 2020. doi: 10.1093/qje/qjaa004.

Simcha Barkai. Declining labor and capital shares. *The Journal of Finance*, 75(5):2421–2463, 2020. doi: 10.1111/jofi.12909.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The wealth distribution in Bewley models with investment risk. 2014 Meeting Papers 617, Society for Economic Dynamics, 2014. URL https://ideas.repec.org/p/red/sed014/617.html.

Michele Boldrin and Raymond J. Deneckere. Sources of complex dynamics in two-sector growth models. *Journal of Economic Dynamics and Control*, 14(3-4):627–653, 1990. doi: 10.1016/0165-1889(90)90036-G.

Markus K. Brunnermeier and Yuliy Sannikov. Macro, money and finance: A continuous-time approach. In John B. Taylor and Harald Uhlig, editors, *Handbook of Macroeconomics*, volume 2B, pages 1497–1546. North-Holland, Amsterdam, 2017.

Markus K. Brunnermeier, Yuliy Sannikov, and Sebastian Merkel. Safe assets: A dynamic retrading perspective. Working paper, 2022. URL https://markus.scholar.princeton.edu/sites/g/files/toruqf2651/files/documents/SafeAsset.pdf.

Marco Cagetti and Mariacristina De Nardi. Entrepreneurship, frictions, and wealth. *Journal of Political Economy*, 114(5):835–870, 2006. doi: 10.1086/508032.

Riccardo A. Cioffi. Heterogeneous risk exposure and the dynamics of wealth inequality. Job market paper, 2021. URL https://rcioffi.com/files/jmp/cioffi_jmp2021_princeton.pdf.

Sebastian Di Tella and Robert Hall. Risk premium shocks can create inefficient recessions. *The Review of Economic Studies*, 89(3):1335–1369, 2022. doi: 10.1093/restud/rdab049.

Andrea L. Eisfeldt, Antonio Falato, and Mindy Z. Xiaolan. Human capitalists. Working Paper 28815, National Bureau of Economic Research, May 2021. URL https://www.nber.org/papers/w28815.

Emmanuel Farhi and Francois Gourio. Accounting for macro-finance trends: Market power, intangibles, and risk premia. *Brookings Papers on Economic Activity*, Fall:147–250, 2018. doi: 10.1353/eca.2018.0024.

Xavier Gabaix, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. The dynamics of inequality. *Econometrica*, 84(6):2071–2111, 2016. doi: 10.3982/ECTA13569.

Matthieu Gomez. Asset prices and wealth inequality. 2017 Meeting Papers 1155, Society for Economic Dynamics, 2017. URL https://ideas.repec.org/p/red/sed017/1155.html.

Matthieu Gomez. Decomposing the growth of top wealth shares. *Econometrica*, 91(3): 979–1024, 2023. doi: https://doi.org/10.3982/ECTA16755.

Matthieu Gomez and Émilien Gouin-Bonenfant. A q-theory of inequality. *R&R at Econometrica*, 2023. URL https://www.matthieugomez.com/files/qtheory.pdf.

Paul Gomme, B. Ravikumar, and Peter Rupert. The return to capital and the business cycle. *Review of Economic Dynamics*, 14(2):262–278, 2011. doi: 10.1016/j.red.2010.11.004.

Paul Gompers and Josh Lerner. The venture capital revolution. *Journal of Economic Perspectives*, 15(2):145–168, 2001. doi: 10.1257/jep.15.2.145.

Will Gornall and Ilya A. Strebulaev. The economic impact of venture capital: Evidence from public companies. Research papers, Stanford University, Graduate School of Business, 2021. URL https://EconPapers.repec.org/RePEc:ecl:stabus:3362.

Daniel Greenwald, Matteo Leombroni, Hanno Lustig, and Stijn Van Nieuwerburgh. Financial and total wealth inequality with declining interest rates. Working Paper 28613, National Bureau of Economic Research, 2021. URL https://www.nber.org/papers/w28613.

Jeremy Greenwood, Pengfei Han, and Juan Sanchez. Venture capital: A catalyst for innovation and growth. *Review*, 104(2):120–130, 2022. URL https://EconPapers.repec.org/RePEc:fip:fedlrv:93937.

Barney Hartman-Glaser, Hanno Lustig, and Mindy Z. Xiaolan. Capital share dynamics when firms insure workers. *The Journal of Finance*, 74(4):1707–1751, 2019. doi: 10.1111/jofi.12773.

Bernard Herskovic, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh. The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics*, 119(2):249–283, 2016. doi: 10.1016/j.jfineco.2015.09.010.

Kathryn Holston, Thomas Laubach, and John C. Williams. Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108(S1): S59–S75, 2017. doi: 10.1016/j.jinteco.2017.01.004.

Joachim Hubmer, Per Krusell, and Anthony A. Smith. Sources of us wealth inequality: Past, present, and future. *NBER Macroeconomics Annual*, 35:391–455, 2021. doi: 10.1086/712332.

Xitong Hui. Asset prices, welfare inequality, and leverage. Working paper, 2023. URL https://drive.google.com/file/d/1kPQTWrMj1yIZ0EMAm8CZzvT_-nnRdlDO/view.

Magnus Irie. Changing valuations and top wealth shares. Working paper, 2023.

Charles I. Jones and Jihee Kim. A Schumpeterian model of top income inequality. *Journal of Political Economy*, 126(5):1785–1826, 2018. doi: 10.1086/699190.

Steven N. Kaplan and Joshua Rauh. It's the market: The broad-based rise in the return to top talent. *Journal of Economic Perspectives*, 27(3):35–56, 2013. doi: 10.1257/jep.27.3.35.

Loukas Karabarbounis and Brent Neiman. Accounting for factorless income. *NBER Macroeconomics Annual*, 33:167–228, 2019. doi: 10.1086/700894.

Baris Kaymak and Markus Poschke. The evolution of wealth inequality over half a century: The role of taxes, transfers and technology. *Journal of Monetary Economics*, 77:1–25, 2016. doi: 10.1016/j.jmoneco.2015.10.004.

Frank Knight. *Risk, Uncertainty and Profit*. Number 14 in Vernon Press Titles in Economics. Vernon Art and Science Inc, new edition, 2013.

Moritz Kuhn, Moritz Schularick, and Ulrike I. Steins. Income and wealth inequality in America, 1949–2016. *Journal of Political Economy*, 128(9):3469–3519, 2020. doi: 10.1086/708815.

Cesaire Meh and Vincenzo Quadrini. Endogenous market incompleteness with investment risks. *Journal of Economic Dynamics and Control*, 30(11):2143–2165, 2006. doi: 10.1016/j.jedc.2005.02.008.

Robert Merton. Lifetime portfolio selection under uncertainty: The continuous-time case. *The Review of Economics and Statistics*, 51(3):247–257, 1969. doi: 10.2307/1926560.

Benjamin Moll, Lukasz Rachel, and Pascual Restrepo. Uneven growth: Automation's impact on income and wealth inequality. Working Paper dp-333, Boston University - Department of Economics, August 2019. URL https://ideas.repec.org/p/bos/iedwpr/dp-333.html.

Alessandra Peter. Equity frictions and firm ownership. *R&R at Review of Economic Studies*, 2021. URL https://drive.google.com/file/d/1sYx-qdexzOyS7bK-RlGTGESSdNBM8qd2/view.

Thomas Piketty, Emmanuel Saez, and Gabriel Zucman. Data for: Distributional national accounts: Methods and estimates for the united states. Data https://gabriel-zucman.eu/usdina/, 2018.

Vincenzo Quadrini. Entrepreneurship, saving and social mobility. *Review of Economic Dynamics*, 3(1):1–40, 2000. doi: 10.1006/redy.1999.0077.

Łukasz Rachel and Lawrence H. Summers. On secular stagnation in the industrialized world. Working Paper 26198, National Bureau of Economic Research, August 2019. URL https://www.nber.org/papers/w26198.

Ricardo Reis. Which r-star, government bonds or private investment? measurement and policy implications. Unpublished, 2022. URL https://personal.lse.ac.uk/reisr/papers/99-ampf.pdf.

Emmanuel Saez and Gabriel Zucman. Wealth inequality in the United States since 1913: Evidence from capitalized income tax data. *The Quarterly Journal of Economics*, 131(2):519–578, 2016. doi: 10.1093/qje/qjw004.

Matt Smith, Owen Zidar, and Eric Zwick. Top wealth in america: New estimates and implications for taxing the rich [revised 10/2021]. Working Paper 29374, National Bureau of Economic Research, 2021. URL https://www.nber.org/papers/w29374.

Matthew Smith, Danny Yagan, Owen Zidar, and Eric Zwick. The rise of pass-throughs and the decline of the labor share. *American Economic Review: Insights*, 4(3):323–340, 2022. doi: 10.1257/aeri.20210268.

A Wealth and Demographics

A.1 Wealth.

The economy is populated by a representative worker endowed with L units of labor, and a continuum $i \in [0, 1]$ of capitalists. The net worth of capitalist i is denoted by n_{it}. Workers have no net worth. For as long as capitalist i is alive, her net worth evolves according to

$$\frac{dn_{it}}{n_{it}} = \left(r_{it} - \frac{c_{it}}{n_{it}}\right) dt + \tilde{\sigma}_{it} dZ_{it} + \sigma_{it} dZ_t \tag{43}$$

where r_{it} is the expected return on the entrepreneurs' portfolio, c_{it} is consumption, $\tilde{\sigma}_{it}$ and σ_{it} are the exposures to the idiosyncratic Brownian motion Z_{it} and the aggregate Brownian motion Z_t, respectively. The expected return, consumption rate, and risk exposures are to be determined in the equilibrium of the model.

A.2 Demographics.

The group of capitalists consists of two types, entrepreneurial capitalists and fully diversified capitalists. These types are denoted by E and D respectively. Entrepreneurial capitalists are in possession of a viable entrepreneurial project and can choose to run a firm based on that project. Diversified capitalists do not have such a project and instead passively invest their wealth. Entrepreneurial capitalists lose their project at rate ϕ^l and then become fully diversified capitalists.

Capitalists die at rate $\tilde{\delta}_d$. When this happens, the capitalist is replaced with an agent who either inherits the wealth and type of their parent, leaving the dynasty intact, or the dynasty breaks and the new agent is reborn with the average wealth level of capitalists. The probability that the dynasty is broken is π_0. In other words, we can define $\delta_d = \tilde{\delta}_d \pi_0$ as the rate at which dynasties are broken. When dynasties are broken, the newborn agent becomes an entrepreneur with probability ψ^0.

The evolution of the fraction of capitalists that have a viable entrepreneurial project, denoted ψ_t, is therefore

$$d\psi_t = \left(-\delta_d \psi_t - \phi^l \psi_t + \delta_d \psi^0\right) dt \tag{44}$$

In steady state, $\psi_t = \bar{\psi} = \frac{\delta_d \psi^0}{\delta_d + \phi^l}$.

B Firms and Technology

B.1 Final good

Final output Y_t is produced by a representative firm using a CES-technology and two types of intermediate goods Y_t^E and Y_t^T:

$$Y_t = \left[\nu \left(Y_t^E \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu) \left(Y_t^T \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{45}$$

where ε is the elasticity of substitution between the intermediate goods. The prices of the intermediate goods are p_t^E and p_t^T respectively. The first-order conditions associated with the final goods producer are

$$p_t^E = \nu \left(\frac{Y_t^E}{Y_t} \right)^{-\frac{1}{\varepsilon}}, \quad p_t^T = (1-\nu) \left(\frac{Y_t^T}{Y_t} \right)^{-\frac{1}{\varepsilon}} \tag{46}$$

B.2 Intermediate goods-producing firms

Intermediate good Y_t^T is produced by a continuum of traditional firms indexed by $j \in [0,1]$. This sector will in the end be captured by a representative traditional firm. Intermediate good Y_t^E is produced by a continuum of entrepreneurial firms indexed by $i \in E$. In other words, the entrepreneurial firms are indexed by their associated entrepreneur. Both types of firm produce output using a Cobb-Douglas technology:

$$\begin{aligned} y_{it} dt &= \bar{A} \left(k_{it} \right)^{\alpha} \left(l_{it} \right)^{1-\alpha} \\ y_{jt} dt &= \underline{A} \left(k_{jt} \right)^{\alpha} \left(l_{jt} \right)^{1-\alpha} \end{aligned} \tag{47}$$

where $\bar{A} > \underline{A}$ so that entrepreneurial firms have higher total factor productivity. Both types of firm own and operate a capital stock.

$$\begin{aligned} dk_{it} &= k_{it} \left(\iota_{it} - \delta \right) dt + y_{it} \tilde{\sigma} dZ_{it} + k_{it}^E \sigma dZ_t + \Delta_{it}^k \\ dk_{jt} &= k_{jt} \left(\iota_{jt} - \delta \right) dt + y_{jt} \tilde{\sigma} Z_{jt} + k_{jt} \sigma dZ_t + \Delta_{jt}^k \end{aligned} \tag{48}$$

where ι_{it}, ι_{jt} are investment rates, δ is the depreciation rate, dZ_{it}, dZ_{jt} are idiosyncratic shocks, dZ_t is an aggregate shock, and $\Delta_{it}^k, \Delta_{jt}^k$ are net capital purchases from other firms. Note that both types of firms have the same level of risk exposures.

Traditional firms' problem and the representative traditional firm. Traditional firms are entirely externally financed. They finance their capital stock by issuing equity (to any capitalist) at the cost of capital $r_t^{\text{out}} = r_t + \varsigma_t \sigma$, where ς_t is the price of aggregate risk in the economy. Equity has the same risk (volatility) as the risk in the capital of the firm. In other words, holding the equity of a traditional firm gives the instantaneous return

$$dR_{jt}^{k,T} = r_t^{\text{out}} dt + \frac{y_{jt}}{k_{jt}} \tilde{\sigma} dZ_{it} + \sigma dZ_t$$

The cost of capital only depends on aggregate risk because the external financiers form diversified portfolios, and so do not care about the idiosyncratic risk in the firm. Traditional firms decide how much capital to raise, how much labor to hire, and how much to invest, to maximize expected profit flows:

$$\pi_{jt} = \max_{\{k_{jt}, l_{jt}, \iota_{jt}\}} p_t^T y_{jt} - w_t l_{jt} - \iota_{jt} k_{jt} + k_{jt} \left(\iota_{jt} - \delta\right) - r_t^{\text{out}} k_{jt}. \tag{49}$$

Profit maximization is consistent with any investment rate. The first-order conditions are

$$\begin{aligned} w_t &= p_t^T (1-\alpha) \frac{y_{jt}}{l_{jt}} \\ r_t^{\text{out}} &= p_t^T \alpha \frac{y_{jt}}{k_{jt}} - \delta. \end{aligned} \tag{50}$$

Maximized profits are $\pi_{jt} = 0$. This means that the expected return to capital in this sector, denoted r_t^T, is equal to the cost of capital in this sector, r_t^{out}. From the first-order conditions, we see that each traditional firm chooses the same production input mix (labor-to-capital ratio). Hence, it is without loss of generality to consider the traditional firms as being represented by a representative traditional firm that produces a flow output

$$Y_t^T dt = \underline{A} \left(K_t^T\right)^{\alpha} \left(L_t^T\right)^{1-\alpha}, \tag{51}$$

and finances a capital stock $K_t^T = \int_j k_{jt} dt$ that evolves according to

$$dK_t^T = \left(\iota_t^T - \delta\right) dt + \sigma dZ_t,$$

which it finances by issuing equity, that pays a return

$$dR_t^{k,T} = r_t^T dt + \sigma dZ_t$$

with first-order conditions

$$\begin{aligned} w_t &= p_t^T(1-\alpha)\frac{Y_t^T}{L_t^T} \\ r_t^{\text{out}} &= p_t^T\alpha\frac{Y_t^T}{K_t^T} - \delta. \end{aligned} \tag{52}$$

using $r_t^{out} = r_t^T$, we can write the labor-to-capital ratio as

$$\frac{L_t^T}{K_t^T} = \frac{1-\alpha}{\alpha}\frac{r_t^T + \delta}{w_t}$$

Entrepreneurial firms. The entrepreneurial firms hire labor on the same labor market as the legacy firm at wage rate w_t. The instantaneous return on the productive assets of an entrepreneurial firm is

$$dr_{it}^k = \left(\frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}\right)dt + \frac{y_{it}}{k_{it}}\tilde{\sigma}dZ_{it} + \sigma dZ_t \tag{53}$$

Entrepreneurial firms are partially financed internally by the associated entrepreneur, and partially financed externally by issuing equity to other capitalists. However, external financing is not unconstrained. In particular, the entrepreneur faces a skin-in-the-game constraint so that at least a fraction χ of the risk in the firm must be retained by the entrepreneur.[25] Letting v_{it}^{out} denote the total value of the liabilities issued to outsiders, the constraint on equity issuance is

$$\frac{k_{it} - v_{it}^{\text{out}}}{k_{it}} \geq \chi. \tag{54}$$

The risk in the liabilities issued to outsiders is the same as the risk in the productive assets of the firm, so the cost of external capital for entrepreneurs is the same as for the traditional firms $r_t^{\text{out}} = r_t + \varsigma_t\sigma = r_t^T$. The total return is therefore

$$dR_{it}^{\text{out}} = r_t^T dt + \frac{y_{it}}{k_{it}}\tilde{\sigma}dZ_{it} + \sigma dZ_t. \tag{55}$$

and the return on a diversified portfolio of the liabilities of all entrepreneurial firms is

$$dR_t^{\text{out}} = r_t^T dt + \sigma dZ_t. \tag{56}$$

Note that the return on investing in the traditional firms' equity, and investing in en-

[25]We technically allow χ to vary over time according to some Ito process, but suppress the dependence on time here.

trepreneurial firms' equity, is identical from the perspective of an outsider. Capitalists are therefore indifferent between these investment opportunities. It is therefore without loss of generality to assume that capitalists hold shares in a mutual fund that buys the liabilities of all entrepreneurial firms and rents out capital to the legacy firm. The return on this mutual fund is

$$dR_t^{\text{fund}} = r_t^T dt + \sigma dZ_t. \tag{57}$$

B.3 Aggregates

The total financial capital in the economy consists of the financial wealth of both types of capitalists, $N_t = N_t^E + N_t^D$. We let η_t denote the fraction of the financial capital in the economy held by capitalists with entrepreneurial projects:

$$\eta_t = \frac{N_t^E}{N_t}. \tag{58}$$

The financial wealth of the economy consists of claims on the productive assets of the economy, in other words the real capital of the economy K_t. Therefore, the balance sheet of the economy is

$$K_t = N_t^E + N_t^D. \tag{59}$$

Recalling that the aggregate capital stock is split between the legacy firm and the entrepreneurial firms, we define κ_t as the fraction of the capital stock in the entrepreneurial sector:

$$\kappa_t = \frac{K_t^E}{K_t} \tag{60}$$

It will turn out to be the case that the labor-to-capital ratio in each firm is the same, and therefore aggregate output can be written as

$$Y_t = A(\kappa_t) K_t^\alpha L^{1-\alpha} \tag{61}$$

where aggregate TFP satisfies

$$A(\kappa_t) = \left[\nu \left(\bar{A} \kappa_t \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu) \left(\underline{A}(1-\kappa_t) \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The aggregate investment in the economy is the output less consumption so that the aggregate capital stock evolves over time according to

	Expected return	Risk
k_{it}:	$\frac{p_t^E y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}} - r_t$	$\frac{y_{it}}{k_{it}} \tilde{\sigma} dZ_{it} + \sigma dZ_t$
v_{it}^{out}:	$\varsigma_t \sigma$	$\frac{y_{it}}{k_{it}} \tilde{\sigma} dZ_{it} + \sigma dZ_t$
v_{it}^{fund}:	$\varsigma_t \sigma$	σdZ_t
b_{it}:	0	0

Table 4: Risk-return profiles

$$dK_t = \left(Y_t - C_t^E - C_t^D - C_t^W - \delta K_t\right) dt + \sigma K_t dZ_t. \tag{62}$$

Finally, since zero-net supply riskless bonds and aggregate risk can be traded without frictions, there is a unique riskless rate r_t and a unique price of aggregate risk ς_t.

B.4 Entrepreneurs problem

The net worth of an individual entrepreneur can be written as

$$n_{it} = \underbrace{k_{it} - v_{it}^{\text{out}}}_{\text{stake in own firm}} + \underbrace{v_{it}^{\text{fund}}}_{\text{mutual fund holdings}} + \underbrace{b_{it}}_{\text{bonds}}. \tag{63}$$

Each of the components of the net worth of an entrepreneur is associated with some expected excess return and some risk. Table 5 summarizes the returns and risk associated with each of these components. Letting $\theta_{it}^k = \frac{k_{it}}{n_{it}}$, $\theta_{it}^{\text{out}} = \frac{v_{it}^{\text{out}}}{n_{it}}$, $\theta_{it}^{\text{fund}} = \frac{v_{it}^{\text{fund}}}{n_{it}}$ and $x_{it} = \frac{y_{it}}{k_{it}}$, we can then write the entrepreneurs problem as a Merton optimal portfolio choice problem:

$$\max_{\{c_{it}, x_{it}, \theta_{it}^k, \theta_{it}^{\text{out}}, \theta_{it}^{\text{fund}}\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_{it}) dt\right] \tag{64}$$

subject to

$$\begin{aligned}
\frac{dn_{it}}{n_{it}} &= \left(r_t + \theta_{it}^k \left(r_{it}^k - r_t\right) - \theta_{it}^{\text{out}} \varsigma_t \sigma + \theta_{it}^{\text{fund}} \varsigma_t \sigma - \frac{c_{it}}{n_{it}}\right) dt \\
&\quad + \left(\theta_{it}^k - \theta_{it}^{\text{out}}\right) x_{it} \tilde{\sigma} dZ_{it} + \left(\theta_{it}^k - \theta_{it}^{\text{out}} + \theta_{it}^{\text{fund}}\right) \sigma dZ_t \\
\text{where} \quad r_{it}^k &= p_t^E x_{it} - w_t \left(\frac{x_{it}}{\bar{A}}\right)^{1/(1-\alpha)} - \delta \\
\text{and} \quad & \frac{\theta_{it}^k - \theta_{it}^{\text{out}}}{\theta_{it}^k} \geq \chi
\end{aligned}$$

Let S_t denote a vector of all state variables except the individuals net worth. We allow ξ

to be such a state variable, and require that it follow an Ito process. The assumption that it follows an Ito process. Then, the HJB equation of this problem can be written as

$$\begin{aligned}
\rho V(n,S) = \max_{\{c,x,\theta^k,\theta^{\text{out}},\theta^{\text{fund}}\}} & \log(c) + V_n n \left(r + \theta^k \left(r^k - r\right) - \theta^{\text{out}} \varsigma\sigma + \theta^{\text{fund}} \varsigma_t \sigma - \frac{c}{n}\right) \\
& + \frac{1}{2} V_{nn} n^2 \left(\left(\theta^k_{it} - \theta^{\text{out}}_{it}\right)^2 (x\tilde{\sigma})^2 + \left(\theta^k_{it} - \theta^{\text{out}}_{it} + \theta^{\text{fund}}_{it}\right)^2 \sigma^2\right) \\
& + \sum_{s\in S} V_s \mu_s s + \frac{1}{2} V_{ss} s^2 \sigma_s^2 + V_{ss'} s s' \sigma_s \sigma_{s'} + V_{sn} s n \sigma_s \sigma_n \\
& + \lambda((1-\chi)\theta^k - \theta^{\text{out}})
\end{aligned}$$

The first-order conditions of this problem are

$$\begin{aligned}
V_n n(r^k - r) &= V_{nn} n^2 \left(\left(\theta^k - \theta^{\text{out}}\right)(x\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2\right) - \lambda(1-\chi) + \sum_{s\in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^k} n s \sigma_s \\
V_n n \varsigma\sigma &= V_{nn} n^2 \left(\left(\theta^k - \theta^{\text{out}}\right)(x\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2\right) - \lambda + \sum_{s\in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^{\text{out}}} n s \sigma_s \\
V_n n \varsigma\sigma &= V_{nn} n^2 \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2 + \sum_{s\in S} V_{sn} \frac{\partial \sigma_n}{\partial \theta^{\text{fund}}} s n \sigma_s \\
V_n n \theta^k \left(p^E - \frac{1}{1-\alpha} w \left(\frac{x}{\bar{A}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\bar{A}}\right) &= V_{nn} n^2 \left(\theta^k - \theta^{\text{out}}\right)^2 x\tilde{\sigma}^2 + \sum_{s\in S} V_{sn} \frac{\partial \sigma_n}{\partial x} s n \sigma_s
\end{aligned}$$

We can guess and verify that the value function takes the form $V(n,S) = \frac{1}{\rho}\log(n) + g(S)$. With this guess we have $V_n n = V_{nn} n^2 = \frac{1}{\rho}$ and all mixed derivatives are $V_{sn} = 0$. The first-order conditions can then be written as

$$\begin{aligned}
r^k - r &= \left(\theta^k - \theta^{\text{out}}\right)(x\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2 - \rho\lambda(1-\chi) \\
\varsigma\sigma &= \left(\theta^k - \theta^{\text{out}}\right)(x\tilde{\sigma})^2 + \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2 - \rho\lambda \\
\varsigma\sigma &= \left(\theta^k - \theta^{\text{out}} + \theta^{\text{fund}}\right)\sigma^2 \\
\theta^k \left(p^E - \frac{1}{1-\alpha} w \left(\frac{x}{\bar{A}}\right)^{\frac{\alpha}{1-\alpha}} \frac{1}{\bar{A}}\right) &= \left(\theta^k - \theta^{\text{out}}\right)^2 x\tilde{\sigma}^2
\end{aligned}$$

Assuming that $\bar{A} > \underline{A}$ ensures that entrepreneurs will want to invest some capital in their firm and so $\theta^k > 0$. Combining the second and third first-order condition we obtain

$$\rho\lambda = \left(\theta^k - \theta^{\text{out}}\right)(x\tilde{\sigma})^2 \tag{65}$$

The skin-in-the-game constraint together with $\theta^k > 0$ ensures that the right-hand side of this equation is positive, which means $\lambda > 0$. Hence, the skin-in-the-game constraint is always binding. We then have $\theta^{\text{out}} = (1-\chi)\theta^k$ and the first-order conditions can be reduced to

$$\begin{aligned}
r^k - r &= \chi\theta^k(x\tilde{\sigma})^2 + \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 - \chi\theta^k(x\tilde{\sigma})^2(1-\chi) \\
\varsigma\sigma &= \chi\theta^k(x\tilde{\sigma})^2 + \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 - \chi\theta^k(x\tilde{\sigma})^2 \\
\varsigma\sigma &= \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma^2 \\
p^E - \frac{1}{1-\alpha}w\left(\frac{x}{\bar{A}}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{\bar{A}} &= \theta^k\chi^2 x\tilde{\sigma}^2
\end{aligned} \tag{66}$$

The second and the third first-order conditions are now identical. We can simplify this further to

$$\begin{aligned}
r^k - r &= \chi^2\theta^k(x\tilde{\sigma})^2 + \varsigma\sigma \\
\varsigma &= \left(\chi\theta^k + \theta^{\text{fund}}\right)\sigma \\
p^E - \frac{1}{1-\alpha}w\left(\frac{x}{\bar{A}}\right)^{\frac{\alpha}{1-\alpha}}\frac{1}{\bar{A}} &= \theta^k\chi^2 x\tilde{\sigma}^2
\end{aligned} \tag{67}$$

we therefore have

$$\begin{aligned}
\theta^k &= \frac{r^k - r^T}{(\chi x\tilde{\sigma})^2} \\
\theta^{\text{fund}} &= \frac{r^T - r}{\sigma^2} - \chi\theta^k \\
\theta^{\text{out}} &= (1-\chi)\theta^k
\end{aligned} \tag{68}$$

Multiplying the last first-order condition by x we obtain the following:

$$p^E x - \frac{1}{1-\alpha}w\left(\frac{x}{\bar{A}}\right)^{\frac{1}{1-\alpha}} = \theta^k(\chi x\tilde{\sigma})^2 = r^k - r^T \tag{69}$$

using the fact that $r^k = p^E x - w\left(\frac{x}{\bar{A}}\right)^{\frac{1}{1-\alpha}} - \delta$ we obtain

$$\frac{1}{1-\alpha}w\left(\frac{x}{\bar{A}}\right)^{\frac{1}{1-\alpha}} = w\left(\frac{x}{\bar{A}}\right)^{\frac{1}{1-\alpha}} + r^T + \delta \tag{70}$$

which implies

$$x = \bar{A}\left(\frac{1-\alpha}{\alpha}\frac{r^T+\delta}{w}\right)^{1-\alpha} \tag{71}$$

Or in terms of the labor-to-capital ratio

$$\frac{l_{it}}{k_{it}} = \frac{1-\alpha}{\alpha}\frac{r^T+\delta}{w} = \frac{L^T}{K^T} \tag{72}$$

confirming that every firm, including the representative traditional firm, has the same labor-to-capital ratio. In conclusion, the decision rules of any entrepreneur is

$$\begin{aligned}
c_{it} &= \rho n_{it} \\
\frac{y_{it}}{k_{it}} &= \bar{A}\left(\frac{1-\alpha}{\alpha}\frac{R_t}{w_t}\right)^{1-\alpha} \\
\theta^k_{it} &= \frac{r^k_t - r^T_t}{(\chi \frac{y_{it}}{k_{it}}\tilde{\sigma})^2} \\
\theta^{\text{fund}}_{it} &= \frac{r^T_t - r_t}{\sigma} - \chi\theta^k_{it} \\
\theta^{\text{out}}_{it} &= (1-\chi)\theta^k_{it}
\end{aligned} \tag{73}$$

where $r^k_t = \frac{p^E_t Y^E_t - w_t L^E_t}{K^E_t} - \delta$. Also note that

$$\theta^k_{it} - \theta^{\text{out}}_{it} + \theta^{\text{fund}}_{it} = \frac{\varsigma_t}{\sigma} \tag{74}$$

which implies that their exposure to aggregate risk is $(\theta^k_{it} - \theta^{\text{out}}_{it} + \theta^{\text{fund}}_{it})\sigma = \varsigma_t$. Because each entrepreneurial firm chooses the same output-to-capital ratio, the aggregate supply of the intermediate good Y^E_t is

$$Y^E_t = \bar{A}\left(K^E_t\right)^{\alpha}(L^E_t)^{1-\alpha} \tag{75}$$

B.5 Diversified capitalists and workers

Diversified capitalists have wealth N^D_t that they invest in the mutual fund and riskless bonds. Diversified capitalists have log utility. Hence, their consumption as a group is $C^D_t = \rho N^D_t$ and the fraction of their wealth invested in the mutual fund is

$$\theta^D_t = \frac{r_t + \varsigma_t\sigma - r_t}{\sigma^2} = \frac{\varsigma_t}{\sigma}. \tag{76}$$

This implies that diversified capitalists net worth exposure to aggregate risk is $\theta^D_t\sigma = \varsigma_t$.

Finally, workers supply labor inelastically and consume their labor income, so that $C_t^W = w_t L$.

B.6 Equilibrium

Given an initial capital stock K_0 and an initial share of wealth held by entrepreneurial capitalists η_0, an equilibrium is a map from histories of the Brownian shocks to price processes w_t, r_t, p_t^E, p_t^T and ς_t, and an allocation of capital between the legacy firm and the entrepreneurial firms κ_t such that:

- All agents solve their respective problems given the prices.
- The markets for capital, labor, and financial assets clear.

$$\begin{aligned} &\int_{i\in E} k_{it}di + K_t^T = K_t, \quad \int_{i\in E} l_{it}di + L_t^D = L \\ &\int_{i\in E} v_{it}^{out}di + K_t^T = \int_i v_{it}^{\text{fund}}di \end{aligned} \tag{77}$$

- The capital stock evolves according to

$$\frac{dK_t}{K_t} = \frac{(Y_t - C_t - \delta K_t)}{K_t}dt + \sigma dZ_t \tag{78}$$

  and $K_t = N_t$.

- The share of wealth held by entrepreneurial capitalists evolves according to

$$\frac{d\eta_t}{\eta_t} = \frac{d\left(\frac{N_t^E}{N_t}\right)}{\frac{N_t^E}{N_t}} \tag{79}$$

  where N_t^E is the total wealth of entrepreneurial capitalists.

B.7 Characterizing the equilibrium

The economy-wide state variables are η_t and K_t. To characterize equilibrium, we now derive an equation that pins down the allocation of capital to the entrepreneurial sector, $K_t^T = \kappa_t K_t$. All objects of interest in the model can then be expressed in terms of η_t, K_t, κ_t and exogenous parameters.

Combining the demand for intermediate goods in (11) with the supply of each intermediate good in equations (51) and (75) we obtain the following intermediate goods prices

$$p_t^T = (1-\nu)\left(\frac{\underline{A}}{A(\kappa_t)}(1-\kappa_t)\right)^{-\frac{1}{\varepsilon}}, \quad p_t^E = \nu\left(\frac{\bar{A}}{A(\kappa_t)}\kappa_t\right)^{-\frac{1}{\varepsilon}} \tag{80}$$

We can then derive an equation that pins down the fraction of the capital operated by the entrepreneurial firms, κ_t, by combining these expressions for the prices with the capital demand of entrepreneurial firms in equation (24). Specifically, using that $\frac{k_{it}}{n_{it}} = \frac{K_t^E}{N_t^E} = \frac{\kappa_t}{\eta_t}$ we obtain from this equation that

$$\frac{\kappa_t}{\eta_t}\left(\bar{A}\left(\frac{L}{K_t}\right)^{1-\alpha}\chi\tilde{\sigma}\right)^2 = r_t^k - r^T = p_t^E\bar{A}\left(\frac{L}{K_t}\right)^{1-\alpha} - w_t\left(\frac{L}{K_t}\right) - R_t \tag{81}$$

Combining this with the first-order conditions of the legacy firm that provide expressions for w_t and R_t we obtain after some tedious algebra

$$\frac{\kappa_t}{\eta_t}\left(\bar{A}\chi\tilde{\sigma}\right)^2\left(\frac{L_t}{K_t}\right)^{1-\alpha} = p_t^E\bar{A} - p_t^T(1-\alpha)\underline{A} - p_t^T\alpha\underline{A} \tag{82}$$

which can be rewritten as

$$\frac{\kappa_t}{\eta_t}\left(\bar{A}\chi\tilde{\sigma}\right)^2\left(\frac{L_t}{K_t}\right)^{1-\alpha} = p_t^E\bar{A} - p_t^T\underline{A} \tag{83}$$

This equation pins down a unique $\kappa_t \in (0,1)$ if $\varepsilon > 0$. To see why, note that the left-hand side is a strictly increasing linear function of κ_t, given positive η_t and K_t. Moreover, using that $A(\kappa_t) = \left[\nu\left(\bar{A}\kappa_t\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(\underline{A}(1-\kappa_t)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$ we can write the prices as

$$p_t^T = (1-\nu)\left[\nu\left(\frac{\bar{A}}{\underline{A}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\kappa_t}{1-\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right]^{\frac{1}{\varepsilon-1}} \tag{84}$$

and

$$p_t^E = \nu\left[\nu + (1-\nu)\left(\frac{\underline{A}}{\bar{A}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{1-\kappa_t}{\kappa_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} \tag{85}$$

We see that if $\varepsilon > 0$, then p_t^T is increasing in $\kappa_t \in (0,1)$ and p_t^E is decreasing. For κ_t close to 0 we will have $p_t^E\bar{A} - p_t^T\underline{A} > 0$, for κ_t close to 1 we will have $p_t^E\bar{A} - p_t^T\underline{A} < 0$. Hence, for some unique $\kappa_t \in (0,1)$, the left-hand side and right-hand side intersect.

B.8 The entrepreneurial appraisal ratio.

In the model, financial innovation raises top wealth inequality only when it raises the so-called appraisal ratio associated with entrepreneurial activity. The appraisal ratio, which is sometimes called the "information ratio," is a close relative of the more widely known Sharpe ratio. In contrast to the Sharpe ratio, it compares the idiosyncratic risk premium of an investment relative to the idiosyncratic risk, instead of the total risk premium relative to the total risk. The appraisal ratio associated with entrepreneurship plays a crucial role because it determines how exposed an entrepreneur wants to be to her firm. Entrepreneurs choose a higher exposure to their firm if the appraisal ratio is high. I obtain tractable formulas for this choice in the model, using well-known tools from Merton (1969) and Angeletos (2007). In particular, entrepreneurs' choice of exposure is going to be proportional to the appraisal ratio associated with entrepreneurship.

B.9 Rates of return

The idiosyncratic volatility of entrepreneurs' wealth is

$$\tilde{\sigma}_t^E = \frac{\kappa_t}{\eta_t}\bar{A}\left(\frac{L}{K_t}\right)^{1-\alpha} \chi\tilde{\sigma} = \frac{p_t^E - p_t^T \frac{\underline{A}}{\bar{A}}}{\chi\tilde{\sigma}} \tag{86}$$

The expected return to entrepreneurs' wealth on the other hand is

$$r_t^E = r_t + \theta_t^k\left(r_t^k - r_t\right) - \theta_t^{\text{out}}\varsigma_t\sigma + \theta_t^{\text{fund}}\varsigma_t\sigma = r_t + \theta_t^k\left(r_t^k - (R_t - \delta)\right) - \chi\theta_t^k\varsigma_t\sigma + \theta_t^{\text{fund}}\varsigma_t\sigma \tag{87}$$

simplifying this using from entrepreneurs' capital demand that

$$\theta_t^k(r_t^k - (R_t - \delta)) = \left(\theta_t^k\chi\tilde{\sigma}\bar{A}\left(\frac{L}{K_t}\right)^{1-\alpha}\right)^2$$

and demand for the mutual fund gives us the following expression

$$r_t^E = r_t + \left(\tilde{\sigma}_t^E\right)^2 + \sigma^2 \tag{88}$$

Similarly, the expected return to diversified capitalists' wealth is

$$r_t^D = r_t + \sigma^2 \tag{89}$$

The overall return to wealth is then

$$r_t^K = \eta_t r_t^E + (1-\eta_t) r_t^D = r_t + \eta_t \left(\tilde{\sigma}_t^E\right)^2 + \sigma^2 \tag{90}$$

The return to the mutual fund and entrepreneurial capital is

$$r_t^T = r_t + \sigma^2, \quad r_t^k = r_t + \frac{\eta_t}{\kappa_t} \left(\tilde{\sigma}_t^E\right)^2 + \sigma^2 \tag{91}$$

Finally, the risk-free rate is pinned down by the first-order condition of the legacy firms' capital demand:

$$r_t = R_t - \delta - \sigma^2 = p_t^T \alpha \underline{A} \left(\frac{L}{K_t}\right)^{1-\alpha} - \delta - \sigma^2 \tag{92}$$

B.10 Labor share and capital-output ratio

The labor share in the legacy firm is

$$\frac{w_t L_t^T}{p_t^T Y_t^T} = 1 - \alpha \tag{93}$$

The labor share in the entrepreneurial firms is

$$\frac{w_t L_t^E}{p_t^E Y_t^E} = \frac{p_t^T (1-\alpha) \underline{A} \left(\frac{K_t}{L}\right)^\alpha}{p_t^E \bar{A} \left(\frac{K_t}{L}\right)^\alpha} = (1-\alpha) \frac{p_t^T}{p_t^E} \frac{\underline{A}}{\bar{A}} \tag{94}$$

using that $r_t^K - r_t^{\text{out}} = (p_t^E \bar{A} - p_t^T \underline{A}) \left(\frac{L}{K}\right)^{1-\alpha}$ This can be rewritten as

$$LS^E = (1-\alpha) \left(1 - \underbrace{\frac{(r_t^K - r_t^{\text{out}}) K_t^E}{p_t^E Y_t^E}}_{\text{“factorless'' share}}\right) \tag{95}$$

The labor share in the overall economy is

$$\frac{w_t L}{Y_t} = (1-\alpha) \frac{p_t^T \underline{A} \left(\frac{K_t}{L}\right)^\alpha L}{A(\kappa_t) \left(\frac{K_t}{L}\right)^\alpha L} = (1-\alpha) p_t^T \frac{\underline{A}}{A(\kappa_t)} \tag{96}$$

We also derive the following expression for the labor share as the weighted average of the labor share in the two sectors. First, note that:

$$LS = \frac{wL}{Y} = \theta \frac{wL^E}{p_t^E Y_t^E} + (1-\theta)\frac{wL^T}{p_t^T Y_t^T} = \theta_t \frac{\kappa_t wL}{\nu_t Y_t} + (1-\theta_t)\frac{(1-\kappa_t)wL}{(1-\nu_t)Y_t}$$

$$= \theta_t \frac{\kappa_t}{\nu_t} LS + (1-\theta_t)\frac{1-\kappa_t}{1-\nu_t} LS$$

Which implies

$$1 = \theta_t \frac{\kappa_t}{\nu_t} + (1-\theta_t)\frac{1-\kappa_t}{1-\nu_t} \tag{97}$$

so that θ_t is

$$\theta_t = \frac{1 - \frac{1-\kappa_t}{1-\nu_t}}{\frac{\kappa_t}{\nu_t} - \frac{1-\kappa_t}{1-\nu_t}} \tag{98}$$

which can be rewritten as

$$\theta_t = \frac{1-\nu_t - 1 + \kappa_t}{\frac{1-\nu_t}{\nu_t}\kappa_t - 1 + \kappa_t} \tag{99}$$

$$\theta_t = \frac{\nu_t(\kappa_t - \nu_t)}{(1-\nu_t)\kappa_t - \nu_t(1-\kappa_t)} \tag{100}$$

$$\theta_t = \nu_t \tag{101}$$

So the overall labor share is the sales weighted labor share between the two sectors. Using the previous expressions for the labor shares in the two sectors is

$$LS = \nu_t LS^E + (1-\nu_t)LS^T = LS^T - \nu_t \left(LS^T - LS^E\right) \tag{102}$$

which means

$$LS = (1-\alpha)\left[1 - \nu_t \left(1 - \frac{p_t^T}{p_t^E}\frac{A}{\bar{\bar{A}}}\right)\right] \tag{103}$$

or differently

$$LS = (1-\alpha)\left[1 - \frac{\nu_t}{p_t^E}\left(p_t^E - p_t^T \frac{A}{\bar{\bar{A}}}\right)\right] \tag{104}$$

Recall that $\tilde{\sigma}_t^E = \frac{p_t^E - p_t^T \frac{A}{\bar{\bar{A}}}}{\chi \tilde{\sigma}}$. We can therefore write this expression of the labor share as

$$LS = (1-\alpha)\left[1 - \frac{\nu_t}{p_t^E}\tilde{\sigma}_t^E \chi \tilde{\sigma}\right] \tag{105}$$

We can go further by noting that

$$\frac{\nu_t}{p_t^E} = \frac{p_t^E Y_t^E}{p_t^E Y_t} = \frac{Y_t^E}{Y_t} = \frac{\bar{A}\left(\frac{L}{K}\right)^{1-\alpha} K_t^E}{A(\kappa_t)\left(\frac{L}{K}\right)^{1-\alpha} K_t} = \frac{\bar{A}\kappa_t}{A(\kappa_t)} \tag{106}$$

We therefore write the labor share as

$$LS = (1-\alpha)\left[1 - \frac{\bar{A}\kappa_t}{A(\kappa_t)}\chi\tilde{\sigma}\tilde{\sigma}_t^E\right] \tag{107}$$

We can use this expression to show that the behavior of the labor share is informative with regard to whether or not the supply of capital to entrepreneurs is high enough for financial innovation to increase the absolute risk exposure of entrepreneurs. To see this, suppose that we are in an economy where the elasticity is not high enough. In such a world, by definition, a fall in χ would lead to a fall in $\kappa_t\chi$ and therefore in $\tilde{\sigma}_t^E$. Noting that $A(\kappa_t)$ always increases when κ_t increases and $\varepsilon > 0$, we see that the expression $\frac{\bar{A}\kappa_t\chi}{A(\kappa_t)}\tilde{\sigma}_t^E\tilde{\sigma}$ must fall in this case. But then the aggregate labor share would go up. Hence, the aggregate labor share would go up in response to improvements in the ability of entrepreneurs to offload risk to financial markets if the supply elasticity was low.

B.10.1 Capital share

The pure capital share in the entrepreneurial firm is

$$\frac{R_t K_t^E}{p_t^E Y_t^E} = \alpha \frac{p_t^T}{p_t^E}\frac{\underline{\underline{A}}}{\bar{A}} \tag{108}$$

From this we see that the pure entrepreneurial share, or "factorless income" of the income in the entrepreneurial firms is

$$1 - \frac{w_t L_t^E}{p_t^E Y_t^E} - \frac{R_t K_t^E}{p_t^E Y_t^E} = 1 - \frac{p_t^T}{p_t^E}\frac{\underline{\underline{A}}}{\bar{A}} = \frac{\tilde{\sigma}\tilde{\sigma}^E}{p_t^E}\chi \tag{109}$$

Moreover, the capital-output-ratio in the economy is

$$\frac{K_t}{Y_t} = \frac{1}{A(\kappa_t)}\left(\frac{K_t}{L}\right)^{1-\alpha} \tag{110}$$

B.11 Evolution of state

The state variable K_t evolves according to

$$\frac{dK_t}{K_t} = \left(r_t^K - \rho\right) dt + \sigma dZ_t \tag{111}$$

and the state variable η_t evolves according to

$$\frac{d\eta_t}{\eta_t} = (1 - \eta_t)\left(\tilde{\sigma}_t^E\right)^2 dt + \left(\frac{\delta_d \psi^0 - \eta_t(\delta_d + \phi^I)}{\eta_t}\right) dt \tag{112}$$

B.12 Summary of key model objects

Key model objects	
r_t:	$p_t^T \alpha \underline{A} \left(\frac{L}{K_t}\right)^{1-\alpha} - \delta - \sigma^2$
r_t^{out}:	$r_t + \sigma^2$
$R_t - \delta$:	$r_t + \sigma^2$
r_t^k:	$r_t + \sigma^2 + \frac{\eta_t}{\kappa_t}\left(\tilde{\sigma}^E\right)^2$
r_t^K:	$r_t + \sigma^2 + \eta_t\left(\tilde{\sigma}^E\right)^2$
r_t^E:	$r_t + \sigma^2 + \left(\tilde{\sigma}^E\right)^2$
r_t^D:	$r_t + \sigma^2$
LS^T:	$1 - \alpha$
LS^E:	$(1-\alpha)\frac{p_t^T}{p_t^E}\frac{\underline{A}}{\overline{A}}$
LS:	$(1-\alpha)p_t^T \frac{\underline{A}}{A(\kappa_t)}$
$\frac{K_t}{Y_t}$:	$\frac{1}{A(\kappa_t)}\left(\frac{K_t}{L}\right)^{1-\alpha}$

Table 5: Summary of key model objects

$$\frac{Y - wL - \delta K}{Y} = \frac{I + C^c - \delta K}{Y} = \frac{I + \rho K - \delta K}{Y} = \rho\frac{K}{Y} = \rho\frac{1}{A(\kappa)}\left(\frac{K}{L}\right)^{1-\alpha}$$

$$\chi K_t^E = \text{internal financing} \Rightarrow \chi\frac{K_t^E}{N_t} = \chi\kappa_t \Rightarrow 1 - \chi\kappa_t = \text{external financing}$$

C Steady State Equilibrium and transition dynamics

A long-run steady state can be defined when setting aggregate shocks $dZ_t = 0$. In this case, a steady state equilibrium is an equilibrium where the state variables K_t and η_t are constant, i.e. where:

$$\begin{aligned}\frac{dK_t}{K_t} &= \left(r^K_{ss} - \rho\right) dt = 0 \\ \frac{d\eta_t}{\eta_t} &= (1-\eta_{ss})\left(\tilde{\sigma}^E_{ss}\right)^2 dt + \left(\frac{\delta_d \psi^0 - \eta_{ss}(\delta_d + \phi^l)}{\eta_{ss}}\right) dt = 0\end{aligned} \tag{113}$$

C.1 Analyzing the steady state

Plugging in the expression for r^K_{ss} in equation (90) evaluated in steady state, the evolution of the economy is described by the following pair of ordinary differential equations:

$$\begin{aligned}\frac{dK_t}{K_t} &= \left(p^T \alpha \underline{A}\left(\frac{L}{K}\right)^{1-\alpha} + \eta\left(\tilde{\sigma}^E\right)^2 - \rho - \delta\right) dt \\ \frac{d\eta_t}{\eta_t} &= \left((1-\eta)\left(\tilde{\sigma}^E\right)^2 + \frac{(\bar{\psi} - \eta)\left(\delta_d + \phi^l\right)}{\eta}\right) dt\end{aligned} \tag{114}$$

where $\tilde{\sigma}^E_{ss} = \frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\bar{A}\right)\left(\frac{L}{K}\right)^{1-\alpha}$, and the equilibrium condition for the allocation of the capital stock:

$$\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}} = \frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\bar{A}\right)\left(\frac{L}{K}\right)^{1-\alpha} \tag{115}$$

where

$$\begin{aligned}p^E = \nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}&, p^T = (1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon} \\ A(\kappa) = \Big[\nu\left(\bar{A}\kappa\right)^{\frac{\varepsilon-1}{\varepsilon}} &+ (1-\nu)\left(\underline{A}(1-\kappa)\right)^{\frac{\varepsilon-1}{\varepsilon}}\Big]^{\frac{\varepsilon}{\varepsilon-1}}\end{aligned} \tag{116}$$

The model admits a steady state if there is a solution to the following system of three equations in the three variables η, K and κ:

$$\begin{aligned}p^T\alpha\underline{A}\left(\frac{L}{K}\right)^{1-\alpha} + \eta\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2 - (\rho+\delta) &= 0 \\ (1-\eta)\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2 + \frac{(\bar{\psi}-\eta)\left(\delta_d + \phi^l\right)}{\eta} &= 0 \\ \frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}} - \frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\bar{A}\right)\left(\frac{L}{K}\right)^{1-\alpha} &= 0\end{aligned} \tag{117}$$

Definition 1. *A non-degenerate steady state equilibrium is a triplet $s = (\eta, K, \kappa)$ that satisfies the equations (117) with $K, \eta > 0$ and $\kappa \in [0,1]$.*

I begin the analysis by proving the following lemmata:

Lemma 4. *If* $\varepsilon > 1$ *then* $\lim_{\kappa\to 0} p^T = \underline{p}^T \equiv (1-\nu)^{\frac{\varepsilon}{\varepsilon-1}}$ *and* $\lim_{\kappa\to 0} p^E = \infty$. *If* $\varepsilon < 1$, *then* $\lim_{\kappa\to 0} p^T = 0$ *and* $\lim_{\kappa\to 0} p^E = \nu^{\frac{\varepsilon}{\varepsilon-1}} < \infty$. *Symmetric limits apply to* $\kappa \to 1$.

Proof.

$$p^T = (1-\nu)\left(\frac{A(\kappa)}{\underline{A}(1-\kappa)}\right)^{\frac{1}{\varepsilon}} = (1-\nu)\left(\nu\left(\frac{\bar{A}\kappa}{\underline{A}(1-\kappa)}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right)^{\frac{1}{\varepsilon-1}}$$

If $\varepsilon > 1$, then the exponent on the ratio inside the bracket is positive and therefore

$$\lim_{\kappa\to 0} p^T = (1-\nu)\left(\nu\left(\frac{\bar{A}\cdot 0}{\underline{A}}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\right)^{\frac{1}{\varepsilon-1}} = (1-\nu)^{\frac{\varepsilon}{\varepsilon-1}}.$$

If $0 < \varepsilon < 1$ then the exponent inside the bracket is negative, and so the argument inside the bracket goes to ∞ as $\kappa \to 0$. But the exponent outside the bracket is also negative and hence $\lim_{\kappa\to 0} p^T = 0$ in this case. For p^E we instead have

$$p^E = \nu\left(\frac{A(\kappa)}{\bar{A}\kappa}\right)^{\frac{1}{\varepsilon}} = \nu\left(\nu + (1-\nu)\left(\frac{\underline{A}(1-\kappa)}{\bar{A}\kappa}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}.$$

If $\varepsilon > 1$, the exponent inside the bracket is positive, and hence the expression inside the bracket goes to ∞ as $\kappa \to 0$. The exponent outside the brackets is positive and so $p^E \to \infty$ as $\kappa \to 0$. If $\varepsilon < 1$ then the exponents are both negative, and so $p^E \to \nu^{\frac{\varepsilon}{\varepsilon-1}}$. $\square$

Lemma 5. *Steady state values of* κ *are bounded above by* $\bar{\kappa} \equiv \frac{\left(\frac{\underline{A}}{\bar{A}}\right)\left(\frac{\bar{A}\nu}{\underline{A}(1-\nu)}\right)^{\varepsilon}}{1+\left(\frac{\underline{A}}{\bar{A}}\right)\left(\frac{\bar{A}\nu}{\underline{A}(1-\nu)}\right)^{\varepsilon}} < 1$

Proof. From equation (115) we see that the numerator on the left-hand side cannot be negative, since all objects on the right-hand side are positive (risk exposure cannot be negative). Therefore

$$\bar{A}p^E - \underline{A}p^F = \bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon} - \underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon} > 0 \tag{118}$$

This implies

$$\frac{\bar{A}\nu}{\underline{A}(1-\nu)} > \left(\frac{\bar{A}\kappa}{\underline{A}(1-\kappa)}\right)^{1/\varepsilon} \Rightarrow \kappa < \bar{\kappa} \equiv \frac{\left(\frac{\underline{A}}{\bar{A}}\right)\left(\frac{\bar{A}\nu}{\underline{A}(1-\nu)}\right)^{\varepsilon}}{1+\left(\frac{\underline{A}}{\bar{A}}\right)\left(\frac{\bar{A}\nu}{\underline{A}(1-\nu)}\right)^{\varepsilon}} < 1 \tag{119}$$

$\square$

Lemma 6. *Aggregate TFP $A(\kappa)$ is increasing in κ for $\kappa \leq \bar{\kappa}$.*

Proof. We can write

$$
\begin{aligned}
A(\kappa) &= \left[\nu\left(\bar{A}\kappa\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(\underline{A}(1-\kappa)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\nu\bar{A}^{\frac{\varepsilon-1}{\varepsilon}}(\kappa)^{-\frac{1}{\varepsilon}}\kappa + (1-\nu)\underline{A}^{\frac{\varepsilon-1}{\varepsilon}}(1-\kappa)^{-\frac{1}{\varepsilon}}(1-\kappa)\right]^{\frac{\varepsilon}{\varepsilon-1}}.
\end{aligned}
\tag{120}
$$

If we can show that $\nu\bar{A}^{\frac{\varepsilon-1}{\varepsilon}}(\kappa_t)^{-\frac{1}{\varepsilon}} > (1-\nu)\underline{A}^{\frac{\varepsilon-1}{\varepsilon}}(1-\kappa_t)^{-\frac{1}{\varepsilon}}$, we are done. This is because what is inside the bracket is a convex combination of $\bar{A}^{\frac{\varepsilon-1}{\varepsilon}}(\kappa_t)^{-\frac{1}{\varepsilon}}$ and $(1-\nu)\underline{A}^{\frac{\varepsilon-1}{\varepsilon}}(1-\kappa_t)^{-\frac{1}{\varepsilon}}$. But using again the non-negativity of risk exposure, we have

$$
\bar{A}p^E - \underline{A}p^F = \bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon} - \underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon} > 0
$$

$$
\begin{aligned}
\Leftrightarrow \bar{A}\nu\left(\bar{A}\kappa\right)^{-1/\varepsilon} - \underline{A}(1-\nu)\left(\underline{A}(1-\kappa)\right)^{-1/\varepsilon} &> 0 \\
\nu\bar{A}^{\frac{\varepsilon-1}{\varepsilon}}(\kappa_t)^{-\frac{1}{\varepsilon}} &> (1-\nu)\underline{A}^{\frac{\varepsilon-1}{\varepsilon}}(1-\kappa_t)^{-\frac{1}{\varepsilon}}
\end{aligned}
$$

which is what we wanted to show. □

Lemma 7. *Steady state values of κ are bounded below by $\underline{\kappa} > 0$ if $\varepsilon > 1$.*

Proof. Through the second steady state equation in (117), we can implicitly define $\eta(\kappa)$ as the value of η that solves this equation for a given value of κ. $\eta(\kappa)$ is a decreasing function of κ for $\kappa < \bar{\kappa}$. This is because, when $\kappa < \bar{\kappa}$ we know by the earlier lemmas that p^E is strictly decreasing in κ, and p^T is strictly increasing in κ. This means that $\bar{A}p^E - \underline{A}p^T$ is strictly decreasing in κ. This implies that higher values of κ means a lower value for $\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2$. With a lower value of $\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2$, the value of η that solves (117) is also lower. Hence $\eta(\kappa)$ is decreasing in κ. Now, we show that there is a lowest admissible value for κ, denoted $\underline{\kappa} > 0$ when $\varepsilon > 1$. As $\kappa \to 0$, then p^E to ∞ when $\varepsilon > 1$. Looking at the equation

$$
p^T\alpha\underline{A}\left(\frac{L}{K}\right)^{1-\alpha} + \eta\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2 - (\rho+\delta) = 0
$$

we see that as $\kappa \to 0$, the second term increases without bound, and hence, it will surpass the value of $\rho + \delta$. Let $\bar{\kappa}$ be the value of for which $\eta\left(\frac{\bar{A}p^E - \underline{A}p^T}{\chi\tilde{\sigma}\bar{A}}\right)^2 - (\rho+\delta) = 0$, then there can be no steady states with $0 < \kappa \leq \bar{\kappa}$, because, then the first term $p^T\alpha\underline{A}\left(\frac{L}{K}\right)^{1-\alpha}$ has to be less than or equal to 0, which cannot happen for $\kappa > 0$. We solve for □

C.2 Existence of steady state

We can now prove the existence of steady state for $\varepsilon > 1$. By substituting the expressions for the prices into the three steady state equations in (117), we obtain

$$
\begin{aligned}
(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}\alpha\underline{A}\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}+\left(\frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\bar{A}\right)\left(\frac{L}{K}\right)^{1-\alpha}\right)^{2}-\frac{\rho+\delta}{\eta}&=0\\
(1-\eta)\left(\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}\right)^{2}+\frac{(\bar{\psi}-\eta)\left(\delta_d+\phi^l\right)}{\eta}&=0\\
\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}-\frac{\kappa}{\eta}\left(\chi\tilde{\sigma}\bar{A}\right)\left(\frac{L}{K}\right)^{1-\alpha}&=0
\end{aligned}
\tag{121}
$$

Note that the first equation does not depend K directly, but only on $\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}$, which can be solved for from the last equation in terms of κ. Specifically,

$$
\frac{1}{\eta}\left(\frac{L}{K}\right)^{1-\alpha}=\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma}\bar{A})^{2}}
\tag{122}
$$

Substituting this into the first equation gives

$$
\begin{aligned}
&(1-\nu)\alpha\underline{A}\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma}\bar{A})^{2}}\\
&+\left(\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}\right)^{2}-\frac{\rho+\delta}{\eta}=0
\end{aligned}
\tag{123}
$$

This equation, together with

$$
(1-\eta)\left(\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}\right)^{2}+\left(\frac{\bar{\psi}}{\eta}-1\right)\left(\delta_d+\phi^l\right)=0
\tag{124}
$$

defines the steady states of the model. Given any κ, the second has exactly one solution $\eta(\kappa)$ on the interval $(0,1)$. To see this latter fact note that $\eta=0$ is not admissible, so we can multiply through by η without affecting the location of the roots. Then the right-hand side is a quadratic function of η. At $\eta=1$, the quadratic function is $(\bar{\psi}-1)\left(\delta_d+\phi^l\right)<0$ and at $\eta = 0$, that quadratic is $\bar{\psi}(\delta_d + \phi^l) > 0$. Hence, it crosses the x-axis once on the interval $\eta \in (0,1)$. Denote this value $\eta(\kappa)$. Note also that $\eta(\kappa)$ is strictly decreasing in κ. If κ rises, then the squared term in the brackets will fall, to maintain equality, η must also fall since the expression is strictly decreasing in η. In summary, candidate steady states are determined by the solutions to the following equation in κ

$$\begin{aligned}&(1-\nu)\alpha\underline{A}\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma}\bar{A})^2}\\&+\left(\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}\right)^2-\frac{\rho+\delta}{\eta(\kappa)}=0\end{aligned}\tag{125}$$

where $\eta(\kappa)$ is a strictly decreasing in κ. We can go further in narrowing down the candidate steady states. To prove that a steady state exists, we define the functions

$$\begin{aligned}h(\kappa)&=\frac{\rho+\delta}{\eta(\kappa)}-\left(\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\chi\tilde{\sigma}\bar{A}}\right)^2\\f(\kappa)&=(1-\nu)\alpha\underline{A}\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}\frac{\bar{A}\nu\left(\frac{\bar{A}\kappa}{A(\kappa)}\right)^{-1/\varepsilon}-\underline{A}(1-\nu)\left(\frac{\underline{A}(1-\kappa)}{A(\kappa)}\right)^{-1/\varepsilon}}{\kappa(\chi\tilde{\sigma}\bar{A})^2}\end{aligned}\tag{126}$$

The steady state equation is then $f(\kappa) = h(\kappa)$. Note that $h(\underline{\kappa}) = 0$ by the definition of $\underline{\kappa}$. Note also that $h(\kappa) \to -\infty$ when $\kappa \to 0$, and $h(\kappa) \to \frac{\rho+\delta}{n(\kappa)} > 0$ when $\kappa \to \bar{\kappa}$. By definition of $\bar{\kappa}$ and $\underline{\kappa}$ we have $f(\bar{\kappa}) = 0$ and $f(\underline{\kappa}) > 0$, while $h(\bar{\kappa}) = \frac{\rho+\delta}{\eta(\underline{\kappa})} > 0$ and $h(\underline{\kappa}) = 0$. By the intermediate value theorem, these lines must cross at least once within the relevant interval, hence a steady state exists. Uniqueness of the steady state can be demonstrated analytically in the case when the goods are perfect substitutes, i.e. $\varepsilon = \infty$. If the goods are perfect substitutes, then note that $\eta(\kappa) = \bar{\eta}$ does not depend on κ since it is pinned down by the equation

$$(1-\eta)\left(\frac{\bar{A}\nu-\underline{A}(1-\nu)}{\chi\tilde{\sigma}\bar{A}}\right)^2+\left(\frac{\bar{\psi}}{\eta}-1\right)\left(\delta_d+\phi^l\right)=0 \tag{127}$$

Given this, we have

$$h(\kappa)=\frac{\rho+\delta}{\bar{\eta}}-\left(\frac{\bar{A}\nu-\underline{A}(1-\nu)}{\chi\tilde{\sigma}\bar{A}}\right)^2 \tag{128}$$

so that $h(\kappa)$ also does not depend on κ. While at the same time

$$f(\kappa) = (1-\nu)\alpha \underline{A} \frac{\bar{A}\nu - \underline{A}(1-\nu)}{\kappa(\chi\tilde{\sigma}\bar{A})^2} \tag{129}$$

is strictly decreasing in κ as long as $\bar{A}\nu - \underline{A}(1-\nu) > 0$ (which ensures that entrepreneurs' idiosyncratic volatility is positive). There is one additional parameter restriction. In particular, the point at which $f(\kappa)$ and $h(\kappa)$ intersect must be such that $\kappa \in (0,1)$. This condition is

$$\kappa = \bar{\eta} \frac{(1-\nu)\alpha \underline{A} \frac{\bar{A}\nu - \underline{A}(1-\nu)}{(\chi\tilde{\sigma}\bar{A})^2}}{\rho + \delta - \bar{\eta}\left(\frac{\bar{A}\nu - \underline{A}(1-\nu)}{\chi\tilde{\sigma}\bar{A}}\right)^2} \in (0,1) \tag{130}$$

This ensures that entrepreneurs are not so much more productive than the traditional sector that they want to hold more than 100% of the capital stock, and that their precautionary savings motive is not so strong that the capital stock grows to without bound.

C.3 Steady state tail coefficient

In this section, I omit the subscript denoting steady state. All variables should be understood as being in steady state. First note that in steady state, the diffusion and drift of the wealth growth of entrepreneurs is

$$\tilde{\sigma}^E, \quad \text{and} \quad \mu^E = r^E - \rho = (1-\eta)\left(\tilde{\sigma}^E\right)^2 \tag{131}$$

The relevant Kolmogorov forward equation for the distribution of entrepreneurs' wealth can therefore be written as

$$0 = -\frac{\partial}{\partial n}\left[\mu^E n f_E(n)\right] + \frac{\partial^2}{\partial n^2}\left[\frac{1}{2}(\tilde{\sigma}^E)^2 n^2 f_E(n)\right] - (\delta_d + \phi^l) f_E(n) \tag{132}$$

Guessing that the distribution takes the form of a double Pareto distribution

$$f_E(n) = A_E \zeta n^{-\zeta-1} \mathbf{1}_{n>N} + (1-A_E)(-\zeta_-) n^{-\zeta_- -1} \mathbf{1}_{n<N}$$

where N is the steady state average wealth level (which is the wealth level at birth) we have the following equation for the tail coefficient for the right tail ζ

$$0 = \zeta \mu^E + \frac{(\tilde{\sigma}^E)^2}{2} \zeta(\zeta - 1) - (\delta_d + \phi^l) \tag{133}$$

This can be rewritten as

$$\zeta^2 + \zeta\left(\frac{2\mu^E}{\left(\tilde{\sigma}^E\right)^2} - 1\right) - \frac{2(\delta_d + \phi^l)}{\left(\tilde{\sigma}^E\right)^2} = 0 \tag{134}$$

using that $\frac{2\mu^E}{\left(\tilde{\sigma}^E\right)^2} = 2(1-\eta)$ we can write this as

$$\zeta^2 + \zeta\,(1 - 2\eta) - \frac{2(\delta_d + \phi^l)}{\left(\tilde{\sigma}^E\right)^2} = 0 \tag{135}$$

the positive solution to this is

$$\zeta = \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{1}{2}\right)^2 + \frac{2(\delta_d + \phi^l)}{\left(\tilde{\sigma}^E\right)^2}} \tag{136}$$

We can go further in characterizing this since the steady state condition for η, $d\eta = 0$ implies

$$\eta\,(1-\eta)\,(\tilde{\sigma}^E)^2 = -\left(\delta_d\psi^0 - \delta_d\eta - \phi^l\eta\right) \tag{137}$$

using that in steady state $\delta_d\psi^0 = \bar{\psi}(\delta_d + \phi^l)$ this implies

$$\eta\,(1-\eta)\,(\tilde{\sigma}^E)^2 = (\eta - \bar{\psi})(\delta_d + \phi^l) \tag{138}$$

so that

$$\frac{2\eta(1-\eta)}{\eta - \bar{\psi}} = \frac{2(\delta_d + \phi^l)}{\left(\tilde{\sigma}^E\right)^2} \tag{139}$$

Note first that this tells us that $\bar{\psi} < \eta$ in steady state, otherwise the left-hand side is negative, which cannot happen since the right-hand side is strictly positive. Moreover, plugging this into the expression for ζ then gives

$$\zeta = \eta - \frac{1}{2} + \sqrt{\left(\eta - \frac{1}{2}\right)^2 + \frac{2\eta(1-\eta)}{\eta - \bar{\psi}}} \tag{140}$$

the left-tail coefficient can be solved similarly, and the weight A_E is determined so that the density integrates to 1. Since switching only occurs from entrepreneurs to diversified, Pareto tail coefficients for the distribution of wealth for diversified capitalists are inherited from the entrepreneurs' distribution.

We now proceed to proving that ζ is strictly decreasing in η. In other words, inequality is increasing in η. First, note that the expression $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$ is strictly decreasing in η. To see this, not that its derivative is

$$\frac{(2(1-\eta)-2\eta)\left(\eta-\bar{\psi}\right)-2\eta(1-\eta)}{(\eta-\bar{\psi})^2}=\frac{2\eta-4\eta^2-2\bar{\psi}+4\bar{\psi}\eta-2\eta+2\eta^2}{(\eta-\bar{\psi})^2}=$$

$$\frac{-2\eta^2+4\bar{\psi}\eta-2\psi^2+2\psi^2-2\bar{\psi}}{(\eta-\bar{\psi})^2}=\frac{-2(\eta-\bar{\psi})^2-2\bar{\psi}(1-\bar{\psi})}{(\eta-\bar{\psi})^2}=-2\left(1+\frac{\bar{\psi}(1-\bar{\psi}))}{(\eta-\bar{\psi})^2}\right)<0. \quad (141)$$

Moreover, clearly $\eta-\frac{1}{2}$ is increasing in η. The question is therefore if the slope of the second term under the bracket, $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$, negative enough to counteract the positive slope coming from the terms $\eta-\frac{1}{2}$ and $(\eta-\frac{1}{2})^2$. To prove this, note that equation (141) implies that slope of $\frac{2\eta(1-\eta)}{\eta-\bar{\psi}}$ is *least* negative (smallest in magnitude) when $\bar{\psi}=0$. Moreover, for any other admissible value of $\bar{\psi}$ (that is $\bar{\psi}<\eta<1$) the slope of this term is more negative. Hence, if we can show that ζ is non-increasing in η when $\bar{\psi}=0$, then it must be the case that ζ is decreasing in η when $\bar{\psi}>0$. If we plug in $\bar{\psi}=0$ in the expression for η, we obtain

$$\eta-\frac{1}{2}+\sqrt{\left(\eta-\frac{1}{2}\right)^2+2(1-\eta)}=\eta-\frac{1}{2}+\sqrt{\eta^2-3\eta+\frac{9}{4}}=$$

$$=\eta-\frac{1}{2}+\sqrt{\left(\eta-\frac{3}{2}\right)^2}=\eta-\frac{1}{2}+|\eta-\frac{3}{2}|$$

Using that $\eta\in(0,1)$, we can write this as

$$=\eta-\frac{1}{2}+\sqrt{\left(\eta-\frac{3}{2}\right)^2}=\eta-\frac{1}{2}-\eta+\frac{3}{2}=1$$

The slope of this is 0. Hence, when $\bar{\psi}=0$, the slope of ζ with respect to η is 0, and we know that the slope is strictly smaller for all other admissible $\bar{\psi}$, hence, ζ is strictly decreasing in η.

Finally, we proceed by showing that ζ is strictly decreasing in $\tilde{\sigma}^E$. This follows from the fact that equation (139) implies that η is strictly increasing in $\tilde{\sigma}^E$, because the left-hand side is decreasing in η and the right-hand side is decreasing in $\tilde{\sigma}^E$. Since ζ is decreasing in η, it must be then that ζ is decreasing in $\tilde{\sigma}^E$ as well.

C.4 Changes in inequality after a fall in χ

Proposition 2 discusses small changes in χ. In this section, I study what happens to inequality for larger changes in χ and we discuss transition dynamics. More specifically, we will consider the transition dynamics of the model in the following type of experiment.

We let the initial values K_0, η_0, and κ_0 be associated with an initial steady state s_0 with skin-in-the-game parameter χ_0. We then examine the transition dynamics of the model in response to a change in χ to $\chi_1 < \chi_0$. For this exercise to make sense, I will assume that there is a unique (non-degenerate) steady state associated with the new value χ_1. I will also have to assume that the transition dynamics ensure that we converge to this new steady state.

Proposition 4. *Consider an initial (non-degenerate) steady state $s_0 = (K_0, \eta_0, \kappa_0)$ with $\kappa_0 \in (0,1)$, associated with skin-in-the-game parameter χ_0, and a different steady state $s_1 = (K_1, \eta_1, \kappa_1)$ with $\kappa_1 \in (0,1)$, associated with $\chi_1 < \chi_0$. All other parameters are fixed, in particular, the parameter ε is the same for both steady states. Then, there exists a $\varepsilon^*_{s_0,s_1}$ such that if $\varepsilon > \varepsilon^*_{s_0,s_1}$, $\eta_1 > \eta_0$ and Pareto inequality is higher in s_1.*

Proof. The initial value of the entrepreneurs' risk exposure is

$$\tilde{\sigma}^E_0 = \frac{\bar{A}p^E(\kappa_0) - \underline{A}p^T(\kappa_0)}{\chi_0 \tilde{\sigma} \bar{A}}.$$

Because the prices can be made arbitrarily insensitive to changes in $\kappa \in (0,1)$ by letting ε be large enough, we know that there exists some $\varepsilon^*_{s_0,s_1}$ such that if $\varepsilon > \varepsilon^*_{s_0,s_1}$ we have

$$\tilde{\sigma}^E_0 = \frac{\bar{A}p^E(\kappa_0) - \underline{A}p^T(\kappa_0)}{\chi_0 \tilde{\sigma} \bar{A}} < \frac{\bar{A}p^E(\kappa_1) - \underline{A}p^T(\kappa_1)}{\chi_1 \tilde{\sigma} \bar{A}} = \tilde{\sigma}^E_1.$$

Because $\tilde{\sigma}^E_1 > \tilde{\sigma}^E_0$, it must be that $\eta_1 > \eta_0$, which means Pareto inequality is higher in the new steady state. □

Now we examine the transition dynamics more closely. Recall that the transition dynamics of the model are determined by the evolution of the state variables

$$\begin{aligned}
\frac{dK_t}{K_t} &= \left(p^T(\kappa_t)\alpha \underline{A}\left(\frac{L}{K_t}\right)^{1-\alpha} + \eta_t \left(\tilde{\sigma}^E_t\right)^2 - \rho - \delta\right) dt \\
\frac{d\eta_t}{\eta_t} &= \left((1-\eta_t)\left(\tilde{\sigma}^E_t\right)^2 + \frac{(\bar{\psi} - \eta_t)\left(\delta_d + \phi^l\right)}{\eta_t}\right) dt
\end{aligned} \tag{142}$$

and the equilibrium condition for the allocation of the capital stock:

$$\frac{\bar{A}p(\kappa_t)^E - \underline{A}p^T(\kappa_t)}{\chi \tilde{\sigma} \bar{A}} = \frac{\kappa_t}{\eta_t}\left(\chi \tilde{\sigma} \bar{A}\right)\left(\frac{L}{K_t}\right)^{1-\alpha} \equiv \tilde{\sigma}^2_t \tag{143}$$

To understand what happens, when χ falls, let's consider a transition between steady state when χ falls.

Lemma 8. *In this experiment, κ_t increases on impact.*

Proof. On impact, K_t and η_t are fixed, so (143) implies that κ_t must increase to maintain equilibrium if χ falls. $\square$

The above lemma tells us what happens on impact. To examine what happens in the transition, we need to study how η_t and K_t evolve. Clearly, for high enough values ε, the idiosyncratic risk exposure of entrepreneurs $\tilde{\sigma}_t^E$ rises on impact. According to the equations describing the evolution of the state variables above, both K_t and η_t will start growing. Looking at the equilibrium condition for the capital allocation, as η_t and K_t start growing, κ_t rises further. The intuition is that as entrepreneurs become wealthier and the operational leverage of the economy (Y/K) becomes smaller, entrepreneurs are better equipped to bear risk and this scale up even more. However, looking at the left-hand side of the equilibrium capital allocation equation (143), we see that this scaling up *after* impact is going to be a reduction in the Sharpe ratio for entrepreneurs (compared to the initial upward jump). In other words, even if the Sharpe ratio jumps upwards on impact, this upward jump is moderated somewhat when η_t and K_t start to grow. However, to be consistent with a new steady state with higher risk exposure for entrepreneurs, $\tilde{\sigma}_t^E$ cannot come back down all the way to its initial value. Looking at the equation describing the evolution of η_t, we see that the growth rate of η_t slows down after impact because $\tilde{\sigma}_t^E$ declines and because a higher η_t makes $d\eta_t$ smaller.

For the capital stock the dynamics are less clear. The increase in κ_t raises the risk-free rate on impact because $p^T(\kappa_t)$ rises. Moreover, the fact that both η_t and $\tilde{\sigma}_t^E$ rise, means the drift of the capital stock becomes even higher. However, as the capital stock rises, the marginal product of capital falls according to the standard neoclassical mechanism. Assuming that the economy actually converges to a new steady state it must be the case the effect from the decreasing marginal product of capital is more powerful than the increase in the price p_t^T in the long run. In particular, we know that there is a maximal κ consistent with steady state, $\bar{\kappa} < 1$, so that the price $p^T(\kappa)$ is bounded above by $p^T(\bar{\kappa})$.

Proving that the economy converges to a unique steady state in a multi-sector growth model is difficult. In fact, Boldrin and Deneckere (1990) shows that multi-sector growth models can display chaotic and cyclical behavior even without aggregate risk. Other than in the limit when $\varepsilon \to \infty$, so that p_t^T is constant, I study the convergence to steady state numerically.

C.5 Decreasing inequality even with perfect substitutes

The relationship between Pareto inequality and χ turns around even when $\varepsilon = \infty$, when χ becomes small enough. This does not contradict Proposition 2 or Proposition 4. These propositions tell us that starting in an initial steady state with interior κ, there exists large enough values of ε, so that inequality increases when χ falls. However, as χ is reduced further, and further the required ε becomes larger and larger. This is because as κ gets closer to 1, there is less room for entrepreneurs to scale up at the expense of the traditional firms. And even in the limit as $\varepsilon \to \infty$, there is a limit to the scaling-up effect coming from the fact that with perfects substitutes, small enough χ implies that in steady state $\kappa = \bar{\kappa} = 1$. Further falls in χ beyond this leads to less inequality. To prove this is straightforward for $\varepsilon = \infty$ because tedious the CES-algebra can be avoided. For $\varepsilon < \infty$, the traditional sector is never fully out competed because prices in that sector increase rapidly when κ gets close to 1. This means intuitively that the limits to the scaling-up effect occur even earlier than with $\varepsilon = \infty$. However, this is more challenging to prove analytically. I therefore produce the proof for $\varepsilon = \infty$ and confirm the intuition numerically.

Proposition 5. *Even with perfect substitutes, $\varepsilon = \infty$, there is a value χ^* such that if $\chi < \chi^*$, a further fall in χ reduces Pareto inequality.*

Proof. Note that with perfect substitutes, the condition in equation (130) must be satisfied for both sectors to be active. However, in that expression, we see that as $\chi \to 0$, this condition does not hold. This is because, with perfect substitutes and low χ, there is no steady state with $\kappa \in (0, 1)$. Instead, the entrepreneurs take over the entire economy. Hence, we let χ^* be the supremum of the values of $\chi \in (0, 1)$ for which (130) does *not* hold. We instead seek an equilibrium where only the entrepreneurs are active. In this economy entrepreneurs' optimal portfolio choice implies

$$\frac{1}{\eta}(\chi\tilde{\sigma}^k)^2 = r^k - r. \tag{144}$$

which follows from plugging in $\kappa = 1$ in entrepreneurs optimal portfolio choice. We recall that $\tilde{\sigma}^k_{ss} = \frac{Y}{K}\tilde{\sigma}$. Moreover, when the traditional firms are not active, the risk-free rate is no longer pinned down by the value of the marginal product in that sector. Instead, we have the following system jointly pinning down the wage rate and the risk-free rate:

$$\begin{aligned} \frac{Y}{K} &= \bar{A}\left(\frac{1-\alpha}{\alpha}\frac{r+\delta}{w}\right)^{1-\alpha} \\ \underbrace{\frac{Y}{K} - w\frac{L}{K} - \delta - r}_{r^k - r} &= \frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2 \end{aligned} \tag{145}$$

Solving this gives us

$$
\begin{aligned}
w &= (1-\alpha)\bar{A}\left(\frac{K}{L}\right)^{\alpha}\left[1-\frac{\chi\tilde{\sigma}}{\eta}\left(\chi\tilde{\sigma}\frac{Y}{K}\right)\right] \\
r &= \alpha\bar{A}\left(\frac{L}{K}\right)^{1-\alpha}\left[1-\frac{\chi\tilde{\sigma}}{\eta}\left(\chi\tilde{\sigma}\frac{Y}{K}\right)\right]-\delta
\end{aligned}
\tag{146}
$$

Plugging this into the equations for the evolution of the state variables and setting these to zero we obtain

$$
\begin{aligned}
\frac{dK_t}{K_t} &= \left(\alpha\frac{Y}{K}+(1-\alpha)\frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2-\rho\right)dt = 0 \\
\frac{d\eta_t}{\eta_t} &= \left((1-\eta)\left(\frac{1}{\eta}\chi\frac{Y}{K}\tilde{\sigma}\right)^2+\frac{(\bar{\psi}-\eta)\left(\delta_d+\phi^l\right)}{\eta}\right)dt = 0
\end{aligned}
\tag{147}
$$

From the steady state for η we obtain

$$
\frac{1}{\eta}\left(\chi\frac{Y}{K}\tilde{\sigma}\right)^2 = -\frac{(\bar{\psi}-\eta)(\delta_d+\phi^l)}{1-\eta}
$$

We can then rewrite the steady state equations for K as

$$
\alpha\frac{Y}{K}-\rho = (1-\alpha)\frac{(\bar{\psi}-\eta)(\delta_d+\phi^l)}{1-\eta}
\tag{148}
$$

The left-hand side is strictly decreasing in η (take derivative). This means that if a fall in χ leads to a fall in η, it must lead to a rise in $\frac{Y}{K}$ for this equation to hold, and vice versa. In other words, it must lead to a fall in K. If a fall in χ leads to a rise in η, then it must similarly lead to a rise in K, and vice versa. In other words, when χ falls, η and K must move in the same direction in steady state. To see that the direction is downward, we look at the steady state equation for η. When χ falls, this equation tells us that either η or K must fall. Since we know that they both move in the same direction, this means they must fall.

What happens to the idiosyncratic risk exposure of entrepreneurs? We have

$$
\tilde{\sigma}^E = \frac{1}{\eta}\chi\frac{Y}{K}\tilde{\sigma}
$$

which has to fall. To see this, note that when χ falls, $1/\eta$ and Y/K rise, but if they rise so much that this offsets the fall in χ in the sense that $\tilde{\sigma}^E$ does not fall, then this contradicts η falling, because if $\tilde{\sigma}^E$ falls, then the steady state equation for η tells us that η should not fall. So $\tilde{\sigma}^E$ must fall. Finally, Pareto inequality falls because $\tilde{\sigma}^E$ falls. This concludes the proof. □

D Measuring entrepreneurial wealth

In this section I discuss how the model presented in this paper can help shed light on the proper measurement of the wealth of an entrepreneur. Clarifying how entrepreneurs' wealth is measured in the context of the model, and how it relates to common ways of measuring entrepreneurs' wealth in practice is also crucial for understanding the quantitative exercise in the next section.

Note that the formulation of how entrepreneurial firms are financed in the model makes no references to the number of shares that the entrepreneurs issue, or the prices of these shares. Instead, the financing of the entrepreneurial firms is expressed in terms of the amount of capital raised from outsiders and the expected return that these outsiders receive. There is of course a link between the two formulations of the financing of the firms. Making this link explicit clarifies the difference of how wealth is commonly measured in practice, and how it is measured in the model.

An entrepreneur who has decided on operating a firm with total capital stock k_{it} must provide χk_{it} of the financing herself, and raise $(1-\chi)k_{it}$ from outsiders. Letting N_0 be the initial number of shares, all owned by the entrepreneur, the number of shares that the entrepreneur has to issue, Δ_{Nt}, is defined by

$$\Delta_{Nt} p_{it} = (1-\chi)k_{it}$$

where p_{it} is the price per share issued. The price per share issued on the other hand is pinned down by the condition that the equilibrium expected return on equity to outsiders is $r_t^{\text{fund}}dt$. In other words,

$$\frac{\left(\frac{\Delta_{Nt}}{N_0+\Delta_{Nt}}\right) k_{it}(1+r_t^k dt)}{p_{it}\Delta_{Nt}} = 1 + r_t^{\text{fund}}dt$$

these equations jointly pin down the price and the number of shares issued in terms of the expected returns and the outside financing fraction $1-\chi$:

$$\Delta_{Nt} = \frac{(1+r_t^{\text{fund}}dt)(1-\chi)}{(r_t^k - r_t^{\text{fund}})dt + \chi(1+r_t^{\text{fund}}dt)} N_0$$

$$p_{it} = \left(\frac{(r_t^k - r_t^{\text{fund}})dt + \chi(1+r_t^{\text{fund}}dt)}{1+r_t^{\text{fund}}dt}\right)\frac{k_{it}}{N_0}$$

Note that measuring outsiders' stake in the firm as $p_{it}\Delta_{Nt}$, the price-per-share times the number of shares they hold, coincides with the model notion of the value of their stake in the firm: $(1-\chi)k_{it}$. That is however not true for the entrepreneur. In particular, the post-money valuation of the entrepreneurs' shares is

$$p_{it}N_0 = \left(\frac{(r_t^k - r_t^{\text{fund}})dt + \chi(1 + r_t^{\text{fund}}dt)}{1 + r_t^{\text{fund}}dt}\right)k_{it} > \chi k_{it} \tag{149}$$

where the inequality follows from the fact that $(r_t^k - r_t^{\text{fund}}) > 0$. This also illustrates that χ should not be confused with the entrepreneurs' ownership share measured as the fraction of the outstanding shares that the entrepreneur holds. Rather, χ is the insider financing share, the share of the financing that the entrepreneur provides.

The discrepancy stems from the fact that p_{it} is the price that an investor with no exposure to the idiosyncratic risk in firm i is willing to pay for a share. This is more than what the entrepreneur associated with that firm is willing to pay for a share. This discrepancy in valuation of a share means that the entrepreneur would like to issue additional shares, but cannot since the constraint is binding. The difference in the pre- and post-money valuations of the entrepreneur's shares reflects the fact that some of the entrepreneur's return from investing in the firm comes directly from selling shares. To see this, note that the expected return to the entrepreneur's stake in the firm coming purely from issuing shares is

$$\frac{p_{it}N_0}{\chi k_{it}} - 1 = \frac{r_t^k - r_t^{\text{fund}}}{\chi}dt > 0 \tag{150}$$

The overall expected return to the entrepreneur's stake, the insider equity return, is

$$r_t^{in}dt = \frac{\frac{N_0}{N_0+\Delta_{Nt}}(1 + r_t^k dt)k_{it}}{\chi k_{it}} - 1 = \left(r_t^{\text{fund}} + \frac{r_t^k - r_t^{\text{fund}}}{\chi}\right)dt \tag{151}$$

In other words, the insider return is the outsider return plus the return that the insider gets from issuing equity.

The fact that χ cannot be mapped to the insider ownership share of the entrepreneur, measured as the fraction of shares outstanding that the entrepreneur holds means that one must look for other sources of data that are informative about the value of χ. To this end, I map the value of χ to the rate at which entrepreneurs issue new shares. Specifically, the growth of the number of shares outstanding when the entrepreneur issues shares to outsiders is

$$\frac{\Delta_{Nt}}{N_0} = \frac{(1 + r_t^{\text{fund}}dt)(1-\chi)}{(r_t^k - r_t^{\text{fund}})dt + \chi(1 + r_t^{\text{fund}}dt)}. \tag{152}$$

Note that this is the growth in the number of shares when the entrepreneur first issues equity to outsiders. It is not the steady growth rate of the number of shares outstanding over time. The growth rate of the total number of shares outstanding only grows after this initial equity issuance if the rates of return or χ change over time. In a steady state, the returns as well as χ are constant, and the annualized average growth rate of the number of shares outstanding over the time that a firm remains entrepreneurial is

$$\left(1+\frac{\Delta_{Nt}}{N_0}\right)^{1/T_l}-1=\left(1+\frac{(1+r_t^{\text{fund}}dt)(1-\chi)}{(r_t^k-r_t^{\text{fund}})dt+\chi(1+r_t^{\text{fund}}dt)}\right)^{1/T_l}-1 \tag{153}$$

where T_l is the average number of years that the firm remains entrepreneurial. The quantity $-\left(1+\frac{\Delta_{Nt}}{N_0}\right)^{1/T_l}$ is the average lifetime buyback yield of an entrepreneurial firm. Gomez and Gouin-Bonenfant (2023) document that this has changed substantially over time for the entrepreneurial firms associated with the members of the Forbes 400. In the quantitative exercise, I map the fall in the parameter χ to the change in this average lifetime buyback yield.