Unpacking Aggregate Welfare in a Spatial Economy *

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Abstract

How do regional productivity shocks or transportation infrastructure affect aggregate welfare? In a general class of spatial equilibrium models, we provide an exact additive decomposition of aggregate welfare changes into (i) technology effects à la Hulten (1978), (ii) spatial dispersion in marginal utility, (iii) fiscal externalities, (iv) technological externalities, and (v) redistribution. We show that Hulten's (1978) theorem is recovered if second-best location-specific transfers are in place, implying that terms (ii)-(v) can be viewed as deviations from second-best transfer policies. We demonstrate the usefulness of our decomposition by analyzing the aggregate welfare changes from regional economic growth and heterogeneous investment returns in transportation infrastructure.

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1 Introduction

How do regional productivity shocks or transportation infrastructure affect aggregate welfare? To answer these questions, there has been significant progress in the development of quantitative spatial general equilibrium models. These frameworks allow researchers to fit the model to geographically disaggregated data and to compute the aggregate welfare of a particular shock or policy. At the same time, these frameworks are highly complex and parameterized, obscuring which forces or parameters in the model govern the aggregate welfare effects.

An alternative approach is to appeal to first-order approximations. Hulten (1978) showed that, in a frictionless economy under perfect competition, the impact on aggregate TFP of microeconomic TFP shocks is equal to the shocked producer's sales as a share of GDP (i.e., Domar weights). In the evaluation of transportation infrastructure, a popular approach has been the "social saving" approach (Fogel 1964), where the benefit of transportation infrastructure is calculated based on the shipment cost saved relative to the second-based alternative. Underlying these approaches is the macro-envelope condition resulting from the first welfare theorem. These approaches have the advantage of being agnostic about the details of the underlying disaggregated equilibrium system. However, whether or how this approach extends to spatial equilibrium models remains an open question.

This paper fills this gap by providing a theory to unpack the first-order aggregate welfare effects of spatially disaggregated shocks in a general class of spatial equilibrium models. We provide an exact additive decomposition of aggregate welfare changes that depends on a minimal set of nonparametric sufficient statistics. Our decomposition clarifies how and why the first-order aggregate welfare gains and losses depart from Hulten's (1978) characterization. We show how our decomposition can be used to assess regional economic growth and returns to investment in transportation infrastructure.

We consider a general class of spatial equilibrium models. Our framework accommodates flexible location-specific utility functions capturing local amenities, production functions, input-output linkages, trade frictions, agglomeration and congestion externalities, ex-ante heterogeneous households, and government transfers across locations and household types. We also introduce idiosyncratic preference shocks to households' location choices that follow arbitrary distribution functions. By accommodating a flexible correlation of preference shocks across alternatives, our model predicts general substitution patterns of location choice decisions. Special cases include the case without preference shocks (Rosen 1979, Roback 1982, Allen and Arkolakis 2014), i.i.d. extreme value distribution (Redding 2016, Diamond 2016), and the generalized extreme value distribution with arbitrary correlations (McFadden 1978).

We start by observing that the competitive equilibrium allocation is suboptimal from the perspective of maximizing households' expected utility. This suboptimality arises regardless of the Pareto weights associated with ex-ante types of households. The suboptimality of the equilibrium allocation arises for two reasons. First, competitive equilibrium does not internalize technological externalities such as agglomeration or congestion externalities. Second, in the competitive equilibrium, the marginal utility of income is not equalized across locations.

The first source of suboptimality is perhaps not surprising. The second source of suboptimality is subtle and warrants a discussion. In equilibrium, agents make location decisions based on the utility *levels* (inclusive of preference shocks). This implies that *marginal* utility from a dollar is not necessarily equalized across these locations. One way to interpret this dispersion of marginal utility is the lack of insurance for the uncertainty associated with location choice. Another interpretation is the lack of redistribution across agents within ex-ante identical households with different preference draws and their choice of locations. The observation that spatial equilibrium models involve suboptimality due to a dispersion of marginal utility is reminiscent of Mirrlees (1972), who points out this issue in a stylized residential location choice model within a city.

The suboptimality of equilibrium implies that Hulten's (1978) analysis does not extend to spatial equilibrium models in general. The main contribution of this paper is to provide a theory to unpack the aggregate welfare changes in spatial equilibrium models and how they depart from Hulten's (1978) characterization. We show that the aggregate welfare changes are exactly additively decomposed into five terms. The first term, (i) technology, is the percentage change in productivity multiplied by the revenue of the region or sector receiving a shock, resonating the characterization by Hulten (1978). The remaining four terms, jointly constituting the reallocation effects, are (ii) marginal utility (MU) dispersion, (iii) fiscal externalities (in the presence of spatial transfers), (iv) technological externalities (such as agglomeration or congestion externalities), and (v) redistribution across ex-ante heterogeneous households. The second term is positive if a shock induces a relative increase in consumption where the marginal utility net of resource cost is high. The third term is positive if a shock induces the reallocation of people to locations with net negative government transfers. The fourth term is positive if a shock induces the reallocation of people to locations that generate higher agglomeration externalities. The fifth term is positive if a shock induces the reallocation of consumption toward the types of households with a higher welfare weight.

We provide several stylized examples to illustrate what model specification affects which component of our welfare decomposition. For example, (ii) MU dispersion term is zero whenever utility is linear and there are no trade frictions, or whenever there are no preference shocks. (iii) Fiscal externality term is zero whenever the government transfers are only specific to household types and they do not depend on locations. If households are immobile across locations, (ii)-(iv) terms disappear.

We also study how the prevailing spatial transfer policies shape the welfare changes from disaggregated shocks. To address this question formally, we first characterize the government's optimal spatial transfer policies, generalizing Fajgelbaum and Gaubert's (2020) results in the presence of an arbitrary form of preference shocks. We then show that, if the optimal transfer policies are implemented in a pre-shock economy, (ii)-(v) terms add up to zero. This result clarifies that (ii)-(v) terms reflect deviations of the competitive equilibrium from optimal spatial transfer policies. This is a careful reminder that the economy being subotpimal does not necessarily lead to systematic deviations from Hulten (1978), and thereby provides a benchmark case. Accordingly, whether and how much Hulten's (1978) theorem over- or under-predict the aggregate welfare changes can be assessed as the deviation of the observed spatial transfers to the optimal ones.

Another advantage of our formula is that it provides a minimal set of nonparametric sufficient statistics to uniquely identify the aggregate welfare changes. In particular, given a minimal subset of baseline equilibrium allocation (prices, population distribution, consumption, and transfers) and the changes in population and consumption, aggregate welfare changes are uniquely pinned down by agglomeration externalities and the spatial dispersion of marginal utility of income. Relying on the econometric literature on discrete choice models (Berry and Haile 2014, Allen and Rehbeck 2019), we argue that the dispersion of marginal utilities is nonparametrically identified from location choice data as long as preference shocks are additively separable. In some contexts, researchers are also interested in the counterfactual changes in welfare, without observing the changes in population distribution and consumption in response to shocks. Together with the existing nonparametric identification result of the factor demand system (Adao, Costinot, and Donaldson 2017), we argue that these objects are also nonparametrically identified, thereby establishing nonparametric identification of welfare changes for a counterfactual shock.

For our baseline analysis, we assume that preference shocks are additively separable. When we depart from additively separable specification, preference shocks directly affect the spatial dispersion of marginal utility. We show that our welfare decomposition is straightforwardly extended to this general case. While straightforward in theory, this extension reveals an identification challenge, as a monotone transformation of utility function changes marginal utility without affecting the location choice decisions. Nonetheless, if preference shocks are multiplicatively separable and follow max-stable multivariate Fréchet distribution with an arbitrary correlation – a predominantly common specification in the literature besides additively separable specification – the aggregate welfare changes are isomorphic to additively separable specification by taking log transformation. Therefore, aggregate welfare changes are nonparametrically identified within this class in addition to the additively separable class mentioned above.

We show that our approach can be further extended to the environment with general agglomeration externalities that depend on the population in surrounding regions and producers' inputs and outputs, shocks to amenity that is not traded in the market, general social welfare function involving paternalistic motive, and models with cross-regional commuting. We also show how our decomposition can be used in applications, including the welfare impact of regional economic growth and returns to investment in transportation infrastructure.

Our paper contributes to the literature on spatial equilibrium models. Building on seminal models of peoples' location choice, trading-off wages, amenities, and cost of living (Rosen 1979, Roback 1982) and models with increasing returns to scale in production (Krugman 1991, Fujita, Krugman, and Venables 2001), there is a recent development in quantitative spatial equilibrium models that incorporates rich geographic heterogeneity in production, amenities, and trade frictions. A growing number of researchers use these frameworks to study the aggregate welfare effects of regional shocks or transportation infrastructure.[1] Our contribution is to provide a nonparametric formula to unpack the aggregate welfare effects of disaggregated shocks in these classes of models.

Our analysis of the aggregate welfare effects of shocks builds on Hulten (1978), who shows that in a perfectly competitive frictionless economy, the first-order effects of disaggregated shock on aggregate welfare are summarized by Domar weights. We show that this characterization generally does not extend to spatial equilibrium models because of the equilibrium suboptimality. Scholars have recognized that agglomeration externality leads to equilibrium suboptimality and hence affects the first-order welfare effects of disaggregated shocks.[2] The equilibrium suboptimality due to the dispersion of marginal utility of income has been pointed out by Mirrlees (1972) in a stylized model of location decisions within a city. However, this point has been less highlighted in the recent quantitative spatial equilibrium literature.[3] Our contribution is to connect these sources of equilibrium suboptimality to the effects of disaggregated shocks on aggregate welfare.

Our analysis of how the equilibrium suboptimality shapes the effects of disaggregated shocks resonates with Baqaee and Farhi (2020), who study this question in an economy with exogenous

[1]See Redding and Rossi-Hansberg (2017), Redding (2022) for recent surveys on quantitative spatial equilibrium models, and Redding and Turner (2015) for the usages of these models for studying the aggregate impacts of transportation infrastructure. Donaldson and Hornbeck (2016) uses a full general equilibrium model to explore the deviation from Fogel (1964) in the aggregate effects of U.S. railways. Tsivanidis (2019) and Zárate (2022) use parameterized quantitative spatial equilibrium models to study the impacts of urban transportation infrastructure and provide a quantitative comparison with Hulten's (1978) characterization. Caliendo, Parro, Rossi-Hansberg, and Sarte (2018) use these frameworks to study the propagation of region- and sector-specific productivity shocks.

[2]See Lebergott (1966) for an early criticism to Fogel (1964) due to an omission of technological externality. Tsivanidis (2019) argues that agglomeration externalities affect the welfare gains from urban transport infrastructure beyond the value of travel time saved (VTTS) (i.e., Small and Verhoef 2007).

[3]See also Wildasin (1986), who explicitly point out that this suboptimality is related to the dispersion of marginal utility of income. Mongey and Waugh (2023) discuss this suboptimality in the context of a broader discrete choice model.

wedges without location choice decisions. In contrast, wedges are endogenously determined in our context. Our contribution is to characterize these endogenous wedges in spatial equilibrium models using nonparametric sufficient statistics. Our paper also relates to Dávila and Schaab (2023), who provide welfare decomposition of general equilibrium models with heterogeneous agents. While the focus on aggregate welfare is similar, our paper is distinct in that we explicitly characterize endogenous wedges in spatial equilibrium models relating to competitive equilibrium allocation.

The remaining paper is organized as follows. Section 2 establishes the general spatial equilibrium framework and discusses its basic equilibrium properties. Section 3 establishes our main theoretical result. Section 4 relaxes our baseline specification of additively separable preference shocks. Section 5 discusses additional extensions. Section 6 discusses our applications.

2 Spatial Equilibrium Framework

We set up a general spatial equilibrium framework for our baseline analysis. Section 2.1 lays out our baseline model and defines the competitive equilibrium. Section 2.2 provides a useful representation of location choice decisions. Section 2.3 shows that competitive equilibrium allocation is suboptimal from the perspective of maximizing households' expected utility.

2.1 General Set-up

There are N locations indexed by $i, j \in \mathcal{N} \equiv \{1, \ldots, N\}$. There are S types of households indexed by $\theta \in \Theta \equiv \{\theta_1, \ldots, \theta_S\}$. The mass of each type is $\bar{l}^\theta$, and we normalize the total measure to one, $\sum_\theta \bar{l}^\theta = 1$. Each household decides its residential location at the beginning. Households who decide to live in location j consume the location-specific final good aggregator specific to household type θ produced using intermediate goods. There are K intermediate goods, some of which can be potentially traded across locations subject to cost (e.g., food or manufacturing) and some of which are not traded across locations (e.g., housing or nontradable services). Intermediate goods are produced using local labor, intermediate goods, and local fixed factors (e.g., land). Households have ownership of these local fixed factors and earn factor income depending on their type θ irrespective of their location choice decisions.

Households of type θ in location j inelastically supply one unit of labor regardless of the location and consume final non-traded goods. Their preferences are given by

$$U_j^\theta(C_j^\theta, \epsilon_j^\theta). \tag{1}$$

Here, the utility function is indexed by j and θ to capture differences in type-and-location-specific amenities. ϵ_j^θ is idiosyncratic household-specific preference shocks associated with location j as we describe further below.

The budget constraint is

$$P_j^\theta C_j^\theta = w_j^\theta + T_j^\theta + \Pi^\theta, \tag{2}$$

where P_j^θ is the price of final goods of type θ household in location j and w_j^θ is the wage of type θ household in location j. T_j^θ is the net government transfers for type θ household in location j. In reality, T_j^θ includes both taxes and transfers explicitly tagged to each location (such as state taxes and transfers in the U.S.) and those set at the national level (such as federal taxes and transfers in the U.S.). We do not impose any additional assumptions about T_j^θ beyond the condition that the net supply of these transfers is zero. Π^θ is the income from fixed factors for household type θ.

Households choose a location that maximizes their utility. Let $j^\theta(\boldsymbol{\epsilon}^\theta)$ denote the choice of location j conditional on their preference draw $\boldsymbol{\epsilon}^\theta = (\epsilon_1^\theta, \ldots, \epsilon_N^\theta)$. The households' optimization implies

$$j^\theta(\boldsymbol{\epsilon}^\theta) \in \arg\max_{m \in \mathcal{N}} u_m(C_m^\theta, \epsilon_m^\theta). \tag{3}$$

Importantly, we do not assume any parametric assumptions of the distribution function for $\boldsymbol{\epsilon}^\theta$ beyond the regularity condition that they have a strictly positive density everywhere in $\mathbb{R}^N$ or are degenerate. This specification nests different assumptions about the location decisions in the literature. For example, Rosen (1979), Roback (1982), and Allen and Arkolakis (2014) consider the case without preference shocks, i.e., where ϵ_m^θ is degenerate at zero for all m; Diamond (2016) considers a case where ϵ_m^θ is distributed according to an i.i.d. type-I extreme value distribution across locations m; and McFadden (1978) considers a case where $\boldsymbol{\epsilon}^\theta$ is distributed according to generalized extreme value distribution with arbitrary correlation across alternatives. By aggregating across the draws of idiosyncratic preference draws, the population size in location j of type θ is given by

$$l_j^\theta = \bar{l}^\theta \mu_j^\theta, \qquad \mu_j^\theta = \int j^\theta(\boldsymbol{\epsilon}^\theta) dG(\boldsymbol{\epsilon}^\theta), \tag{4}$$

where μ_j^θ is the probability that type θ chooses location j and $G(\boldsymbol{\epsilon}^\theta)$ is the distribution function of preference shocks $\boldsymbol{\epsilon}^\theta$.

Final goods in location j are produced using a constant returns to scale technology over intermediate goods

$$C_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta),$$

where $\mathbf{c}_j^\theta \equiv \{c_{ij,k}^\theta\}_{i,k}$ denotes a vector of intermediate goods used for final goods production for type θ household in j, where k indexes intermediate goods and i indexes the origin location of these intermediate goods.

Intermediate good k produced in location i sold in location j is produced using the following technology

$$y_{ij,k} = A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}),$$

where $\mathbf{l}_{ij,k} \equiv \{l_{ij,k}^\theta\}_\theta$ denotes an input of labor, $h_{ij,k}$ denotes an input of local fixed factor, $A_{ij,k}$ is Hicks-neutral technology (including iceberg trade costs), $f_{ij,k}$ is strictly increasing, concave, differentiable, and homogeneous of degree one function, and $\mathbf{x}_{ij,k} \equiv \{x_{ij,k}^{l,m}\}_{l,m}$ denotes a vector of intermediate inputs, where m indexes the intermediate goods for inputs and l indexes the location of origin.

We assume that the supply of local fixed factor at location j is given exogenously by $\bar{h}_j$. We assume that each of type θ household owns α^θ share of fixed factors, where $\sum_\theta \bar{l}^\theta \alpha^\theta = 1$. We also denote the price of the local fixed factor by r_j. Then, the aggregate per-capita return from the fixed factor is given by

$$\Pi^\theta = \alpha^\theta \sum_j r_j \bar{h}_j. \tag{5}$$

The net government transfer is zero such that

$$\sum_\theta \sum_j T_j^\theta l_j^\theta = 0. \tag{6}$$

Finally, we assume that productivity $\{A_{ij,k}\}$ is subject to agglomeration spillovers depending on the local population density of various household types:[4]

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ij,k}(\{l_i^\theta\}_\theta), \tag{7}$$

where $g_{ij,k}(\cdot)$ are the spillover functions, and $\tilde{A}_{ij,k}$ is fundamental exogenous components of the productivity. Note that we allow for the flexible functional form of spillovers arising from the population size of different household types θ for different locations and sectors i, j, k. For

[4]By interpreting some intermediate goods k as type θ's labor services, this specification nests general agglomeration spillovers from type θ to another type $\tilde{\theta}$ households' labor productivity, nesting the framework of Fajgelbaum and Gaubert (2020). In Section 5.1, we show that it is straightforward to extend the agglomeration externalities beyond local population size, e.g., introducing cross-region productivity spillovers (e.g., Ahlfeldt, Redding, Sturm, and Wolf 2015) or agglomeration/congestion externality specific to the sector's inputs and outputs (e.g., Allen and Arkolakis 2022).

notational convenience, we define the elasticity of agglomeration spillover evaluated at $\{l_i^\theta\}_\theta$ as

$$\gamma_{ij,k}^\theta \equiv \frac{\partial \ln g_{ij,k}(\{l_i^\theta\}_\theta)}{\partial \ln l_i^\theta}. \tag{8}$$

We define the decentralized equilibrium of this economy as follows.

Definition 1. The decentralized equilibrium consists of prices $\{P_j^\theta, p_{ij,k}, w_j^\theta, r_j\}$, quantities $\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, \mu_j^\theta, l_j^\theta\}$, transfer policies $\{T_j^\theta\}$, productivity $\{A_{ij,k}\}$, such that

(i) $\{C_j^\theta\}_{j,\theta}$ satisfies households' budget constraint (2) and $\{\mu_j^\theta, l_j^\theta\}_{j,\theta}$ solves households' optimal location choice problem (3) and (4).

(ii) firms maximize profits:

$$\mathbf{c}_j^\theta \in \arg\max_{\tilde{\mathbf{c}}_j^\theta} P_j^\theta \mathcal{C}_j^\theta(\tilde{\mathbf{c}}_j^\theta) - \sum_{i,k} p_{ij,k}\tilde{c}_{ij,k}^\theta \tag{9}$$

and

$$\begin{aligned}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \in \arg \max_{\mathbf{l}_{ij,k}, \tilde{h}_{ij,k}, \tilde{\mathbf{x}}_{ij,k}} & p_{ij,k}A_{ij,k}f_{ij,k}(\tilde{\mathbf{l}}_{ij,k}, \tilde{h}_{ij,k}, \tilde{\mathbf{x}}_{ij,k}) \\ & - \sum_\theta w_i^\theta \tilde{l}_{ij,k}^\theta - r_i\tilde{h}_{ij,k} - \sum_{l,m} p_{li,m}\tilde{x}_{ij,k}^{l,m}\end{aligned} \tag{10}$$

(iii) goods markets clear

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{i,jl}^{k,m} = A_{ij,k}f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{11}$$

$$C_j^\theta l_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta) \tag{12}$$

(iv) labor markets clear

$$\sum_{i,k} l_{ji,k}^\theta = \bar{l}^\theta \mu_j^\theta \tag{13}$$

(iv) fixed factor markets clear

$$\sum_{i,k} h_{ji,k} = \bar{h}_j \tag{14}$$

(v) aggregate factor payment Π^θ satisfies (5)

(vi) government budget constraint (6) holds

(vii) productivity $\{A_{ij,k}\}$ is subject to agglomeration spillovers given by (7).

We also define the aggregate welfare of this economy as follows:

Definition 2. Aggregate welfare W is given by a social welfare function that takes the expected utility of each household type θ:

$$W = \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}), \qquad W^\theta = \mathbb{E}[\max_j\{U_j^\theta(C_j^\theta, \epsilon_j^\theta)\}]. \tag{15}$$

Without loss of generality, we assume $\mathcal{W}$ is homogenous of degree one.

In a special case with ex-ante homogenous household types ($S = 1$), the objective function is simply the expected utility of the household, or equivalently, the utilitarian social welfare function with respect to preference shocks. One restriction of Definition 2 is that the aggregate welfare only depends on the expected utilities of households, and not directly on allocations. In Section 5.4, we show that our results straightforwardly extend to these alternative welfare criteria, capturing the case where the social welfare function involves a paternalistic motive.

We refer to the welfare weights attached to households type θ, Λ^θ, as a contribution of the welfare of households type θ to the aggregate welfare relative to their population size:

$$\Lambda^\theta \equiv \frac{\partial \mathcal{W}(\{W^\theta\}_{\theta\in\Theta})}{\partial W^\theta}\frac{1}{\bar{l}^\theta}. \tag{16}$$

With a linear social welfare function, $\{\Lambda^\theta\}_{\theta\in\Theta}$ corresponds to what is often referred to as Pareto weights. Under utilitarian social welfare function, $\Lambda^\theta = 1$. Note that $\sum_{\theta\in\Theta}\bar{l}^\theta\Lambda^\theta = 1$ by the assumption that $\mathcal{W}$ is homogenous of degree one.

Our theoretical results in the remaining part of the paper do not rely on further restrictions of primitives and fundamentals. However, for expositional purposes, it is instructive to first focus on the case where the utility function is additively separable between the common location-specific component and an idiosyncratic component:

$$U_j^\theta(C_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta) + \epsilon_j^\theta. \tag{17}$$

The key implication of this assumption is that the marginal utility of consumption is not affected by the idiosyncratic preference shocks. In Section 4, we revisit how this additional consideration influences our results. In Section 4.2, we show that in a common alternative specification in the literature where the preference shocks enter multiplicatively and follow max-stable multi-variate Frét distribution, all of our results remain isomorphic to the additively separable specification.

For an expositional reason that will become clear below, we normalize prices so that the population-weighted average of the inverse of the marginal utility of income $u_j^{\theta\prime}(C_j^\theta)$ is one:

$$\sum_\theta \sum_j l_j^\theta \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} = 1. \tag{18}$$

Although this merely amounts to a choice of numeraire, this normalization simplifies our exposition.

We also focus on the case where decentralized equilibrium is unique and interior ($l_j^\theta > 0$ for all j and θ). Since our approach relies on the first-order approximation, this assumption avoids dealing with the case where equilibrium is non-differentiable with respect to the shock and the possibility that a small shock leads to a switch to a different equilibrium.[5]

2.2 Isomorphic Representation of Location Choice

In this section, we introduce a convenient alternative representation of location choice decisions. Following Hofbauer and Sandholm (2002), the discrete location choice decision under additive preference shocks (17) can be isomorphically represented by households jointly choosing the population size subject to a cost function, as summarized in the following lemma:

Lemma 1 (Hofbauer and Sandholm 2002)**.** Under additively separable utility function (17), the share of households of type θ living in each location, $\{\mu_j^\theta\}_j$ can be represented as the solution to the following problem given a vector of equilibrium consumption $\{C_j^\theta\}_j$:

$$\begin{aligned} W^\theta = &\max_{\{\mu_j^\theta\}_j} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \\ &\text{s.t.} \quad \sum_j \mu_j^\theta = 1 \end{aligned} \tag{19}$$

for some function $\psi^\theta(\{\mu_j^\theta\}_j)$, which we provide an explicit expression for in Appendix A.1. Moreover, W^θ coincides with the expected utility in Equation (15), i.e., $W^\theta = \mathbb{E}[\max_j\{u_j^\theta(C_j^\theta) + \epsilon_j^\theta\}]$.

The proofs of this lemma and the subsequent propositions of this paper are found in Appendix A. Importantly, $\psi^\theta(\{\mu_j^\theta\}_j)$ summarizes the influence of preference shocks on households' location decisions. If there are no preference shocks, we have $\psi^\theta(\{\mu_j^\theta\}_j) = 0$. If the preference shocks follow i.i.d. type-I extreme value distribution with shape parameter ν, then $\psi^\theta(\{\mu_j^\theta\}_j) = \frac{1}{\nu}\sum_j l_j^\theta \ln l_j^\theta$ (Anderson, De Palma, and Thisse 1988). When $\{\epsilon_j^\theta\}_j$ follows type-I generalized extreme value (GEV) with an arbitrary correlation (i.e., McFadden 1978), $\psi^\theta(\{\mu_j^\theta\}_j) = \frac{1}{\nu}\sum_j \mu_j^\theta \ln S_j^\theta(\{\mu_i^\theta\}_i)$,

[5] See Allen and Arkolakis (2014) and Allen, Arkolakis, and Li (2020) for sufficient conditions for the equilibrium uniqueness in spatial equilibrium models.

where function $S^\theta_j(\cdot)$ depends on the correlation function of $\{\epsilon^\theta_j\}_j$ across alternatives j (see Appendix C for details).[6]

2.3 Suboptimality of Equilibrium Allocation

The following lemma shows that the competitive equilibrium allocation is represented as a solution to the "pseudo-planning" problem.

Lemma 2. Any decentralized equilibrium $\{\check{P}^\theta_j, \check{p}_{ij,k}, \check{w}^\theta_j, \check{r}_j, \check{C}^\theta_j, \check{\mathbf{c}}^\theta_j, \check{\mathbf{x}}_{ij,k}, \check{\mathbf{l}}_{ij,k}, \check{h}_{ij,k}, \check{\mu}^\theta_j, \check{l}^\theta_j, \check{T}^\theta_j, \check{A}_{ij,k}\}$ solves the following pseudo-planning problem

$$W = \max_{\{W^\theta, C^\theta_j, \mathbf{c}^\theta_j, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu^\theta_j, A_{ij,k}\}} \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \tag{20}$$

$$\text{s.t.} \qquad W^\theta = \sum_j \mu^\theta_j u^\theta_j(C^\theta_j) - \psi^\theta(\{\mu^\theta_j\}_j) \tag{21}$$

$$C^\theta_j \bar{l}^\theta \mu^\theta_j = \mathcal{C}^\theta_j(\mathbf{c}^\theta_j) \tag{22}$$

$$\sum_\theta c^\theta_{ij,k} + \sum_{l,m} x^{l,m}_{ij,k} = A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{23}$$

$$\sum_{i,k} l^\theta_{ji,k} = \bar{l}^\theta \mu^\theta_j, \qquad \sum_{i,k} h_{ji,k} = \bar{h}_j \tag{24}$$

$$\{\mu^\theta_j\}_j \in \arg\max_{\{\tilde{\mu}_j\}:\sum_j \tilde{\mu}_j = 1} \sum_j \tilde{\mu}^\theta_j u^\theta_j(C^\theta_j) - \psi^\theta(\{\tilde{\mu}^\theta_j\}_j) \tag{25}$$

$$C^\theta_j = \check{C}^\theta_j \tag{26}$$

$$A_{ij,k} = \check{A}_{ij,k} \tag{27}$$

with the Lagrangian multipliers on (22)-(24), $P^{\theta,L}_j$, $p^L_{ij,k}$, $w^{\theta,L}_j$, and r^L_j, satisfying $P^{\theta,L}_j = \check{P}^\theta_j$, $p^L_{ij,k} = \check{p}_{ij,k}$, $w^{\theta,L}_j = \check{w}^\theta_j$ and $r^L_j = \check{r}_j$.

The objective function is what we define as the aggregate welfare. The constraints (22)-(24) correspond to resource constraints. The constraints (25), (26), and (27) restrict that location choice, consumption, and productivity follow equilibrium allocations.

An immediate implication of this lemma is that the equilibrium is suboptimal. This is regardless of the welfare weights associated with ex-ante heterogenous household types. To see this, it is instructive to compare the pseudo-planning problem and the first-best planning problem, where the planner specifies the labor and consumption allocations only subject to resource constraints (22)-(24). (See Appendix B for the full characterization of the first-best planning problem).

[6] An alternative interpretation of $\psi^\theta(\cdot)$ is that it captures congestion externality. For example, the model with preference shocks following i.i.d. type-I extreme value distribution with shape parameter ν is isomorphic to the model without preference shocks and utility is given by $u^\theta_j(C^\theta_j) - \frac{1}{\nu}\ln\mu^\theta_j$. See Section 5.3 for further discussion about this isomorphism.

There are two key differences from the frist-best allocation. First, the pseudo-planning problem does not internalize the agglomeration externalities in productivity (27). Second, the pseudo-planning problem takes the equilibrium population distribution and consumption allocation as given by (25) and (26). Instead, the first-best planning problem allocates population distribution and consumption only subject to adding up constraint $\sum_j \mu_j^\theta = 1$ for all θ.

The first source of suboptimality is perhaps not surprising; it is simply an externality that the market does not internalize. The second source of suboptimality is subtle and warrants a discussion. In equilibrium, agents make location decisions based on utility *levels* (inclusive of preference shocks). This implies that *marginal* utility of income is not necessarily equalized across these locations given household type θ. In contrast, in the first-best allocation, the Planner exactly equalizes the marginal utility of income across locations. At the same time, the Planner directly controls population movement by breaking the incentive compatibility constraint of households' location decisions.

There are two ways to interpret this equilibrium suboptimality due to the dispersion of marginal utility of income. The first interpretation is the lack of insurance for the uncertainty associated with location choice. Depending on the preference draws, or depending on the random sunspot process of location assignment in the absence of preference shocks, individual households may end up in a variety of locations that differ in terms of their associated marginal utility of income. Ex-ante, households can benefit by committing to making transfers from a state where they end up in a location with a low marginal utility of income to a state with a high marginal utility of income. However, there is no security that allows for such a transfer. The second interpretation is the lack of redistribution across agents within household type θ depending on location preference shocks and where they reside. By taking the expected utility as a welfare criterion, we effectively attach equal social marginal weight to individuals with different preference draws within household type θ.

The purpose of this discussion is not to discuss particular policy tools to achieve the first-best allocation. In most scenarios, it is unrealistic to consider a policy that directly controls population movement by breaking the incentive compatibility constraint.[7] Instead, this discussion aims to show that this suboptimality could lead to the departure from Hulten's (1978) characterization of the aggregate welfare effects of disaggregated technological shocks. In the next section, we use Lemma 2 to unpack the first-order effects of disaggregated shocks on aggregate welfare.

[7]A few examples where such policy could be realistic are the Hukou system in China or refugee settlement policies.

3 Unpacking Welfare Effects of Disaggregated Shocks

How do regional productivity shocks or transportation infrastructure improvements affect aggregate welfare? This section provides our main theoretical result of the decomposition of the first-order effects of disaggregated regional shocks. Section 3.1 provides our decomposition, where the first term corresponds to Hulten's (1978) characterization and the remaining terms correspond to reallocation effects. Section 3.2 provides several stylized examples to illustrate what model specification affects which component of our welfare decomposition. Section 3.3 shows that Hulten's theorem is recovered if second-best location-specific transfers are in place, implying that the sign and magnitudes of the reallocation effects can be assessed as the deviations from second-best policies. Section 3.4 discusses how our framework can be used for ex-ante and ex-post welfare evaluations in applications. Section 3.5 uses our decomposition to discuss the nonparametric identification of aggregate welfare changes in spatial equilibrium models.

For expositional purposes, we introduce the following expectation and covariance operators. The first set of operators takes the expectation and covariance of statistics associated with location j for each household type θ, weighted by population share, μ_j^θ:

$$\mathbb{E}_{j|\theta}[X_j^\theta] = \sum_j \mu_j^\theta X_j^\theta, \qquad \mathrm{Cov}_{j|\theta}(X_j^\theta, Y_j^\theta) \equiv \mathbb{E}_{j|\theta}[X_j^\theta Y_j^\theta] - \mathbb{E}_{j|\theta}[X_j^\theta]\mathbb{E}_{j|\theta}[Y_j^\theta]. \tag{28}$$

The second set of operators takes the expectation and covariance of statistics associated with household type θ, weighted by population share of household type θ, $\bar{l}^\theta$:

$$\mathbb{E}_\theta[X^\theta] \equiv \sum_\theta \bar{l}^\theta X^\theta, \qquad \mathrm{Cov}_\theta(X^\theta, Y^\theta) \equiv \mathbb{E}_\theta[X^\theta Y^\theta] - \mathbb{E}_\theta[X^\theta]\mathbb{E}_\theta[Y^\theta]. \tag{29}$$

3.1 Main Results

Consider small changes in the exogenous components of productivity specific to origin location, destination location, and sector, $\{d\ln \tilde{A}_{ij,k}\}$. These shocks can represent region-sector TFP shocks (e.g., Caliendo et al. 2018) or transportation infrastructure changes (e.g., Allen and Arkolakis 2014, Donaldson and Hornbeck 2016).[8] We also allow for the possibility that the structure of the transfers may change simultaneously, denoted by $\{dT_j^\theta\}$, are affected, either because of the exogenous policy changes or as endogenous responses to the productivity shocks.

[8]In some context, researchers are interested in the shocks to amenity instead of productivity. Our analysis includes those cases by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices of the amenities, which is often unobserved and needs to be calibrated or estimated. For example, if transportation infrastructure also brings amenity benefits by shortening commuting time, one can use the value of time for $p_{ij,k}$ and commuting time for $y_{ij,k}$ (i.e., Small and Verhoef 2007). In Section 5.2, we provide an alternative expression for Proposition 1 without using amenity prices.

By applying the envelope theorem to the pseudo-planning problem in Lemma 2, we obtain the following expression for welfare changes:

Proposition 1. *Consider an arbitrary set of small shocks to exogenous components of productivity, $\{d\ln\tilde{A}_{ij,k}\}$, and transfers $\{dT_j^\theta\}$, in a decentralized equilibrium. The first-order impact on welfare defined by Definition 2 can be expressed as*

$$\begin{aligned} dW = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln\tilde{A}_{ij,k}}_{\text{(i) Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta\left[Cov_{j|\theta}\left(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right)\right]}_{\text{(ii) MU Dispersion } (\Omega_{MU})} \\ & + \underbrace{\mathbb{E}_\theta\Big[Cov_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\Big]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta\left[Cov_{j|\theta}(\sum_{l,k} p_{jl,k} y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta, d\ln l_j^\theta)\right]}_{\text{(iv) Technological Externality } (\Omega_{TE})} \\ & + \underbrace{Cov_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right], \mathbb{E}_{j|\theta}\left[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right]\right)}_{\text{(v) Redistribution } (\Omega_R)}. \end{aligned} \tag{30}$$

The first term of Proposition 1, which we refer to as (i) technology (Ω_T), captures the effects of productivity changes absent the reallocation of resources. The coefficient in front of $d\ln\tilde{A}_{ij,k}$, $p_{ij,k}y_{ij,k}$, corresponds to the total sales of intermediate inputs k produced in i and sold in j. The observation that the total sales summarize the aggregate effects of a shock is reminiscent of Hulten (1978).[9]

If the equilibrium maximizes aggregate welfare W, the first term is sufficient for the welfare consequence of disaggregated shock to a first-order. However, since the equilibrium is generally suboptimal, the reallocation of resources additionally enters on top of this technology term.

The second term, which we refer to as (ii) MU (marginal utility) dispersion (Ω_{MU}), captures the fact that shocks reallocate resources across locations that differ in the marginal utility of income. A shock leads to an increase of utility $u_j^{\theta\prime}(C_j^\theta)$ in each location j for household type θ. The covariance inside $\mathbb{E}_\theta[\cdot]$ is positive if this utility change is higher in a location where the marginal utility of income, $u_j^{\theta\prime}(C_j)/P_j^\theta$, is higher for household type θ. The expectation $\mathbb{E}_\theta[\cdot]$ takes the weighted average of this covariance across household types θ.

The third term, which we label as (iii) fiscal externality (Ω_{FE}), comes from the fact that shock affects the government budget. If the shock induces the population to move toward a location

[9] Note that our choice of numeraire (equation 18) implies that all prices are in the unit of the population-weighted average of marginal utilities.

that receives net transfers (higher T_j^θ), this term induces additional negative effects on welfare.[10] This term is absent whenever there are no transfers ($T_j^\theta = 0$ for all j and θ), or the shock does not induce any labor reallocation ($dl_j^\theta = 0$ for all j and θ).

The fourth term, which we label as (iv) technological externality (Ω_{TE}), captures the agglomeration externalities in productivity. If the shock induces the population to move toward a location with a higher agglomeration externality $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta$, this term induces additional positive effects on welfare.

The fifth term, which we label as (v) redistribution (Ω_R), is the covariance between the marginal increase of expected utility of type θ household $\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta]$ and the utility weight on each household type θ, $\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right]$. The first term of the utility weight is the welfare weight defined by Equation (16) and the second term is the expected inverse marginal utility of income.

Proposition 1 provides a characterization of the changes in aggregate welfare. In many contexts, researchers may wish to convert the welfare changes into alternative measurable units. For example, one may wish to compute how much uniform productivity changes $d \ln \tilde{A}$ for all i, j, k induces equivalent changes in expected utility. To answer this question, one can use Proposition 1 again to ask what changes in $d \ln \tilde{A}$ can achieve the equivalent changes in dW.

3.2 Stylized Examples

We provide several stylized examples to illustrate what model specification affects which component of our welfare decomposition. Table 1 summarizes which terms of our decomposition in Proposition 1 are generically non-zero in each special case covered below. We go through each case in turn.

1. Single Type If all households are ex-ante homogenous ($S = 1$), (v) redistribution term becomes zero. Moreover, all the expectation operators with respect to household types $\mathbb{E}_\theta[\cdot]$ drops out from the terms (ii)-(iv).

2. No preference shocks Following the tradition of Rosen (1979) and Roback (1982), researchers often abstract preference shocks (Allen and Arkolakis 2014, Fajgelbaum and Gaubert 2020), i.e., $\epsilon_m^\theta = 0$ for all location m for all households. In this case, assuming an interior solution, shocks induce the same changes in utility in every location ($u_j^{\theta\prime}(C_j^\theta) dC_j^\theta$ are equalized across all

[10] In some existing models, researchers assume that some fraction of fixed factor income is rebated to local households directly (such as through local governments' ownership of local fixed factors), which implies that Π_i depends on i (e.g., Caliendo et al. 2018). In such a case, the fiscal externality term is simply modified to capture these local rebates, i.e., $w_j^\theta - P_j^\theta C_j^\theta = -(T_j^\theta + \Pi_j^\theta)$.

Table 1: Decomposition in Special Cases

	Ω_T	Ω_{MU}	Ω_{FE}	Ω_{TE}	Ω_R
1. Single type	✓	✓	✓	✓	
2. No preference shocks	✓		✓	✓	✓
3. Linear utility and no trade frictions	✓		✓	✓	✓
4. No location-specific transfers	✓	✓		✓	✓
5. No population mobility	✓				✓
6. No technological externality	✓	✓	✓		✓
7. Second-best transfers	✓				✓
8. ...with redistribution	✓				

j). Therefore, the second term of the covariance in (ii) MU dispersion term is constant across j, and (ii) MU dispersion term becomes zero.

3. Linear utility and no trade frictions Another special case where (ii) MU dispersion term is zero arises if the marginal utility of income ($u_j^{\theta\prime}(C_j^\theta)/P_j^\theta$) is equalized across locations. This case arises under linear utility (i.e., $u_j^\theta(C_j^\theta) = C_j^\theta + B_j^\theta$ for some B_j^θ) *and* no trade frictions such that final prices P_j are equalized across locations j. The primitive assumptions that deliver the equalizations of final prices correspond to $A_{ij,k} = A_i^k$, $f_{ij,k}(\cdot) = f_i^k(\cdot)$, and $\mathcal{C}_j^\theta(\cdot) = \mathcal{C}^\theta(\cdot)$. Kline and Moretti (2014) consider this special case under ex-ante homogeneous households and argue that expected utility is maximized in the competitive equilibrium without technological externalities.

4. No location-specific transfers We next consider when (iii) fiscal externality term becomes zero. A sufficient condition for this term to be zero is that transfers only depend on ex-ante household types and there are no location-specific components ($T_j^\theta = T^\theta$ for all j and θ). Note that this does not necessarily imply that the net transfers in each region are zero, because households of different types θ with net positive or negative transfers T^θ may sort into different locations.

5. No population mobility In some cases, researchers model locations but do not model population mobility. This is nested in our framework by setting $S = N$ and preference shocks are such that type θ_i households always locate themselves in location i, $\mu_i^{\theta_i} = 1$. Examples of these specifications arise in international trade models where researchers typically abstract international migrations or when studying the short-run effects of an acute shock.[11] If the population

[11] For example, Fajgelbaum, Goldberg, Kennedy, and Khandelwal (2020) study the short-run welfare effects of the trade war in the U.S. in 2018 by restricting population mobility across U.S. states.

is immobile, i.e., $d\ln l_j^\theta = 0$ for all j, θ, (iii) fiscal externality and (iv) technological externality also become zero. Since there is no location choice, the covariance of (ii) MU dispersion term (conditional on being type θ) also becomes zero as well.

6. No technological externality Finally, (iv) technological externality term becomes zero if there are no technological externalities in the pre-shock equilibrium, i.e., $\gamma_{ij,k}^\theta = 0$ for all i, j, k, θ.

Importantly, assuming constant elasticity agglomeration externality ($\gamma_{ij,k}^\theta = \gamma$) alone does not ensure that (iv) technological externality term is zero. To see why, consider a special case with a single sector $K = 1$, single type $S = 1$, and no fixed factor $\bar{h}_j = 0$ for all j. We further assume that there are no intermediate inputs used in production ($y_{ij} = A_{ij} f_{ij}(l_{ij})$, dropping subscript k and θ). In this case, from profit maximization and labor market clearing condition, $\sum_l p_{jl} y_{jl} = w_j l_j$, and the term $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{ij,k}^\theta$ simplifies to $w_j \gamma$. Therefore, the reallocation of the population toward a location with a higher nominal marginal product of labor generates positive effects on aggregate welfare. This result is consistent with the observation by Fajgelbaum and Gaubert (2020), who show that competitive equilibrium involves misallocation of the population even under constant elasticity of agglomeration externality as long as the marginal product of labor is not equalized (e.g., due to compensating variation).

3.3 Welfare Changes if Optimal Spatial Transfers are in Place

So far, we have remained agnostic about how transfers T_j^θ are endogenously determined in the equilibrium. In reality, national governments may set spatial transfers T_j^θ to correct for agglomeration externalities (Fajgelbaum and Gaubert 2020, Rossi-Hansberg, Sarte, and Schwartzman 2019) or to address spatial inequalities (Gaubert, Kline, and Yagan 2021). While Proposition 1 embraces any endogenous response of T_j^θ to shocks, it is instructive to consider how each term in Proposition 1 is affected in those cases. We also argue below that understanding the government's incentive facilitates the interpretation of Proposition 1.

Specifically, we consider a scenario where the government sets spatial transfers T_j^θ to trace the Pareto frontier subject to competitive equilibrium constraint. The government's optimal transfer policy problem is

$$\max_{\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, l_j^\theta, \mu_j^\theta, A_{ij,k}, w_j^\theta, r_j, p_{ij,k}, P_j^\theta, T_j^\theta\}} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{31}$$

subject to (2)-(14) and the constraints such that

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) - \psi^{\tilde{\theta}}(\{\mu_j^{\tilde{\theta}}\}_j) \geq \underline{W}^{\tilde{\theta}}, \qquad \forall \tilde{\theta} \neq \theta. \tag{32}$$

Tracing for all feasible values of $\underline{W}^{\tilde{\theta}}$ for all $\tilde{\theta}$ defines the set of optimal transfers.

To solve this problem, we follow the primal approach in the public finance literature. That is, we focus on a relaxed planning problem where the Planner picks an incentive-compatible consumption and population allocation and later confirms that the solution to the relaxed problem is also a solution to the original one. The relaxed planning problem is defined as follows.

$$\max_{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, l_{ij,k}^\theta, h_{ij,k}, l_j^\theta, A_{ij,k}} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{33}$$

$$\text{s.t.} \qquad C_j^\theta \bar{l}^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta) \tag{34}$$

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{ij,k}^{l,m} = A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{35}$$

$$\sum_{i,k} l_{ji,k}^\theta = \bar{l}^\theta \mu_j^\theta, \qquad \sum_{i,k} h_{ji,k} = \bar{h}_j \tag{36}$$

$$\{\mu_j^\theta\}_j \in \arg \max_{\{\tilde{\mu}_j\}: \sum_j \tilde{\mu}_j = 1} \sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j) \tag{37}$$

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ij,k}(\{l_i^\theta\}_\theta) \tag{38}$$

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) - \psi^{\tilde{\theta}}(\{\mu_j^{\tilde{\theta}}\}_j) \geq \underline{W}^{\tilde{\theta}}, \forall \tilde{\theta} \neq \theta \tag{39}$$

Compared to the pseudo-planning problem in Lemma 2, the relaxed planning problem internalizes the agglomeration externality (equation (38) in place of (27)) and chooses consumption instead of taking the equilibrium allocation as given (equation (26)). We also simply assume that the government traces the Pareto frontier under constraints (39), instead of maximizing the social welfare function defined by Definition 2. At the same time, this problem is also different from the first-best planning problem as considered in Section 2.3 and Appendix B, as the Planner must choose an incentive compatible location decision (37). For this reason, we refer to these policies as the second-best policies. The following proposition provides our key characterization of the second-best transfer policy.

We let $\{\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\}$ denote the location choice function that maps a vector of consumption in each location to location choice probabilities as the solution to (37).

Proposition 2. *Assume that preference shocks are not degenerate at zero. If the second-best transfer policy is implemented, the allocation* $\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu_j^\theta\}$ *must satisfy* (34)-(38) *and*

$$\mu_j^\theta \left[\tilde{\Lambda}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta\right] = -\sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right], \qquad \forall j, \theta, \tag{40}$$

for some $\tilde{\Lambda}^\theta > 0$ *that satisfies* $\sum_\theta \bar{l}^\theta \tilde{\Lambda}^\theta = 1$, *and* $\frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}$ *is the location choice response to consump-*

tion. Furthermore, this allocation can be implemented with transfers $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$.

This proposition summarizes the key trade-off associated with optimal spatial transfer policy. The left-hand side of this expression summarizes the marginal benefit from transferring one unit of consumption to location j for type θ. In particular, if the marginal utility $u_j^{\theta\prime}(C_j^\theta)$ is high and the associated price P_j^θ is low in location j relative to other locations, the net benefit of transfer to location j tends to be high. On the right-hand side of this equation, we summarize the marginal cost of this transfer through fiscal and technological externalities. In particular, a unit increase of consumption in location j increases population by $\frac{\partial \mu_i^\theta}{\partial C_j^\theta}$ in location i. Notice that this relocation happens in all locations, not only in location j. This population relocation is associated with fiscal externality $w_i^\theta - P_i^\theta C_i^\theta$ and technological externality $\sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta$.

The above formula has a strong connection to optimal unemployment insurance literature (Baily 1978, Chetty 2006). In this literature, the trade-off is between the benefits of unemployment insurance and the fiscal externality of unemployment. In our context, the fiscal externality instead arises due to location decisions.

Proposition 2 is a strict generalization of Fajgelbaum and Gaubert (2020), who study the same problem in the special cases where there are no preference shocks and where preference shocks follow i.i.d. type-I extreme value distribution. In particular, if we take the limit of taking the variance of preference shocks to zero, the elasticity of population with respect to consumption diverges to infinity, i.e., $|\frac{\partial \mu_i^\theta}{\partial C_j^\theta}| \to \infty$. By noting that $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$, the only way to satisfy equation (40) is to set $w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta = E^\theta$ for some constant E^θ.[12] Therefore, the cross-location component of transfers only addresses technological externalities, and the cross-type component of transfers addresses redistribution concerns. Our formula generalizes their cases under flexible preference shocks and highlights key nonparametric statistics for optimal transfers.

We now consider how the implementation of these second-best policies affects the aggregate welfare effects of disaggregated shocks in Proposition 1. Note that, if we multiply equation (40) by $\bar{l}^\theta dC_j^\theta$ and sum up across j and θ, the second term of the left-hand side of equation (40) coincides with (ii) MU dispersion term in Proposition 1, and the right-hand side of this equation coincides with the negative of (iii) fiscal externality term and (iv) technological externality term. Therefore, optimal spatial transfer policy offsets these three distortions, and welfare changes are summarized solely by (i) technology term and (v) redistribution term.

Proposition 3. *Suppose that second-best transfers* $\{T_j^\theta\}_{j,\theta}$ *are implemented according to Proposition*

[12] See Appendix A.4 for a more formal treatment of this limit case.

2 in the pre-shock equilibrium. Then, Proposition 1 comes down to

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k}}_{\text{(i) Technology } (\Omega_T)} + \underbrace{Cov_\theta\left(\Lambda^\theta - \tilde{\Lambda}^\theta, \mathbb{E}_{j|\theta}\left[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right]\right)}_{\text{(v) redistribution } (\Omega_R)}. \tag{41}$$

Furthermore, if the implied Pareto weights of the second-best policy $\tilde{\Lambda}^\theta$ coincide with the welfare weights in the social welfare function Λ^θ, (v) redistribution term also disappears, thereby obtaining Hulten's theorem in a spatial economy.

Corollary 1. *Suppose that transfers $\{T_j^\theta\}_{j,\theta}$ are set so that Proposition 2 holds with $\tilde{\Lambda}^\theta = \Lambda^\theta$ for all θ in the pre-shock equilibrium. Then, Proposition 1 comes down to*

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k}}_{\text{(i) Technology } (\Omega_T)}. \tag{42}$$

Interestingly, despite the policy being the second-best but not the first-best, the reallocation effects become zero. This is because the incentive compatibility constraint of households' location decisions in Lemma 1 is not directly affected by shocks in technology ($d\ln A_{ij,k}$), and therefore the reallocation effects from location choices remain as second-order. This observation resonates with Costinot and Werning (2018), who show that Hulten's theorem holds under the second-best policies, although the environment they consider is very different from ours.

In reality, it is unlikely the case that the government implements optimal transfers. Nevertheless, Proposition 3 and Corollary 1 are careful reminders that the spatial equilibrium models being suboptimal do not necessarily imply systematic deviations from Hulten (1978) and thereby provide an important benchmark case. Moreover, Proposition 3 and Corollary 1 are helpful to facilitate the assessment of how the aggregate welfare changes depart from Hulten's (1978) theorem. In particular, by assessing which of the left-hand side and right-hand side of Proposition 2 is greater under observed transfers $\{T_j\}$, one can conclude whether Hulten's (1978) theorem over- or under-predict the aggregate welfare changes.

3.4 From Theory to Applications

In the previous sections, we show how Proposition 1 can be useful in understanding the sources of welfare gains and losses in special cases of spatial equilibrium models. This section discusses how one can use Proposition 1 in more general, applied cases.

One usage of our formula is for *ex-post* welfare accounting. For example, suppose we are interested in the aggregate welfare changes in the U.S. economies from regional economic growth for the last decade. Suppose we observe the subset of baseline equilibrium prices $\{P_j^\theta, p_{ij,k}, w_j^\theta\}$, quantities $\{C_j^\theta, l_j^\theta, y_{ij,k}\}$, and transfers $\{T_j^\theta\}$. Suppose also that we observe the changes in productivity $\{d\ln\tilde{A}_{ij,k}\}$ and associated consumption and population changes $\{dC_j^\theta, d\ln l_j^\theta\}$ during the period of interest.[13] Then, given the knowledge of agglomeration externalities $\{\gamma_{ij,k}^\theta\}$ and spatial dispersion of marginal utility $\{u_j^{\theta\prime}(C_j^\theta)\}$ evaluated around the baseline equilibrium, as well as the assumption about the welfare weights $\{\Lambda^\theta\}$, one can use Proposition 1 to compute the aggregate welfare change and its decomposition. Importantly, this ex-ante welfare accounting does not require specifying additional model structure beyond the sufficient statistics discussed above.

Our formula can also be used for *ex-ante* welfare accounting. For example, suppose we are interested in understanding the heterogeneous returns from transportation infrastructure across different locations. Researchers often answer this question through counterfactual simulations using a parameterized structural model.[14] In these situations, researchers can use Proposition 1 to unpack the sources of the welfare changes in their structural model. This exercise is particularly useful when researchers have less confidence about the parametric assumptions of the full structural model. Specifically, by knowing which term of Proposition 1 is most relevant for the heterogeneous investment returns, one can undertake sensitivity analysis for the parameters associated with each term. For example, if (ii) MU dispersion term turns out as most relevant for their quantitative results, they can focus on the estimation or sensitivity analysis of the utility function specification $\{u_j^\theta(\cdot)\}$.

3.5 Nonparametric Identification of Welfare Changes

Another benefit of Proposition 1 is that it clarifies the minimal set of sufficient statistics to uniquely identify the aggregate welfare changes. In this section, we discuss the nonparametric identification of these sufficient statistics, and hence the aggregate welfare changes.

We first consider the case where the changes in productivity, consumption, and population $\{d\ln\tilde{A}_{ij,k}, dC_j^\theta, d\ln l_j^\theta\}$ are observed, corresponding to the case of *ex-post* welfare evaluation discussed in the previous section. Suppose we observe the subset of baseline equilibrium prices $\{P_j^\theta, p_{ij,k}, w_j^\theta\}$, quantities $\{C_j^\theta, l_j^\theta, y_{ij,k}\}$, and transfers $\{T_j^\theta\}$. Suppose we also take a stand on welfare weights $\{\Lambda^\theta\}$. The only remaining two statistics are the agglomeration externality elasticities $\{\gamma_{ij,k}^\theta\}$ and the spatial dispersion of marginal utility $\{u_j^{\theta\prime}(C_j^\theta)\}$ evaluated around the baseline equilibrium. For the former, identification requires the causal effect of exogenous popula-

[13]The changes in productivity $\{d\ln\tilde{A}_{ij,k}\}$ may not be directly observed, and one may need to back out from the changes using the goods demand system. See, for example, Allen and Arkolakis (2014) for an example of this procedure.

[14]For example, Allen and Arkolakis (2022) use a fully specified parametric spatial equilibrium model to undertake a counterfactual simulation of the improvement of transportation infrastructures in various locations, assuming that the transportation infrastructure affects productivity (through trade costs) $\{d\ln\tilde{A}_{ij,k}\}$.

tion changes on productivity. The long-standing literature on agglomeration economies provides plausible values for these parameters.[15]

The identification of the spatial dispersion of marginal utility is highlighted less in the context of spatial equilibrium models. Fortunately, existing econometrics literature on discrete choice models provides a way to nonparametrically identify these objects from location choice data. Let us focus on the case where preference shocks are additively separable and not degenerate.[16] In this case, location choice decisions are summarized by the function $\{\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)\}_i = \arg\max_{\{\mu_j^\theta\}:\sum_j \mu_j^\theta=1} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j)$. Suppose that we have a sufficiently long period of data observations. Suppose also that we have an exogenous variation of consumption in each location, so that we can credibly identify the response of the population size in i to consumption change in j, $\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)/\partial C_j^\theta$, for all location pairs i, j and household types θ. Berry and Haile (2014) establish sufficient conditions for the nonparametric identification of such a discrete choice system (see Appendix Appendix E for further detail).

Once we have the discrete choice system, Allen and Rehbeck (2019) show that the dispersion of marginal utility is obtained using Lemma 1. Namely, denoting the expected utility of type θ household as a function of location-specific utility $\hat{W}^\theta(\boldsymbol{u}^\theta) = \max_{\{\mu_j^\theta\}_j:\sum_j \mu_j^\theta=1} \mu_j^\theta u_j^\theta - \psi^\theta(\{\mu_j^\theta\}_j)$, from the envelope theorem and the chain rule,

$$\frac{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} = \frac{\partial^2\hat{W}^\theta(\boldsymbol{u}^\theta)}{\partial u_i^\theta \partial u_j^\theta} u_j^{\theta\prime}(C_j^\theta). \tag{43}$$

Taking the ratio between arbitrary pair (i, j), we have

$$\frac{u_j^{\theta\prime}(C_i^\theta)}{u_i^{\theta\prime}(C_j^\theta)} = \frac{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \Big/ \frac{\partial\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta}. \tag{44}$$

Intuitively, if the marginal utility is higher in j than i, a marginal increase of consumption in location j induces a larger effect of population reallocation away from i, compared to the other way around (consumption increase in i on population reallocation away from j).

We next consider the case where we only know the changes in productivity $\{d\ln\tilde{A}_{ij,k}\}$, corresponding to the *ex-ante* welfare evaluation in the previous section. In this case, we additionally need to identify the changes in consumption $\{dC_j^\theta\}$ and population size $\{d\ln l_j^\theta\}$ as a response to counterfactual shocks $\{d\ln\tilde{A}_{ij,k}\}$. These equilibrium responses are uniquely determined by the factor supply and demand systems. Factor supply system, i.e., how population $\{d\ln l_j^\theta\}$ responds to the vector of consumption $\{dC_j^\theta\}$, can be nonparametrically identified following Berry and

[15]For example, see Melo, Graham, and Noland (2009) for a meta-analysis of the agglomeration externality.

[16]If the preference shocks are degenerate, (ii) MU dispersion term is zero as discussed in Section 3.2. Therefore, the relative marginal utility does not directly influence aggregate welfare changes in Proposition 1.

Haile (2014) as discussed above. Factor demand system, i.e., how the changes in consumption $\{dC_j^\theta\}$ affect each location's labor demand $\{d\ln l_j^\theta\}$, can be nonparametrically identified following Adao et al. (2017), who establish the nonparametric identification of factor demand system in general equilibrium trade models. Together, $\{dC_j^\theta, d\ln l_j^\theta\}$ can be nonparametrically identified for a counterfactual shocks $\{d\ln \tilde{A}_{ij,k}\}$.

While it is reassuring that the welfare changes are in principle nonparametrically identified, the data requirement for the nonparametric identification is unrealistic in most applications. For example, identifying the factor supply system, $\{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)/\partial C_j^\theta\}_{i,j}$ for all i and j, requires a long period of data and exogenous variation of consumption at every location. Therefore, the purpose of this section is not to suggest a practical estimation procedure in applications. Instead, this discussion aims to establish a clear mapping between nonparametric welfare-relevant sufficient statistics and data moments. Such results are useful because they point toward the data moments that discipline the welfare conclusions drawn from spatial equilibrium models.

4 Beyond Additively Separable Preference Shocks

So far, we have focused on specifications where preference shocks are additively separable. This section relaxes this assumption. Section 4.1 discusses the general case. Section 4.2 discusses a special case with the multiplicatively separable preference shocks following max-stable multivariate Fréchet distribution.

4.1 General Case

We now assume that utility in location i is given by $U_i^\theta(C_i^\theta, \epsilon_i^\theta)$. Compared to the additively separable specification, marginal utility in each location now depends on the preference shock draws. To see this, the average marginal utility for households deciding to live in location j is given by

$$\mathbb{E}\left[\frac{\partial}{\partial C_j^\theta} U_j^\theta(C_j^\theta, \epsilon_j^\theta) | j = \arg\max_l U_l^\theta(C_l^\theta, \epsilon_l^\theta)\right]. \tag{45}$$

Unlike the additively separable specification, i.e., $\frac{\partial}{\partial C_j^\theta} U_j^\theta(C_j^\theta, \epsilon_j^\theta) = u_j^{\theta\prime}(C_j^\theta)$, the selection of preference shocks influences the marginal utility of consumption in each location.

Under this general preference specification, the isomorphic representation of households' location decisions in Lemma 1 is modified as

$$\max_{\{\mu_j^\theta\}:\sum_j \mu_j^\theta = 1} \mathcal{U}^\theta(\{C_j^\theta\}, \{\mu_j^\theta\}), \tag{46}$$

where we give the explicit expression for $\mathcal{U}^\theta s$ in Appendix A.6. Under additively separable specification, $\mathcal{U}^\theta(\{C_j^\theta\},\{\mu_j^\theta\}) = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j)$, and $\partial\mathcal{U}^\theta(\{C_j^\theta\},\{\mu_j^\theta\})/\partial C_j^\theta = \mu_j^\theta u_j^{\theta\prime}(C_j^\theta)$, i.e., marginal expected utility only depends on j's population and consumption. In the general case, it is affected by the entire vector of population distribution $\{\mu_j^\theta\}_j$ and consumption $\{C_j^\theta\}_j$ beyond location j through the selection of preference draws.

It is straightforward to extend our theory to this general case. Proposition 1 is simply modified by replacing the marginal utility per household $u_j^{\theta\prime}(C_j^\theta)$ with the one under this general specification. In particular, (ii) MU dispersion term becomes

$$\Omega_{MU} = \mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}\left(-\frac{P_j^\theta}{\mathcal{MU}_j^\theta}, \frac{1}{l_j^\theta}\mathcal{MU}_j^\theta dC_j^\theta\right)\right], \qquad \mathcal{MU}_j^\theta = \frac{1}{l_j^\theta}\frac{\partial\mathcal{U}^\theta(\{C_j^\theta\},\{\mu_j^\theta\})}{\partial C_j^\theta}. \tag{47}$$

Conditional on the price normalization using this marginal utility (18), all other terms are unaffected.

While this extension is straightforward in theory, it poses a challenge to the identification of aggregate welfare. To understand this challenge, consider a monotone transformation of the utility function from the additively separable class: $U_j^\theta(C_j^\theta,\epsilon_j^\theta) = m(u_j^\theta(C_j^\theta)+\epsilon_j^\theta)$ for some strictly increasing function $m(\cdot)$. This transformation does not affect the model's *positive* prediction because of the ordinal nature of the utility function for location choice decisions. However, the expected marginal utility in each location becomes

$$\mathcal{MU}_j^\theta = u_j^{\theta\prime}(C_j^\theta)\mathbb{E}\left[g'(u_j^\theta(C_j^\theta)+\epsilon_j^\theta)|j = \arg\max_l g(u_l^\theta(C_l^\theta)+\epsilon_l^\theta)\right]. \tag{48}$$

Therefore, the function $m(\cdot)$ generally affects the marginal utility in each location. This discussion implies that the *normative* prediction, i.e., aggregate welfare, generically depends on the choice of $m(\cdot)$. Since $m(\cdot)$ cannot be identified from data, aggregate welfare also cannot be identified from location choice data alone.

This lack of identification is worrisome, as it indicates that welfare predictions are not uniquely pinned down from data. Even when the two models match the same data moments, the welfare conclusions drawn from these two models can be arbitrarily different. However, we show below that, under a common parametric assumption in the existing literature, such a concern is not warranted.

4.2 Multiplicative Shocks with Multivariate Fréchet Distribution

We now focus on a special case of nonseparable preference shocks. Specifically, we assume that preference shocks are multiplicatively separable and follow max-stable multivariate Fréchet distribution. Formally, we assume that preference for households living in location j of type θ is given by

$$\begin{aligned} &\tilde{U}_j^\theta(C_j^\theta, \tilde{\epsilon}_j^\theta) = \tilde{u}_j^\theta(C_j^\theta)\tilde{\epsilon}_j^\theta, \\ \text{with} \qquad &\mathbb{P}(\tilde{\epsilon}_1^\theta \leq \bar{\epsilon}_1, \ldots, \tilde{\epsilon}_N^\theta \leq \bar{\epsilon}_N) = \exp(-G^\theta(K_1^\theta(\bar{\epsilon}_1)^{-\nu^\theta}, \ldots, K_N^\theta(\bar{\epsilon}_N)^{-\nu^\theta})), \end{aligned} \tag{49}$$

where G^θ is a function of homogeneous degree one, which we call "correlation function." The key implication of this specification is the max-stability property, where the distribution of the maximum is Fréchet with shape parameter ν^θ.[17]

Specification (49) covers many specifications that appear in the previous literature besides the additively separable specification. For example, Redding (2016) is a special case with i.i.d. Fréchet distribution, which corresponds to the case with $G^\theta(x_1, \ldots, x_N) = \sum_{j=1}^N x_j$. Some researchers introduce nested Fréchet distribution to accommodate richer substitution patterns of location choice. More broadly, this preference specification delivers a generalized extreme value (GEV) demand system with flexible substitution patterns as introduced by McFadden (1978). Dagsvik (1995) shows that GEV demand systems can approximate arbitrary demand system generated by random utility models.

To consider the property of this specification, consider the log transformation of this utility specification: $u_j^\theta(C_j^\theta) = \ln(\tilde{u}_j^\theta(C_j^\theta))$, and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$. It is straightforward to show that ϵ_j follows multivariate Gumbel distribution with the same correlation function $G(\cdot)$ such that

$$\begin{aligned} &U_j^\theta(C_j, \epsilon_j) = u_j^\theta(C_j^\theta) + \epsilon_j^\theta, \\ \text{with} \quad &\mathbb{P}(\epsilon_1 \leq \bar{\epsilon}_1^\theta, \ldots, \epsilon_N^\theta \leq \bar{\epsilon}_N) = \exp(-G^\theta(K_1^\theta(\exp(-\nu^\theta\bar{\epsilon}_1)), \ldots, K_N^\theta(\exp(-\nu^\theta\bar{\epsilon}_N)))). \end{aligned} \tag{50}$$

Since $\ln(\cdot)$ is a monotone transformation, the system (49) and (50) have isomorphic *positive* predictions. The following proposition shows that these two models also deliver isomorphic *normative* predictions.

Proposition 4. *Consider the spatial equilibrium with max-stable multivariate Fréchet preference shocks (49). Let $\tilde{W} \equiv \tilde{\mathcal{W}}(\{\tilde{W}^\theta\}_{\theta\in\Theta})$ be the welfare in this economy. Consider another economy with the log transformation of utility specification (50) without changing remaining equilibrium conditions in Definition 1. Let $W \equiv \mathcal{W}(\{W^\theta\}_{\theta\in\Theta})$ be the welfare in this economy with $\mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \equiv \ln\tilde{\mathcal{W}}(\{\exp(W^\theta)\}_{\theta\in\Theta})$ being the social welfare function.*

1. *Equilibrium allocations are identical in both economies.*

2. *The welfare decomposition according to Proposition 1 is identical in both economies up to*

[17] See McFadden (1978) for further properties of this demand system and the correlation function. See also Lind and Ramondo (2023) for the application of this demand system for Ricardian trade models.

multiplicative constant. Formally, let $d\tilde{W} = \tilde{\Omega}_T + \tilde{\Omega}_{MU} + \tilde{\Omega}_{FE} + \tilde{\Omega}_{TE} + \tilde{\Omega}_R$ be the decomposition in the economy with multiplicative Fréchet preference shocks, and let $dW = \Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R$ be the decomposition in an economy with additive separable preference shocks counterpart. Then,

$$d\tilde{W} = \tilde{W} dW, \quad \tilde{\Omega}_c = \tilde{W}\Omega_c, \quad \textit{for } c \in \{T, MU, FE, TE, R\}. \tag{51}$$

Proposition 4 establishes that an economy with multiplicative Fréchet shocks is isomorphic to an economy to its additively separable counterpart both for positive *and* normative implications. An important corollary of Proposition 4 is that all the welfare relevant sufficient statistics of an economy with multiplicative Fréchet shocks are identified, provided that they are identified in an economy with additively separable preference shocks, as in Section 3.5. The possibility of identification is enlightening. As discussed in the previous section, outside of additively separable preference shocks, it is generally not possible to identify the marginal utility of consumption from location choice data, and thereby our decomposition lacks an empirical content. However, Proposition 4 shows such a concern is not warranted under a class of models with nonseparable preference shocks that cover almost all the applications in the literature.

What is the reason behind the equivalence under multiplicative Fréchet? As discussed in the previous section, the transformation of the utility function matters only through the differences in marginal utility. The marginal utility of consumption of households in location j in the system (49) is given by

$$\begin{aligned} \mathcal{MU}_j^\theta &= u_j^{\theta\prime}(C_j^\theta)\mathbb{E}\big[\tilde{u}_j^\theta(C_j^\theta)\tilde{\epsilon}_j^\theta | j = \arg\max_l \tilde{u}_l^\theta(C_l^\theta)\tilde{\epsilon}_l^\theta\big] \\ &= u_j^{\theta\prime}(C_j^\theta)\tilde{W}^\theta, \end{aligned} \tag{52}$$

where the first transformation used the fact that $u_j(C_j) = \ln(\tilde{u}_j^\theta(C_j^\theta))$ and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$. The second transformation of equation (52) follows from the max-stable property of $\tilde{\epsilon}_j$: The distribution of the maximum follows the same distribution irrespective of the chosen option (McFadden 1978, Lind and Ramondo 2023). Therefore, the marginal utility under specification is identical to its log transformation (50) up to scale $\tilde{W}$. Given that all terms in our welfare decomposition in Proposition 1 scale up by marginal utility under price normalization (18), we have the isomorphism in the aggregate welfare.[18]

It is worth stressing again that this result depends both on the multiplicatively separable specification and the max-stable multivariate Fréchet distribution of preference shocks. A failure of

[18] Another way to interpret this result is through a particular property of the Fréchet distribution: the expectation of the log coincides with the log of the expectation.

either assumption implies that the *normative* implications are not isomorphic under log transformation.[19] Despite this sensitivity, it is reassuring that specifications (49) and (50) – predominantly common specifications in the literature – agree in their aggregate welfare predictions.

5 Additional Extensions and Discussions

This section provides additional extensions and discussions to our baseline analysis.

5.1 General Externality

In our main model, we assumed that agglomeration externality is purely a function of local population size (7). In some contexts, researchers specify that a higher population size in the surrounding regions also generates agglomeration spillovers (e.g., Ahlfeldt et al. 2015). In other contexts, researchers also specify that the externality arises from the specific producers' input use (e.g., free entry model with labor fixed cost such as Krugman 1991) or the producers' output (e.g., congestion cost from shipment, as in Allen and Arkolakis 2022). To capture these general externalities, we extend the spillover function (7) such that

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ij,k}(\{l_\ell^\theta\}_{\ell,\theta}, \{l_{ij,k}^\theta\}_\theta, y_{ij,k}), \tag{53}$$

where the first argument of $g_{ij,k}(\cdot)$ corresponds to the population size across types and locations, the second argument corresponds to labor input in production, and the third argument corresponds to output. We also denote the spillover elasticities such that

$$\gamma_{ij,k}^{P,\ell\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_\ell^\theta}, \qquad \gamma_{ij,k}^{L,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_{ij,k}^\theta}, \qquad \gamma_{ij,k}^{Y,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln y_{ij,k}} \tag{54}$$

Under this extension, the only modification in Proposition 1 is the (iv) technological externality term, which is modified as:

$$\Omega_{TE} = \sum_{j,l,k} p_{jl,k} y_{jl,k} \left(\sum_{\ell,\theta} \gamma_{\ell l,k}^{P,\ell\theta} d\ln l_j^\theta + \sum_\theta \gamma_{jl,k}^{L,\theta} d\ln l_{ij,k}^\theta + \gamma_{jl,k}^{Y} d\ln y_{ij,k} \right). \tag{55}$$

This expression comes down to (iv) technological externality term in Proposition 1 if the spillover function only depends on local population size. The only difference here is that the reallocation of

[19] An important exception where preference shocks do not follow max-stable multivariate Fréchet distribution is the mixed logit model. See Davis and Gregory (2021) for a discussion of the lack of identification of optimal transfer policy under multiplicatively separable specifications with additional random shocks.

population in surrounding regions and other quantities may have first-order effects on aggregate welfare through additional technological externalities.

5.2 Shocks to Amenity and Amenity Externality

In Section 3, we analyzed the effects of productivity shocks on aggregate welfare. In some contexts, researchers are interested in the shocks to amenity instead of productivity. The analysis in Section 3 embraces this possibility by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices associated with amenities, which is often unobserved. Below, we provide an alternative expression for Proposition 1 without using prices for the amenities.

To consider this extension, we explicitly introduce amenity as an argument of utility function as follows:

$$U_j^\theta(C_j^\theta, B_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta, B_j^\theta) + \epsilon_j^\theta, \tag{56}$$

where B_j^θ is the amenity in region j. Furthermore, we assume that these amenities take the following form:

$$B_i^\theta = \tilde{B}_i^\theta g_i^{B,\theta}(\{l_i^\theta\}_\theta), \qquad \gamma_i^{B,\tilde{\theta}\theta} = \frac{\partial \ln g_i^{B,\tilde{\theta}\theta}(\{l_i^\theta\}_\theta)}{\partial \ln l_i^{\tilde{\theta}}}, \tag{57}$$

$\tilde{B}_i^\theta$ is the fundamental exogenous component of amenity, $g_i^{B,\theta}(\{l_i^\theta\}_\theta)$ is the spillover function, and $\gamma_i^{B,\tilde{\theta}\theta}$ is the amenity spillover elasticity from type $\tilde{\theta}$ to type θ in location i.

Under this extension, Proposition 1 is modified as follows. Consider an arbitrary set of small shocks to exogenous components of productivity, $\{d \ln \tilde{A}_{ij,k}\}$, and amenity, $\{d \ln \tilde{B}_i^\theta\}$. The first-order impact of microeconomic shocks on welfare in utility terms can be expressed as

$$
\begin{aligned}
dW = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta}_{\text{(i) Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta \left[\text{Cov}_{j|\theta} \left(-\frac{P_j^\theta}{\partial_C u_j^\theta}, \partial_C u_j^\theta dC_j^\theta \right) \right]}_{\text{(ii) MU Dispersion } (\Omega_{MU})} \\
& + \underbrace{\mathbb{E}_\theta \left[\text{Cov}_{j|\theta}(-T_j^\theta, d\ln l_j^\theta) \right]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta \left[Cov_{j|\theta} (\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_j^{\tilde{\theta}} B_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}}, d\ln l_j^\theta) \right]}_{\text{(iv) Technological Externality } (\Omega_{TE})} \\
& + \underbrace{\text{Cov}_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{\partial_C u_j^\theta (C_j^\theta)} \right], \mathbb{E}_{j|\theta} \left[\partial_C u_j^\theta dC_j^\theta \right] \right)}_{\text{(v) Redistribution } (\Omega_R)} .
\end{aligned}
\tag{58}
$$

where $\partial_B u_j^\theta \equiv \frac{\partial u_j^\theta}{\partial B_j^\theta}$ and $\partial_C u_j^\theta \equiv \frac{\partial u_j^\theta}{\partial C_j^\theta}$.

The main difference from Proposition 1 is the additional terms in (i) technology and (iv) technological externality. The second term inside (i) technology captures the effects of exogenous amenity terms absent reallocation effects. The coefficient in front of $d\ln \tilde{B}_i^\theta$, $l_i \partial_B u_i^\theta B_i^\theta$, is the population-weighted sum of the marginal utility of amenity. This term strongly resembles the technology effect on productivity (the first term). In particular, if the amenity is traded and priced in the market, $\partial_B u_i$ corresponds to the competitive price of the amenity, and hence $l_i \partial_B u_i B_i$ is the total sales of the amenity, corresponding to $p_{ij,k} y_{ij,k}$. The second term inside (iv) technological externality term has the same feature: if the amenity is traded, the changes of amenity from externality collapses to the same form as the productivity externality term.

5.3 Isomorphism between Amenity Externality and Preference Shocks

In quantitative spatial equilibrium literature, researchers often argue that amenity congestion externality is isomorphic to amenity externality and use these specifications interchangably.[20] This section discusses this isomorphism through the lens of our framework.

For expositional convenience, we assume a single type and drop superscript θ. Consider the following utility specification with amenity externality without preference shocks:

$$
U_j(C_j, B_j, \epsilon_j) = u_j(C_j) + B_j, \qquad B_j = g_j(\{l_i\}_i) = -\frac{1}{\nu} \ln S_j(\{l_i\}_i), \tag{59}
$$

[20] See, for example, Allen and Arkolakis (2014) and Desmet, Nagy, and Rossi-Hansberg (2018), for papers that mention the isomorphism between the two specifications.

where $S_j(\{l_i\}_i)$ satisfies the following property

$$\frac{1}{\nu}\sum_j l_j \frac{\partial \ln S_j(\{l_i\}_i)}{\partial l_i} = 1. \tag{60}$$

Note that this specification accommodates that the population in i generates externality in other regions. A special case of this example is when $S_j(\{l_i\}_i) = l_j^{\nu}$, i.e., amenity is iso-elastic to local population size with elasticity $-\nu$.

It is straightforward to see that this specification is isomorphic to the case where there are no amenity externality and preference shocks follow max-stable multivariate Gumbel distribution with shape parameter ν, i.e., $U_j(C_j, B_j, \epsilon_j) = u_j(C_j) + \epsilon_j$ and $\{\epsilon_j\}$ follows Specification (50).[21] It is also straightforward to see that both specifications deliver the same households' expected utilities, thereby delivering identical normative predictions.

This isomorphism arises because this particular form of congestion externality does not induce misallocation. In particular, the (iv) amenity externality term in Equation (58) comes down to

$$\mathrm{Cov}_j(-\sum_i l_i \frac{\ln S_i(\{l_j\}_i)}{\partial \ln l_j}, d\ln l_j) = \mathrm{Cov}_j(-\nu, d\ln l_j) = 0, \tag{61}$$

where we used (60) and $\partial_B u_i = 1$. Given that all other terms in Equation (58) are identical between the two specifications, the aggregate welfare predictions are also isomorphic.

This discussion also clarifies this isomorphism holds only when preference shocks follow max-stable multivariate Gumbel distribution, or equivalently, when the congestion externality takes the specific functional form given by (59) and (60). Outside these special cases, congestion externality generates a source of misallocation, and hence the isomorphism does not hold in general.[22]

5.4 Arbitrary Social Welfare Function

Our baseline analysis focuses on expected utility as the welfare criterion. In some contexts, researchers may want to consider an alternative welfare criterion. For example, consider a scenario

[21]See Appendix C for the derivation that $\psi(\{l_j\})$ in Lemma 1 takes the form of $\psi(\{l_j\}) = \frac{1}{\nu}\sum_j l_j \ln S_j(\{l_i\}_i)$ in this case.

[22]Fajgelbaum and Gaubert (2020) show that, under multiplicative utility specification, competitive equilibrium involves misallocation even under iso-elastic amenity externality. Through the lens of Equation (58), this source of misallocation appears in (ii) MU dispersion term. The multiplicative amenity without preference shocks implies that the marginal utility of income is not equalized across locations. Furthermore, unlike our baseline model abstracting direct effects of shocks on utility $u_j(\cdot)$, the utility changes from consumption changes dC_j are not equalized because of the changes of utility from amenity. Therefore the term (ii) is not zero. Note that this specification is isomorphic to the specification with multiplicative max-stable Fréchet shocks (without amenity externality) as discussed in Section 4.2. In this case, the dispersion of marginal utility instead arises from preference shock draws.

where researchers alternatively interpret idiosyncratic preference shocks $\{\epsilon_j^\theta\}$ as "mistakes". Under this interpretation, one may desire to exclude this component of ϵ_j from aggregate welfare. To capture these cases, in Appendix D.2, we consider a scenario with a general social welfare function $W^{SP} = \mathcal{W}(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta\}_j, \{l_j^\theta\}_j)\}_\theta)$, where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the distribution of consumption and population of household type θ. Then, our decomposition in Proposition 1 only adds an additional term (vi) paternalistic motive, which is defined as

$$\Omega_{PM} = \mathbb{E}_\theta\left[\Lambda^\theta \mathbb{E}_{j|\theta}\left[\left(\mathcal{MU}_j^{SP,\theta} - u_j^{\theta\prime}(C_j)\right) dC_j^\theta\right]\right], \qquad \mathcal{MU}_j^{SP,\theta} = \frac{\partial}{\partial C_j}\mathcal{U}^{SP,\theta}(\{C_i^\theta\}, \{l_i^\theta\}). \tag{62}$$

This additional term captures the misalignment between the social planner's welfare assessment of a marginal value of consumption may not coincide with the private agents' assessment (marginal utility).

5.5 Commuting

Our baseline model assumes that households supply labor at the same location as their residential location. In the urban economics literature, it is typically assumed that households make separate decisions about their residential and employment location decisions (e.g., Ahlfeldt et al. 2015, Tsivanidis 2019, Zárate 2022). Our framework can be straightforwardly extended to such a framework by reinterpreting household's location decisions j as a combination of residential and work locations, (j_1, j_2), where the first index captures the residential location and the second index captures the work location. For example, the utility of agents deciding home location j_1 and work location j_2 is given by $U_{j_1 j_2}^\theta(C_{j_1 j_2}^\theta, \epsilon_{j_1 j_2}^\theta)$, where $\epsilon_{j_1 j_2}^\theta$ is home-and-work-specific preference shocks.[23] Consequently, Proposition 1 remains unchanged by simply replacing j with (j_1, j_2) combinations.

6 Applications

TBA

[23]This extension accommodates the specification where households consume different consumption bundles depending on the home-work combination, as studied by Miyauchi, Nakajima, and Redding (2022).

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Online Appendix for "Aggregate Welfare in a Spatial Economy"

Eric Donald, Masao Fukui, Yuhei Miyauchi

A Proofs

A.1 Proof of Lemma 1

For expositional simplicity, we prove the lemma under ex-ante homogenous types and drop superscript θ. Note also that we have $l_j = \mu_j$ given the normalization of population size $\bar{l} = 1$.

Economy with Heterogeneous Preferences. Consider the problem of households deciding where to live. We index each individual by $\omega \in [0, 1]$, and $\{\epsilon_k(\omega)\}_k$ denote the preference draw of individual ω. Each individual solves the following problem:

$$\begin{aligned} v(\omega) = \max_{\{\mathbb{I}_j(\omega)\}_j} \sum_j \mathbb{I}_j(\omega) \left[u_j(C_j) + \epsilon_j(\omega)\right] \\ \text{s.t.} \quad \sum_j \mathbb{I}_j(\omega) = 1, \end{aligned} \tag{A.1}$$

where $\mathbb{I}_j(\omega) \in \{0, 1\}$ is an indicator function for location choice of individual ω, and $C_j = w_j/P_j$. The fraction of individuals living in location j is given by

$$l_j = \int_0^1 \mathbb{I}_j(\omega) d\omega. \tag{A.2}$$

Economy with Representative Agent. Define the following function:

$$\begin{aligned} \psi(\{l_j\}_j) = - \max_{\{\mathbb{I}_j(\omega)\}_{\omega,j}} \int_0^1 \sum_j \epsilon_j(\omega) \mathbb{I}_j(\omega) d\omega \\ \text{s.t.} \quad \int_0^1 \mathbb{I}_j(\omega) d\omega = l_j \\ \sum_j \mathbb{I}_j(\omega) = 1. \end{aligned} \tag{A.3}$$

The representative agent solves

$$W = \max_{\{l_j\}_j : \sum_j l_j = 1} \sum_j l_j u_j(C_j) - \psi(\{l_j\}_j) \tag{A.4}$$

Equivalence Result. We formally restate the equivalence result of Lemma 1 as follows.

Lemma. *Suppose $\{\mathbb{I}_j(\omega)\}_j$ solves (A.1) for all ω. Then, $\{l_j\}_j$, given by (A.2), solves (A.4). Conversely, suppose $\{l_j\}_j$ solves (A.4). Then $\{\mathbb{I}_j(\omega)\}_{\omega,j}$, given by the solution to (A.3) associated with $\{l_j\}_j$, solves (A.1) for almost all ω. Moreover, the expected utility in the economy with heterogeneous* *preferences equals the utility of the representative agent:*

$$\int_0^1 v(\omega)d\omega = W$$

Proof. We prove the first part. Suppose to the contrary, there exists $\{\tilde{l}_j\}_j$ such that

$$\sum_j \tilde{l}_j u_j(C_j) - \psi(\{\tilde{l}_j\}_j) > \sum_j l_j u_j(C_j) - \psi(\{l_j\}_j). \tag{A.5}$$

Let $\{\tilde{\mathbb{I}}_j(\omega)\}_{\omega,j}$ denote the solution to (A.3) associated with $\{\tilde{l}_j\}_j$. Plugging into (A.5),

$$\int_0^1 \sum_j \tilde{\mathbb{I}}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega > \int_0^1 \sum_j \mathbb{I}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega, \tag{A.6}$$

where $\sum_j \tilde{\mathbb{I}}_j(\omega) = 1$ and $\sum_j \mathbb{I}_j(\omega) = 1$ for all ω. However, this is a contradiction because by our presumption, for any ω,

$$\sum_j \mathbb{I}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] \geq \sum_j \tilde{\mathbb{I}}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right]$$

for all $\tilde{\mathbb{I}}_j(\omega)$, which would imply

$$\int_0^1 \sum_j \tilde{\mathbb{I}}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega \leq \int_0^1 \sum_k \mathbb{I}_k(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega. \tag{A.7}$$

Now we prove the converse. Suppose to the contrary, there exists $\{\tilde{\mathbb{I}}_j(\omega)\}_j$ such that

$$\sum_j \tilde{\mathbb{I}}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] > \sum_j \mathbb{I}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] \tag{A.8}$$

and $\sum_j \tilde{\mathbb{I}}_j(\omega) = 1$ hold for all $\omega \in \Omega$, where $\Omega \subset [0, 1]$ and $|\Omega| > 0$. Define

$$\tilde{l}_j = \int_0^1 \tilde{\mathbb{I}}_j(\omega)d\omega. \tag{A.9}$$

Then

$$\begin{aligned}\sum_j l_j u_j(C_j) - \psi(\{l_j\}_j) &= \int_0^1 \sum_j \mathbb{I}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega \\ &< \int_0^1 \sum_j \tilde{\mathbb{I}}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega \\ &\le \sum_j \tilde{l}_j u_j(C_j) - \psi(\{\tilde{l}_j\}_j).\end{aligned}$$

This is a contradiction that $\{l_j\}_j$ is a solution to (A.4).

We need to show that the expected utility coincides with each other in the two economies. This immediately follows given the above result. Let $\{\mathbb{I}_j(\omega)\}_{\omega,j}$ be the solution to (A.1) for all ω, and let $\{l_j\}_j$ denote the solution to (A.4). Then

$$\int_0^1 \sum_j \mathbb{I}_j(\omega)\left[u_j(C_j) + \epsilon_j(\omega)\right] d\omega = \sum_j l_j u_j(C_j) - \psi(\{l_j\}_j). \tag{A.10}$$

□

A.2 Proof of Lemma 2

Decentralized equilibrium solves (22)-(27) and firms' optimality conditions, given by

$$P_j^\theta \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^\theta, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}, \tag{A.11}$$

The first-order conditions of the pseudo planner's problem with respect to $c_{ij,k}$, $l_{ij,k}^\theta$, $h_{ij,k}$, $x_{ij,k}^{l,m}$ are

$$P_j^{L,\theta} \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}^L, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{L,\theta}, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^L, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^L, \tag{A.12}$$

The pseudo planner's solution solves (22)-(27) as well as (A.12). When $P_j^{L,\theta} = P_j^\theta$, $p_{ij,k}^L = p_{ij,k}$, $w_j^{L,\theta} = w_j^\theta$, and $r_j^L = r_j$, these conditions are identical to the equilibrium conditions. Therefore the decentralized equilibrium satisfies the optimality conditions for the pseudo planner's problem with associated Lagrangian multipliers $P_j^{L,\theta} = P_j^\theta$, $p_{ij,k}^L = p_{ij,k}$, $w_j^{L,\theta} = w_j^\theta$, and $r_j^L = r_j$.

A.3 Proof of Proposition 1

To solve the pseudo planning problem, we first note that the constraint (25) is simply rewritten as $\mu_j^\theta = \breve{\mu}_j^\theta$, where $\breve{\mu}_j^\theta$ is the equilibrium population distriubtion.

The first order condition of pseudo planning problem with respect to C_j^θ is given by

$$\chi_j^\theta = l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta], \tag{A.13}$$

where χ_j^θ and P_j^θ correspond to the Lagrange multipliers for constraints (25) and (22), respectively. Furthermore, the first order condition with respect to μ_j^θ is given by

$$\eta_i^\theta = \bar{l}^\theta \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right], \tag{A.14}$$

where η_j^θ, w_j^θ and $p_{il,k}$ correspond to the Lagrange multipliers for constraints (22), (24) and (23), respectively.

By applying the Envelope theorem to pseudo planning problem (20),

$$\begin{aligned}
\frac{dW}{d\ln \tilde{A}_{il,k}} =& p_{il,k} y_{il,k} + \sum_\theta \sum_j \left[\chi_j^\theta \frac{dC_j^\theta}{d\ln \tilde{A}_{il,k}} + \eta_j^\theta \frac{d\mu_j^\theta}{d\ln \tilde{A}_{il,k}} \right] \\
=& p_{il,k} y_{il,k} + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta] \frac{dC_j^\theta}{d\ln \tilde{A}_{il,k}} \\
&+ \sum_\theta \sum_j \bar{l}^\theta \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right] \frac{d\mu_j^\theta}{d\ln \tilde{A}_{il,k}}.
\end{aligned}$$

Multiplying both hand side by $d\ln \tilde{A}_{il,k}$, we have

$$\begin{aligned}
dW =& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_\theta \sum_j \chi_j^\theta P_j^\theta l_j^\theta dC_j^\theta \\
=& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta] dC_j^\theta \\
&+ \sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta.
\end{aligned} \tag{A.15}$$

Now,

$$\begin{aligned}
&\sum_\theta \sum_j l_j^\theta[\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta]dC_j^\theta \\
=&\sum_\theta \bar{l}^\theta \sum_j \mu_j^\theta[\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}]u_j^{\theta\prime}(C_j^\theta)dC_j^\theta \\
=&\sum_\theta \bar{l}^\theta \left[\sum_j Cov_{j|\theta}(\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta) + \mathbb{E}_{j|\theta}[\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}]\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta]\right] \\
=&\sum_\theta \bar{l}^\theta \left[Cov_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta) + \left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}]\right)\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta]\right] \\
=&\mathbb{E}_\theta[Cov_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta)] + Cov_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}], \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta]\right) \\
&+ \mathbb{E}_\theta\left[\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}]\right]\mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta]\right] \\
=&\mathbb{E}_\theta[Cov_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta)] + Cov_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}], \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta]\right),
\end{aligned}$$

where the last equation used the fact that $\mathbb{E}_\theta\left[\Lambda^\theta\right] = 1$ under our normalization of Pareto weights (??) and $\mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}]\right] = \mathbb{E}_\theta\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right] = 1$ under our price normalization (18). The two terms correspond to (ii) MU dispersion and (v) redistribution in Proposition 1.

Similarly,

$$\begin{aligned}
&\sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta]dl_j^\theta \\
=&\sum_\theta \bar{l}^\theta \sum_j \mu_j^\theta[w_j^\theta - P_j^\theta C_j^\theta]d\ln l_j^\theta \\
=&\sum_\theta \bar{l}^\theta \left[Cov_{j|\theta}(w_j^\theta - P_j^\theta C_j^\theta, d\ln l_j^\theta) + \mathbb{E}_{j|\theta}[w_j^\theta - P_j^\theta C_j^\theta]\underbrace{\mathbb{E}_{j|\theta}[d\ln l_j^\theta]}_{=0}\right] \\
=&\mathbb{E}_\theta\left[Cov_{j|\theta}(w_j^\theta - P_j^\theta C_j^\theta, d\ln l_j^\theta)\right] \\
=&\mathbb{E}_\theta\left[Cov_{j|\theta}(-\Pi^\theta - T_j^\theta, d\ln l_j^\theta)\right] \\
=&\mathbb{E}_\theta\left[Cov_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\right],
\end{aligned}$$

which corresponds to (iii) fiscal externality term. Finally,

$$
\begin{aligned}
&\sum_{\theta}\sum_{j}\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta} dl_j^{\theta} \\
=&\sum_{\theta} \bar{l}^{\theta} \sum_{j} \mu_j^{\theta} \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta} d\ln l_j^{\theta} \\
=&\mathbb{E}_{\theta}\left[Cov_{j|\theta}\left(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta}, d\ln l_j^{\theta}\right)\right],
\end{aligned}
$$

which corresponds to (iv) technological externality term.

A.4 Proof of Proposition 2

We first characterize the first-order condition of the relaxed problem (33). The first-order conditions of Problem with respect to $c_{ij,k}^{\theta}, l_{ij,k}^{\theta}, h_{ij,k}, x_{ij,k}^{l,m}$ are given by

$$
P_j^{SB,\theta} \frac{\partial \mathcal{C}_j^{\theta}}{\partial c_{ij,k}^{\theta}} = p_{ij,k}^{SB}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^{\theta}} = w_i^{SB,\theta}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{SB}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{SB}. \tag{A.16}
$$

When $P_j^{SB,\theta} = P_j^{\theta}, p_{ij,k}^{SB} = p_{ij,k}$, $w_j^{SB,\theta} = w_j^{\theta}$, and $r_j^{SB} = r_j$, these conditions are identical to the equilibrium conditions (A.11). Finally, the first-order condition with respect to C_j^{θ} is given by equation (40).

It remains to be shown that all the equilibrium conditions are satisfied under $T_j^{\theta} = P_j^{\theta} C_j^{\theta} - w_j^{\theta} - \Pi^{\theta}$ where C_j^{θ} satisfies (40) with supporting prices $\{P_j^{SB,\theta}, p_{ij,k}^{SB}, w_j^{SB,\theta}, r_j^{SB}\}$. First, it is immediate to see that market clearing conditions coincide by comparing (11)-(14) with (34)-(36). The constraint (37) implies that the population distribution solves (19). Given prices $\{P_j^{SB,\theta}, p_{ij,k}^{SB}, w_j^{SB,\theta}, r_j^{SB}\}$, the firm's optimality conditions (A.12) are satisfied because they are identical to (A.16).

Finally, it remains to show that the government budget (6) and price normalization (18) are satisfied. Multiplying $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$ by $l_j^{SB,\theta}$ and summing across j and θ,

$$
\begin{aligned}
&\sum_\theta \sum_j T_j^\theta l_j^{SB,\theta} \\
&= \sum_\theta \sum_j P_j^{SB,\theta} C_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= \sum_\theta \sum_{i,j,k} p_{ij,k}^{SB} c_{ij,k}^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k}^{SB} \left[A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}^{SB}, h_{ij,k}^{SB} \mathbf{x}_{ij,k}^{SB}) - \sum_{l,m} p_{ij,k}^{SB} x_{jl,m}^{SB,i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k}^{SB} \left[\sum_\theta A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}} l_{ij,k}^{SB,\theta} + A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} h_{ij,k}^{SB} + \sum_{l,m} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} x_{ij,k}^{SB,l,m} - \sum_{l,m} p_{ij,k}^{SB} x_{jl,m}^{SB,i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= \sum_{i,j,k} \left[\sum_\theta w_i^{SB,\theta} l_{ij,k}^{SB,\theta} + r_i^{SB} h_{ij,k}^{SB} + \sum_{l,m} p_{li,m}^{SB} x_{ij,k}^{SB,l,m} - \sum_{l,m} p_{ij,k}^{SB} x_{jl,m}^{SB,i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB} + \sum_j r_j^{SB} \bar{h}_j^{SB} - \sum_\theta \sum_j w_j^{SB,\theta} l_j^{SB,\theta} - \sum_\theta \bar{l}^\theta \Pi^\theta \\
&= 0.
\end{aligned}
$$

Finally, by dividing Equation (40) by $u_j^{SB,\theta\prime}(C_j^{SB,\theta})$ and summing up across j and θ with weights $\bar{l}^\theta$,

$$
\begin{aligned}
&\sum_\theta \bar{l}^\theta \sum_j \mu_j^\theta \left[\tilde{\Lambda}^\theta - \frac{P_j^{SB,\theta}}{u_j^{SB,\theta\prime}(C_j^{SB,\theta})} \right] \\
&= -\sum_\theta \bar{l}^\theta \sum_j \sum_i \frac{1}{u_j^{SB,\theta\prime}(C_j^{SB,\theta})} \frac{\partial \mu_i^\theta}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right] \\
&= -\sum_\theta \bar{l}^\theta \sum_i \underbrace{\sum_j \frac{1}{u_j^{SB,\theta\prime}(C_j^{SB,\theta})} \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=0} \left[w_i - P_i C_i + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right] \\
&= 0,
\end{aligned}
$$

Therefore, by using the normalization $\sum_\theta \bar{l}^\theta \tilde{\Lambda}^\theta = 1$,

$$\sum_\theta \sum_j l_j^\theta \frac{P_j^{SB,\theta}}{u_j^{SB,\theta\prime}(C_j^{SB,\theta})} = 1, \tag{A.17}$$

satisfying our price normalization (18).

Without preference shocks. We also discuss the case without preference shocks as considered by Fajgelbaum and Gaubert (2020). To do so, we rewrite the second-best problem as follows:

$$W = \max_{\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu_j^\theta, A_{ij,k}, W^\theta\}} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) \tag{A.18}$$

$$\text{s.t.} \qquad C_j^\theta \bar{l}^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta) \tag{A.19}$$

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{ij,k}^{l,m} = A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{A.20}$$

$$\sum_{i,k} l_{ji,k}^\theta = \bar{l}^\theta \mu_j^\theta, \qquad \sum_{i,k} h_{ji,k} = \bar{h}_j \tag{A.21}$$

$$\bar{l}^\theta \mu_j^\theta \left[u_j^\theta(C_j^\theta) - W^\theta\right] = 0 \tag{A.22}$$

$$\sum_j \mu_j^\theta = 1 \tag{A.23}$$

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ijk}(\{l_i^\theta\}_\theta) \tag{A.24}$$

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) \geq \underline{W}^{\tilde{\theta}}, \forall \tilde{\theta} \neq \theta \tag{A.25}$$

Note that we rewrote households' incentive compatibility constraint for location choice (25) with utility equalization (A.22) and adding up constraint (A.23). Note also that $\psi^\theta(\cdot) = 0$ without preference shocks.

The first-order condition for μ_j^θ is given by

$$\bar{l}^\theta \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta\right] + \delta^\theta + \underbrace{l_j^\theta \kappa_j^\theta \left[u_j^\theta(C_j^\theta) - \overline{u}\right]}_{=0} = 0$$

$$\iff w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta = \tilde{\delta}^\theta, \tag{A.26}$$

where κ_j^θ and δ^θ denote the Lagrange multipliers for (A.22) and (A.23), respectively. By noting that $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$, the cross-location component of transfers only addresses technological externalities, and the cross-type component of transfers addresses redistribution concerns, as highlighted by Fajgelbaum and Gaubert (2020).

A.5 Proof of Proposition 3 and Corollary 1

By multiplying equation (40) by $dC_j^\theta / u_j^{\theta\prime}(C_j^\theta)$ and summing up across j and θ, we have

$$\begin{aligned}
&\sum_j \sum_\theta l_j^\theta \left[\tilde{\Lambda}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta\right] dC_j^\theta \\
&= -\sum_j \sum_\theta \sum_i \bar{l}^\theta dC_j^\theta \frac{\partial \mu_i^\theta}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right] \\
&= -\sum_\theta \sum_i \bar{l}^\theta \underbrace{\sum_j dC_j^\theta \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=d\mu_i^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right] \qquad \text{(A.27)}
\end{aligned}$$

By following the same procedure as in the Proof of 1, we prove the statement in the proposition.

Given Proposition 3, Corollary 1 is immediate. One can also prove Corollary 1 is also obtained by directly applying the envelope theorem to the relaxed planning problem (33). Despite the presence of incentive compatibility constraint of households' location decisions, there are no reallocation effects because technological effects do not directly affect this constraint.

A.6 General Non-Separable Preference Shocks

We follow the same notation and setup as in Appendix A.1. Now we define $\mathcal{U}$ as follows.

$$\begin{aligned}
\mathcal{U}(\{C_j\}_j, \{l_j\}_j) = &\max_{\{\mathbb{I}_j(\omega)\}_{\omega,j}} \int_0^1 \sum_j u_j(C_j, \epsilon_j(\omega)) \mathbb{I}_j(\omega) d\omega \\
\text{s.t.} \quad &\int_0^1 \mathbb{I}_j(\omega) d\omega = l_j \qquad \text{(A.28)} \\
&\sum_j \mathbb{I}_j(\omega) = 1.
\end{aligned}$$

Then, Lemma 1 straightforwardly generalizes to this environment by replacing $\sum_j l_j u_j(C_j) - \psi(\{l_j\})$ with $\mathcal{U}(\{C_j\}_j, \{l_j\}_j)$.

A.7 Proof of Proposition 4

Since $\ln(\cdot)$ is a monotone transformation, it is immediate that the system (49) and (50) have isomorphic *positive* predictions. This proves the first statement.

We now prove the second statement. We first note that, given the assumption that $\mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \equiv$

$\ln \tilde{\mathcal{W}}(\{\exp(W^\theta)\}_{\theta\in\Theta})$, we have

$$dW = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} dW^\theta, \qquad d\ln \tilde{W} = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} d\ln \tilde{W}^\theta \tag{A.29}$$

Therefore, to show $dW = d\ln \tilde{W}$, it is sufficient to show the same isomorphism for the expected utility for each type θ, i.e., $dW^\theta = d\ln \tilde{W}^\theta$. The expected households' utility in (49) is given by

$$W^\theta = \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu^\theta u_N^\theta(C_N^\theta))) \tag{A.30}$$

and that in (50) is given by

$$\tilde{W}^\theta = G^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})^{1/\nu^\theta}. \tag{A.31}$$

See Appendix C for detailed mathematical derivation. Therefore, under $u_j^\theta(C_j^\theta) = \ln(\tilde{u}_j^\theta(C_j^\theta))$, and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$, we have $W^\theta = \ln \tilde{W}^\theta$.

Finally, we prove that the decomposition is also identical. To do so, we obtain the marginal utility of consumption in each specification. From the envelope condition, $\frac{\partial}{\partial C_j^\theta} \mathcal{U}^\theta(\{C_i^\theta\}_i, \{\mu_i^\theta\}_i)$ $= \frac{\partial}{\partial C_j^\theta} \max_{\{\mu_i^\theta\}:\sum_i \mu_i^\theta = 1} \mathcal{U}^\theta(\{C_i^\theta\}, \{\mu_i^\theta\})$. Therefore,

$$\frac{\partial W^\theta}{\partial C_j^\theta} = \frac{G_j^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu^\theta u_N^\theta(C_N^\theta)))}{G^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu^\theta u_N^\theta(C_N^\theta)))} u_j^{\theta\prime}(C_j^\theta) = \mu_j^\theta u_j^{\theta\prime}(C_j^\theta), \tag{A.32}$$

where G_j^θ indicates the derivative of function G^θ with respect to its j-th argument and the second transformation uses the property of the GEV demand system (see Appendix C). Furthermore,

$$\frac{\partial \tilde{W}^\theta}{\partial C_j^\theta} = \tilde{W}^\theta \frac{G_j^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})}{G^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})} \tilde{u}_j^{\theta\prime}(C_j^\theta) = \tilde{W}^\theta \mu_j^\theta \frac{\tilde{u}_j^{\theta\prime}(C_j^\theta)}{\tilde{u}_j^\theta(C_j^\theta)} = \tilde{W}^\theta \mu_j^\theta u_j^{\theta\prime}(C_j^\theta), \tag{A.33}$$

where the final transformation used the fact that $\ln u_j^\theta(C_j^\theta) = \tilde{u}_j^\theta(C_j^\theta)$. Therefore,

$$\frac{\partial W^\theta}{\partial C_j^\theta} = \frac{1}{\tilde{W}^\theta} \frac{\partial W^\theta}{\partial C_j^\theta}. \tag{A.34}$$

That is, the marginal utility is identical up to scale $\tilde{W}^\theta$. By noting that each term of our decomposition in Proposition 1 scales up by marginal utility under price normalization (18), the statement in the proposition holds.

B First-Best Allocation

In this section, we discuss the first-best planning problem, where the Planner can specify the consumption allocation and location decisions based on the draw of preference shocks $\boldsymbol{\epsilon}$. The problem is given by follows:

$$W = \max_{\{W^\theta, C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, l_{ij}^{k,\theta}, h_{ij,k}, l_j^\theta\}} \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \tag{B.1}$$

$$\text{s.t.} \qquad W^\theta = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{B.2}$$

$$C_j^\theta l_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta) \tag{B.3}$$

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{ij,k}^{l,m} = A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{B.4}$$

$$\sum_{i,k} l_{ji,k}^\theta = l_j^\theta, \qquad \sum_{i,k} h_{ji}^k = \overline{h}_j \tag{B.5}$$

$$\sum_j l_j^\theta = \overline{l}^\theta \tag{B.6}$$

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ij,k}(\mathbf{l}_i) \tag{B.7}$$

Compared to the pseudo planning problem in Lemma 2, there are two major differences. First, the first-best planning problem does not have incentive compatibility constraints for households' location decisions (25) and the competitive consumption allocation (26), and instead freely choose population only subject to add up constraint (B.6). Second, the first-best planning problem internalizes the agglomeration externalities in productivity and amenity (B.7), while the pseudo-planning problem take the values in the competitive equilibrium as given (27).

The first-order conditions with respect to $c_{ij,k}, l_{ij,k}^\theta, h_{ij,k}, x_{ij,k}^{l,m}$ are

$$P_j^{FB,\theta} \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}} = p_{ij,k}^{FB}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{FB,\theta}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{FB}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{FB}, \tag{B.8}$$

where $P_j^{FB,\theta}, p_{ij,k}^{FB}, w_i^{FB,\theta}, r_i^{FB}$ are the Lagrangian multipliers on (B.3), (B.4), and (B.5), respectively. We let all the variables with FB superscript denote those of the planner's solution. Therefore, the relative quantities of inputs are not distorted in equilibrium.

The Planner's solution deviates from the equilibrium when we consider optimality conditions for C_j^θ and l_j^θ. The first-order condition with respect to C_j^θ gives

$$\Lambda^\theta u_j'(C_j^{FB,\theta})/P_j^{FB,\theta} = 1, \tag{B.9}$$

i.e., the marginal utility of income is equalized across locations conditional on type θ. The optimality condition for l_j^θ is

$$u_j^\theta\left(C_j^{FB,\theta}\right)+w_j^{FB,\theta}+\sum_{i,k}p_{ji}^{k,FB}\frac{\partial}{\partial l_j^\theta}g_{ji,k}(\{l_j^\theta\}_\theta)f_{ji}^k(\mathbf{l}_{ji}^k,h_{ji}^k,\mathbf{x}_{ji}^k)-P_j^{FB,\theta}C_j^{FB,\theta}=\frac{\partial\psi^\theta(\{l_k^\theta\}_k)}{\partial l_j^\theta}+u^{FB,\theta}, \tag{B.10}$$

where $u^{FB,\theta}$ is a Lagrangian multiplier on (B.6). Now, with $\{P_j^{FB,\theta},p_{ij,k}^{FB},w_i^{FB,\theta},r_i^{FB}\}$ coinciding with $\{P_j^\theta,p_{ij,k},w_i^\theta,r_i\}$ up to scale, the only way in which Equation (B.10) is satisfied in the equilibrium is that there is no transfer so that $C_j^\theta=w_j^\theta/P_j^\theta$ and there are no agglomeration externalities, i.e., $\frac{\partial}{\partial l_j^\theta}g_{ji,k}(\{l_j^\theta\}_\theta)=0$. However, there is no guarantee in general that (B.9) is satisfied for this C_j^θ, except for the knife-edge case where the marginal utility is equalized across all locations. We summarize the results as follows.

Proposition B.1. *Decentralized equilibrium is suboptimal for any Pareto weights Λ^θ except for the case that marigal utility of income is equalized across locations ($\frac{1}{P_j^\theta}u_j^{\theta\prime}\left(w_j^\theta/P_j^\theta\right)$ is equalized across j for all θ), there are no agglomeration externalities ($\gamma_{ij,k}^\theta=0$ for all i,j,k,θ), and there is no transfer ($T_j^\theta=0$ for all j and θ).*

What is the reason for the suboptimality of the equilibrium? There are two reasons. First, market does not internalize the agglomeration externalities in productivity (27). Second, the market does not equalize the marginal utility of income across locations.

The first source of suboptimality is perhaps not surprising; it is simply an externality that the market does not internalize. The second source of suboptimality is subtle and warrants a discussion. In equilibrium, agents make location decisions based on utility *levels* (inclusive of preference shocks). This implies that *marginal* utility of income is not necessarily equalized across these locations. In contrast, in the first-best allocation, the Planner equalizes the marginal utility of income across locations, while simultaneously controlling for population movement by breaking the incentive compatibility constraint of households' location decisions.

There are two ways to interpret this suboptimality of the dispersion of marginal utility of income. The first interpretation is the lack of insurance for the uncertainty associated with location choice. Depending on the preference draws, or depending on the random sunspot process of location assignment in the absence of preference shocks, individual households may end up in a variety of locations that differ in terms of their associated marginal utility of income. Ex-ante, households can benefit by committing to making transfers from a state where they end up in a location with a low marginal utility of income to that with a high marginal utility of income, but there is not security that allows for such a transfer. The second interpretation is the lack of redistribution across agents depending on location preference and where they reside. By taking the expected utility as a welfare criterion, we effectively attach equal social marginal weight to individuals with different preference draws. The observation that spatial equilibrium models involve suboptimality due to dispersion of marginal utility is reminiscent of Mirrlees (1972), who show this issue in the context of location decisions within a city.

C Location Choice under Generalized Extreme Value (GEV) Preference Shocks

For expositional simplicity, we assume that households are ex-ante homogenous and drop the superscript θ indexing household types.

C.1 Additively Separable Case

Consider the additively separable utility function of the form

$$U_j(C_j, \epsilon_j) = u_j(C_j) + \epsilon_j, \tag{C.1}$$

and ϵ_j follows type-I generalized extreme value distribution:

$$\mathbb{P}[\epsilon_1 \leq \bar{\epsilon}_1, \ldots, \epsilon_N \leq \bar{\epsilon}_N] = \exp(-G(\exp(-\nu\bar{\epsilon}_1), \ldots, \exp(-\nu\bar{\epsilon}_N)), \tag{C.2}$$

and G is a correlation function for its definition), and we normalize the location so that the unconditional mean is zero. As is well known since McFadden (1978), this yields the following location choice probability.

$$l_j = \frac{G_j(V_1, \ldots, V_N)V_j}{\sum_l G_l(V_1, \ldots, V_N)V_l}, \quad \text{where} \quad V_j \equiv \exp(\nu u(C_j)). \tag{C.3}$$

where

$$G_j = \frac{\partial G(V_1, \ldots, V_N)}{\partial V_j}. \tag{C.4}$$

Now we construct a representative agent formulation that is isomorphic to the above model. Define a mapping $V_j = S_j(\mathbf{l})$ that satisfies the following condition for all j:

$$G_j(V_1, \ldots, V_N)V_j = l_j. \tag{C.5}$$

The representative agent solves

$$W = \max_{\{l_j\}:\sum_k l_k=1} \sum_j u_j(C_j)l_j - \frac{1}{\nu}\sum_j l_j \ln S_j(\mathbf{l}). \tag{C.6}$$

The first-order optimality condition is given by

$$u_i(C_i) - \frac{1}{\nu}\ln S_i(\mathbf{l}) - \frac{1}{\nu}\sum_j l_j \frac{\partial \ln S_j(\mathbf{l})}{\partial l_i} - \bar{u} = 0, \tag{C.7}$$

where $\bar{u}$ is the Lagrangian multiplier on the adding up constraint, $\sum_k l_k = 1$. Note that

$$\sum_j l_j \frac{\partial \ln S_j(\mathbf{l})}{\partial l_i} = 1, \tag{C.8}$$

for all j. To see this, we add up (C.5) across j to have $G(S_1(\mathbf{l}),\ldots,S_N(\mathbf{l})) = \sum_j l_j$. Taking the derivative with respect to l_i gives

$$\sum_j G_j(S_1(\mathbf{l}),\ldots,S_N(\mathbf{l}))S_j(\mathbf{l})\frac{\partial \ln S_j(\mathbf{l})}{\partial l_i} = 1 \tag{C.9}$$

$$\Leftrightarrow \quad \sum_j l_j \frac{\partial \ln S_j(\mathbf{l})}{\partial l_i} = 1, \tag{C.10}$$

where we used (C.5) in the second line. Therefore the optimality condition collapses to

$$S_i(\mathbf{l}) = \exp(-\nu\bar{u} - 1 + \nu u_i(C_i)). \tag{C.11}$$

Since $V_i = S_i(\mathbf{l})$ satisfies (C.5) by its definition, we plug back $V_i = S_i(\mathbf{l})$ into (C.5) to obtain

$$l_i = \exp(-\nu\bar{u} - 1)G_i\big(\exp(\nu u(C_1)),\ldots,\exp(\nu u_N(C_N))\big)\exp(\nu u_i(C_i)). \tag{C.12}$$

The adding up constraint, $\sum_i l_i = 1$, implies that

$$\exp(\nu\bar{u} + 1) = \sum_j G_j\big(\exp(\nu u(C_1)),\ldots,\exp(\nu u_N(C_N))\big)\exp(\nu u_j(C_j)). \tag{C.13}$$

Therefore we obtain

$$l_j = \frac{G_j(V_1,\ldots,V_N)V_j}{\sum_l G_l(V_1,\ldots,V_N)V_l}, \quad \text{where} \quad V_j \equiv \exp(\nu u(C_j)), \tag{C.14}$$

coinciding with the solution to the discrete choice problem (C.20).

Finally, we confirm that the indirect utility coincides with each other. In the discrete choice problem, the indirect utility is given by (see McFadden (1978))

$$W \equiv \mathbb{E}\left[\max_j \{u_j(C_j) + \epsilon_j\}\right] \tag{C.15}$$
$$= \frac{1}{\nu} \ln G(\exp(\nu u_1(C_1)), \ldots, \exp(\nu u_N(C_N))). \tag{C.16}$$

In the representative agent model, substituting (C.7) and (C.13) into (C.6), we obtain

$$W = \frac{1}{\nu} \ln G(\exp(\nu u_1(C_1)), \ldots, \exp(\nu u_N(C_N))), \tag{C.17}$$

verifying that the indirect utility also coincides with the original discrete choice formulation.

C.2 Multiplicatively Separable Case

Consider the multiplicatively separable utility function of the form

$$\tilde{U}_j(C_j, \epsilon_j) = \tilde{\epsilon}_j \tilde{u}_j(C_j) \tag{C.18}$$

and $\tilde{\epsilon}_j$ follows type-II generalized extreme value distribution (multi-variate Fréchet):

$$\mathbb{P}[\tilde{\epsilon}_1 \leq \bar{\epsilon}_1, \ldots, \tilde{\epsilon}_N \leq \bar{\epsilon}_N] = \exp(-G((\bar{\epsilon}_1)^{-\nu}, \ldots, (\bar{\epsilon}_N)^{-\nu})), \tag{C.19}$$

and G is a correlation function. This yields the following location choice probability.

$$l_j = \frac{G_j(V_1, \ldots, V_N)V_j}{\sum_l G_l(V_1, \ldots, V_N)V_l} = \frac{G_j(V_1, \ldots, V_N)V_j}{G(V_1, \ldots, V_N)}, \quad \text{where} \quad V_j \equiv \tilde{u}(C_j)^\nu. \tag{C.20}$$

where

$$G_j(V_1, \ldots, V_N) = \frac{\partial G(V_1, \ldots, V_N)}{\partial V_j}. \tag{C.21}$$

The indirect utility is

$$\tilde{W} = G(V_1, \ldots, V_N)^{1/\nu} \text{where} \quad V_j \equiv \tilde{u}(C_j)^\nu. \tag{C.22}$$

Now we construct a representative agent formulation that is isomorphic to the above model.

The representative agent solves

$$\tilde{W} = \max_{\{l_j\}:\sum_k l_k=1} \sum_j (l_j)^{\frac{\nu-1}{\nu}} G_j\Big(\tilde{u}_1(C_1)^\nu, \dots, \tilde{u}_N(C_N)^\nu\Big)^{1/\nu} \tilde{u}_j(C_j). \tag{C.23}$$

The first-order condition is

$$(l_j)^{-1/\nu} G_j\Big(\tilde{u}_1(C_1)^\nu, \dots, \tilde{u}_N(C_N)^\nu\Big)^{1/\nu} \tilde{u}_j(C_j) - \bar{u} = 0, \tag{C.24}$$

where $\bar{u}$ is the Lagrangian multiplier on the adding up constraint, $\sum_k l_k = 1$. Solving the set of first-order conditions together with the adding up constraint, we have

$$l_j = \frac{G_j(V_1, \dots, V_N)V_j}{\sum_l G_l(V_1, \dots, V_N)V_l} = \frac{G_j(V_1, \dots, V_N)V_j}{G(V_1, \dots, V_N)}, \quad \text{where} \quad V_j \equiv \tilde{u}_j(C_j)^\nu, \tag{C.25}$$

as desired. We can plug the above expression into the objective to confirm that the indirect utility also coincides with the original discrete choice formulation:

$$\tilde{W} = G(V_1, \dots, V_N)^{1/\nu} \text{where} \quad V_j \equiv \tilde{u}_j(C_j)^\nu. \tag{C.26}$$

D Details on Extensions

D.1 Amenity Shocks and Amenity Externality

We follow the same steps in the proof of Proposition 1. The first order condition of pseudo planning problem with respect to C_j^θ is given by

$$\chi_j^\theta = l_j^\theta[\Lambda^\theta \partial_C u_j^\theta - P_j^\theta] \tag{D.1}$$

where χ_j^θ and P_j^θ correspond to the Lagrange multipliers for constraints (25) and (22), respectively. Furthermore, the first order condition with respect to μ_j^θ is given by

$$\eta_i^\theta = \bar{l}^\theta \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_i^{\tilde{\theta}} B_i^{\tilde{\theta}} \gamma_i^{B,\theta\tilde{\theta}} \right] \tag{D.2}$$

where η_j^θ, w_j^θ and $p_{il,k}$ correspond to the Lagrange multipliers for constraints (22), (24) and (23), respectively.

By applying the Envelope theorem to pseudo planning problem (20),

$$
\begin{aligned}
dW &= \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta + \sum_\theta \sum_j \chi_j^\theta P_j^\theta l_j^\theta dC_j^\theta \\
&= \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta \partial_C u_j^\theta - P_j^\theta] dC_j^\theta \\
&\quad + \sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \left[\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_j^{\tilde{\theta}} B_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}} \right] dl_j^\theta,
\end{aligned}
$$

which delivers the results in the main text.

D.2 Arbitrary Social Welfare Function

Consider a scenario with a general social welfare function

$$
W^{SP} = \mathcal{W}(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta\}_j, \{l_j^\theta\}_j)\}_\theta)
$$

, where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the distribution of consumption and population of household type θ. Then, by applying the envelope theorem to the pseudo planner's problem as in the proof of Proposition 1 yields

$$
\begin{aligned}
dW &= \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_\theta \sum_j \left[\frac{\partial}{\partial l_j^\theta} \mathcal{U}^{SP}(\{l_j^\theta\}, \{C_j^\theta\}) - l_j^\theta P_j^\theta \right] dC_j^\theta \\
&\quad + \sum_\theta \sum_j \left[w_j^\theta - P_j^\theta C_j^\theta \right] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta
\end{aligned} \tag{D.3}
$$

The only difference from our main proposition is the second term. By denoting $\frac{\partial}{\partial C_j} \mathcal{U}^{SP,\theta}(\{C_i^\theta\}, \{l_i^\theta\})$,

$$
\begin{aligned}
&\sum_\theta \sum_j \left[\Lambda^\theta \mathcal{MU}_j^{SP,\theta} - P_j^\theta \right] dC_j^\theta \\
=& \sum_\theta \bar{l}^\theta \sum_j \mu_j^\theta [\Lambda^\theta \left(\frac{\mathcal{MU}_j^{SP,\theta}}{u_j^{\theta\prime}(C_j)} - 1 \right) + \Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j)}] u_j^{\theta\prime}(C_j) dC_j^\theta \\
=& \mathbb{E}_\theta \left[Cov_{j|\theta} \left(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j)}, u_j^{\theta\prime}(C_j) dC_j^\theta \right) \right] + Cov_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j)} \right], \mathbb{E}_{j|\theta} \left[u_j^{\theta\prime}(C_j) dC_j^\theta \right] \right) \\
&+ \mathbb{E}_\theta \left[\Lambda^\theta \mathbb{E}_{j|\theta} \left[\left(\mathcal{MU}_j^{SP,\theta} - u_j^{\theta\prime}(C_j) \right) dC_j^\theta \right] \right],
\end{aligned} \tag{D.4}
$$

which gives the expression in our main text.

E Nonparametric Identification of Location Choice System

We discuss the conditions under which the location choice system, $\{\mu_j(\boldsymbol{C})\}_j$ are nonparametrically identified. To do so, we build on the existing results of the nonparametric identification of discrete choice models (Berry and Haile 2014). We abstract household types and drop superscript θ.

We start by formalizing our econometric environment. Consider a dataset generated by the model of Section 2. We assume that we observe equilibrium configurations under different sets of fundamentals, indexed by $t = 0, 1, \ldots, \mathcal{T}$. A natural interpretation of t is time, while one could also interpret them as types of individuals or demographic groups. We assume that $\{w_{j,t}, l_{j,t}, P_{j,t}, T_{j,t}, \Pi_t\}$ are observed to the econometrician, so that consumption $C_{j,t} = (w_{j,t} + T_{j,t} + \Pi_t)/P_{j,t}$ is observed as well.

We specify the utility of residing in location by $u_j(C_{j,t}, \zeta_{j,t}) + \epsilon_{j,t}(\omega)$, where $\zeta_{j,t}$ is a scalar variable that is unobserved to the econometrician. The unobserved location heterogeneity, $\zeta_{j,t}$, captures amenity that varies over t. Analogous to Assumption 1 of Berry and Haile (2014), we assume that $\zeta_{j,t}$ only affects location choice through the utility index $u_j(C_{j,t}, \zeta_{j,t})$, but it does not affect the distribution of $\{\epsilon_{j,t}\}$.

Assumption E.1. The distribution function of preference shocks $\epsilon_{j,t}$ is independent of $\{\zeta_{j,t}\}$ and t, i.e.,

$$\mathbb{P}(\epsilon_{1,t} \leq \bar{\epsilon}_1, \ldots, \epsilon_{N,t} \leq \bar{\epsilon}_N | \{\zeta_{j,t}\}) = H(\bar{\epsilon}_1, \ldots, \bar{\epsilon}_N). \tag{E.1}$$

While this assumption is restrictive, we are not imposing any parametric assumption for the distribution function $H(\cdot)$, allowing for flexible correlation of preference shocks across locations.

For the sake of expositional clarity, we also assume in the main text that the unobserved heterogeneity enters into the utility function in as a multiplicative of consumption, $u_j(C_{j,t}, \zeta_{j,t}) = \bar{u}_j(\zeta_{j,t} C_{j,t})$. As is demonstrated by Berry and Haile (2014), this assumption can be relaxed but it requires more technically invovled assumptions.

Importantly, we assume that there are vectors of instruments $\mathbf{z}_t$ that are mean independent of unobserved component of location choice, $\ln \zeta_{j,t}$, for all j and t (Assumption E.2), and there is a sufficient variation of $\mathbf{z}_t$ to induce the changes in consumption vector (Assumption E.3).

Assumption E.2. $\mathbb{E}[\ln \zeta_{j,t} | \mathbf{z}_t] = 0 \quad$ for all j, t.

Assumption E.3. For all functions $B(C_{jt})$ with finite expectation, if $E[B(C_{jt})|\mathbf{z}_t] = 0$ almost surely, then $B(C_{jt}) = 0$ almost surely.

Assumption E.2 is the standard exclusion restriction. Assumption E.3 requires the completeness of the joint distribution $\{C_{jt}, \mathbf{z}_t\}$, capturing the idea that instruments $\mathbf{z}_t$ induces sufficient variation in C_{jt}. Under these assumptions, Berry and Haile (2014) show that the location choice system $\mu_{j,t}(\mathbf{C}_t)$ is identified.

Lemma E.1. (Berry and Haile 2014). Suppose Assumptions E.1, E.2, E.3 hold. Then the location choice system $\mu_{j,t}(\mathbf{C}_t)$ is identified.

Therefore, location choice system $\{\mu_{j,t}(\boldsymbol{C}_t)\}_j$ are, at least in principle, nonparametrically identified. At the same time, the data requirement of the excluded instruments $\mathbf{z}_t$ (Assumptions E.2 and E.3) is substantial. Importantly, to fully identify the flexible substitution patterns for location choice, we need instruments $\mathbf{z}_t$ that induce independent variation in consumption levels in each location $C_{j,t}$. More fundamentally, we need independent observation of equilibrium configurations across different fundamentals (t).