Hayek Meets Leontief: Incomplete Information in Production Networks

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Abstract

Production of goods and services in modern economies rely on firms that participate in complex supply chains, but which often have a limited understanding of the economy's underlying shocks. In this paper, I introduce incomplete information in a general equilibrium model of production networks and study its implications for macroeconomic fluctuations. Following Hayek (1945), firms use input prices as endogenous market signals that help inform their production decisions. Theoretically, I characterize how sectoral shocks affect aggregate output and how this depends on economy-wide uncertainty. When calibrated to historical measures of uncertainty in the US economy, the model gives rise to measures of sectoral importance that deviate significantly from the complete information benchmark.

1 Introduction

The production of goods and services in any modern economy relies on a complex web of transactions in which firms act as both suppliers and customers — often with only a limited understanding of the broader economy's underlying shocks. In the "*Use of Knowledge in Society*", Hayek (1945) argues that these networked and complex economies can efficiently "coordinate" to respond to disturbances in spite of the "unavoidable imperfection of man's knowledge". But how exactly do our networked economies respond to macroeconomic fluctuations when their participants have limited information?

This paper studies the joint role of incomplete information and firm-to-firm linkages in shaping macroeconomic fluctuations. To do this, I embed incomplete information in an otherwise standard general equilibrium model of production networks. Although firms are only partially aware of the economy's underlying productivity and demand shocks, they interact with their suppliers through input markets. In the spirit of Hayek (1945), this puts the informational role of markets at the forefront: input prices act as endogenous market signals which inform firms of pay-off relevant disturbances to other sectors of the economy.

This informational role of prices gives rise to a novel theory of fluctuations in which the transmission of shocks is determined by the interaction between firms' (endogenous) uncertainty and the production network structure of the economy. First, I show theoretically that the introduction of incomplete information changes how *responsive* firms are to input prices relative to the benchmark case of complete information. This change in responsiveness occuring at the *firm-level* has crucial macroeconomic consequences for how sectoral shocks aggregate over firm linkages in general equilibrium. Intuitively, a higher firm-level responsiveness makes it easier for sectoral shocks to spill over to other sectors in the economy through these linkages. Consequently, the impact of incomplete information on the *aggregate* economy is to "weight" different firm-to-firm linkages in a way that depends on firms' relative uncertainty about demand and productivity shocks.

When calibrated to the US input-output network and historical measures of productivity and demand uncertainty, I find that incomplete information significantly changes the impact of productivity shocks on prices and output. First, sectors with low systemic importance under complete information become systemically risky when one accounts for firm-level uncertainty. Second, the systemic importance of sectors is state-dependent and time-varying: sectors that are critical in the transmission of shocks during times of high productivity uncertainty might be relatively unimportant in times of high demand uncertainty. Taken together, the results emphasize the importance of accounting for firm-level uncertainty and sectoral linkages when designing industrial policy or aggregate demand-management.

Model. To study the macroeconomic implications of incomplete information in production networks, embed production networks in a general equilibrium monetary macroeconomic model. Firm-to-firm linkages follow standard microfoundations following Long and Plosser (1983) or Acemoglu et al. (2016). Firms make static production decisions, but face time-varying volatility with respect to the two kinds of fundamental shocks that drive the economy: productivity and aggregate demand shocks.

At each point in time, past productivity and demand shocks are common knowledge. However, firms make input choices while facing uncertainty about the *contemporaneous* realizations of household demand, their own productivity, and the productivity of other sectors. When a firm chooses an input, it takes the price of that input as given. For this reason, firms can condition their demand on the price of that input, but not separately on the contemporaneous realizations of demand or productivity. Nevertheless, following Lucas (1972) or Grossman and Stiglitz (1980), firms can use input prices to form inferences about these payoff relevant fundamental shocks.

Incomplete Information at the Firm-Level. Under incomplete information, firms choose each input x to maximize their expected profits, conditional on the price of that input p. First, I show that all firms optimally choose a log-linear demand function for their inputs given by $\log x = \omega_0 - \omega_1 \log p$. Thus, firms' responsiveness to input price changes is given by the elasticity of this demand function ω_1. Next, I show that this responsiveness depends on firms' perceived covariance between their revenues and the input price, as well as firms' unconditional input price volatility. Intuitively, the presence of these terms formally capture the Hayekian idea that prices convey valuable information to economic agents.

Two comparative statics are particularly important for my analysis. As the covariance between input prices and revenues increases, firms' responsiveness to input prices *decreases*. Intuitively, firms become more hesitant to decrease production in response to higher input costs, because they believe production will be associated with greater revenues. Moreover, as firms' unconditional input price volatility increases, firms' responsiveness to input prices *increases* if and only if revenues positively covary with input prices. This is because firms are less able to make inferences about their revenues conditional on observed input prices. Of course, in order to fully characterize the statistical properties of revenues and input prices, one must study the determinants of these objects in general equilibrium.

Incomplete Information in General Equilibrium. I begin my general equilibrium analysis by first studying how changes in responsiveness at the firm-level affects aggregate outcomes. I show that firm-level responsiveness affects shock transmission by changing the relative importance of different firm-to-firm linkages in the economy. Formally, I show that the effect of a sectoral productivity shock on aggregate output is summarized by that sector's *Augmented-by-Uncertainty Domar Index* (AUDI). This AUDI index closely corresponds to the sector's *alpha centrality*, which captures how "central" a sector is in a network, where connections to distant sectors are penalized by some attenuation factor (Katz, 1953). I show that this attenuation factor is monotonically increasing in firm-level input responsiveness. As firms become more responsive to input price changes, "distant" firm-to-firm linkages become relatively more important in transmitting productivity shocks to output. In contrast, as firm-level responsiveness decreases, direct links become relatively more important in shaping the transmission of productivity shocks. In the limit — as firms become entirely unresponsive to input price changes — productivity shocks of non-final good producing sectors do not propagate through the economy's input-output network and have no effect on household aggregate consumption. As such, the effect of productivity shocks on output is generically increasing in firm-level responsiveness.

However, the opposite reasoning pertains to the transmission of *demand* shocks to output. When firm-level responsiveness is high, firms respond to the inflationary impact of demand shocks by *decreasing* inputs, which attenuates the effect of the demand shock on output. Hence, incomplete information changes the transmission mechanism of different economic shocks in qualitatively different ways.

Next, I study the determinants of *optimal* firm-level responsiveness in general equilibrium. This reveals the presence of *feedback loops*: responsiveness shapes the transmission of shocks to output, and therefore the statistical properties of input prices and revenues. Hence, input responsiveness shapes the informational role of prices, which in turn feeds back into the optimal level of responsiveness to begin with. I show that high demand uncertainty relative to productivity uncertainty, is associated with *lower* firm-level input responsiveness. The logic for this is intimately tied to Lucas (1972): as demand uncertainty dominates, firms perceive all input price changes to be nominal, and therefore forecast greater nominal revenues in response to a greater input price increase. Conversely, high productivity uncertainty relative to demand uncertainty increases firm-level responsiveness, thereby increasing the importance of firm-to-firm linkages in propagating shocks. As a consequence, it is precisely when productivity uncertainty is high that the pass-through of productivity shocks to output is also high.

Quantitative Analysis. In the final part of the paper, I leverage my theoretical results to undertake a quantitative analysis and measure the systemic importance of each sector in the presence of incomplete information. When calibrating the model to the US input-output structure and the covariance matrix of sectoral productivity and demand shocks, I find that incomplete information predicts a substantially different systemic sectoral landscape than what is implied by complete information. Intuitively, sectors that are of high systemic importance under complete information (*i.e.* that have a high Domar index) have a substantially different AUDI weight when one takes uncertainty into account. Moreover, because AUDI is a time-varying index that depends on contemporaneous levels of uncertainty, the systemic importance of sectors is also time-varying.

Using a GARCH model to estimate historical time-variation in productivity uncertainty, the model predicts that the AUDI weights of all sectors increase during the financial crisis (a time of high productivity uncertainty), but decrease during Covid (a time of high demand uncertainty). Finally, counterfactual exercises suggest that increasing the *correlation* of sectoral productivity shocks (due to a greater relative importance of common shocks, such as climate change), or changes in *systematic* monetary policy that increase the correlation between demand and productivity can raise the pass-through of productivity shocks to output. Overall, the design of industrial policies that target aggregate output crucially depends on what shocks are most important in driving the cycle: demand shocks or productivity shocks.

Literature. This paper relates to two strands of literature. First, it relates to the literature that studies how production networks function as a source of risk in the macroeconomy and shape the transmission of macroeconomic shocks (Long and Plosser, 1983; Carvalho, 2014; Acemoglu et al., 2016; Baqaee and Farhi, 2019; Bigio and La'o, 2020; Dew-Becker, 2023; Rubbo, 2023; Liu and Tsyvinski, 2024). I show that incomplete information qualitatively alters the propagation mechanism of production networks relative to complete information, and gives rise to rich state-dependence in measures of sectoral importance that is shaped by macroeconomic uncertainty.

Related to this work is also a more recent literature that explores incomplete information in the context of production networks (Bui et al., 2022; Kopytov et al., 2022; La'O and Tahbaz-Salehi, 2022; Pellet and Tahbaz-Salehi, 2023). These works assume that firm choices are contingent on an *exogenous* information structure. For this reason, they abstract from the informational role of prices in shaping firms' input choices, which is the focus of my analysis. A key methodological contribution to this literature is to show how one can incorporate incomplete information in a production network setting, while preserving Hayek's notion that prices serve as information signals that can coordinate economic outcomes. Moreover, allowing for *price-contingent* input demand schedules makes the model robust to the "re-contracting" critique of Grossman (1989), in which firms would re-contract with their suppliers upon observation of the terms of trade.

Second, this paper relates to the classical literature on rational expectations equilibrium (Lucas, 1972, 1973, 1975; Grossman and Stiglitz, 1980; Grossman, 1981, 1989). A shared methodological premise with this literature is that agents act on what they learn from endogenous objects. In turn, the information conveyed by these economic objects shapes aggregate outcomes. This paper contributes to this literature by modelling rational expectations in a production network setting. By allowing firms to learn from the interactions with their suppliers in the *input market*, the inference problem that links uncertainty to input choices arises without reference to migration or the physically separated markets of Lucas (1972). Instead, firms use input prices to make optimal forecasts about their potential revenues. A related work by Flynn et al. (2023) studies how firms use information in the *output market* to make optimal pricing decisions. This paper studies how endogenous market signals emerging from the *input market* shape shock propagation across firm-to-firm linkages.

Outline. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the main theoretical results. Section 4 presents the main quantitative findings when the model is calibrated to the US economy. Section 5 concludes.

2 Incomplete Information in Production Networks

In this section, I embed production networks in a general equilibrium monetary macroeconomic model. The model follows standard microfoundations in modeling production networks with firm market power (e.g. Afrouzi and Bhattarai 2023; Basu, 1994; La'O and Tahbaz-Salehi, 2022), and only deviates from the literature by assuming that some inputs have to be chosen under incomplete information about the realization of shocks.

2.1 Primitives

Time is discrete and infinite and indexed by $t \in \mathbb{N}$. The economy consists of a representative household and N sectors with input-output linkages, indexed by $n \in [N] = \{1, \dots, N\}$. In each sector $n \in [N]$, a continuum of intermediate good producers indexed by $in \in [0, 1]$ use labor and final goods from other sectors to produce an intermediate good under monopolistic competition. They sell these goods to final good producers within the same sector. Final good producers, in turn, sell these goods to households and intermediate good producers.

2.2 Households

The representative household has standard (Golosov and Lucas, 2007) expected discounted utility preferences with a discount factor $\beta \in (0, 1)$ and per-period utility defined over a consumption aggregate $\mathcal{C}_t$; holdings of real money balances $M_t/\mathcal{P}_t$, and total labor supplied to each sector L_{nt}:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln \mathcal{C}_t + \ln \frac{M_t}{\mathcal{P}_t} - L_t \right) \right] \tag{1}$$

The consumption aggregator C_t is defined by:

$$\mathcal{C}_t = \prod_{n=1}^{N} C_{nt}^{\gamma_n} \tag{2}$$

where C_{nt} is total consumption of sector n and $\gamma_n \geq 0$ are positive constants that satisfy $\sum_{n=1}^{N} \gamma_n = 1$.

Households can save in either money or risk-free one-period bonds B_t (in zero net supply) that pay an interest rate of $(1 + i_t)$. The household owns the firms in the economy, which earn total profits Π_t. Thus, the household faces the following budget constraint:

$$M_t + B_t + \mathcal{P}_t \mathcal{C}_t = M_{t-1} + (1 + i_{t-1}) B_{t-1} + w_t L_t + \Pi_t + T_t \tag{3}$$

where $\mathcal{P}_t$ is the dual price index to $\mathcal{C}_t$, w_t is the nominal wage, and T_t are lump-sum taxes. The aggregate money supply follows an exogenous random walk with drift μ_M and time-dependent volatility σ_t^M:

$$\log M_t = \log M_{t-1} + \mu_M + \delta \sigma_t^M \varepsilon_t^M \tag{4}$$

where the money innovation is an IID random variable that follows $\varepsilon_t^M \sim N(0, 1)$. Moreover, $\delta > 0$ is a strictly positive scalar that parameterizes the total amount of *uncertainty* in the economy. Furthermore, so that interest rates remain strictly positive, we assume that $\frac{1}{2}(\delta \sigma_t^M)^2 \leq \mu_M$ for all $t \in \mathbb{N}$.

Finally, I assume that wages are determined according to the following equation:

$$w_t = (w_{t-1})^{\chi} (w_t^*)^{1-\chi} \tag{5}$$

where $0 < \chi < 1$ parameterizes aggregate wage rigidities and w_t^* denotes the frictionless nominal wage rate, *i.e.* the wage rate that would prevail when $\chi_{nt} = 0$ (to be determined below). Households therefore supply sufficient labor to meet firms' labor demand. This specification of the real wage rate allows the model to parsimoniously capture the cyclicality of nominal wages. To ease notation, we also define the N-sized vectors of consumption shares $\gamma = [\gamma_n]$.

2.3 Final Good Producers

Competitive final good producers purchase intermediate firms from its own industry n, indexed by $in \in [0, 1]$. Final good producers produce a final good Q_{nt}, which they sell to households and intermediate producers at a price of P_{nt}. The competitive final good producer therefore solves:

$$\max_{\{x_{in,t}\}_{in\in[0,1]}} P_{nt}Q_{nt} - \int_{in\in[0,1]} p_{in,t}x_{in,t}\mathrm{d}in \tag{6}$$

$x_{in,t}$ is the amount of variety in purchased by the final good producer at time t at a given price of $p_{in,t}$ and Q_{nt} is a standard CES aggregator defined as:

$$Q_{nt} = \left(\int_{in\in[0,1]} x_{in,t}^{\frac{\eta_n-1}{\eta_n}} \mathrm{d}in\right)^{\frac{\eta_n}{\eta_n-1}} \tag{7}$$

where $\eta_n > 1$ is the elasticity of substitution between intermediate goods in sector n. Note that this specification defines a demand function for each intermediate good given by:

$$x_{in,t} = \left(\frac{p_{in,t}}{P_{nt}}\right)^{-\eta_n} Q_{nt} \tag{8}$$

where P_{nt} satisfies:

$$P_{nt} = \left(\int_{in\in[0,1]} p_{in,t}^{1-\eta_n}\mathrm{d}in\right)^{\frac{1}{1-\eta_n}} \tag{9}$$

Given that final good producers are perfectly competitive under constant returns to scale, they earn zero profits and have zero value added. The purpose of final good producers is therefore to define a unified final good for each industry.

2.4 Intermediate Good Producers

There exist a continuum of $in \in [0, 1]$ intermediate good producers in sector $n \in [N]$. Each intermediate good producer in purchases final goods $X_{in,n',t}$ from sectors $n' \in [N]$ at a price of $P_{n't}$ as well as labor $L^d_{in,t}$ at the prevailing wage rate w_{nt} to produce an intermediate good $x_{in,t}$. It then sells this intermediate good monopolistically at a price of $p_{in,t}$ to a final good producer in its own sector. Intermediate good producers in sector n produce with Cobb-Douglas technology given by:

$$q_{in,t} = c_n A_{nt} \left(L^d_{in,t}\right)^{\alpha_{nl}} \prod_{n'=1}^{N} X_{in,n',t}^{\alpha_{nn'}} \tag{10}$$

where c_n is a normalizing constant[1], $\alpha_{nl} > 0$, $\alpha_{nn'} \geq 0$, with $\alpha_{nl} + \sum_{n'=1}^{N} \alpha_{nn'} = 1$. We also define the economy's input-output matrix as $\mathbf{A} = [\alpha_{nn'}]$.

A_{nt} is a sector-specific technology shifter that follows an AR(1) process with time-varying volatility $\delta\sigma_{nt}^A$ given by:

$$\log A_{nt} = \rho_n^A \log A_{nt-1} + \delta\sigma_{nt}^A \varepsilon_{nt}^A \tag{11}$$

where $\varepsilon_{nt} \sim N(0,1)$ and ρ_n^A is a constant. I allow the ε_{nt}^A to be potentially correlated across sectors.

In order to isolate the role of uncertainty in shaping the transmission of shocks, I also assume that the government imposes an ad-valorem subsidy τ_n to all intermediate good producers in sector n in proportion to their monopolistic mark-up. This implies that the steady-state of the economy (with no wage rigidity) is constrained efficient, but otherwise has no bearing on the main results. The firm's profits are therefore given by:

$$\Pi_{in,t} = (1+\tau_n)p_{in,t}q_{in,t} - w_{nt}L_{in,t}^d - \sum_{n' \in [N]} P_{n't}X_{in,n',t} \tag{12}$$

2.5 Input Choice with Informational Frictions

I assume that some input choices need to be made under incomplete information of the shock realizations $\{\{A_{nt}\}_{n\in[N]}, M_t\}$. This assumption is motivated by the fact that firms that operate in supply chains often have to make input choices without perfect knowledge of all underlying shocks in the economy. Concretely, let $\mathcal{S}_n^R \subseteq [N]$ denote the set of inputs in sector n that are purchased under incomplete information, and let the complementary set $\mathcal{S}_n^F = [N]/\mathcal{S}_n^R$ denote the set of inputs purchased under perfect information. For brevity, I will refer to these two distinct types of inputs as *rigid* and *flexible*, respectively. I assume that labor is chosen under perfect information, which ensures that at least one input can adjust instantaneously to shocks. This assumption is sufficient to ensure market clearing, but is otherwise inessential for the main results.

Intermediate good producers that purchase inputs under incomplete information take the per-unit price of an input as given and choose $X_{in,n',t}$ maximize their expected real,

[1] c_n is defined as:

$$c_n = \left(\alpha_{nl}^{-\alpha_{nl}} \prod_{n' \in [N]} \alpha_{nn'}^{-\alpha_{nn'}} \right)$$

risk-adjusted profits. They therefore solve the following maximization problem:

$$\max_{X_{in,n't}} \mathbb{E}_{in,t}\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\Pi_{in,t}\middle| P_{n't}\right] \quad \text{for all} \quad n' \in \mathcal{S}_n^R \tag{13}$$

where the expectation is over firm in's information set at time t and $1/(\mathcal{P}_t\mathcal{C}_t)$ is the representative household's stochastic discount factor.

I assume that all past shocks $\{\{A_{nk}\}_{n\in[N]}, M_k\}_{k=0}^{t-1}$ are common knowledge at time t, but that contemporaneous shocks are not known. The presence of incomplete information therefore imposes a measurability constraint on firms' input choices: the choice of $X_{in,n',t}$ can be contingent on its own price $P_{n't}$ and all past shock realizations, but not the *current* time t shock realizations $\{\{A_{nt}\}_{n\in[N]}, M_t\}$

Input choices made under *perfect information*, in contrast, can be contingent on the contemporaneous realizations of productivity and demand shocks. These inputs are to be interpreted as inputs that can be scaled up quickly and frictionlessly in response to contemporaneous economic conditions. Because these inputs are chosen under perfect information, the firm's problem is to simply choose the level of inputs that maximize its profits, taking the prices of its inputs and demand as given:

$$\max_{X_{in,n',t}} \Pi_{in,t} \quad \text{for all} \quad n' \in \mathcal{S}_n^F \tag{14}$$

Sources of Incomplete Information. I interpret input choices under incomplete information as input choices that require advance planning and therefore cannot perfectly incorporate future demand or productivity conditions. There are many reasons why these frictions arise in practice. As emphasized by Pellet and Tahbaz-Salehi (2023), firms may be subject to large *lead times* when acquiring inputs. In January 2024, for example, the average lead time for production materials in the US was 84 days (Institute for Supply Management, 2024). Lead times can therefore create frictions in quantity adjustment that prevent firms from instantaneously adjusting their inputs in response to future realized shocks.

In other cases, the terms of trade pertaining to a particular input are often mediated through contracts which are negotiated in advance. In practice, these contracts tend to be incomplete in the sense that the terms of trade specified in the contract are not fully contingent on future shock realizations (Battigalli and Maggi, 2002). Bajari and Tadelis (2001) provide examples of such contracts in the building and construction industry and show that the "vast majority" of contracts are "simple" contracts of a *cost-contingent* nature. In Appendix B.1, I show that the model can equivalently be recast to a setting in which intermediate good producers negotiate input purchases ex-ante through contracts. Formally, the demand schedules that are implemented under incomplete information are equivalent to those that arise under a cost-contingent contract negotiated via Nash bargaining.

Discussion of Informational Structure. Note that Equation 13 implicitly assumes that the demand for rigid inputs cannot be contingent on the prices of *other* inputs at time t. This assumption is maintained because it imposes the lowest possible degree of informational sophistication on firms' input choices. Importantly, the analysis does not rely on the (potentially unrealistic) assumption that firms know the prices of all inputs in the economy when making input choices under incomplete information (Angeletos and Sastry, 2019). Moreover, Appendix B.1 shows that this measurability restriction on prices arises *endogenously* when transactions are mediated through bilateral contracts.

2.6 Rational Expectations General Equilibrium

We are now ready to define the equilibrium of this economy. At the beginning of time period t, firms observe $\{A_{nt-1}\}_{n=1}^{N}$ and M_{t-1}. Intermediate good producers purchase their rigid inputs under incomplete information to maximize their expected profits, taking the prices of those inputs as given; their flexible inputs to maximize their profits under perfect information, taking the prices of those inputs and their realized demand as given; final good producers operate; and households make their consumption and savings decisions. Formally, we define an equilibrium as follows:

Definition 1 (Rational Expectations General Equilibrium)**.** *An equilibrium is a collection of variables*

$$\{\{\{p_{in,t}, x_{in,t}, L_{in,t}, q_{in,t}, \{X_{in,n',t}\}_{n'\in[N]}\}_{i\in[0,1]}, P_{nt}, Q_{nt}, C_{nt}\}_{n\in[N]}, \mathcal{C}_t, \mathcal{P}_t, L_t, w_t\}_{t\in\mathbb{N}}$$

and a collection of exogenous variables

$$\{\{A_{nt}\}_{n\in[N]}, M_t, B_t\}_{t\in\mathbb{N}}$$

such that:

1. (Rigid Input Optimality) *Intermediate good producers choose their rigid inputs to maximize their expected profits given their information according to Equation 13, taking the price of inputs as given.*

2. (Flexible Input Optimality) *Intermediate good producers choose their flexible inputs to maximize their profits according to Equation 14, taking the price of inputs and their own demand as given.*

3. (Final Good Producer Optimality) *Final good producers choose intermediate goods to maximize their profits according to Equation 6.*

4. (Household Optimality) *The household chooses consumption C_{nt}, labor supply L_t, money holdings M_t, and bond holdings B_t to maximize their expected discounted utility 1 subject to their budget constraint 3, taking prices and the nominal interest rate as given.*

5. (Exogenous Stochastic Processes) *The money supply M_t, wages w_t, and sector-specific productivity shocks A_{nt} evolve exogenously according to Equations 4, 5, and 11, respectively.*

6. (Rational Expectations) *The expectations of intermediate good producers and households are consistent with all variables' equilibrium law of motion.*

7. (Market Clearing) *The markets for intermediate goods, final goods, and labor clear:*

$$q_{in,t} = x_{in,t}, \quad Q_{nt} = C_{nt} + \sum_{n' \in [N]} \int_{i \in [0,1]} X_{in',n,t} \mathrm{d}i, \quad L_t = \sum_{n' \in [N]} \int_{i \in [0,1]} L^d_{in,t} \mathrm{d}i \tag{15}$$

   and the markets money balances, and bonds clear.

Equilibrium conditions (1)-(2) ensure that intermediate good producers choose inputs optimally given the available information that they possess. Equilibrium conditions (3)-(7) are standard and impose optimality, rational expectations, and market clearing. Finally, note that intermediate good producers take the price of their inputs as given when choosing their inputs under incomplete information. This observation, in conjunction with the assumption of rational expectations, implies that input prices can serve as an endogenous signal that informs firms' decisions. This equilibrium definition therefore mirrors the canonical rational expectation formulation of Lucas' "island" economy (Lucas, 1972) in a production network setting.

3 Shock Propagation in Production Networks

In this section, I characterize how incomplete information affects the macroeconomic transmission of shocks in a production network economy. First, I show that the level of firm responsiveness to input prices is shaped by the nature of uncertainty that firms face: responsiveness increases in the perceived covariance between input prices and sectoral revenues. I then show that input responsiveness shapes the general equilibrium transmission of shocks:

greater input responsiveness amplifies the role of higher-order linkages in propagating shocks. Finally, I characterize firms' optimal input responsiveness in general equilibrium and show how this depends on firms' relative uncertainties about demand and productivity shocks. Throughout the section, I relate the main results to illustrative network structures to highlight intuition.

3.1 Characterizing Input Choice under Uncertainty

I first characterize how incomplete information shapes the *responsiveness* of inputs to their own price changes. We first use the final producer's demand for intermediate inputs 8, as well as the intermediate good producer's production function (10) and their profits (12) to derive an expression for the firm's *flexible* inputs:

$$X_{in,n',t} = \alpha_{nn'} \frac{P_{nt} Q_{nt}^{\frac{1}{\eta_n}} q_{in,t}^{\frac{\eta_n - 1}{\eta_n}}}{P_{n't}} \quad \text{for} \quad n' \in \mathcal{S}_n^f \tag{16}$$

Since firms in each sector are homogeneous, we have $q_{in,t} = Q_{nt}$. The optimal input purchased is therefore proportional to industry-level revenues over the price of that input:

$$X_{in,n',t} = \alpha_{nn'} \frac{R_{nt}}{P_{n't}} \quad \text{for} \quad n' \in \mathcal{S}_n^f \tag{17}$$

Hence, inputs respond both to *realized* revenues, as well as prices. Inputs that are chosen under incomplete information, in contrast, satisfy the following demand schedule:

$$X_{in,n',t} = \alpha_{nn'} \frac{\mathbb{E}_{nt}[(\mathcal{P}_t \mathcal{C}_t)^{-1} R_{nt} | P_{n't}]}{\mathbb{E}_{nt}[(\mathcal{P}_t \mathcal{C}_t)^{-1} P_{n't} | P_{n't}]} \quad \text{for} \quad n' \in \mathcal{S}_n^r \tag{18}$$

where recall that the inverse of nominal expenditures $\mathcal{P}_t \mathcal{C}_t$ is the firm's (nominal) stochastic discount factor. In the presence of incomplete information, firms use input prices to predict their real, risk-adjusted revenues from input prices. This informational role that prices play can therefore alter the responsiveness of inputs to price changes relative to the perfect information benchmark.

To isolate the informational role of prices in shaping the responsiveness of inputs, it is useful to first *assume* that intermediate good producers believe nominal expenditures, industry-level revenues, and input prices to be unconditionally log-normally distributed: $(\mathcal{P}_t \mathcal{C}_t, R_{nt}, P_{n't}) \sim \log N(\mu_t, \Sigma_t)$, with mean μ_t and a variance-covariance matrix $\mathbf{\Sigma_t}$. The following proposition characterizes the responsiveness of inputs to prices under incomplete information under this assumption of log-normality.

Proposition 1. *Suppose revenues and prices are jointly log-normally distributed. The optimal input quantity for $n' \in \mathcal{S}_n^r$ given price $P_{n't}$ satisfies:*

$$\log X_{in,n',t} = \omega_{0,nn't} - \omega_{nn't} \log P_{n't} \tag{19}$$

where $\omega_{0,nn't}$ is independent of $P_{n't}$ and the responsiveness of inputs to their price is given by

$$\omega_{nn't} = 1 - \frac{\mathrm{Cov}(\log R_{nt}, \log P_{n't})}{\mathrm{Var}(\log P_{n't})} \tag{20}$$

Proof. See Appendix A.1 □

First, this proposition demonstrates that under the assumption of a log-normal structure on uncertainty, firms' optimal demand schedules are optimally log-linear. Second, the responsiveness of inputs to their price is mediated by the covariance between revenues and costs. When firms believe that revenues and costs positively co-move, the responsiveness of inputs to prices will be *dampened* (relative to the perfect information benchmark). In contrast, a negative covariance between revenues and costs will *amplify* the responsiveness of inputs to prices. Of course, the covariance between revenues and prices enters into responsiveness $\omega_{nn't}$ precisely because inputs are chosen under incomplete information: firms use input prices to infer their realized revenues. As I will show in the next section, the magnitude of this responsiveness will be crucial to understand the transmission of shocks to prices and output.

Of course, in general equilibrium, the covariances between revenues and input prices are endogenous objects that primitively depend on the exogenous stochastic processes of the economy, as well as on the responsiveness of inputs to prices themselves. In order to understand the endogenous nature of uncertainty in this economy, we must therefore first understand the determinants of equilibrium prices and revenues. In what follows, I assume that all intermediate inputs are chosen under incomplete information (*i.e.* $\mathcal{S}_n^f = \emptyset$), so that labor is the only input chosen under perfect information. This helps to simplify the exposition and highlight the role of incomplete information in shaping the transmission of shocks. Appendix B.2 generalizes all proceeding results to the case in which $\mathcal{S}_n^f$ is an arbitrary non-empty subset of intermediate inputs, at the cost of some additional notational complexity.

3.2 Characterizing the Equilibrium Price System

In this subsection, I characterize equilibrium prices and revenues in terms of a system of equation. This representation will highlight the endogeneous interaction between input responsiveness and uncertainty in general equilibrium.

I first begin by characterizing nominal expenditures and flexible wages:

Lemma 1. *Nominal expenditures and flexible wages in each sector satisfy*

$$\mathcal{P}_t\mathcal{C}_t = \iota_t M_t \quad \textit{and} \quad w_{nt}^* = \iota_t M_t \tag{21}$$

where $\iota_t = \frac{i_t}{1+i_t} > 0$ *is independent of the sequence of productivity* M_t *and demand shocks* A_{nt}.

Proof. See Appendix A.2 □

The above lemma shows that we can express both nominal expenditures and flexible wages as a function of exogenous variables at time t. The household's problem therefore collapses to a static one given knowledge of the demand shocks M_t. Intuitively, the presence of money in the utility function allows to derive an additional intertemporal trade-off between savings and consumption, thereby allowing us to express nominal expenditures in terms of the total money supply.

I interpret an increase in the money supply as an aggregate demand shock due to expansionary monetary policy. The major benefit of this formulation is that we can study these demand shocks tractably. Baqaee and Farhi (2022) show that these demand shocks can equivalently arise through a reduction in the discount factor or an increase in expected future output.

We now use this result to obtain a characterization of revenues and prices in terms of exogenous variables.

Proposition 2. *Equilibrium prices* $\{P_{nt}\}_{n\in[N]}$ *and revenues* $\{R_{nt}\}_{n\in[N]}$ *satisfy the following system of equations:*

$$R_{nt} = \tilde{c}_{nt} A_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi}}\right)^{\alpha_{nl}} \prod_{n'\in[N]} \left(\frac{\mathbb{E}_t\left[M_t^{-1}R_{nt}\middle|P_{n't}\right]}{\mathbb{E}_t\left[M_t^{-1}P_{n't}\middle|P_{n't}\right]}\right)^{\alpha_{nn'}} \tag{22}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n'\in[N]} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t\left[M_t^{-1}R_{n't}\middle|P_{nt}\right]}{\mathbb{E}_t\left[M_t^{-1}P_{nt}\middle|P_{nt}\right]} \tag{23}$$

where $\tilde{c}_{nt} = (\iota_t)^{-\alpha_{nl}(1-\chi_n)}(w_{nt-1})^{\alpha_{nl}\chi_n}$.

Proof. See Appendix A.3 □

This proposition obtains by substituting firms' demand schedules 13 into firms' production technology 10 and the market clearing condition 15. The result shows that the determinants of prices and revenues (and therefore output in every industry) depend primitively on the input-output structure of the economy and firm's uncertainty about fundamentals. As in Lucas (1972) or Grossman and Stiglitz (1980), this uncertainty is *endogenous* and depends on the signalling role that prices play within the economic system. This interdependence reveals the presence of a *feedback loop*: the responsiveness of prices to macroeconomic shocks depends on firm's uncertainty – which is in turned determined by the responsiveness of prices to shocks to begin with.

In order to understand the mechanics of how uncertainty shapes shock transmission, it is instructive to consider the case in which revenues are *assumed* to be fully revealing of firm's revenues (*i.e.* $\mathbb{E}_t[R_{nt}|P_{nt}] = R_{nt}$). Then, taking logarithms of Equation 22 and rearranging yields:

$$\log P_t = cons + (1-\chi)\log(M_t) + (\mathbf{I}-\mathbf{A})^{-1}\log A_t \tag{24}$$

where P_t and A_t is the N-sized vector of prices and productivity shocks, respectively, and the constant is independent of time t shocks. This is the standard result of benchmark production network models: the elasticity of prices to a productivity shock is given by the Leontief inverse matrix $\mathbf{L} \equiv (\mathbf{I}-\mathbf{A})^{-1} = \sum_{k=0}^{\infty}\mathbf{A}^k$, which captures the propagation that arises through the input-output linkages in the economy.[2] Note that we may also use Equation 23 to obtain a closed form expression for revenues:

$$\log R_t = cons + \log M_t \tag{25}$$

where R_t is the N-sized vector of revenues.

When is the assumed condition that prices are perfectly revealing of revenues correct? Given the dependence of prices and revenues on productivity shocks and demand shocks characterized by Equations 24 and 25, prices are perfectly revealing of revenues if and only if (i) there are no productivity shocks, or (ii) there is no prior demand uncertainty. Outside of these two extreme cases, the benchmark result obtained above will no longer apply. The effect of demand and productivity shocks on output will instead be shaped by productivity

[2] It is well known that the Leontief matrix is invertible and can be decomposed into the power sum of the input-output matrix: $\mathbf{L} = \sum_{k=0}^{\infty}\mathbf{A}^k$ (Carvalho and Tahbaz-Salehi, 2019).

uncertainty, demand uncertainty, and the interaction of these two forces with the network structure of the economy. In particular, the responsiveness of output to input prices will largely depend on how *informative* prices are of current economic conditions.

The nature of this shock transmission will then determine how *informative* prices are of revenues, which will in turn feed back into the transmission of shocks by changing the *responsiveness* of inputs to their price. In order to disentangle how responsiveness and information shape the transmission of macroeconomic shocks, I proceed by studying the economy's *temporary equilibrium*, in which the intermediate good producers' demand schedules for inputs are assumed fixed.

3.3 Shock Transmission under Incomplete Information

In order to understand how firm responsiveness to shocks shapes the transmission of macroeconomic shocks, I analyze the economy's equilibrium in which firm's rigid demand functions are held fixed. Concretely, I assume that the demand function for a rigid input takes the following log-linear form:

$$\log X_{in,n',t} = \omega^*_{0,nn't}(\omega_{nn't}) - \omega_{nn't} \log P_{n't} \tag{26}$$

where $\omega_{nn't}$ parameterizes the responsiveness of inputs from sector n' to n and $\omega^*_{0,nn't}$ is firm in's profit-maximizing intercept given the choice of $\omega_{nn't}$. This choice is motivated by Proposition 1, which shows that firm's optimal demand schedules are log-linear when fundamentals are distributed log-normally. Later, we will verify that the general equilibrium model endogenously gives rise to a log-normal structure on uncertainty.

We can now recast the equilibrium dynamics of the system in terms of firms' exogenously given demand schedules. To keep the analysis tractable, we work with a log-linearization of the above economy as $\delta \to 0$, where recall that δ parameterizes the extent of uncertainty about aggregate shocks. To this end, we define the *revenue share matrix* $\mathbf{S}$, which captures the share of revenues attributed to a customer as a fraction of total sales to customers and households. Formally, this is defined as:

$$\mathbf{S} = [s_{nn'}] = \left[\alpha_{n'n} \frac{\lambda_{n'}}{\lambda_n}\right] \tag{27}$$

where λ_n is the sectoral Domar weight of industry n under perfect information, defined as the ratio of revenues to aggregate consumption.[3] Finally, we define the *demand-adjusted*

[3] The Appendix characterizes these Domar weights in terms of model primitives and shows that they are time-invariant.

input-output matrix and revenue-share matrix as $\mathbf{A}(\omega_t) = [\alpha_{nn'} \times \omega_{nn't}]$ and $\mathbf{S}(\omega_t) = [s_{nn'} \times (1-\omega_{n'nt})]$. These matrices capture how the responsiveness of firms' demand schedules shape the transmission of shocks to prices and revenues. We denote log-linearized variables with a hat.

Proposition 3. *The first-order response of demand and productivity shocks to prices* $\hat{P}_t$ *and revenues* $\hat{R}_t$ *is given by:*

$$\Big[\underbrace{\mathbf{I} - \mathbf{A}(\omega_t)}_{\substack{\text{demand-adjusted} \\ \text{Leontief matrix}}} - \underbrace{\mathbf{D}(\omega_t)}_{\substack{\text{revenue impact} \\ \text{matrix}}} \Big] \hat{P}_t = -\hat{A}_t + \mathbf{\Phi} \hat{M}_t \tag{28}$$

$$\hat{R}_t = (\mathbf{I} - \text{diag}(\mathbf{S1}))\hat{M}_t + \text{diag}(\mathbf{S}(\omega_t)\mathbf{1})\hat{P}_t \tag{29}$$

where the matrices $\mathbf{D}(\omega_t)$ *and* $\mathbf{\Phi}$ *are given by*

$$\mathbf{D}(\omega_t) = \text{diag}(\mathbf{A1})\text{diag}(\mathbf{S}(\omega_t)\mathbf{1}) \tag{30}$$

$$\mathbf{\Phi} = (1-\chi)(\mathbf{I} - \mathbf{A}) + \text{diag}(\mathbf{A1})(\mathbf{I} - \text{diag}(\mathbf{S1})) \tag{31}$$

Proof. See Appendix A.4 □

In order to understand the above result, consider how prices respond to a productivity shock. First, a productivity shock directly lowers prices by lowering firms' marginal costs. In turn, this reduction in prices lowers marginal costs further through input-output linkages: a reduction in the price of input n' for firm n reduces firm n's price by the share of that input in the production function $\alpha_{nn'}$ *multiplied* by the responsiveness of firm n's input purchases to this price reduction: $\omega_{nn't}$. This effect is captured by the economy's *demand-adjusted* input-output matrix $\mathbf{A}(\omega_t)$. Of course, this price reduction leads to a new round of propagation through input-output linkages. The cumulative effect for the reduction in prices that occurs through this channel is therefore $[\mathbf{I} - \mathbf{A}(\omega_t)]^{-1}$, which the standard inverse Leontief matrix, adjusted for firms' demand schedule slope.

This reduction in prices, however, will also affect a firm's revenues by changing the total quantity demanded from its customers. If the elasticity of demand schedules is less than one, the increase in quantity demanded will not offset the impact of the per-unit price reduction on firm's revenues. To the extent that the firm cannot reduce its inputs in response to this reduction in revenues, this channel necessitates a further price reduction in order to equate the supply of inputs with their demand. The total price reduction from this channel is captured by the revenue impact matrix $\text{diag}(\mathbf{A1})\text{diag}(\mathbf{S}(\omega_t)\mathbf{1})$. This matrix naturally consists of the overall rigidity within the firm's production function ($\text{diag}(\mathbf{A1})$) and the total share of revenues that are attributed to other rigid customers ($\text{diag}(\mathbf{S}(\omega_t)\mathbf{1})$. A similar propagation mechanism characterizes the economy's response to a monetary shock, with the key difference being that a monetary shock directly raises firms' marginal costs by a factor of $(1-\chi)(\mathbf{I}-\mathbf{A})$ (through wages) and directly affects firm revenues by a factor $(\mathbf{I}-\text{diag}(\mathbf{S1}))$ (through nominal expenditures).

3.4 Unpacking the Propagation Mechanism

In this subsection, I analyze how incomplete information interacts with the economy's input-output structure. The following theorem shows that incomplete information dampens the role of higher-order firm linkages in propagating shocks, where the strength of this dampening depends on firms' responsiveness ω^*.

Theorem 1. *Suppose $\omega_{nn't} = \omega^*$. Let*

$$\boldsymbol{\epsilon}(\omega^*)' \equiv \boldsymbol{\gamma}' \left[\sum_{k=0}^{\infty} \big(\underbrace{\omega^* \left(\mathbf{I} - \mathbf{D}(\omega^*)\right)^{-1}}_{dampening} \big)^k \mathbf{A}^k \right] \left(\mathbf{I} - \mathbf{D}(\omega^*)\right)^{-1} \tag{32}$$

The first-order effect of productivity shocks and demand shocks on consumption $\hat{\mathcal{C}}_t$ and the price level $\hat{\mathcal{P}}_t$ is given by

$$\hat{\mathcal{C}}_t = \boldsymbol{\epsilon}(\omega^*)'\hat{A}_t + \left[1 - \boldsymbol{\epsilon}(\omega^*)'\boldsymbol{\Phi}\right]\hat{M}_t \tag{33}$$

$$\hat{\mathcal{P}}_t = -\boldsymbol{\epsilon}(\omega^*)'\hat{A}_t + \boldsymbol{\epsilon}(\omega^*)'\boldsymbol{\Phi}\hat{M}_t \tag{34}$$

If $\omega^* \leq 1$, the matrix $\omega^*(\mathbf{I}-\mathbf{D}(\omega^*))^{-1}$ has eigenvalues weakly less than unity. Moreover, all eigenvalues are strictly increasing in ω^*.

Proof. See Appendix A.5. □

This theorem shows that the diagonal matrix $\omega^*(\mathbf{I}-\mathbf{D}(\omega^*))$ *dampens* the importance of higher-order links in propagating macroeconomic shocks relative to perfect information benchmark (Equation 24). The elements of this matrix depend primitively on the production network structure of the economy as well as on the responsiveness of inputs to prices given by ω^*. Moreover, these elements are strictly increasing ω^*. Hence, higher-order links play a proportionately larger role in propagating productivity shocks as the responsiveness of firms' inputs to input prices increases. Intuitively, inputs (and therefore output) are more responsive to price reductions, which facilitate further price reductions through the economy's input-output linkages.

Although the effect of productivity shocks on consumption is *increasing* in ω^*, the effect of demand shocks on output is *decreasing* in ω^*. Intuitively, a larger firm-level responsiveness to prices implies that firms respond more to the inflationary effect of a demand shock, thereby reducing output. Incomplete information therefore has countervailing forces on the propagation of productivity and demand to consumption in a networked economy. Similarly, the *deflationary* impact of productivity shocks is *decreasing* in ω^*, while the *inflationary* impact of demand shocks is *increasing* in ω^*. In order to summarize these findings formally, we define the operator $\Delta[x(\omega^*)] = x(1) - x(0)$ as the difference of a function x evaluated at $\omega^* = 1$ and $\omega^* = 0$. The following proposition demonstrates how the effect of productivity and demand shocks on consumption and the price level vary with the level of input responsiveness.

Proposition 4. *Suppose all sectors have a common labor share* $\alpha_{nl} = \alpha_l \in (0, 1)$. *Then:*

$$\Delta\left[\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}\right] > 0 \quad \textit{and} \quad \Delta\left[\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{A}_t}\right] < 0 \tag{35}$$

where $\hat{A}_t$ *is a common productivity shock to all sectors:* $\hat{A}_{nt} = \hat{A}_t$. *Moreover,*

$$\Delta\left[\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t}\right] < 0 \quad \textit{and} \quad \Delta\left[\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}\right] > 0 \tag{36}$$

Proof. See Appendix A.6. □

Connection to Alpha Centrality. The alpha centrality of a sector for $\alpha \in [0, 1]$ is defined as:

$$C_{alpha} = \boldsymbol{\gamma}'(\mathbf{I} - \alpha\mathbf{A})^{-1} \tag{37}$$

Intuitively, alpha centrality captures how "central" a sector is in an economy, where connections to distant sectors are penalized by an attenuation factor α (as represented by the power series of the input-output matrix $\mathbf{A}$):

$$C_{alpha} = \boldsymbol{\gamma}' \sum_{k=0}^{\infty} \alpha^k \mathbf{A}^k \tag{38}$$

The relevant attenuation factor in our framework is captured through the dampening matrix $\omega^*(\mathbf{I} - \mathbf{D}(\omega^*))$. However, if we assume that all sectors have a common labor share $\alpha_{nl} = \alpha_l$ and a common revenue share of consumption $1 - \sum_{n'} s_{nn'} = s_c$, we can write this attenuation factor in terms of a scalar $\alpha(\omega^*)$:

$$\alpha(\omega^*) \equiv \frac{\omega^*}{1-(1-\alpha_l)(1-s_c)(1-\omega^*)} \tag{39}$$

where it is easy to check that $\alpha(\omega^*) \in [0,1]$ if and only if $\omega^* \in [0,1]$. The systemic importance of a sector under incomplete information is therefore given by its alpha centrality, where the alpha centrality parameter is determined endogenously by the production network structure of the economy and the nature of firms' uncertainty. If $\omega^* = 1$, then $\boldsymbol{\epsilon}(1)' = \boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1}$ and we recover the standard result in perfect information economies that the effect of productivity disturbances on aggregate consumption is given by the economy's inverse Leontief matrix (cf. Equation 24). Of course, there is no *a priori* economic restriction that $\omega^* \in [0,1]$. If $\omega^* > 1$, then incomplete information attaches *more* weight on higher-order linkages relative to the perfect information benchmark. The vector $\epsilon(\omega^*)'$ is still well defined as long as the infinite sum in Equation 32 converges.

Complete *vs.* Incomplete Information. Recall from Equations 24 and 25 that the benchmark response of productivity shocks on prices and output is given by the economy's inverse Leontief matrix $(\mathbf{I}-\mathbf{A})^{-1}$. Moreover, when there are no wage rigidities ($\chi = 0$), demand shocks in the presence of complete information increase all prices one-to-one have no effect on output. The following corollary shows that there is an equivalence between the transmission mechanism of the complete *vs.* incomplete information economies when $\omega^* = 1$ and $\omega^* = 0$.

Corollary 1. *Suppose there are no wage rigidities* ($\chi = 0$)*. Then, the following statements are true:*

$$\textit{If } \omega^* = 1: \quad \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}' = \boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1} \quad \textit{and} \quad \frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{A}_t}' = -\boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1} \tag{40}$$

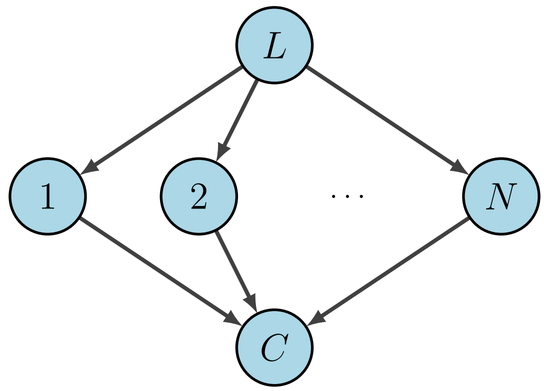
$$\textit{If } \omega^* = 0: \quad \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t} = 1 \tag{41}$$

where the n-th element of $\mathrm{d}\hat{\mathcal{C}}_t/\mathrm{d}\hat{M}_t{}'$ and $\mathrm{d}\hat{\mathcal{C}}_t/\mathrm{d}\hat{A}_t{}'$ is the passthrough of sector's n productivity shock to consumption and the price level, respectively.

Proof. See Appendix A.7. □

Hence, if the responsiveness of firms' inputs to prices has a unitary elasticity ($\omega^* = 1$), we obtain the complete information effect of productivity on output. In contrast, it is when firms are entirely unresponsive to nominal input price changes ($\omega^* = 0$) that we obtain the canonical effect that demand shocks are neutral for output. Of course, in light of Proposition

Figure 1: A Horizontal Production Network Economy



4, this is exactly when the impact of productivity shocks on output is *dampened* relative to the complete information benchmark.

3.5 Illustrative Examples

In this subsection, I use Theorem 1 to explicitly show how different production network structures affect the response of output and prices under incomplete information.

Example 1: A Horizontal Production Network Economy. Consider first a horizontal production network economy, depicted in Figure 1. Here, labor is the only factor or production for all sectors. Since there are no input-output linkages, the presence of incomplete information is irrelevant and the effect of a productivity shock to output in each sector is constant and equal to that sector's Domar weight (which, in this context, is simply the weight in the representative household's consumption bundle γ_n).

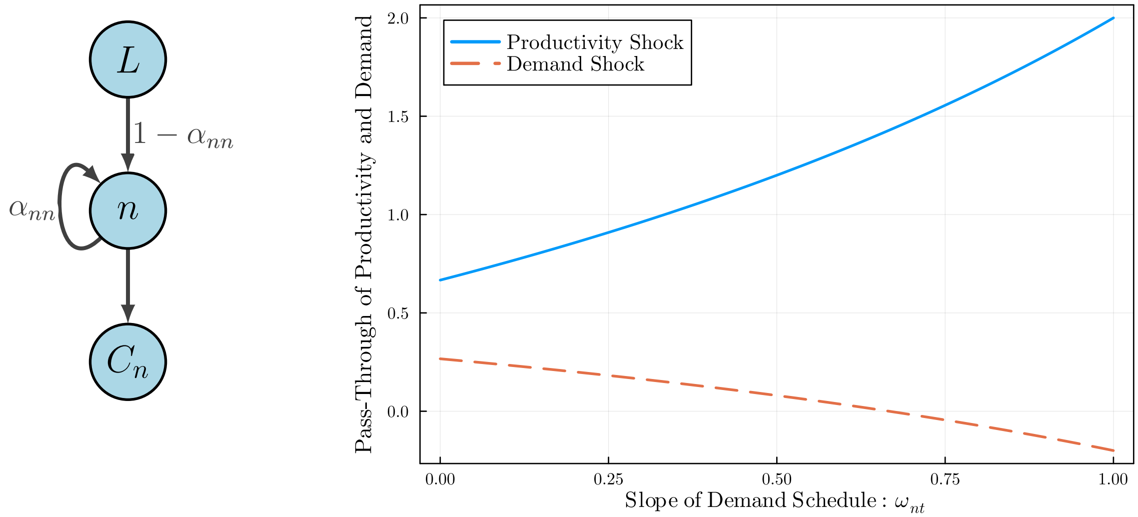
Example 2: Roundabout Production Networks. We can obtain further insights about the economy's temporary equilibrium by focusing on "roundabout" production networks, depicted in Figure 2a. Firms in sector n produce using a rigid input from their own sector (with input share α_{nn}) and labor (with input share $1 - \alpha_{nn}$). The dynamics of this economy can be analyzed through a scalar ω_{nt}, which is the slope of the demand schedule for a firm's inputs from its own sector.

Lemma 2. *$\boldsymbol{\epsilon}(\omega^*)'$ in the roundabout economy is a scalar given by:*

$$\boldsymbol{\epsilon}(\omega_{nt})' = \frac{1}{1-\alpha^2(1-\omega)} \sum_{k=0}^{\infty} \frac{\omega_{nt}}{1-\alpha_{nn}^2(1-\omega_{nt})} \alpha_{nn} = \left(\frac{1}{1+\alpha_{nn}(1-\omega_{nt})} \right) \frac{1}{1-\alpha_{nn}} \tag{42}$$

Figure 2: Shock Pass-Throughs in a Roundabout Network Economy

(a) Roundabout Network **(b)** Effect of Shocks on Output in Roundabout Network



Note: Figure 2a depicts a roundabout network economy. Sector n's input share for goods produced in its own sector is α_{nn}. The input share of labor is $(1 - \alpha_{nn})$. Figure 2b plots the pass-through of productivity (blue solid line) and demand shocks (orange dotted line) to the output of sector n as a function of the demand schedule slope ω_{nt} using the expressions derived in Lemma 2.

The effect of productivity and demand on output is given by:

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_{nt}} = \left(\frac{1}{1 + \alpha_{nn}(1 - \omega_{nt})} \right) \frac{1}{1 - \alpha_{nn}} \quad \textit{and} \quad \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = \frac{1 + \alpha_{nn} - \chi}{1 + \alpha_{nn}(1 - \omega_{nt})} \tag{43}$$

Proof. See Appendix A.8 □

Note that the pass-through of productivity shocks to output is given by the economy's inverse Leontief matrix $(1 - \alpha_{nn})^{-1}$ when $\omega_{nt} = 1$, as claimed in Corollary 1. When $\omega_{nt} = 0$, the pass-through of productivity shocks to output is *lower* and is equal to $(1 + \alpha_{nn})^{-1}$. In contrast, the pass-through of demand shocks to output is higher when $\omega_{nt} = 0$ (relative to $\omega_{nt} = 1$). Figure 2a plots the pass-throughs of productivity and demand shocks as a function of the demand-schedule slope ω_{nt}. The pass-through of productivity to output is *increasing* in ω_{nt}, while the pass-through of demand shocks to output is *decreasing* in ω_{nt} (Proposition 4). Again, these results are driven by the impact of firm-level responsiveness on the economy's higher-order linkages.

Example 3: Vertical Supply Chains. We next consider shock transmission for a vertical supply chain. Sector N uses labor to supply an input to sector $N-1$, which combines sector

N's input with labor. This production process is repeated until we reach sector 1, which sells the final good to the representative household. We let $1-\alpha_n$ denote labor's share in sector n, and let ω_{nt} denote the demand schedule for sector $n+1$'s input. Figure 3a depicts the network structure of this economy graphically.

Lemma 3. *The effect of a productivity shock in sector $n \in \{2, \ldots, N\}$ on consumption is given by*

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_{nt}} = \prod_{i=1}^{n-1} \frac{\alpha_i \omega_{i+1}}{1-\alpha_{i+1}(1-\omega_{i+1})} \quad \text{for} \quad n \in \{2, \ldots, N\} \tag{44}$$

The effect of a demand shock on consumption is given by

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = (1-\alpha_1)\chi - \sum_{k=1}^{N-1} \left[\prod_{i=1}^{k} \left(\frac{\alpha_i \omega_{i+1}}{1-\alpha_{i+1}(1-\omega_{i+1})} \right) \right] (1-\alpha_{k+1}(1-\chi)) \tag{45}$$

Proof. See Appendix A.9. □

The above Lemma characterizes the transmission of shocks in a vertical supply chain. The pass-through of a productivity in sector n to consumption depends on the responsiveness of all firms *downstream* to sector n. This explicitly shows that the downstream propagation of productivity shocks that is present in complete information Cobb-Douglas economies persists under incomplete information Acemoglu et al. (2016). Hence, the input responsiveness of downstream sectors is relatively more important in shaping the transmission of shocks to output. To formalize this observation, we define the *demand pass-through difference* Δ_n^M as the difference in the pass-through of demand shocks to output when the slope of sector n's demand schedule ω_{nt} moves to zero, holding all other responsiveness parameters fixed. We may similarly define the *productivity pass-through difference* Δ_n^A by further assuming that all productivity shocks are driven by a common component (i.e. $\hat{A}_{nt} = \hat{A}_t$).

Corollary 2. *Fix a tuple of demand schedules $(\omega_{2t}^*, \ldots, \omega_{Nt}^*)$ and assume it is element-wise weakly positive. Define the demand pass-through difference as:*

$$\Delta_n^Z = \left| \frac{\mathrm{d}\hat{C}_t}{\mathrm{d}\hat{Z}_t}(\omega_{2t}^*, \ldots, 0, \ldots, \omega_{Nt}^*) - \frac{\mathrm{d}\hat{C}_t}{\mathrm{d}\hat{Z}_t}(\omega_{2t}^*, \ldots, \omega_{nt}^*, \ldots, \omega_{Nt}^*) \right| \tag{46}$$

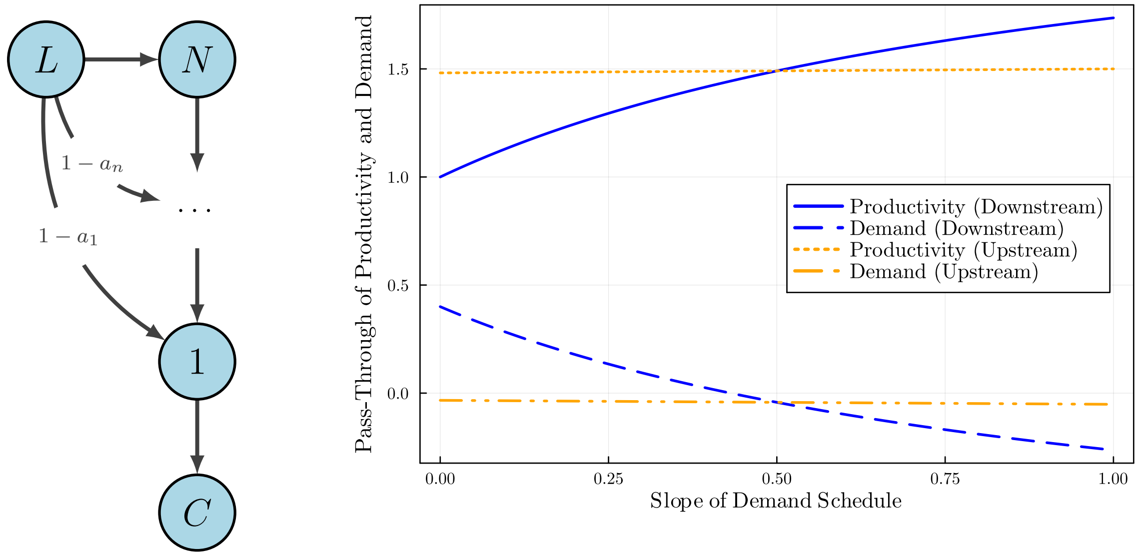
where $n \in \{2, \ldots, N\}$ and $Z_t \in \{A_t, M_t\}$. Then:

$$\Delta_2^Z \geq \ldots \geq \Delta_n^Z \geq \ldots \geq \Delta_N^Z \tag{47}$$

Proof. See Appendix A.10. □

Figure 3: Shock Pass-Throughs in a Vertical Supply Chain

(a) A Vertical Supply Chain **(b)** Pass-Through of Shocks to Output



Note: Figure 3a depicts a vertical supply chain. Sector N is the most upstream sector and uses labor to produce a final good, which it sells to sector $N-1$. In general, sector $n \in \{1, \ldots, N-1\}$ uses labor with input share $1-\alpha_n$ and final goods produced by sector $n+1$ to produce a final good. Sector 1 sells the final good to the representative household. Figure 3b plots the pass-through of productivity and demand shocks to output (blue solid and blue dashed lines) when varying the demand schedule slope of the most downstream sector, holding all other demand schedules fixed. The two orange lines (dotted and dash-dot-dot) plot the pass-through of productivity and demand shocks to output when varying the demand schedule of the most upstream sector, holding all other demand schedules fixed. The parameterization for Figure 3b is $N = 5$ with $a_n = 0.5$ for $n \in \{1, \ldots, N-1\}$, $\omega_n^* = 0.5$ for $n \in \{2, \ldots N\}$, and $\chi = 0.8$.

The above corollary therefore suggests that the pass-through of both demand and productivity shocks is relatively more sensitive to the demand schedules of *downstream* sectors. Figure 3b plots these pass-throughs for a simple parameterization of the model, varying the demand schedule slope for a downstream and a relatively more upstream sector. It is evident from Lemma 3 that the pass-through of productivity shocks is *increasing* in the slope of these demand schedules, while the pass-through of demand shocks is *decreasing* in the demand schedule slope.

Remark: The Limits of Linearization. Most of the results in this section have been derived by linearizing the system of equations that characterize the equilibrium (Proposition 2). However, it is important to note that firms' best responses are already log-linear given their Cobb-Douglas production function. Consequently, the results are derived by *only* linearizing the market clearing condition (Equation 23), thus preserving the global accuracy for firms' best responses. Moreover, the analysis of vertical supply chains above is globally accurate because the market clearing condition is log-linear when the customers of each firm all operate within a single sector.

3.6 Shock Transmission in Equilibrium: The Role of Uncertainty

So far, we have analyzed the exogenous choice of firm's demand schedules shapes the propagation of macroeconomic shocks. In this section, we analyze how this choice is endogenously determined by the nature of firms' prior uncertainty and the network structure of the economy. To this end, I define the *uncertainty ratio* u_{nt} of productivity to demand in sector n as:

$$u_{nt} = \frac{\sigma_{nt}^A}{\sigma_t^M} \tag{48}$$

The ratios $\{u_{nt}\}_{n=1}^N$ parameterize the extent of prior uncertainty about productivity shocks *relative* to demand shocks. We now establish firms' *optimal* responsiveness as either source of uncertainty dominates.

Theorem 2. *All firms optimally use log-linear demand schedules with a responsiveness parameter given by:*

$$\omega_{nn't}^* = \frac{\text{Cov}\left(\hat{R}_{nt}, \hat{P}_{n't}\right)}{\text{Var}\left(\hat{P}_{n't}\right)} \tag{49}$$

Moreover,

1. *If $u_{nt} \to 0$ for all $n \in [N]$, then:*

$$\omega_{nn't}^* = -\frac{\chi}{1-\chi} \quad \text{and} \quad \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = \chi \tag{50}$$

2. *Suppose further that the matrix $\mathbf{A}$ is irreducible. If $u_{nt} \to \infty$ for some $n \in [N]$, then:*

$$\omega_{nn't}^* = 1 \quad \text{and} \quad \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}' = \boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1} \tag{51}$$

Proof. See Appendix A.11. □

Theorem 2 shows that the pass-through of demand shocks to output is equal to the full information benchmark as uncertainty about demand becomes dominant. Intuitively, as prior uncertainty about the money supply increases, firms believe that prices are driven by *nominal* disturbances. For this reason, firms also expect their own (nominal) prices and revenues to increase when faced with higher marginal costs. This force induces firms' demand schedules to endogenously flatten. Furthermore, observe that $\omega_{nn't} = 0$ when there are no wage rigidities. In this case, nominal disturbances have no effect on output.

When uncertainty about productivity shocks becomes dominant, the effect of productivity on consumption is given by the standard Leontief inverse. As prior uncertainty about productivity shocks increase, firms believe that prices are driven by *real* disturbances. For this reason, firms expect their revenues to remain unchanged when faced with higher marginal costs. This force induces firms' responsiveness to endogenously increase. Of course, a higher responsiveness increases the effect of productivity shocks on output, but reduces the effect of demand shocks on output. Finally, the irreducibility condition on the input-output matrix $\mathbf{A}$ ensures that *all* prices in the economy serve as informative signals for real disturbances. Intuitively, this condition ensures that the network structure of the economy is sufficiently connected so that information can "flow" to every sector through input-output linkages. In sum, this theorem highlights the role of uncertainty in shaping the transmission of macroeconomic shocks in a networked economy. It is exactly when uncertainty about an underlying shock is highest that it has the greatest effect on output.

Roundabout Economy Revisited. It is instructive to consider the roundabout economy from Section 3.5. From Theorem 2, the firm's optimal responsiveness is given by:

$$\omega^*_{nt} = 1 - \frac{\text{Cov}(\hat{R}_{nt}, \hat{P}_{nt})}{\text{Var}(\hat{P}_{nt})} \tag{52}$$

since a firm's input price is also equalt o its output price. Moreover, firms in a roundabout economy derive a revenue share of $(1 - \alpha_{nn})$ from final sales to households and a share α_{nn} from sales in their own sector. Using Proposition 3, the first-order response of revenues to shocks is therefore equal to

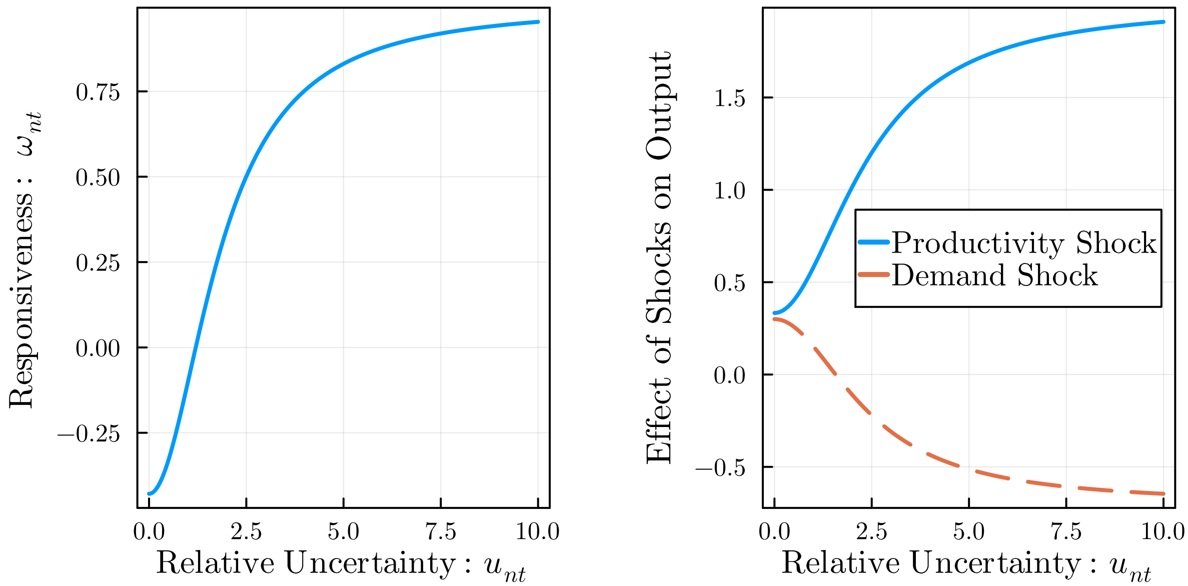
$$\hat{R}_{nt} = (1 - \alpha_{nn})\hat{M}_t + \alpha_{nn}(1 - \omega^*_{nt})\hat{P}_{nt} \tag{53}$$

The firm therefore uses input prices to form an inference regarding its demand $\hat{M}_t$ from the nominal expenditures of households. Substituting for the dynamics of $\hat{P}_{nt}$ derived in Lemma 2, then yields a fixed point for the optimal ω^*_{nt}.

Proposition 5. *Optimal responsiveness ω^*_{nt} in the roundabout economy solves the unique fixed point*

$$\omega^*_{nt} = 1 - \alpha_{nn}(1 - \omega^*_{nt}) - \frac{(1 - \alpha_{nn})(1 + \alpha_{nn} - \chi)}{(1 - \alpha_{nn})^{-2} u_{nt}^{-2} + (1 + \alpha_{nn} - \chi)^2}(1 + \alpha_{nn}(1 - \omega^*_{nt})) \tag{54}$$

Figure 4: Uncertainty in a Roundabout Economy



Note: This figure plots the optimal responsiveness in a roundabout economy given by Proposition 5 (left panel) and the associated pass-through of a demand and productivity shock to consumption (right panel).

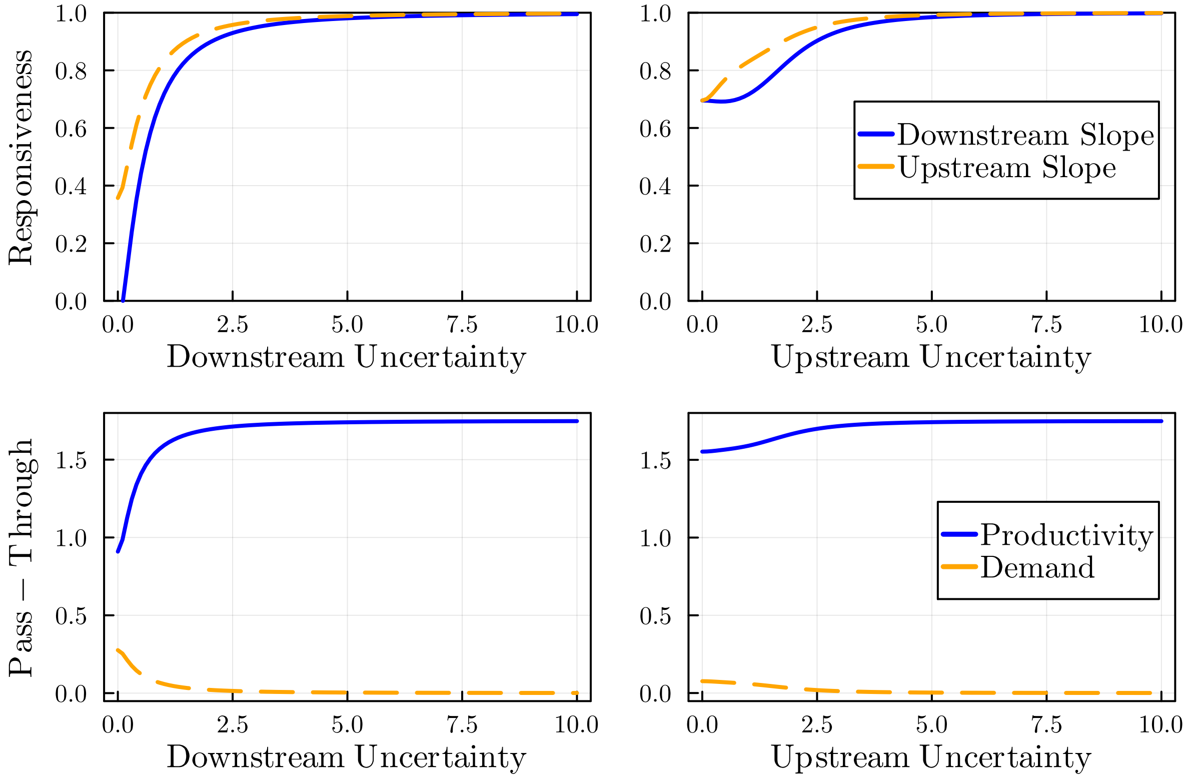
Moreover, $\partial \omega^*_{nt} / \partial u_{nt} > 0$*.*

Proof. See Appendix A.12. □

Proposition 5 shows that there exists a unique firm responsiveness to input prices that is increasing in relative uncertainty about productivity. Figure 4 plots firms' optimal responsiveness as a function of relative uncertainty (left panel) and the associated dynamics for consumption in response to a productivity and demand shock (right panel). As productivity uncertainty dominates, firm's respond one-to-one to input price changes and the pass-through of productivity shocks to output is equal to the perfect information benchmark, while the pass-through of demand shocks to output decreases.

Supply Chain Economy Revisited. We can revisit the vertical supply chain economy of Figure 3a to explore how uncertainty in different parts of the chain shapes the transmission of macroeconomic shocks. Although productivity shocks travel downstream (Corollary 3), uncertainty *about* productivity shocks has bidirectional effects across the entire chain. Intuitively, uncertainty about the productivity of downstream sectors will directly change the responsiveness of those downstream sectors by affecting the stochastic properties of their revenues through Theorem 2. However, this change in responsiveness will induce additional

Figure 5: Uncertainty in a Vertical Supply Chain



Note: This figure plots responsiveness and shock pass-throughs as a function of downstream and upstream uncertainty in a vertical supply chain given by Figure 3a. Higher downstream uncertainty increases σ_{2t}^{A} and holds all other parameters fixed; higher upstream uncertainty increases σ_{Nt}^{A} and holds all other parameters fixed.

variability in the revenues of their *suppliers*, in turn inducing these suppliers to change how they respond to input price variation.

Figure 5 formalizes this intuition by plotting firms' optimal responsiveness for the most upstream sector (orange line) and the most downstream sector (blue line) as a function of productivity uncertainty for different sectors. The figure shows that downstream productivity uncertainty is the most important determinant of firms' responsiveness along the entire supply chain. Larger downstream productivity uncertainty directly increases the responsiveness of downstream firms, but has ripple effects across the entire supply chain: it induces revenue variation for upstream *suppliers*, thereby causing them to increase their input responsiveness as well.

4 Quantitative Analysis

In this section, I study the model's implications for the transmission of productivity and demand shocks when calibrated to the input-output structure and historical volatility of the US economy.

My analysis relies on three sources of data. First, I use the 2022 input-output tables constructed by the Bureau of Economic Analysis (BEA) to determine the intermediate input shares of each industry. Second, I use the March 2024 release of the BEA/BLS Integrated Production Level Accounts (ILPA) which contains data on industry-level productivities at an annual frequency over the 1987-2023 period. Finally, I use data on nominal GDP to provide estimates of demand uncertainty at the quarterly frequency. I merge the the BEA input-output data with the ILPA data the 3-digit NAICS industry level, while excluding industries that correspond to federal, state, and local governments. This obtains a matched data set of 66 industries.

4.1 Calibration

I interpret each period as a quarter. I calibrate the input-output matrix $\mathbf{A}$ and labor expenditures $\{\alpha_{nl}\}$ of each industry so as to match the intermediate good expenditures and compensation of employees in the BEA input-output data. I also calibrate the final consumption shares γ to match the corresponding final consumption expenditures in the data.[4]

Next, I use the ILPA data to calculate the implied productivity variance-covariance matrix over the 1987-2023 period. The ILPA data is only available at an annual frequency. For this reason, I linearly interpolate the productivity data between quarters, as in La'O and Tahbaz-Salehi (2022). In order to obtain an estimate of aggregate demand uncertainty σ_t^M, I make use of the fact that model-implied nominal GDP is equal to $\iota_t M_t$. Under the assumption that demand uncertainty is constant, the time-series variation in nominal GDP is exactly equal the variance of M_t. I use this as my estimate of $\sigma_t^M = \bar{\sigma}^M$.

Finally, I set $\chi = 0.9$ following the measure of aggregate nominal wage adjustments on the extensive and intensive margin by Grigsby et al. (2021).

Extending the Information Structure. In practice, firms may receive additional information about contemporaneous shocks beyond what is conveyed by the price system as in Angeletos et al. (2016) or La'O and Tahbaz-Salehi (2022). In order to allow for this possibility in a parsimonious way, I extend the baseline model to allow for an interim public signal

[4]Under incomplete information, the input expenditures of each industry are time-varying and stochastic. For this reason, I calibrate these expenditures under the assumption that the model is in a perfect information steady-state.

Table 1: Calirbated Parameters

Parameter	Interpretation	Method	Value
$\mathbf{A}$	Input Shares	Match BEA Input-Output Tables	
$\{\alpha_{nl}\}$	Labor Shares	Match BEA Input-Output Tables	
$\boldsymbol{\gamma}$	Final Expenditure Shares	Match BEA Input-Output Tables	
$\boldsymbol{\Sigma}$	Productivity Covariance Matrix	ILPS	
$\bar{\sigma}^M$	Demand Uncertainty	Nominal GDP Std. Dev.	0.012
σ^p	Interim Information	Match Ball and Mazumder (2011)	0.004
χ	Wage Rigidities	Match Grigsby et al. (2021)	0.9

Note: Description of model parameters, how I interpret them, how I estimate them, and their values.

on the realization of the demand shock:

$$s_t^p = \log M_t + \sigma^p \varepsilon_t^p \tag{55}$$

where $\varepsilon_t^p \sim N(0,1)$. The scalar σ^p parameterizes the *posterior* uncertainty about the demand shock. As the discussion in Section 3 makes clear, this interim public signal does not only parameterize posterior uncertainty about demand shocks, but uncertainty about *all* aggregate shocks. Indeed, as $\sigma^p \to 0$, the economy converges to the perfect information benchmark, since prices become perfectly revealing of productivity shocks.

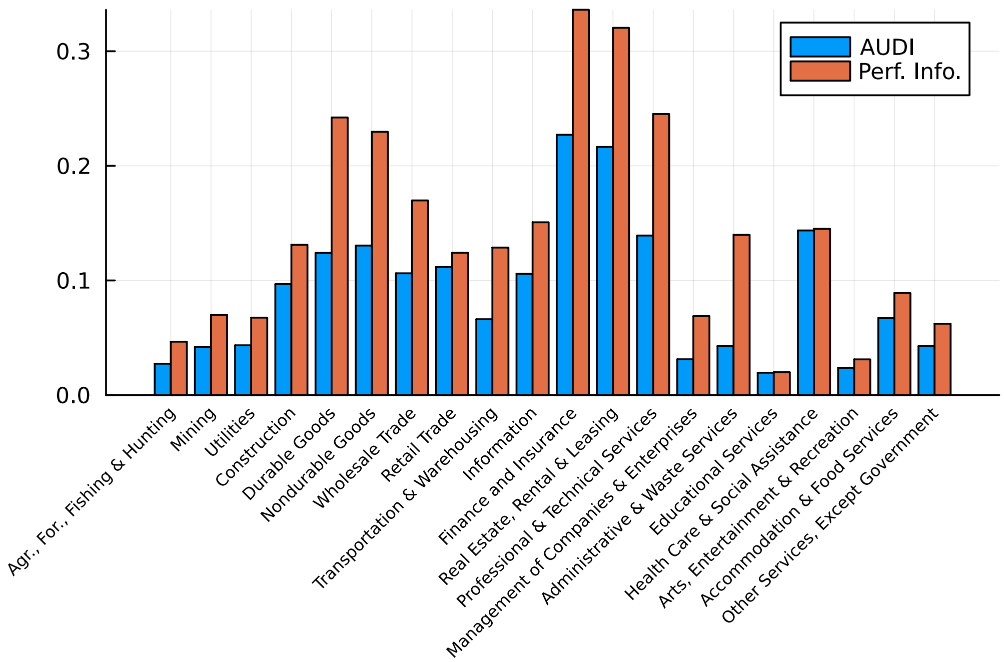
I calibrate σ_p to match the relative responsiveness of the aggregate price index to aggregate output in the model to external estimates of the slope of aggregate supply over the 1987-2023 period from Ball and Mazumder (2011). This gives an estimated signal variance of $\sigma_p = 0.004$. The signal to noise ratio defined as $\sigma_p/\bar{\sigma}_M = 0.32$, which suggests that about two-thirds of demand fluctuations are known ex-ante. Table 1 gives an overview of the calibrated parameters.

4.2 Uncertainty-Adjusted Domar Weights in the US Economy

Figure 6 plots the Uncertainty-Adjusted Domar Weights for the calibrated model, aggregated at the two-digit NAICS level. The height of the blue bars reflect the contemporaneous impact of a sectoral productivity shock on output under incomplete information, while the height of the red bars reflect the impact of a productivity shock on output under complete information.

In all cases, incomplete information dampens the effect of productivity shocks on output. This is because the presence of demand uncertainty implies that firms become *less* responsive to input price changes relative to complete information. For this reason, productivity

Figure 6: Uncertainty Adjusted Domar Weights in the US



shocks have a lower pass-through to output. In spite of this dampening, there is sectoral heterogeneity in how *much* incomplete information dampens the sectoral productivity shock of each sector. This is a consequence of differing alpha centralities across sectors. For example, "Health Care and Social Assistance" is a large industry, but with few connections to other sectors. As a consequence, any changes in its productivity are not "dampened" through firm-to-firm linkages, but are transmitted directly to consumers. In contrast, the "Finance and Insurance" sector, or the "Real Estate, Rental, and Leasing" sector is connected to many different sectors in the US economy, which are in turn connected to further sectors. These links are discounted relative to the perfect information benchmark under incomplete information. For this reason, their uncertainty-adjusted domar index differs from their complete information Domar index.

5 Conclusion

This paper shows how to embed incomplete information in a general equilibrium model of production networks. Because firms are only partially aware of the economy's underlying shocks, input prices serve as endogenous market signals which guide production decisions. This informational role of prices formally captures the notion that prices coordinate economic outcomes in decentralized networked economies (Hayek, 1945).

I show that incomplete information qualitatively alters the role of production networks in macroeconomic shock transmission by changing firm-level responsiveness to input prices. Firm-level responsiveness primitively depends on the statistical covariance between their revenues and input prices, which depends on the relative volatilties of the economy's underlying productivity and demand shocks. I have shown that incomplete information gives rise to a measure of sectoral importance that is given by the Augmented-by-Uncertainty Domar Index (AUDI) and which depends on economy-wide uncertainty.

When calibrated to the US economy, the model gives rise to quantitatively important deviations of the AUDI index relative to complete information measures of sectoral importance. Taken together, the results emphasize the importance of taking into account underlying economic uncertainty when desiging industrial interventions and aggregate demand management policies.

References

Acemoglu, D., Akcigit, U., and Kerr, W. (2016). Networks and the macroeconomy: An empirical exploration. *Nber macroeconomics annual*, 30(1):273–335.

Afrouzi, H. and Bhattarai, S. (2023). Inflation and gdp dynamics in production networks: A sufficient statistics approach. Technical report, National Bureau of Economic Research.

Angeletos, G.-M., Iovino, L., and La'o, J. (2016). Real rigidity, nominal rigidity, and the social value of information. *American Economic Review*, 106(01):200–227.

Angeletos, G.-M. and Sastry, K. (2019). Inattentive economies. Technical report, National Bureau of Economic Research.

Bajari, P. and Tadelis, S. (2001). Incentives versus transaction costs: A theory of procurement contracts. *Rand journal of Economics*, pages 387–407.

Ball, L. M. and Mazumder, S. (2011). Inflation dynamics and the great recession. Technical report, National Bureau of Economic Research.

Banerjee, A. V. and Duflo, E. (2000). Reputation effects and the limits of contracting: A study of the indian software industry. *The Quarterly Journal of Economics*, 115(3):989–1017.

Baqaee, D. and Farhi, E. (2022). Supply and demand in disaggregated keynesian economies with an application to the covid-19 crisis. *American Economic Review*, 112(5):1397–1436.

Baqaee, D. R. and Farhi, E. (2019). The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203.

Basu, S. (1994). Intermediate goods and business cycles: Implications for productivity and welfare.

Battigalli, P. and Maggi, G. (2002). Rigidity, discretion, and the costs of writing contracts. *American Economic Review*, 92(4):798–817.

Bigio, S. and La'o, J. (2020). Distortions in production networks. *The Quarterly Journal of Economics*, 135(4):2187–2253.

Bui, H., Huo, Z., Levchenko, A. A., and Pandalai-Nayar, N. (2022). Noisy global value chains. Technical report, National Bureau of Economic Research.

Carvalho, V. M. (2014). From micro to macro via production networks. *Journal of Economic Perspectives*, 28(4):23–48.

Carvalho, V. M. and Tahbaz-Salehi, A. (2019). Production networks: A primer. *Annual Review of Economics*, 11:635–663.

Crocker, K. J. and Reynolds, K. J. (1993). The efficiency of incomplete contracts: An empirical analysis of air force engine procurement. *The RAND journal of economics*, pages 126–146.

Dew-Becker, I. (2023). Tail risk in production networks. *Econometrica*, 91(6):2089–2123.

Flynn, J. P., Nikolakoudis, G., and Sastry, K. A. (2023). A theory of supply function choice and aggregate supply.

Golosov, M. and Lucas, R. E. (2007). Menu costs and phillips curves. *Journal of Political Economy*, 115(2):171–199.

Grigsby, J., Hurst, E., and Yildirmaz, A. (2021). Aggregate nominal wage adjustments: New evidence from administrative payroll data. *American Economic Review*, 111(2):428–471.

Grossman, S. (1989). The informational role of prices. *MIT Press Books*, 1.

Grossman, S. J. (1981). An introduction to the theory of rational expectations under asymmetric information. *The Review of Economic Studies*, 48(4):541–559.

Grossman, S. J. and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408.

Hayek, F. A. (1945). The use of knowledge in society. *American Economic Review*, 35:519–530. Reprinted in F.A. Hayek (ed.), Individualism and Economic Order. London: Routledge and Kegan Paul.

Hiller, J. R. and Tollison, R. D. (1978). Incentive versus cost-plus contracts in defense procurement. *The Journal of Industrial Economics*, pages 239–248.

Holthausen, R. W. and Leftwich, R. W. (1983). The economic consequences of accounting choice implications of costly contracting and monitoring. *Journal of accounting and economics*, 5:77–117.

Institute for Supply Management (2024). January 2024 Manufacturing ISM Report on Business. https://www.ismworld.org/supply-management-news-and-reports/reports/ism-report-on-business/pmi/january/.

Katz, L. (1953). A new status index derived from sociometric analysis. *Psychometrika*, 18(1):39–43.

Kopytov, A., Mishra, B., Nimark, K., and Taschereau-Dumouchel, M. (2022). Endogenous production networks under supply chain uncertainty. *Available at SSRN*.

La'O, J. and Tahbaz-Salehi, A. (2022). Optimal monetary policy in production networks. *Econometrica*, 90(3):1295–1336.

Liu, E. and Tsyvinski, A. (2024). A dynamic model of input–output networks. *Review of Economic Studies*, page rdae012.

Long, J. B. and Plosser, C. I. (1983). Real business cycles. *Journal of political Economy*, 91(1):39–69.

Lucas, R. E. (1972). Expectations and the neutrality of money. *Journal of economic theory*, 4(2):103–124.

Lucas, R. E. (1973). Some international evidence on output-inflation tradeoffs. *The American economic review*, 63(3):326–334.

Lucas, R. E. (1975). An equilibrium model of the business cycle. *Journal of political economy*, 83(6):1113–1144.

Meyer, C. D. (2023). *Matrix analysis and applied linear algebra*, volume 188. Siam.

Pellet, T. and Tahbaz-Salehi, A. (2023). Rigid production networks. *Journal of Monetary Economics*.

Rubbo, E. (2023). Networks, phillips curves, and monetary policy. *Econometrica*, 91(4):1417–1455.

A Omitted Proofs

A.1 Proof of Proposition 1

Proof. The demand schedule for inputs chosen under incomplete information satisfies

$$X_{in,n',t} = \alpha_{nn'} \frac{\exp\left\{\mu_{R_{nt}|P_{n't}} + \frac{1}{2}\sigma^2_{\mathcal{P}_t\mathcal{C}_t|P_{n't}} + \frac{1}{2}\sigma^2_{R_{nt}|P_{n't}} + \sigma_{\mathcal{P}_t\mathcal{C}_t,R_{nt}|P_{n't}}\right\}}{\exp\left\{\mu_{P_{n't}|P_{n't}} + \frac{1}{2}\sigma^2_{\mathcal{P}_t\mathcal{C}_t|P_{n't}} + \frac{1}{2}\sigma_{P_{n't}|P_{n't}} + \sigma_{\mathcal{P}_t\mathcal{C}_t,P_{n't}|P_{n't}}\right\}} \tag{56}$$

where $\sigma_{X,Y|Z} = \text{Cov}(\log X, \log Y \,|\, \log Z)$ and $\mu_{X|Z} = \mathbb{E}[\log X \,|\, \log Z]$. Using standard Gaussian formulas for conditional expectations, we have:

$$\mu_{R_{nt}|P_{n't}} = \mu_{R_{nt}} + \frac{\sigma_{R_{nt},P_{n't}}}{\sigma^2_{P_{n't}}}(\log P_{n't} - \mu_{P_{n't}}) \tag{57}$$

$$\mu_{P_{n't}|P_{n't}} = \log P_{n't} \tag{58}$$

All other variances and covariances are independent of $\log P_{n't}$. Collecting terms with $\log P_{n't}$ then yields the claimed expression for $\omega_{nn't}$.

□

A.2 Proof of Lemma 1

Proof. We use the households' first-order condition to derive an expression for the real stochastic discount factor $(\mathcal{P}_t\mathcal{C}_t)^{-1}$ in terms of the money supply M_t. From the intratemporal Euler equation for consumption demand *vs.* labor supply, we can obtain an expression for the *frictionless* wage rate w^*_{nt}:

$$w^*_{nt} = \mathcal{P}_t\mathcal{C}_t \tag{59}$$

From Equation 5, the wage rate therefore satisfies the recursion:

$$w_{nt} = (w_{nt-1})^{\chi}(\mathcal{P}_t\mathcal{C}_t)^{1-\chi} \tag{60}$$

From the intertemporal Euler equation between consumption and money today, the cost of holding an additional dollar today equals the benefit of holding an additional dollar today plus the value of an additional dollar tomorrow:

$$\frac{1}{\mathcal{P}_t\mathcal{C}_t} = \frac{1}{M_t} + \beta\mathbb{E}_t\left[\frac{1}{\mathcal{P}_{t+1}\mathcal{C}_{t+1}}\right] \tag{61}$$

Further, from the intertemporal choice between bonds, the cost of saving an additional dollar today equals the nominal interest rate $1+i_t$ times the value of an additional dollar tomorrow:

$$\frac{1}{\mathcal{P}_t\mathcal{C}_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1}{\mathcal{P}_{t+1}\mathcal{C}_{t+1}}\right] \tag{62}$$

Combining these two equations, we obtain that aggregate consumption follows:

$$\mathcal{P}_t\mathcal{C}_t = \iota_t M_t \tag{63}$$

where $\iota_t = i_t/(1+i_t)$. This equation implies that nominal expenditures are proportional to money balances. Note further that the Cobb-Douglas aggregator over final sectoral goods in household preferences implies that expenditure shares are constant:

$$P_{nt}C_{nt} = \gamma_n\mathcal{P}_t\mathcal{C}_t = \gamma_n\iota_t M_t \tag{64}$$

Finally, the nominal interest rate adjusts to clear the bond market. Substituting Equation 63 back into Equation 62, we obtain a recursion that interest rates must satisfy:

$$\frac{1+i_t}{i_t} = 1+\beta\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{M_t}{M_{t+1}}\right] \tag{65}$$

As money follows a random walk, solving this equation forward and employing the household's transversality condition, we obtain that:

$$\frac{1+i_t}{i_t} = 1+\beta\exp\left\{-\mu_M+\frac{1}{2}(\sigma^M_{t+1})^2\right\}\left(1+\sum_{i=1}^{\infty}\prod_{j=1}^{i}\beta\exp\left\{-\mu_M+\frac{1}{2}(\sigma^M_{t+j+1})^2\right\}\right) \tag{66}$$

which is deterministic, but depends on the full future path of monetary volatility.

□

A.3 Proof of Proposition 2

Proof. We prove the claim for general sets $\mathcal{S}^f_n$. The demand for rigid inputs $n' \in \mathcal{S}^r_n$ is given by

$$X_{in,n',t} = \alpha_{nn'}\frac{\mathbb{E}_t\left[(P_tC_t)^{-1}R_{nt}\middle|P_{n't}\right]}{\mathbb{E}_t\left[(P_tC_t)^{-1}P_{n't}\middle|P_{n't}\right]} \tag{67}$$

The demand function for flexible inputs as:

$$X_{in,n',t} = \alpha_{nn'} \frac{R_{nt}}{P_{n't}} \tag{68}$$

for $n' \in \mathcal{S}_n^f$.

Note that the intermediate good's demand schedules (67) and (68) allow us to obtain a fixed point in terms of the firm's revenues using the definition of Q_{nt} from Equation 10:

$$R_{nt} = c_n A_{nt} P_{nt} \left(\frac{R_{nt}}{w_{nt}} \right)^{\alpha_{nl}} \prod_{n' \in S_n^r} \left(\frac{\mathbb{E}_t \left[(P_t C_t)^{-1} R_{nt} \middle| P_{n't} \right]}{\mathbb{E}_t \left[(P_t C_t)^{-1} P_{n't} \middle| P_{n't} \right]} \right)^{\alpha_{nn'}} \prod_{n' \in S_n^f} \left(\frac{R_{nt}}{P_{n't}} \right)^{\alpha_{nn'}} \tag{69}$$

We can combine market clearing with firm's optimal demand schedules for inputs (Equations 67 and 68) to obtain:

$$R_{nt} = P_{nt} C_{nt} + \sum_{n' : n \in S_{n'}^r} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t \left[(P_t C_t)^{-1} R_{n't} \middle| P_{nt} \right]}{\mathbb{E}_t \left[(P_t C_t)^{-1} P_{nt} \middle| P_{nt} \right]} + \sum_{n' : n \in S_{n'}^f} \alpha_{n'n} R_{n'} \tag{70}$$

We can then combine equations 69 and 70 with the expression for nominal expenditures 63 and 64 to obtain a characterization of revenues and prices in terms of exogenous variables:

$$R_{nt} = \tilde{c}_{nt} A_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi}} \right)^{\alpha_{nl}} \prod_{n' \in S_n^r} \left(\frac{\mathbb{E}_t \left[M_t^{-1} R_{nt} \middle| P_{n't} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{n't} \middle| P_{n't} \right]} \right)^{\alpha_{nn'}} \prod_{n' \in S_n^f} \left(\frac{R_{nt}}{P_{n't}} \right)^{\alpha_{nn'}} \tag{71}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n' : n \in S_{n'}^r} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t \left[M_t^{-1} R_{n't} \middle| P_{nt} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{nt} \middle| P_{nt} \right]} + \sum_{n' : n \in S_{n'}^f} \alpha_{n'n} R_{n't} \tag{72}$$

where $\tilde{c}_{nt} = (\iota_t)^{-\alpha_{nl}(1-\chi)} w_{nt-1}^{\alpha_{nl}\chi}$ and w_{nt-1}^{χ} is independent of time t shocks. This proves the claim.

□

A.4 Proof of Proposition 3

We begin by proving the following Lemma.

Lemma 4. *The first-order dynamics of the economy are characterized by the following system* *of equations:*

$$(1-\alpha_n^f)\hat{R}_{nt} = \hat{A}_{nt} + \hat{P}_{nt} - \alpha_{nl}(1-\chi)(\hat{M}_t) - \sum_{n' \in S_n^r} \alpha_{nn'}\omega_{nn't}\hat{P}_{n't} - \sum_{n' \in S_n^f} \alpha_{nn'}\hat{P}_{n't} \tag{73}$$

$$\hat{R}_{nt} = s_{nc}\hat{M} + \sum_{n' : n \in S_{n'}^r} s_{nn'}(1-\omega_{n'nt})\hat{P}_{nt} + \sum_{n' : n \in \mathcal{S}_n^f} s_{nn'}\hat{R}_{n't} \tag{74}$$

where $\alpha_n^f = \alpha_{nl} + \sum_{n' \in S_n^f} \alpha_{nn'}$ *is sector n's flexible input share and the revenue-shares* $\{s_{nc}, \{s_{nn'}\}_{n' \in [N]}\}$ *are time-invariant positive scalars with* $\sum_{n \in [N]} s_{nn'} = 1$.

Proof. We first substitute the exogenous log-linear demand schedule into the intermediate firm's profits:

$$\Pi_{in,t} = (1+\tau_n)P_{nt}Q_{nt}^{\frac{1}{\eta_n}} q_{int}^{\frac{\eta_n - 1}{\eta_n}} - w_{nt}L_{nt} - \sum_{n' \in S_n^r} P_{n't}\tilde{\omega}_{in,n',t}P_{n't}^{-\omega_{nn't}} - \sum_{n' \in S_n^f} P_{n't}X_{in,n',t} \tag{75}$$

where

$$q_{in,t} = A_{nt}L_{in,t}^{\alpha_{nl}} \prod_{n'=1}^{N} \tilde{\omega}_{in,n',t}^{\alpha_{nn'}} P_{n't}^{-\alpha_{nn'}\omega_{nn't}} \tag{76}$$

and where $\tilde{\omega}_{in,n',t} = \exp[\omega_{in,n',t}]$ We can solve for the profit-maximizing intercept $\omega_{in,n',t}$ for rigid inputs using the firm's first-order condition to obtain:

$$\tilde{\omega}_{in,n',t}^{*} = \alpha_{nn'} \frac{\mathbb{E}_t\left[(P_tC_t)^{-1}P_{nt}Q_{nt}^{\frac{1}{\eta_n}}\left(A_{nt}L_{in,t}^{\alpha_{nl}}\prod_{n' \in [N]} X_{in,n',t}^{\alpha_{nn'}}\right)^{\frac{\eta_n - 1}{\eta_n}}\right]}{\mathbb{E}_t\left[(P_tC_t)^{-1}P_{n't}^{1-\omega_{nn't}}\right]} \tag{77}$$

Note that this is a constant that is independent of shocks at time t, as the expectation operator is not conditional on the realized price of input n'. We may therefore express the firm's demand function for its rigid inputs as:

$$X_{in,n',t} = \tilde{\omega}_{in,n',t}^{*} P_{n't}^{-\omega_{n'nt}} \tag{78}$$

Furthermore, the firm's flexible inputs continue to be given by the expression:

$$X_{in,n',t} = \alpha_{nn'}\frac{R_{nt}}{P_{n't}} \tag{79}$$

Substituting Equations 78 and 79 into the production function 76 and log-linearizing then yields Equation 73 directly.

To derive Equation 74, we may substitute the demand for rigid inputs into the market clearing equation 15. This yields:

$$R_{nt} = \gamma_n \iota_t M_t + P_{nt} \sum_{n':n \in \mathcal{S}^r_{n'}} \tilde{\omega}^*_{in,n',t} P_{nt}^{-\omega_{n'nt}} + \sum_{n':n \in \mathcal{S}^f_{n'}} \alpha_{n'n} R_{n't} \tag{80}$$

Log-linearizing this expression yields around $\delta = 0$ yields

$$\hat{R}_{nt} = s_{nc}\hat{M}_t + \sum_{n':n \in \mathcal{S}^r_{n'}} s_{nn'}(1 - \omega_{nn't})\hat{P}_{nt} + \sum_{n':n \in \mathcal{S}^f_{n'}} \hat{R}_{n't} \tag{81}$$

where

$$s_{nc} = \frac{\gamma_n}{\sum_{n' \in [N]} \alpha_{n'n}\lambda_{n'} + \gamma_n} \quad \text{and} \quad s_{nn'} = \frac{\alpha_{n'n}\lambda_{n'}}{\sum_{n' \in [N]} \alpha_{n'n}\lambda_{n'} + \gamma_n} \tag{82}$$

and where λ_{nt} are the full-information Domar weights given by:

$$\lambda_t = (\mathbf{I} - \mathbf{A}')^{-1}\gamma \tag{83}$$

□

We can now prove the Proposition.

Proof. We consider first general subsets $\mathcal{S}^r_n \subset [N]$ and then prove the Proposition as a special case in which $\mathcal{S}^r_n = [N]$, as considered in the main text.

In addition to the revenue share matrix $\mathbf{S} = [s_{nn'}]$ considered in the main text, we also define the rigid revenue share matrix as:

$$\mathbf{S}^r \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^r_{n'}\}] \tag{84}$$

We also define the flexible revenue share matrix as:

$$\mathbf{S}^f \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^f_{n'}\}] \tag{85}$$

We can also similarly define the *rigid* input-output matrix as $\mathbf{A}^r = [\alpha_{nn'} \times \mathbb{1}\{n' \in S^r_n\}]$ and the flexible input out matrix $\mathbf{A}^f = \mathbf{A} - \mathbf{A}^r$. Finally, we define the *demand-adjusted* input-output matrix and revenue-share matrix as $\mathbf{A}^r(\omega_t) = [\alpha_{nn'} \times \omega_{nn't} \times \mathbb{1}\{n' \in S^r_n\}]$ and $\mathbf{S}(\omega_t) = [s_{nn'} \times (1 - \omega_{n'nt}) \times \mathbb{1}\{n \in \mathcal{S}^r_{n'}\}]$. We further assume that $\sum_{n'} s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}^f_{n'}\} < 1$. This is clearly satisfied if $\mathcal{S}^r_n = [N]$.

Observe that Equation 74 can be written in matrix form as:

$$\hat{R}_t = (\mathbf{I} - (\mathbf{I} - \mathbf{S}^f)^{-1}\mathbf{S}^r)\hat{M}_t + (\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})\hat{P}_t \tag{86}$$

Next, observe that Equation 73 can be expressed as:

$$\text{diag}(\mathbf{A}^r\mathbf{1})\hat{R}_t = \hat{A}_t + (\mathbf{I} - \mathbf{A}^r(\omega_t) - \mathbf{A}^f)\hat{P}_t - (1-\chi)(\mathbf{I} - \mathbf{A})\hat{M}_t \tag{87}$$

We may substitute for $\hat{R}_t$ to obtain:

$$\begin{aligned}\text{diag}(\mathbf{A}^r\mathbf{1})\left[(\mathbf{I} - (\mathbf{I} - \mathbf{S}^f)^{-1}\mathbf{S}^r)\hat{M} + (\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})\hat{P}_t\right] = \\ \hat{A}_t + (\mathbf{I} - \mathbf{A}^r(\omega_t) - \mathbf{A}^f)\hat{P}_t + (1-\chi)(\mathbf{I} - \mathbf{A})\hat{M}_t\end{aligned} \tag{88}$$

The special case of $\mathcal{S}_n^r = [N]$ considered in Proposition 3 obtains by setting $\mathbf{S}^f = \mathbf{0}$, $\mathbf{A}^f = \mathbf{0}$, and $\mathbf{A}^r = \mathbf{A}$. Rearranging the above expression then yields the result directly. □

A.5 Proof of Theorem 1

Proof. We first note that we can write

$$\mathbf{I} - \mathbf{A}(\omega^*) - \mathbf{D}(\omega^*) = (\mathbf{I} - \mathbf{D}(\omega^*))\left(\mathbf{I} - (\mathbf{I} - \mathbf{D}(\omega^*))^{-1}\mathbf{A}(\omega^*)\right) \tag{89}$$

where $\mathbf{I} - \mathbf{D}(\omega^*)$ is invertible for $\omega^* \in [0,1]$ given the assumption that $\sum_{n'\in[N]} \alpha_{nn'} < 1$ and $\sum_{n'\in[N]} s_{nn'} \leq 1$.

Next, we show that $\mathbf{I} - (\mathbf{I} - \mathbf{D}(\omega^*))^{-1}\mathbf{A}(\omega^*)$ is an M-matrix. To this end, observe that the diagonal elements of this matrix are given by

$$1 - \frac{\omega^*\alpha_{nn}}{1 - (1-\omega^*)\sum_{n'\in[N]}\alpha_{nn'}\sum_{n'\in[N]}s_{nn'}} \tag{90}$$

We therefore require that

$$1 - \omega^*\alpha_{nn} - (1-\omega^*)\sum_{n'\in[N]}\alpha_{nn'}\sum_{n'\in[N]}s_{nn'} > 0 \tag{91}$$

Note that this expression is strictly positive for $\omega^* = 0$ and $\omega^* = 1$ (given the assumption that $\sum_{n'\in[N]}\alpha_{nn'} < 1$ and $\sum_{n'\in[N]}s_{nn'} \leq 1$). Since this is a linear function of ω^* it is therefore strictly positive for all $\omega^* \in [0,1]$.

Next, observe that all elements of $(\mathbf{I} - \mathbf{D}(\omega^*))^{-1}\mathbf{A}(\omega^*)$ are weakly positive. Moreover, the sum of each row of this matrix is given by

$$\frac{\omega^*\sum_{n'\in[N]}\alpha_{nn'}}{1 - (1-\omega^*)\sum_{n'\in[N]}\alpha_{nn'}\sum_{n'\in[N]}s_{nn'}} < 1 \tag{92}$$

By the Gershgorin Circle Theorem, the norm of the principal eigenvalue of this matrix is less than unity. Hence, $(\mathbf{I}-\mathbf{D}(\omega^*))^{-1}\mathbf{A}(\omega^*)$ is an M-matrix. Its inverse can therefore be written as the power sum:

$$\left[\mathbf{I}-(\mathbf{I}-\mathbf{D}(\omega^*))^{-1}\mathbf{A}(\omega^*)\right]^{-1}=\sum_{k=0}^{\infty}\left(\omega^*(\mathbf{I}-\mathbf{D}(\omega^*))^{-1}\mathbf{A}\right)^k \tag{93}$$

Moreover, the matrix $\omega^*(\mathbf{I}-\mathbf{D}(\omega^*))^{-1}$ is diagonal with elements given by

$$\frac{\omega^*}{1-(1-\omega^*)x_n} \tag{94}$$

for $x_n < 1$. This ratio is therefore increasing for all $\omega^* \in \mathbb{R}$. Hence, the eigenvalues of this matrix are increasing in ω^*.

Finally, observe that the ideal price index is given by

$$\mathcal{P}_t = c_P \prod_{n=1}^{N} P_{nt}^{\gamma_n} \tag{95}$$

where $c_P = \prod_{n=1}^{N} \gamma_n^{-\gamma_n}$. The first-order change in the price level and consumption to a shock is therefore given by

$$\hat{\mathcal{P}}_t = \gamma' \hat{P}_t \tag{96}$$

$$\hat{\mathcal{C}}_t = \hat{M}_t - \gamma' \hat{P}_t \tag{97}$$

The Theorem then follows directly from Proposition 3 and the definition of $\boldsymbol{\epsilon}(\omega^*)$. □

A.6 Proof of Proposition 4

Proof. If $\omega^* = 1$, we have $\boldsymbol{\epsilon}(1)' = \boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1}$

The first-order response of consumption to a common productivity shock $\hat{A}_t$ is therefore given by

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t} = \boldsymbol{\gamma}'(\mathbf{I}-\mathbf{A})^{-1}1 \tag{98}$$

This can be written as

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}(\omega^*=1) = \boldsymbol{\gamma}'\left(1+\mathbf{A}1+\mathbf{A}^2 1+\mathbf{A}^3 1+\ldots\right) \tag{99}$$

Next, observe that if all sectors have a common labor share $\alpha_{nl} = \alpha_l$, the sum of the rows of

$\mathbf{A}$ sum to $1-\alpha_l$ given the assumption of constant returns to scale. Hence,

$$\mathbf{A}1 = (1-\alpha_l)1 \tag{100}$$

But this implies that the unit vector 1 is an eigenvector of $\mathbf{A}$ with associated eigenvalue $1-\alpha_l$. Hence,

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}(\omega^* = 1) = \boldsymbol{\gamma}' \left(1 + (1-\alpha_l)1 + (1-\alpha_l)^2 1 + (1-\alpha_l)^3 1 + \ldots\right) = \frac{1}{\alpha_l} \tag{101}$$

Next, suppose $\omega^* = 0$. In this case, we have $\boldsymbol{\epsilon}(0)' = \boldsymbol{\gamma}'(\mathbf{I} - \mathrm{diag}(\mathbf{A1})\mathrm{diag}(\mathbf{S1}))^{-1}$. Hence, we have that the first-order impact of a common productivity shock on consumption is given by

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}(\omega^* = 0) = \boldsymbol{\gamma}'(\mathbf{I} - \mathrm{diag}(\mathbf{A1})\mathrm{diag}(\mathbf{S1}))^{-1}1 = \sum_{n\in[N]} \frac{\gamma_n}{1-(1-s_{nc})(1-\alpha_l)} \tag{102}$$

where s_{nc} is the consumption revenue share of sector n. It then follows that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}(\omega^* = 1) - \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t}(\omega^* = 0) > 0 \tag{103}$$

The result for the price level follows by noting that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{A}_t} = -\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{A}_t} \tag{104}$$

Next, we have that the first-order effect of a demand shock on the price level when $\omega^* = 1$ is given by:

$$\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 1) = \boldsymbol{\epsilon}(1)'\boldsymbol{\Phi} = \boldsymbol{\gamma}' \left[(1-\chi)\mathbf{I} + (\mathbf{I}-\mathbf{A})^{-1}\mathrm{diag}(\mathbf{A1})(\mathbf{I} - \mathrm{diag}(\mathbf{S1}))\right] 1 \tag{105}$$

Similarly, we have that the first-order effect of a demand shock on the price level when $\omega^* = 0$ is given by:

$$\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 0) = \boldsymbol{\gamma}'(\mathbf{I} - \mathrm{diag}(\mathbf{A1})\mathrm{diag}(\mathbf{S1}))^{-1} \left[(1-\chi)(\mathbf{I}-\mathbf{A}) + \mathrm{diag}(\mathbf{A1})(\mathbf{I} - \mathrm{diag}(\mathbf{S1}))\right] 1$$

Note that $\mathbf{A}1 = \text{diag}(\mathbf{A1})1$. Hence, this can be simplified to:

$$\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 0) = \boldsymbol{\gamma}'(\mathbf{I} - \text{diag}(\mathbf{A1})\text{diag}(\mathbf{S1}))^{-1}\left[-\chi(\mathbf{I} - \mathbf{A}) + \mathbf{I} - \text{diag}(\mathbf{A1})\text{diag}(\mathbf{S1})\right] \tag{106}$$

$$= \boldsymbol{\gamma}'\left[-\chi(\mathbf{I} - \text{diag}(\mathbf{A1})\text{diag}(\mathbf{S1}))^{-1}(\mathbf{I} - \text{diag}(\mathbf{A1})) + \mathbf{I}\right]1 \tag{107}$$

Observe now that

$$\chi(\mathbf{I} - \text{diag}(\mathbf{A1})\text{diag}(\mathbf{S1}))^{-1}(\mathbf{I} - \text{diag}(\mathbf{A1})) \geq \chi\mathbf{I} \tag{108}$$

where the ordering above denotes the element-wise order. Moreover, the inequality is strict for at least one element if element nn has a positive consumption share ($\gamma_n > 0$), since $\sum_{n'} s_{nn'} < 1$. Hence, we have that

$$\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 0) < (1 - \chi)\mathbf{I} \tag{109}$$

It follows that:

$$\frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 1) - \frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 0) > 0 \tag{110}$$

Finally, note that:

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = 1 - \frac{\mathrm{d}\hat{\mathcal{P}}_t}{\mathrm{d}\hat{M}_t} \tag{111}$$

which implies that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 1) - \frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t}(\omega^* = 0) < 0 \tag{112}$$

This completes the proof.

□

A.7 Proof of Corollary 1

Proof. If $\omega^* = 1$, we have $\mathbf{D}(1) = 0$. If $\omega^* = 0$, we have $\epsilon(\omega^*) = 0$. The proof then follows. □

A.8 Proof of Lemma 2

Proof. From Proposition 2, we can write s_{nn} as α_{nn} in the roundabout network economy. Substituting for s_{nn} into the definition of ϵ_{nt} then yields Equation 42. Equation 43 follows from Theorem 1. □

A.9 Proof of Lemma 3

We use Lemma 4 to derive the equilibrium dynamics of the system. Using Equations 73 and 74 for sector N, we obtain the following:

$$\hat{X}_{Nt} = -\omega_{Nt}\hat{P}_{Nt} \tag{113}$$

and

$$\hat{P}_{Nt} = (1-\chi)\hat{M}_t - \hat{A}_{Nt} \tag{114}$$

For sector $n \in \{2, \ldots, N-1\}$, we obtain:

$$\hat{X}_{nt} = -\omega_{nt}\hat{P}_{nt} \tag{115}$$

$$\hat{P}_{nt} = \frac{1}{1-\alpha_n(1-\omega_{nt})}\left[\hat{A}_{nt} - (1-\alpha_n)(1-\chi)\hat{M} - \alpha_n\omega_{n+1t}\hat{P}_{n+1t}\right] \tag{116}$$

For sector 1, we obtain:

$$\hat{X}_{1t} = \hat{C}_t = \hat{M}_t - \hat{P}_{1t} \tag{117}$$

$$\hat{P}_{1t} = \alpha_1\hat{M}_t - \hat{A}_{1t} + (1-\alpha_1)(1-\chi)\hat{M}_t + \alpha_1\omega_{2t}\hat{P}_{2t} \tag{118}$$

Solving this difference equation using $\hat{P}_N$ as a terminal condition yields the claim.

A.10 Proof of Corollary 2

From Lemma 3, Δ_n^M is given by:

$$\Delta_n^M = \sum_{k=n}^{N-1}\prod_{i=1}^{k}\left(\frac{\alpha_i\omega_{i+1}^*}{1-\alpha_{i+1}(1-\omega_{i+1}^*)}\right)(1-\alpha_{k+1})(1-\chi) \tag{119}$$

which is an increasing sequence for $n = \{2, \ldots, N\}$ under the assumption that $\omega_{nt}^* \geq 0$. Similarly, Δ_n^A is given by:

$$\Delta_n^A = \sum_{k=n}^{N-1}\prod_{i=1}^{k}\left(\frac{\alpha_i\omega_{i+1}^*}{1-\alpha_{i+1}(1-\omega_{i+1}^*)}\right)(1-\alpha_{k+1})(1-\chi) \tag{120}$$

which is also a decreasing sequence for $n = \{2, \ldots, N\}$ under the assumption that $\omega_{nt}^* \geq 0$.

A.11 Proof of Theorem 2

Proof. We first prove the first statement of the Theorem. From equations 73 and 74, the linearized dynamics of the economy are given by:

$$\left(\sum_{n'\in S_n^r} \alpha_{nn'}\right) \hat{R}_{nt} = \hat{A}_{nt} + \hat{P}_{nt} - \alpha_{nl}(1-\chi)\hat{M}_t + \sum_{n'\in S_n^r} \alpha_{nn'}\mathbb{E}[\hat{R}_{nt}|\hat{P}_{n't}] - \sum_{n'\in[N]} \alpha_{nn'}\hat{P}_{n't} \quad (121)$$

$$\hat{R}_{nt} = s_{nc}\hat{M}_t + \sum_{n':n\in S_{n'}^r} s_{nn'}\mathbb{E}[\hat{R}_{n't}|\hat{P}_{nt}] + \sum_{n':n\in S_{n'}^f} s_{nn'}\hat{R}_{n'} \quad (122)$$

We suppress dependence on time indices for notational simplicity. We guess that there exists a solution to the above system of equations in which $\hat{R}_{nt}$ is linear in shocks:

$$\hat{R}_n = \chi_{nM}^R \hat{M} + \sum_{n'\in[N]} \chi_{nn'}^R \hat{A}_{n'} \quad (123)$$

for scalars $\{\chi_{nn'}^R\}_{n'\in[N]}$ and χ_{nM}^R. We similarly guess that prices are linear in shocks:

$$\hat{P}_n = \chi_{nM}^P \hat{M} + \sum_{n'\in[N]} \chi_{nn'}^P \hat{A}_{n'} \quad (124)$$

We further assume that $\chi_{nM}^P > 0$ for all $n \in [N]$, a guess that will be verified later. Observe that due to the log-normality of the aggregate shocks, we have that:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{\tilde{n}}] = \frac{\text{Cov}(\hat{R}_n, \hat{P}_{\tilde{n}})}{\text{Var}(\hat{P}_{\tilde{n}})}\hat{P}_{\tilde{n}} \quad (125)$$

for $n, \tilde{n} \in [N]$. Further, observe that if $u_n \to 0$, then:

$$\mathbb{E}[\hat{R}_n|\hat{P}_{\tilde{n}}] = \frac{\chi_{nM}^R}{\chi_{\tilde{n}M}^P} P_{\tilde{n}} \quad (126)$$

We substitute our guess in Equation 122 and collect coefficients on $\hat{M}$:

$$\chi_{nM}^R = s_{nc} + \sum_{n':n\in S_{n'}^r} s_{nn'}\chi_{n'M}^R + \sum_{n':n\in S_{n'}^f} s_{nn'}\chi_{n'M}^R \quad (127)$$

We can rewrite this in matrix form as:

$$\chi_M^R = (\mathbf{I}-\mathbf{S})^{-1}(\mathbf{I}-\mathbf{S})1 = 1 \quad (128)$$

where χ_M^R is the N-sized vector of χ_{nM}^R. Hence, $\chi_{nM}^R = 1$ for all $n \in [N]$ is the unique solution to this equation when $u_{nt} \to 0$. We now need to solve for χ_{nM}^P to obtain the firms' demand schedules. Using Equation 121 and χ_{nM}^R, we obtain:

$$0 = \chi_{nM}^P - \alpha_{nl}(1-\chi) - \sum_{n' \in [N]} \alpha_{nn'} \chi_{n'M}^P \tag{129}$$

which can be expressed in matrix form as:

$$\chi_M^P = 1 - \chi \tag{130}$$

where χ_M^P is the N-sized vector of χ_{nM}^P. Note this vector has strictly positive elements given the assumption that $\chi < 1$. Hence, using the definition of a demand schedule, we have: $\omega_{nn'} = 1 - \frac{1}{\chi_{n'M}^P}$. The linearity of revenues and prices then follows from Proposition 3. Finally, note that

$$\frac{\mathrm{d}\hat{\mathcal{C}}_t}{\mathrm{d}\hat{M}_t} = 1 - \boldsymbol{\gamma}' \hat{P}_t = 1 - (1-\chi) = \chi \tag{131}$$

This proves the first statement of the Theorem.

We now prove the second statement of the Theorem. We guess that there exists a solution to the system of equations 121 and 122:

$$\hat{R}_n = \chi_{nM}^R \hat{M} + \sum_{n' \in [N]} \chi_{nn'}^R \hat{A}_{n'} \tag{132}$$

for scalars $\{\chi_{nn'}^R\}_{n' \in [N]}$ and χ_{nM}^R. We similarly guess that prices are linear in shocks:

$$\hat{P}_n = \chi_{nM}^P \hat{M} + \sum_{n' \in [N]} \chi_{nn'}^P \hat{A}_{n'} \tag{133}$$

We now guess that $\chi_{nn'}^R = 0$ for all $(n, n') \in [N]^2$. Using the log-linearity of the shocks, this implies that:

$$\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = \frac{\chi_{nM}^R \chi_{\tilde{n}M}^P \sigma_M^2}{\mathrm{Var}\left(\sum_{n' \in [N]} \chi_{\tilde{n}n}^P A_{n'}\right) + \chi_{\tilde{n}M} \hat{M}} \tag{134}$$

Note that if $u_n \to \infty$, $\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = 0$ if $\chi_{\tilde{n}n}^P > 0$, a condition which we verify momentarily. Using Proposition 3, we observe that $\mathbb{E}[\hat{R}_n | \hat{P}_{\tilde{n}}] = 0$ is equivalent to $\omega_{nn'} = 1$ for all $(n, n') \in [N]^2$. From Proposition 3, we have that $\mathbf{S}(\omega^*) = \mathbf{0}$ for $\omega^* = 1$ (and so revenues are indeed independent of productivity shocks). This verifies the guess that $\chi_{nn'}^R = 0$. We now need to verify that $\chi^P_{n'n} > 0$, which will prove that $\mathbb{E}[\hat{R}_n|\hat{P}_n] = 0$. Note that

$$(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k \tag{135}$$

Hence, the pass-through of a productivity shock from sector n to all other sectors is non-zero if, for each n', there exists some finite $k \in \mathbb{N}$ such that $[\mathbf{A}^k]_{nn'} > 0$. By Theorem 8.3.5 in Meyer (2023), this condition is satisfied if and only if the matrix $\mathbf{A}$ is irreducible. Finally, note that

$$\frac{\hat{\mathcal{C}}_t}{\hat{M}_t} = -\boldsymbol{\gamma}'\hat{P}_t = \boldsymbol{\gamma}'(\mathbf{I} - \mathbf{A})^{-1} \tag{136}$$

This proves the second statement of the Theorem.

□

A.12 Proof of Proposition 5

Proof. Substituting the first-order response of revenues into Equation 52, we obtain:

$$\omega^* = 1 - \alpha_{nn}(1 - \omega^*_{nt}) - (1 - \alpha_{nn})\frac{\text{Cov}(\hat{M}_t, \hat{P}_{nt})}{\text{Var}(\hat{P}_{nt})} \tag{137}$$

From Lemma 2, we have that

$$\hat{P}_{nt} = \frac{1}{1 + \alpha_{nn}(1 - \omega_{nt})}\left[-(1 - \alpha_{nn})^{-1}\hat{A}_{nt} + (1 + \alpha_{nn} - \chi)\hat{M}_t\right] \tag{138}$$

Substituting for $\hat{P}_{nt}$ into Equation 137 yields the desired expression.

□

B Extensions

B.1 Equivalence to Bilateral Contracting

The model in the main text assumed that input purchases are made under incomplete information in a spot market. Below, I show that the informational structure in the main text arises *endogenously* in a model in which intermediate good producers negotiate input purchases with final good producers bilaterally. Formally, these *cost-contingent* bilateral contracts implement demand schedules that are equivalent to Proposition 1.

Contracting under Uncertainty. Intermediate good producers in negotiate bilaterally with final good producers to purchase inputs $X_{in,n',t}$. The structure on technology and demand for the firm is otherwise the same as in Section 2. We assume that the final good producer's cost of producing that input (given by P_{nt} in light of Equation 6) is potentially stochastic, but is *ex-post* verifiable. This assumption is motivated by the fact that a company's costs can often easily verified at the time of input delivery takes place.[5]

Prior to the realization of $\{A_{nt}\}_{n\in[N]}$ and M_t, the firm and the supplier negotiate a cost-contingent transfer $\tau_{in,n't}(P_{n't})$ as well as cost-contingent input delivery $X_{in,n',t}(P_{n't})$ via Nash bargaining. The firm and supplier therefore choose transfers and input deliveries to maximize the generalized joint surplus:

$$\left(\mathbb{E}_t\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\left(\tilde{\Pi}_{in,t}(\mathbf{P}_t)\right)\right]-O_f\right)^{\beta}\left(\mathbb{E}_t\left[\frac{1}{\mathcal{P}_t\mathcal{C}_t}\left(\tau_{in,n',t}(P_{n't})-P_{n't}X_{in,n',t}(P_{n't})\right)\right]-O_s\right)^{1-\beta} \tag{139}$$

where β parameterizes the firm's bargaining power, and O_f, O_s are (exogenous) positive constants that capture the firm's and supplier's disagreement value, respectively. The variable $\tilde{\Pi}_{in,t}$ denotes the firm's nominal profits post-transfer:

$$\tilde{\Pi}_{in,t}(\mathbf{P}_t)=(1+\tau_n)P_{nt}Q_{nt}^{\frac{1}{\eta_n}}\left[q_{in,t}(\mathbf{P}_t)\right]^{\frac{\eta_n-1}{\eta_n}}-w_tL^d_{in,t}-\sum_{k\in[N]}\tau_{in,k,t}(P_{k't}) \tag{140}$$

Note that profits depend on the entire vector of costs $\mathbf{P}_t$ through input deliveries (thereby changing the quantity produced $q_{in,t}$) and transfer payments made to final good producers. However, the contract between firm in and final good producer in sector n' is only contingent on final good producer n''s costs and not the on the costs of other sectors. This is an intuitive theoretical and practical restriction: the terms of trade of the contract do not depend on the costs of *other* suppliers.

We can solve this contracting problem for the *functions* $X_{in,n',t}(P_{n't})$ and $\tau_{in,n',t}(P_{in,n',t})$ using variational methods. The proposition below characterizes the optimal cost-contingent input production:

[5] Holthausen and Leftwich (1983) argue that firms that engage in bilateral contracting change their accounting practices so as to make their costs more transparent and reduce monitoring costs. Moreover, Bajari and Tadelis (2001) find little evidence that adverse selection is an influential component of the contracting process in the engineering and construction industry. They write:

> *While carefully examining the literature and speaking with industry participants, we have found little evidence that either the contractor or the buyer has private information at the onset of a procurement project.*

Proposition 6. *The optimal cost-contingent contract specifies production as a function of input costs given by:*

$$\log x(P) = \omega_{0,nn't} - \omega_{nn't} \log P_{n't} \tag{141}$$

where $\omega_{0,nn't} \in \mathbb{R}$ *is a constant independent of* $P_{n't}$ *and the responsiveness of inputs to the supplier's costs is given by:*

$$\omega_{nn't} = 1 - \frac{\text{Cov}(\log R_{nt}, \log P_{n't})}{\text{Var}(\log P_{n't})} \tag{142}$$

where R_{nt} *are the total (common) revenues of intermediate good firms in sector* n.

Proof. See Appendix C.1 □

The *responsiveness* of total inputs purchased to supplier *costs* is therefore the same as the responsiveness of total inputs to *prices* considered in the main text and formalized in Proposition 1. Note that this responsiveness is independent of firms' relative bargaining power. Intuitively, the efficiency property inherent in Nash bargaining implies that firms will choose inputs to maximize total revenues minus costs. Of course, how firms split the surplus amongst each other will depend on firms' relative bargaining power. Nevertheless, because all firms are owned by households, this split (mediated by the transfer $\tau_{in,n',t}$) is unimportant for *real* outcomes. The basic properties of the demand schedule in Proposition 1 are therefore a general feature of both spot market and arms-length transactions.

In light of Proposition 3, the responsiveness $\omega_{in,n't}$ (in conjunction with firms' revenue shares) is a sufficient statistic that disciplines the first-order effect of productivity and demand shocks to aggregates. Hence, all theoretical results in the main text go through when one "microfounds" the model's incomplete information structure through contract incompleteness.

Discussion of Contracting Assumptions. The contingency assumption on contracts in this section were based on theoretical and empirical grounds. The preceding discussion assumed that the contract is contingent on the supplier's costs, but not on other stochastic variables (such as the supplier's revenues).[6] This assumption is meant to capture the idea that the supplier's costs can be verified at the time of input delivery, while other random variables (such as the firm's revenues from the use of those inputs) are still uncertain. Of course, *in theory*, there is nothing that prevents the firm and the supplier from specifying a fully contingent contract ex-ante which would allow the parties to implement the perfect information allocation ex-post. Such contracts, however, are complex and costly to write

[6] Note that the contract can be contingent on the firm's and the supplier's *beliefs* about these variables.

(Battigalli and Maggi, 2002) and are therefore not commonly observed in practice. Indeed, the "vast majority" of contracts in the building and construction industry, for example, are "simple" contracts of a *cost-contingent* nature (Bajari and Tadelis, 2001). Moreoever, the literature suggests that simple cost-contingent contracts are widely used in other sectors, such as air force engine procurement (Crocker and Reynolds, 1993), defense (Hiller and Tollison, 1978), or the Indian software industry (Banerjee and Duflo, 2000).

B.2 Arbitrary Flexible Inputs

Characterizing Prices with Arbitrary Flexible Inputs. This section develops the results for general rigid subsets $\mathcal{S}_n^r$. Note that the demand schedule for flexible inputs is given by:

$$X_{in,n',t} = \alpha_{nn'} \frac{R_{nt}}{P_{n't}} \tag{143}$$

for $n' \in \mathcal{S}_n^f$. We therefore obtain the following alternative to Proposition 2.

Proposition 7. *Equilibrium prices $\{P_{nt}\}_{n\in[N]}$ and revenues $\{R_{nt}\}_{n\in[N]}$ satisfy the following system of equations:*

$$R_{nt} = \tilde{c}_{nt} A_{nt} P_{nt} \left(\frac{R_{nt}}{M_t^{1-\chi_n}} \right)^{\alpha_{nl}} \prod_{n' \in \mathcal{S}_n^r} \left(\frac{\mathbb{E}_t \left[M_t^{-1} R_{nt} \middle| P_{n't} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{n't} \middle| P_{n't} \right]} \right)^{\alpha_{nn'}} \prod_{n' \in \mathcal{S}_n^f} \left(\frac{R_{nt}}{P_{n't}} \right)^{\alpha_{nn'}} \tag{144}$$

$$R_{nt} = \gamma_n \iota_t M_t + \sum_{n' : n \in \mathcal{S}_{n'}^r} \alpha_{n'n} P_{nt} \frac{\mathbb{E}_t \left[M_t^{-1} R_{n't} \middle| P_{nt} \right]}{\mathbb{E}_t \left[M_t^{-1} P_{nt} \middle| P_{nt} \right]} + \sum_{n' : n \in \mathcal{S}_{n'}^f} \alpha_{n'n} R_{n't} \tag{145}$$

where $\tilde{c}_{nt} = (\iota_t \phi_n)^{\alpha_{nl}(1-\chi_n)} (w_{nt-1})^{\alpha_{nl}\chi_n}$.

Proof. Follows from the proof of Proposition 2. □

We can also characterize the dynamics of the system as we did in the main text. In addition to the revenue share matrix $\mathbf{S} = [s_{nn'}]$ considered in the main text, we also define the rigid revenue share matrix as:

$$\mathbf{S}^r \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}_{n'}^r\}] \tag{146}$$

We also define the flexible revenue share matrix as:

$$\mathbf{S}^f \equiv [s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}_{n'}^f\}] \tag{147}$$

We can also similarly define the *rigid* input-output matrix as $\mathbf{A}^r = [\alpha_{nn'} \times \mathbb{1}\{n' \in S_n^r\}]$ and the flexible input out matrix $\mathbf{A}^f = \mathbf{A} - \mathbf{A}^r$. Finally, we define the *demand-adjusted* input-output matrix and revenue-share matrix as $\mathbf{A}^r(\omega_t) = [\alpha_{nn'} \times \omega_{nn't} \times \mathbb{1}\{n' \in S_n^r\}]$ and $\mathbf{S}(\omega_t) = [s_{nn'} \times (1 - \omega_{n'nt}) \times \mathbb{1}\{n \in \mathcal{S}_{n'}^r\}]$.

Proposition 8. *Assume* $\sum_{n' \in [N]} s_{nn'} \times \mathbb{1}\{n \in \mathcal{S}_{n'}^f\} < 1$. *Then, the first-order response of demand and productivity shocks to prices and revenues is given by:*

$$\mathbf{Z}(\omega_t)\hat{P}_t = -\hat{A}_t + [(1-\chi)(\mathbf{I} - \mathbf{A}) + \text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - (\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1}))]\hat{M}_t \tag{148}$$

$$\hat{R}_t = (\mathbf{I} - (\mathbf{I} - \mathbf{S}^f)^{-1}\mathbf{S}^r)\hat{M}_t + (\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1}))\hat{P}_t \tag{149}$$

where the propagation matrix $\mathbf{Z}(\omega_t)$ *is given by*

$$\mathbf{Z}(\omega_t) = \underbrace{\mathbf{I} - \mathbf{A}^r(\omega_t) - \mathbf{A}^f}_{\substack{\text{demand-adjusted} \\ \text{Leontief matrix}}} - \underbrace{\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})}_{\substack{\text{revenue impact} \\ \text{matrix}}} \tag{150}$$

Proof. Follows from the proof of Proposition 3. □

Note that we obtain Proposition 3 by setting $\mathbf{S}^f = \mathbf{0}$ and $\mathbf{A}^f = \mathbf{0}$. It is instructive to consider the general equilibrium transmission of shocks to understand this more general result, as we have done in the main text. Consider how prices respond to a productivity shock. First, a productivity shock directly lowers prices by lowering firms' marginal costs. In turn, this reduction in prices lowers marginal costs further through input-output linkages: a reduction in the price of flexible input n' for firm n reduces the firm's marginal cost by the share of that input in the production function $[\mathbf{A}^f]_{nn'}$. In contrast, a reduction in the price of rigid input n' for firm n reduces the firm's marginal cost by the share of that input in the production function *multiplied* by firm n's demand slope for that input: $[\mathbf{A}^r(\omega_t)]_{nn'}$. The cumulative effect for the reduction in prices that occurs through this channel is therefore $[\mathbf{I} - \mathbf{A}^r(\omega_t) - \mathbf{A}^f]^{-1}$, which the standard inverse Leontief matrix, adjusted for firms' demand schedule slope.

This reduction in prices, however, will also affect a firm's revenues by changing the quantity demanded of its product. From Equation 149, the impact of a change in prices on firms' revenues is given by the matrix $(\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})$. If the elasticity of firm's demand schedules in not equal to unity, this effect will be non-zero. This change in revenues will in turn increase the quantity of flexible inputs that a firm purchases through its flexible and rigid demand schedules. However, this quantity adjustment will not occur one-to-one for all inputs if some of the firm's inputs are rigid. For this reason, marginal costs will increase by

$\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r(\omega_t)\mathbf{1})$, where $\text{diag}(\mathbf{A}^r\mathbf{1})$ captures the extent of these rigidities in a firm's production function. The *total* change in prices in response to a productivity shock is therefore parameterized by the firm's demand-adjusted Leontief matrix, as well as the revenue impact matrix described above. A similar propagation mechanism characterizes the economy's response to a monetary shock, with the key difference being that a monetary shock directly raises firms' marginal costs by a factor of $(1-\chi)(\mathbf{I}-\mathbf{A})$ (through wages) and directly affects firm revenues by a factor $\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1}))$ (through nominal expenditures).

General Propagation Mechanism. For a common responsiveness parameter $\omega_{nn't}=\omega^*$, we can write the propagation matrix $\mathbf{Z}(\omega^*)$ as

$$\mathbf{Z}(\omega^*)=\mathbf{I}-\omega^*\mathbf{A}-(1-\omega^*)\left(\mathbf{A}^f+\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\right) \tag{151}$$

Under the assumption that $\mathbf{Z}(\omega^*)$ is an M-matrix, we can write the effect of a productivity shock on prices as the inverse of this matrix given by:

$$\sum_{k=0}^{\infty}\left[\omega^*\mathbf{A}+(1-\omega^*)\left(\mathbf{A}^f+\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\right)\right]^k \tag{152}$$

In this more general set-up, we see that the effect of productivity shocks to output is given by a convex combination of the economy's input-output matrix and its revenue impact matrix. When $\omega^*=1$, we recover the complete information result that the effect of productivity shocks on prices is given by the Leontief inverse matrix $\mathbf{L}=(\mathbf{I}-\mathbf{A})^{-1}$.

The below corollaries provide a counterpart to Corollary 1 in recovering the complete information pass-through when $\chi=0$.

Corollary 3. *Suppose $\chi=0$ and $\omega^*=1$. Then, the first-order effect of productivity and demand on prices and output is given by:*

$$\hat{P}_t=-(\mathbf{I}-\mathbf{A})^{-1}\hat{A}_t+\left(\mathbf{I}+(\mathbf{I}-\mathbf{A})^{-1}\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\right)\hat{M}_t \tag{153}$$

$$\hat{Q}_t=(\mathbf{I}-\mathbf{A})^{-1}\hat{A}_t-(\mathbf{I}-\mathbf{A})^{-1}\text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I}-\mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\hat{M}_t \tag{154}$$

Proof. Follows directly from Proposition 8 □

Hence, when all demand schedules have unitary elasticity, the pass-through of productivity shocks to prices is equal to its full information benchmark and given by the inverse Leontief matrix. In contrast to its full information counterpart, however, demand shocks are no longer neutral: when there are no wage rigidities, demand shocks have a *contractionary* effect on output, where this response is mediated through the inverse Leontief matrix and the extent of input rigidities.

We may also consider the other "extreme" case in which demand schedules are not responsive to price changes: $\omega^* = 0$:

Corollary 4. *Suppose $\chi_n = 0$ for all $n \in [N]$ and $\omega_{nn't} = 0$ for all $(n, n') \in [N]^2$. Then, the equilibrium dynamics of the economy are given by:*

$$\hat{P}_t = -\left(\mathbf{I} - \mathbf{A}^f - \text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\right)^{-1}\hat{A}_t + \hat{M}_t \tag{155}$$

$$\hat{Q}_t = (\mathbf{I} - (\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1}))\left(\mathbf{I} - \mathbf{A}^f - \text{diag}(\mathbf{A}^r\mathbf{1})(\mathbf{I} - \mathbf{S}^f)^{-1}\text{diag}(\mathbf{S}^r\mathbf{1})\right)^{-1}\hat{A}_t \tag{156}$$

Proof. Follows directly from Proposition 8 □

Hence, if firm's demand schedules are not responsive to prices, demand shocks are *neutral* – they have no effect on output and pass through one-to-one to prices. Intuitively, firms' production decisions do not respond to price changes that arise from nominal wage changes (and therefore changes in nominal money balances). Of course, this implies that firms also do not respond to price changes that arise from variation in marginal costs due to productivity shocks. For this reason, the responsiveness of output to productivity shocks is no longer given by the standard inverse Leontief matrix. The demand schedule of firms' rigid inputs can shape the transmission mechanism of productivity and demand shocks in qualitatively important ways.

C Additional Proofs

C.1 Proof of Proposition 6

Proof. We first note that all firms in each sector continue to be homogeneous. Hence, we can write firms' profits as

$$\tilde{\Pi}_{in,t}(\mathbf{P}_t) = (1 + \tau_n)R_{nt} - w_t L^d_{in,t} - \sum_{n' \in [N]} \tau_{in,n',t}(P_{n't}) \tag{157}$$

where $R_{nt} = P_{nt}Q_{nt}$ are sector-level revenues (pre-subsidy).

Suppose now that a given contract $\{\tau^*_{in,n',t}(P_{n't}), X^*_{in,n',t}(P_{n't})\}$ is optimal. Consider a variation $\tilde{\tau}_{in,n',t}(P_{n't}) = \tau^*_{in,n't}(P_{n't}) + \varepsilon h(P_{n't})$ for some $\varepsilon > 0$. The logarithm of the joint surplus generated from this variation is given by

$$
\begin{aligned}
J(\varepsilon, h) = \beta \log \mathbb{E}_t & \left(\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t) - \varepsilon h(P_{n't})) - O_f \right) \\
& + (1-\beta) \mathbb{E}_t \left(\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) + \varepsilon h(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right)
\end{aligned} \tag{158}
$$

A necessary condition for optimality is that $J_\varepsilon(\varepsilon, h)$ achieves its maximum at $\varepsilon = 0$ for all $h(P_{nt})$. Hence, we have the following necessary first-order condition:

$$
\frac{-\beta}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t)) - O_f \right]} + \frac{(1-\beta)}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right]} = 0 \tag{159}
$$

which holds for all $h(P_{nt})$. We can also consider an identical variation for the input delivery $\tilde{X}_{in,n't} = X^*_{in,n',t}(P_{n't}) + \varepsilon h_x(P_{nt})$. The necessary first-order condition that is obtained from this variation is:

$$
\frac{-\beta \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} h_x(P_{n't}) \right]}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tilde{\Pi}_{in,t}(\mathbf{P}_t)) - O_f \right]} + \frac{(1-\beta) \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) h_x(P_{n't}) \right]}{\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} (\tau^*_{in,n',t}(P_{n't}) - P_{n't} X^*_{in,n',t}(P_{n't})) - O_s \right]} = 0 \tag{160}
$$

Combining the two first-order conditions yields the necessary condition

$$
\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} h_x(P_{n't}) \right] = \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) h_x(P_{n't}) \right] \tag{161}
$$

This is true for any function $h_x(P_{n't})$. We can therefore consider the Dirac function $\delta_{\hat{P}_{n't}}$ for some $\hat{P}_{n't} \in \mathbb{R}_{++}$. We then obtain

$$
\mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} \alpha_{nn'} R_{nt} \middle| \hat{P}_{n't} \right] = \mathbb{E}_t \left[\frac{1}{\mathcal{P}_t \mathcal{C}_t} P_{n't} X^*_{in,n',t}(P_{n't}) \middle| \hat{P}_{n't} \right] \tag{162}
$$

Since $X^*_{in,n',t}$ is adapted to $P_{n't}$ by assumption, we obtain

$$
X^*_{in,n',t}(\hat{P}_{n't}) = \frac{\mathbb{E}_t \left[(\mathcal{P}_t \mathcal{C}_t) R_{nt} | \hat{P}_{n't} \right]}{\mathbb{E}_t \left[(\mathcal{P}_t \mathcal{C}_t) P_{n't} | \hat{P}_{n't} \right]} \tag{163}
$$

This traces out a demand schedule for any realization of P_{nt}. The form of the demand schedule then follows directly from Proposition 1. □