How Tariffs Affect Trade Deficits*

Iván Werning ⓡ Arnaud Costinot ⓡ
MIT

We study the positive (not normative) effect of a permanent import tariff on trade deficits. We consider a two-period trade model with general preferences and technology. We first develop an aggregation result showing one can work with induced preferences over aggregate imports and exports. Our main results provide conditions under which tariffs are neutral as well as conditions under which they reduce deficits. A central theme is that the *static* Engel curve for aggregate imports and exports holds the key to this *dynamic* question.

1 Introduction

How do import tariffs affect trade imbalances? Setting welfare consequences aside, can a permanent increase in tariffs reduce an ongoing trade deficit?

The answer depends on whom you ask. Politicians and the general public often assume that tariffs, by discouraging imports, will narrow the trade deficit. The great trade policy disaster of the 1930s is a case in point. As Irwin (2011) convincingly argues, trade wars over that period were not primarily driven by lobbying and other forms of redistributive politics, but rather by countries' desire to correct trade imbalances via a rise in trade protection. The "reciprocal tariffs" put forward on April 2, 2025 by the Trump administration seem to derive from a similar belief that an increases in US tariffs can lower trade deficits by choking off imports.

Economists are quick to point out that this is only part of the story. Everything else being equal, tariffs may reduce imports, but why would exports be unaffected? Trade economists may note that import tariffs are equivalent to export taxes, an expression of Lerner symmetry (Costinot and Werning, 2019). Macroeconomists may add that, following textbook analyses, trade imbalances are fundamentally shaped by national savings and investment decisions that are orthogonal to trade policy. Tariffs affect the extent and

*This version: June 2025. Author contacts: costinot@mit.edu and iwerning@mit.edu. We thank Ariel Burstein, Pablo Fajgelbaum, and Jon Vogel for helpful questions and comments and Kazuatsu Shimizu for valuable research assistance. All remaining errors are ours.

nature of intratemporal trade, but the trade balance issue is one of intertemporal trade (Economic Expert Panel, 2025).[1]

Intuitions aside, formal analyses of the impact of tariffs on trade imbalances are scarce. An early exception using an intertemporal approach is Razin and Svensson (1983). They pointed out that temporary tariffs do not satisfy Lerner symmetry and differentially affect the cost of living over time, which then affects borrowing and lending. Relatedly, if exogenous economic conditions are not stationary and, instead, vary over time, then even a permanent tariff may have a different incidence on the cost of living today versus tomorrow, with implications for trade imbalances.

The question we address in this paper is whether one might still expect a systematic effect of permanent tariffs on the trade deficit in a stationary scenario. Our main finding is that, perhaps surprisingly, there may indeed be such a systematic effect. Our analysis is based on the notion that even if economic primitives are unchanged between today and tomorrow, the fact that a country is currently running a trade deficit implies that aggregate consumption is different today and tomorrow. This opens the door, endogenously, for the non-neutrality of a permanent tariff on the trade deficit.

We consider a neoclassical trade model over two periods. We allow for an arbitrary number of goods, general preferences and general technology. Imports may be used as final goods or as inputs into production. International prices are taken as given in our baseline model; we later extend the analysis. The government levies a uniform tariff on all imports in both periods and rebates the revenue back to households. We assume expectations are rational, so there is perfect foresight.

The starting point of our analysis is a new aggregation result. We show that in each period, one can summarize all the relevant implications of our general trade model for trade deficits into a preference relation over aggregate imports and aggregate exports only. The existence of this aggregate preference relationship turns out to be key to simplify our analysis and to generate novel insights.

As a warm up, we provide sufficient conditions for two extreme scenarios: neutral tariffs and autarky-inducing tariffs. The trade balance is locally unaffected by tariffs in an economy with preferences that are appropriately homothetic when the equilibrium feature strictly positive imports and exports of all goods, with no non-tradables. These assumptions are strong and, it turns out, always violated for large enough tariffs. Indeed, we prove that large enough tariffs drive the economy to autarky and thus, in the extreme,

[1] Link to the Clark Center Economic Expert Panel Poll: `https://www.kentclarkcenter.org/surveys/tariffs-reciprocal-and-retaliatory-2/`. Expressions of the conventional wisdom that tariffs are unlikely to affect trade deficits can also be found in Baldwin (2024) and Krugman (2024).

reduce trade deficits to zero.

As our main contribution, we then offer a general analysis of the impact of tariffs around free trade. We show that whether a tariff reduces a trade deficit depends on a single sufficient statistic: the slope of the Engel curve for aggregate imports and exports. If this slope is higher in the first period, then tariffs increase the marginal cost of consumption disproportionately more in this period. This raises the real domestic interest rate, which creates incentives to save and reduces the deficit. When all economic primitives are fixed over time, and only aggregate consumption varies across period, the slope of the Engel curve today versus tomorrow is determined by the curvature of the Engel curve, i.e. whether aggregate imports are a luxury good. If the Engel curve is linear, then they are not, and tariff neutrality holds. If the Engel curve is strictly convex, then they are, and tariffs reduce trade deficits.

This begs the question: What determines the curvature of the Engel curve? One possibility focuses entirely on the non-homotheticity of preferences over goods. If consumers' preferences are such that imported goods have elasticities higher than one, then the Engel curve will tend to be convex. Interestingly, though, non-homothetic preferences over individual goods are not necessary.[2] It is so because the relevant preferences over aggregate imports and exports also capture technological considerations.

In particular, we show that the curvature of the Engel curve may capture an active extensive margin of trade. In the context of a general Armington model with fixed export and import prices, the Engel curve turns out to be linear if there is no action at the extensive margin, no shifting of goods between imported and non-traded, or between non-traded and exported (explaining our earlier neutrality result). In contrast, under the same assumptions, when goods do shift between these categories, the Engel curve becomes strictly convex, implicitly revealing changes in the prices of these goods. Likewise, differences in the slope of the Engel curve across periods may arise from terms-of-trade considerations. In the context of a Ricardian model, home-bias creates another systematic relationship between aggregate consumption and prices and, in turn, another reason for tariffs to incentivize savings and reduce trade deficits.

We conclude with a series of analytical and simulation results focusing on large tariffs. Away from free trade, tariffs create distortions that are no-longer second order. This creates a wealth effect distinct from the substitution effect that we have emphasized up to this point. Interestingly, when only this distortion channel is active, tariffs again tend to reduce trade deficits. They do so by making the economy poorer in both periods, but

[2] In the case of the United States, Borusyak and Jaravel (2021) document that import shares are flat throughout the income distribution.

disproportionately less so when aggregate consumption is high. Quantitatively, we explore the range of possible effects that might emerge under typical calibrations, both in the case of a general Armington economy (with fixed prices) and a Ricardian economy (with terms-of-trade effects). Our simulation results suggest that very different predictions about the impact of tariffs on deficits may arise from models that have been calibrated to the same levels of openness to trade, both intra and inter-temporally, as well as the same trade elasticity. In line with our analytical results, these differences arise because different models imply very different Engel curves. We are not aware of empirical work that has directly estimated the slope of Engel curves for aggregate imports and exports, but we hope that our analysis may provide further motivation.

Related Literature

In their influential work, Obstfeld and Rogoff (2000) have shown that permanent trade costs may dampen both intratemporal and intertemporal trade. Our analysis elevates and extends their mechanism and applies it to the case of import tariffs. In the context of a two-good endowment economy, Obstfeld and Rogoff (2000) show that trade costs create a wedge between the real interest rates faced by borrowing and lending countries. Furthermore, they show that the magnitude of this wedge is larger when trade imbalances are larger as well. Based on these two observations, they argue that trade costs, by creating this wedge, may keep trade imbalances in a modest range, thereby explaining the high correlation between domestic savings and investment and offering a solution to the Feldstein-Horioka puzzle.[3] Although Obstfeld and Rogoff (2000) never formally study how trade costs or tariffs affect trade imbalances, the interest rate channel that they emphasize is at the heart of our analysis. As we show, the shape of the Engel curves encodes all the information that is relevant for the interest rate channel.

Motivated by Obstfeld and Rogoff's original work, Eaton, Kortum and Neiman (2016) offer a quantitative exploration of the role of trade costs. At their preferred calibration, the find that very large changes in trade costs, going all the way to zero trade costs, can raise the US trade deficit to 20% of US GDP. Using a related model, Reyes-Heroles (2016) concludes that US trade deficits observed in the late 2000s could have been three times smaller absent his estimated changes in trade costs. Although we study a tariff, not iceberg trade costs, our analysis can be applied with some adjustments. In particular, it highlights the (implicit) role played by the shape of the Engel curves at different points in

[3] A similar emphasis on the relationship between trade costs and trade imbalances can be found in Dornbusch (1983). In his paper, it is the existence of non-tradable goods that create a wedge between domestic and world real interest rates whose magnitude varies with aggregate consumption.

time, driven by terms-of-trade effects in the context of a Ricardian model.[4]

A related literature centered on wealth effects discusses the impact of terms-of-trade shocks, such as oil shocks, either temporary or permanent, on the current account. Classic references include Harberger (1950) and Laursen and Mezler (1950). Obstfeld (1982) and Svensson and Razin (1983) offer formal treatments of this issue. We show that if wealth effects derive from tariff distortions, they create another systematic reasons why permanent tariffs can reduce deficits, even in a stationary environment.

Recent events have created a surge of interest in the macroeconomic implications of tariffs and their impact on trade deficits in particular. Aguiar et al. (2025) and Itskhoki and Mukhin (2025) study how tariffs may affect net foreign asset positions. Auclert et al. (2025) consider the effects of a temporary tariff in a New-Keynesian model. Ignatenko et al. (2025) and Rodríguez-Clare et al. (2025) focus on the incidence of the "Liberation Day" tariffs in quantitative trade models with exogenous trade deficits. We propose instead a new aggregate approach to study the effect of a permanent tariff on current trade deficits.

Our crucial aggregation result combines elements of the perspectives put forward by Hicks (1936) and Meade (1952). It emphasizes preferences over exports and imports, as in Meade (1952), and further creates aggregate composites of the two, as in Hicks (1936). This approach allows us to study trade deficits in a tractable way.[5] Despite the fact that we keep preferences and technologies general, our analysis is no more complex than in a simple two-good economy. Through the lens of our aggregation result, richer economies with a continuum of goods and active extensive margins of trade implicitly give rise to non-homothetic induced preferences in the space of aggregate exports and imports, which is what the impact of tariffs on deficits depends on.

2 A Neoclassical Model of Trade Imbalances and Tariffs

To study the causal relationship between tariffs and trade imbalances, we start from a rich static neoclassical trade model and extend it to two periods. A representative household with general preferences makes consumption choices. Production is handled by firms using a general technology, with any number of factors. Imports may be used as final goods

[4]Their quantitative results also derive from the time-varying nature of trade costs, which are falling over the period they consider. Our analysis of permanent tariffs has nothing to say about this issue. Other quantitative analysis of the impact of changes in trade costs on trade balances include Fitzgerald (2012) and Alessandria and Choi (2021).

[5]Different questions may, of course, call for different aggregate approaches. For instance, to study how factor prices respond to changes in trade costs, one may want to construct aggregate preferences over factor services instead, as in Adao et al. (2017).

or as inputs into production. We abstract from capital investment decisions. The government levies a uniform tariff τ on all imports in both periods, and rebates the revenue back to consumers. For now, we consider a small open economy that takes international prices as given and delay the discussion of terms-of-trade effects to subsequent sections.

2.1 Environment

Preferences. The representative household has utility

$$U(C_1, C_2),$$

where U is increasing and concave in aggregate consumption (C_1, C_2). Some of our analysis applies without further restriction on U, but other results rely on U being homothetic, so that the marginal rate of substitution $U_1(C_1, C_2)/U_2(C_1, C_2)$ is a function of C_1/C_2. This is a common benchmark assumption for intertemporal decision problems.

Aggregate consumption C_t is given by an aggregator

$$C_t = G_t(c_t),$$

where c_t is a finite or infinite dimensional vector representing all consumption goods, and G_t is assumed to be increasing and concave. In some cases it will be useful to specialize further and assume that G_t is homogeneous of degree one, so that preferences are homothetic within each period.

Technology. Technology is described by an aggregate production set Y_t for $t = 1, 2$. Given vectors of imports $m_t \geq 0$ and exports $x_t \geq 0$, the domestic consumption vector c_t is feasible if

$$(c_t, m_t, x_t) \in Y_t.$$

This formulation captures general production technologies, using any number of factors of production owned or hired by firms. Domestic firms import m_t as inputs, produce c_t for domestic consumption and x_t for foreign consumption.[6] The three vectors c_t, m_t, and x_t may, in principle, have different dimensions. As is standard in general equilibrium theory, factors can be incorporated into c_t as negative entries or implicitly folded into Y_t.

The setup allows for general trade costs, which may derive from the transportation services associated with international shipping as well as from the specific inputs required

[6] This implicitly assumes that all imports (and exports) are performed by firms, not directly by consumers. This is realistic and without loss of generality, as one can always introduce importing firms that produce final goods one-for-one from imports.

for domestic distribution and retail. A non-tradable good i is one for which technology dictates that $x_{it} = m_{it} = 0$. An economy with so-called iceberg trade costs is a special case where Y_t is given by requiring

$$y_i = c_i + (1 + \delta_{i,x})x_i - m_i/(1 + \delta_{i,m}), \text{ with } y \in \Omega_t, \tag{1}$$

where $\delta_{i,x} \geq 0$ and $\delta_{i,m} \geq 0$ denote exporting and importing costs, respectively, and Ω_t represents a domestic net-production set.[7]

International Prices and Intertemporal Trade Balance. All international prices are fixed. International prices consist of import prices p^*_{mt}, export prices p^*_{xt}, and the world interest rate R^*. Import and export prices may simply differ because imports and exports are different goods—recall that we do not even require m_t and x_t to have the same dimension—or more generally because of foreign tariffs and transportation costs—which create a wedge between the price at which domestic firms can sell a good abroad and the price at which foreign firms would be willing to sell the same good to them.

Evaluated at international prices, the trade deficit in period t is

$$D_t = p^*_{mt} \cdot m_t - p^*_{xt} \cdot x_t.$$

The intertemporal trade balance condition is then

$$D_1 + \frac{1}{R^*} D_2 = NFA, \tag{2}$$

where NFA represents an inherited net foreign asset position, which we take as given.[8]

Tariffs. It is clear that time-varying tariffs may affect the incentives to borrow and lend. Here, we focus on the more interesting situation where tariffs are permanent. That is, all imports are subject to an ad-valorem tariff τ. Each period, tariff revenues are rebated to the representative household via a lump-sum transfer

$$T_t = \tau \times (p^*_{mt} \cdot m_t).$$

[7] In equation (1), both x_i and m_i measure units of good i available at the "foreign dock." To sell x_i units abroad, domestic firms need to produce $(1 + \delta_{i,x})x_i$ units at home. Likewise, if they buy m_i units from abroad, they can only offer consumption of $m_i/(1 + \delta_{i,m})$ units at home. This is one among many possible conventions. For instance, we could have also chosen to measure goods at the "domestic dock," in which case $y_i = c_i + x_i - m_i$, with iceberg trade costs $\delta_{i,x}$ and $\delta_{i,m}$ now folded into export and import prices.

[8] In general, even a small open economy may be subject to revaluation effects, since trade taxes can directly affect domestic prices and, in turn, the value of domestic assets and liabilities. We discuss such considerations in Costinot and Werning (2019).

2.2 Definition of a Competitive Equilibrium

In a competitive equilibrium, households choose (c_1, c_2) subject to their budget constraint, taking as given domestic consumption prices p_{ct}, the world interest rate R^*, firm profits, and government transfers. Firms choose (c_t, m_t, x_t) to maximize profits, taking as given the tariff rate τ as well as domestic consumption prices p_{ct}, import prices p^*_{mt}, and export prices p^*_{xt} as given. Total government transfers T are equal to the present discounted value of tariff revenues. Finally, market clearing in international markets requires the intertemporal budget balance condition (2). These conditions are standard and their formal description is relegated to Appendix A.

Note that a competitive equilibrium not only includes the import and export prices, p^*_{mt} and p^*_{xt}, but also a vector of domestic prices p_{ct} for all goods that enter the vector of consumption c_t. Domestic prices are not necessarily equal to international prices, both because of the tariff τ as well as other trade costs. More importantly, the possibility that some goods are not traded in equilibrium implies that domestic prices may not be fully pinned down by a no-arbitrage condition. In the rest of this section, we develop an aggregate approach that, for the purposes of studying how a change in the tariff rate τ affects trade deficits, will allow us to set aside the endogenous determination of domestic prices.

2.3 Static Equilibrium Conditions: Meade meets Hicks

Our approach focuses on the determination of aggregate imports and exports in a competitive equilibrium. We start here with a discussion of static equilibrium conditions. Dynamic equilibrium conditions will be discussed in Section 2.4. The formal derivation of both sets of conditions can be found in Appendix B. Here, we offer a heuristic argument.

Conditional on the vector of imports and exports, domestic consumption and production choices in a competitive market are efficient. The tariff makes imports artificially more expensive than exports, so imports and exports will not be chosen efficiently overall. However, since the tariff is uniform across imports, it does not affect relative prices within imports nor relative prices within exports and equilibrium choices are efficient within each of these categories. Thus, the competitive equilibrium is efficient conditional on aggregate imports and aggregate exports, even though it is inefficient in its choice of aggregate imports and aggregate exports.

Preferences Over Aggregate Imports and Exports. The previous reasoning suggests to subsume both intra-period preferences G_t and technology Y_t by defining

$$\mathcal{C}_t(M_t, X_t) \equiv \max_{(c_t, m_t, x_t) \in Y_t} G_t(c_t),$$

subject to

$$p^*_{mt} \cdot m_t = M_t,$$

$$p^*_{xt} \cdot x_t = X_t.$$

Here $M_t \geq 0$ and $X_t \geq 0$ are scalars representing the international value of aggregate imports and aggregate exports, respectively. The trade deficit, in turn, is simply

$$D_t = M_t - X_t.$$

All considerations that shape international trade in a given period t, either coming from preferences (G_t), technology (Y_t), or international prices (p^*_{mt}, p^*_{xt}), are encoded in the induced utility function $\mathcal{C}_t$ over (M, X). As we will argue, this is all one needs to know in order to study the impact of tariffs on trade deficits.[9] Figure 1 provides a graphical illustration of the aggregate preferences, with three indifference curves and an Engel curve. The latter is defined as the locus of points (M, X) where $\mathcal{C}_{tM}/\mathcal{C}_{tX}$ is constant. The shape of these static Engel curves, which differ in Figures 1a and 1b, will play a key role below.

Hicksian Demand for Aggregate Imports and Exports. We will make extensive use of the expenditure function associated with $\mathcal{C}_t$,

$$e_t(C, \tau) \equiv \min_{M, X \geq 0} \{(1+\tau)M - X\}$$

$$s.t: \mathcal{C}_t(M, X) \geq C.$$

The static equilibrium conditions require import and exports to solve this problem. We let $M_t(C, \tau)$ and $X_t(C, \tau)$ denote its solution. We can then define the deficit function

$$\mathcal{D}_t(C, \tau) \equiv M_t(C, \tau) - X_t(C, \tau).$$

Under free trade, the expenditure and deficit functions coincide: $e_t(C, 0) = \mathcal{D}_t(C, 0)$. But if tariffs are positive, the deficit function differs from the expenditure function, with $\mathcal{D}_t(C, \tau) = e_t(C, \tau) - \tau M_t(C, \tau)$, a reflection of the distortionary effect of tariff.

[9] Since the existence of the induced utility function $\mathcal{C}_t$ derives from the efficiency of the competitive equilibrium, we conjecture that our analysis extends, without further qualifications, to monopolistically competitive environments in which the equilibrium is also efficient, as in Krugman (1980) or Melitz (2003).

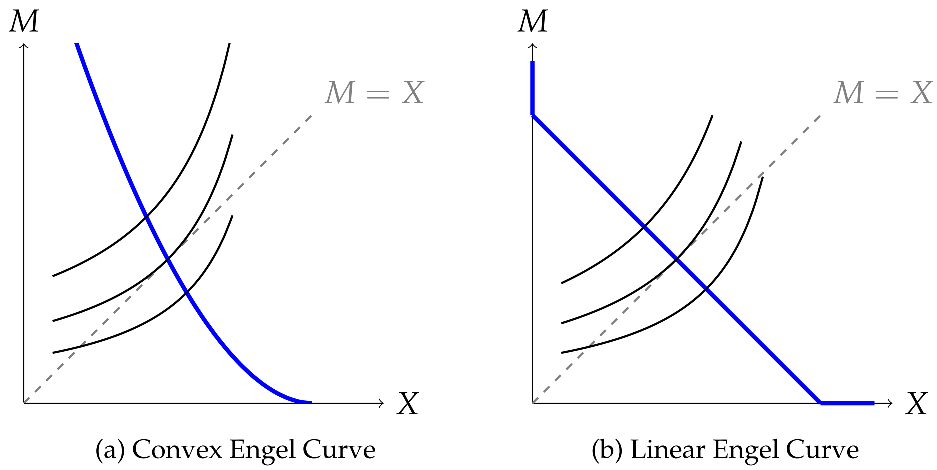


Figure 1: Preferences over Aggregate Imports and Exports

Notes: Each panel plots three indifference curves (in solid black) associated with the utility $\mathcal{C}_t$ over (M, X) along with the Engel curve for $\tau = 0$ (in solid blue) and the zero deficit condition $M = X$ (in dashed grey). In Figure 1a, the Engel curve is strictly convex. In Figure 1b, it is locally linear.

Historical Note. The notion that preferences and domestic production can be combined, exploiting domestic efficiency, follows and extends a perspective introduced by Meade (1952) and further formalized by Dixit and Norman (1980). However, because of the questions they were studying, they worked with the vectors of net imports $x - m$. In contrast, we do not net out and keep imports and exports separate. We do so because the import tariff and other general trade costs may create different prices for the same good depending on whether it is imported or exported. In addition, we aggregate the import vector m to the scalar M and the exports vector x to the scalar X. This reflects our interest in the impact of a uniform tariff τ that affects the price of (all) imports relative to (all) exports, which allows us to apply what Deaton and Muellbauer (1980) refer to as the Composite Commodity Theorem due to Hicks (1936). Thus, our analysis extends and combines elements of Meade and Hicks.

2.4 Dynamic Equilibrium Conditions

Using the expenditure function associated with the static equilibrium conditions, one can show that the household problem is

$$\max_{C_1, C_2} U(C_1, C_2)$$

$$e_1(C_1, \tau) + \frac{1}{R^*} e_2(C_2, \tau) = NFA + T,$$

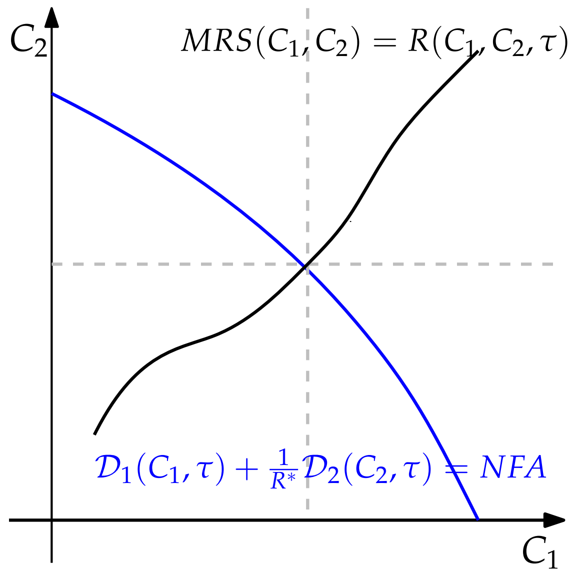


Figure 2: How tariffs affect trade deficits.

Notes: This figure plots the Euler condition 3(in solid black) and intertemporal trade balance condition 4 (in solid blue). The dashed-grey lines represent iso-deficit curves in each period $t = 1, 2$.

where the lump-sum transfer $T = \tau M_1(C_1, \tau) + \frac{1}{R^*}\tau M_2(C_2, \tau))$ is taken as given. Despite the fact that domestic prices p_{ct} enter the original budget constraint of the household, they can be conveniently omitted from the previous problem.

The dynamic equilibrium conditions are then

$$MRS(C_1, C_2) = R(C_1, C_2, \tau), \tag{3}$$

$$\mathcal{D}_1(C_1, \tau) + \frac{1}{R^*}\mathcal{D}_2(C_2, \tau) = NFA, \tag{4}$$

where $MRS(C_1, C_2)$ denotes the marginal rate of substitution between aggregate consumption in the two periods and $R(C_1, C_2, \tau)$ their relative marginal cost, which is also the domestic real interest rate,

$$MRS(C_1, C_2) \equiv U_1(C_1, C_2)/U_2(C_1, C_2),$$
$$R(C_1, C_2, \tau) \equiv R^* e_{1C}(C_1, \tau)/e_{2C}(C_2, \tau),$$

with the convention $e_{tC} \equiv \partial e_t / \partial C$.

We are ready to describe the equilibrium trade deficit $\mathcal{D}_t(\tau)$ in each period $t = 1, 2$ as a function of the tariff τ.

Proposition 1. *For a given tariff τ, the trade deficit in period t is equal to $D_t(\tau) = \mathcal{D}_t(C_t(\tau), \tau)$ with the equilibrium consumption levels $(C_1(\tau), C_2(\tau))$ a solution to* (3) *and* (4).

Proposition 1 encapsulates our aggregate approach. It shows how the preferences $\mathcal{C}_t$

over aggregate imports and exports—as well as its dual e_t and the associated functions $\mathcal{D}_t$, and R—shape the causal relationship between tariffs and trade imbalances in a general neoclassical model. This relationship is illustrated in Figure 2, which plots both the Euler condition (3) (in solid black) and trade balance condition (4) (in solid blue). Given a tariff τ, the equilibrium deficit $D_t(\tau)$ in period t then only depends on the value of aggregate consumption in this period, as captured by the dashed grey lines.

Whether or not tariffs reduce trade deficit boils down to a horse race between two potential effects. First, a change in τ may affect the domestic real interest rate, as captured by the partial derivative $\partial R/\partial\tau$. This is a substitution effect, which corresponds to a shift in the Euler condition. Second, a tariff, because it is distortionary, may also raise the level of the deficit, i.e. the transfer from the rest of the world required, to achieve a given level of consumption, as captured by the partial derivatives $\partial\mathcal{D}_t/\partial\tau$. This is an income effect, which corresponds to a shift in the trade balance condition.

3 Do Tariffs Reduce Trade Deficits?

We now use Proposition 1 to conduct comparative static and study how trade deficits $D_t(\tau)$ vary with the tariff τ.

3.1 Warming Up

We start with two extreme results that illustrate the range of effects that a permanent increase in tariffs may have on trade deficits, from neutral to closing them entirely.

Neutrality. Our first comparative static result provides sufficient conditions under which a marginal tariff increase is neutral around free trade.

Proposition 2 (Neutrality)**.** *Suppose static preferences are homothetic: G_t is homogeneous of degree one; the environment is stationary: $G_1 = G_2$, $Y_1 = Y_2$, $p^*_{m1} = p^*_{m2}$, and $p^*_{x1} = p^*_{x2}$; the equilibrium has each good i either strictly imported ($m_{it} > 0$) or strictly exported ($x_{it} > 0$) subject to iceberg trade costs (equation 1); and the sets of goods imported and exported are the same across periods. Then, starting from free trade ($\tau = 0$), tariffs do not affect trade deficits: $D'_t(\tau) = 0$.*

Proposition 2 resonates well with the broad intuition that unlike temporary tariffs, permanent tariffs may have no effect on a country's incentives to borrow and save. It should also be clear that even a permanent tariff may affect a country's incentives to borrow and save if primitives differ in periods $t = 1$ and $t = 2$. Proposition 2 therefore requires preferences, technology, trade costs, and prices to be stationary.[10] Interestingly, though, the proof of Proposition 2 also requires assumptions about the tradability of different goods. We will explain in details why in the next section. But before doing so, we turn to another extreme scenario in which tariffs are far from neutral.

Autarky. Our next result provides conditions under which a large enough tariff leads to zero aggregate imports, zero aggregate exports, or both. In the special case where both imports and exports are zero, the economy is under autarky and tariffs fully close the deficit. This occurs when $NFA = 0$. Otherwise the economy runs persistent deficits, if $NFA > 0$, or persistent surpluses, if $NFA < 0$, as the next proposition demonstrates.

Proposition 3 (Autarky)**.** *Suppose U and $\mathcal{C}_t$ are such that all aggregate commodities, C_t, M_t, and $-X_t$, are normal; the environment is stationary: $\mathcal{C}_1 = \mathcal{C}_2 = \mathcal{C}$; and $\mathcal{C}$ has bounded derivatives. Then there exists a $\hat{\tau}$ such that for all $\tau \geq \hat{\tau}$: (i) $M_t = X_t = 0$ and $D_t = 0$, if $NFA = 0$; (ii) $X_t = 0$ and $D_t > 0$, if $NFA > 0$; and (iii) $M_t = 0$ and $D_t < 0$, if $NFA < 0$.*

We view the technical conditions imposed in Proposition 3 as very mild.[11] The normality of all aggregate commodities is only required to establish that autarky is the unique competitive equilibrium. The stationarity of the environment could be dispensed with.

At this point, one may be tempted to interpret Propositions 2 and 3 as establishing that small permanent tariffs are neutral, whereas large enough tariffs are not. Next, we will show that this interpretation is incorrect and that there are systematic reasons why even a small permanent tariff may reduce the deficit.

3.2 A Sufficient Statistic: Slope of the Engel Curve!

As previously discussed, tariffs have two types of effects: an interest rate channel, via $\partial R/\partial\tau$, and a distortion channel, via $\partial\mathcal{D}_t/\partial\tau$. For our main result, we focus on a small change in tariff around free trade ($\tau = 0$). This implies that the distortionary effect of the tariff is second-order and that only the interest rate channel is active.

[10] Razin and Svensson (1983) show how neutrality breaks down when the same goods are exported and imported in both periods, but the environment is not stationary.

[11] We assume the derivatives of $\mathcal{C}$ are finite, even at $M = 0$ or $X = 0$ for simplicity, to avoid the need for a limit $\tau \to \infty$ argument. Economically, this represents the realistic assumption of finite choke prices for supply and demand. Note, also, that G_t may still satisfy the Inada condition $G_{tc_i} \to \infty$ as $c_{ti} \to 0$, since $m_i = 0$ does not necessarily imply $c_i = 0$ if good i can be produced domestically.

Interest Rate Channel. Consider the domestic real interest rate,

$$R(C_1, C_2, \tau) = R^* \frac{e_{1C}(C_1, \tau)}{e_{2C}(C_2, \tau)}.$$

How does a change in the tariff τ affect the real interest rate, holding fixed aggregate consumption in the two periods? To answer this question, we can take logs and differentiate the previous expression,

$$\frac{\partial \ln R(C_1, C_2, \tau)}{\partial \ln(1+\tau)} = \frac{\partial \ln e_{1C}(C_1, \tau)}{\partial \ln(1+\tau)} - \frac{\partial \ln e_{2C}(C_1, \tau)}{\partial \ln(1+\tau)}. \tag{5}$$

In any given period, the first and cross-derivatives of the expenditure function satisfy

$$\begin{aligned} e_C(C, \tau) &= (1+\tau) M_C(C, \tau) - X_C(C, \tau) \geq 0 \\ e_\tau(C, \tau) &= M(C, \tau) \geq 0, \\ e_{C\tau}(C, \tau) &= M_C(C, \tau) \geq 0, \end{aligned}$$

where we have dropped the subscript t for notational convenience. It follows that

$$\frac{\partial \ln e_C(C, \tau)}{\partial \ln(1+\tau)} = \frac{(1+\tau) M_C(C, \tau)}{(1+\tau) M_C(C, \tau) - X_C(C, \tau)}. \tag{6}$$

The right-hand side is a decreasing function of $M_C(C, \tau)/X_C(C, \tau)$, which is the slope of the Engel curve pictured in Figure 1,

$$\frac{dM}{dX} = \frac{M_C(C, \tau)}{X_C(C, \tau)} \leq 0.$$

Equations (5) and (6) imply that an increase in tariff shifts up the real interest rate, $\partial R / \partial \tau \geq 0$, if and only if the slope of the static Engel curve, dM/dX is lower at $t = 1$ than at $t = 2$.

No Distortion Channel. Would a change in tariff τ have any other effect? In general, the answer is yes. Tariffs may also shift the deficit function, $\mathcal{D}_t(C, \tau)$. To see this, note that

$$\mathcal{D}_{t\tau}(C, \tau) = e_{t\tau}(C, \tau) - M_t(C, \tau) - \tau M_{t\tau}(C, \tau) = -\tau M_{t\tau}(C, \tau) \geq 0.$$

This captures the usual welfare loss due to a fiscal externality measured as a change in the area of the "Harberger triangle" under the imports demand curve. This distortion channel will be the focus of our analysis in Section 4.1. At $\tau = 0$, however, this channel is inactive and $\mathcal{D}_{t\tau} = 0$. This then implies that the shift in the equilibrium value of the deficit $D_t(\tau) = \mathcal{D}_t(C(\tau), \tau)$ is entirely driven by the change in the equilibrium value of

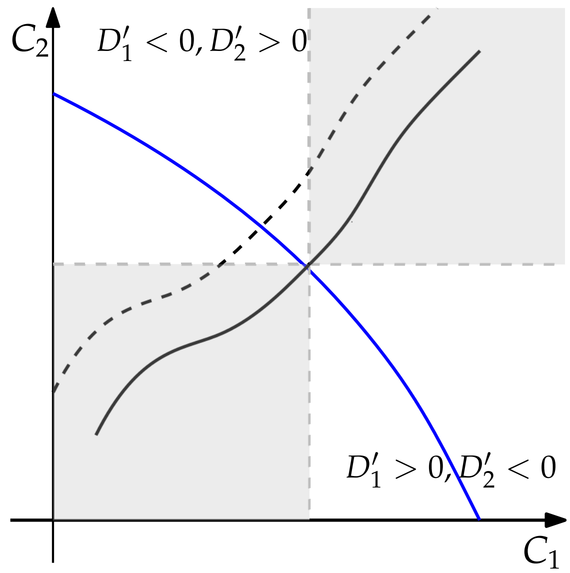


Figure 3: The Interest Rate Channel.

Notes: This figure illustrates how a shift in the interest rate, $\partial R/\partial\tau > 0$, causes a shift in the Euler condition (3) (in dashed black) and, in turn, affects the equilibrium size of the trade deficits. In this example, a small increase in τ lowers the deficit in period 1.

consumption $C_t(\tau)$ caused by the shift in the interest rate, $\partial R/\partial\tau$. If $\partial R/\partial\tau > 0$, then $C_1'(\tau) < 0$ and $D_1'(\tau) < 0$. The opposite happens if $\partial R/\partial\tau < 0$, as illustrated in Figure 3.

This leads to our next proposition.

Proposition 4 (Sufficient Statistic)**.** *Starting from free trade ($\tau = 0$), an increase in tariffs reduces the deficit in period* 1*,*

$$D_1'(\tau) < 0,$$

if and only if the Engel curve is steeper in this period,

$$\left|\frac{dM_1}{dX_1}\right| > \left|\frac{dM_2}{dX_2}\right|. \tag{7}$$

Intuitively, Inequality (7) captures a higher reliance, at the margin, on imports in the first period. A tariff then raises the cost of consumption more in the first period, creating a substitution effect away from C_1 towards C_2, i.e. an incentive to save, which reduces the trade deficit. A naive intuition might have been that what matters is the direct incidence of the tariff on an aggregate consumption price index (CPI). Our result shows that this intuition, however, is generally lacking. First, the use of a CPI presumes homotheticity of preferences, but our result does not invoke such an assumption. Thus, our result allows for inferior, superior, or luxury goods. Second, in general, imports may be used in production as inputs, rather than directly consumed as final goods in which case a CPI-based approach is doomed or requires more work to trace out the effect. Third and most surprising, according to our sufficient statistic, the consumption price vector p_{ct} does not directly enter the picture. Neither does the share of expenditure on non-traded goods. According to Proposition 4, only the relative expansion of imports to exports matters. In short, the static Engel curve holds the key to the dynamic question of interest.

Interestingly, Proposition 4 implies that a permanent tariff may reduce trade deficits even though preferences, technology, and, in turn, Meade preferences over imports and exports, $\mathcal{C}_t$, are invariant over time. This can happen merely because the Engel curve is strictly convex, as described in Figure 1. In this case, if there is a deficit in period 1, then $C_1 > C_2$ further implies $|dM_1/dX_1| > |dM_2/dX_2|$. This is sufficient for the same tariff τ to have a different incidence in the two periods.[12]

Corollary 1. *Consider a stationary economy with invariant Meade preferences, $\mathcal{C}_1 = \mathcal{C}_2 = \mathcal{C}$, and strictly convex Engel curves for $\mathcal{C}$. Suppose the economy runs a trade deficit in period 1. Then starting from free trade ($\tau = 0$), an increase in tariffs reduces the deficit in this period, $D_1'(\tau) < 0$.*

3.3 What Shape for the Engel Curve?

In a stationary economy, the shape of the Engel curve is sufficient to evaluate the impact of a small change in tariff. But this begs the question: What shape do we expect Engel curves to take?

Non-homotheticity in the preferences over goods (represented by G_t) may lead to non-homotheticity in the preferences over aggregate imports and exports (represented by $\mathcal{C}_t$). For instance, aggregate imports may be a luxury in developing countries, whereas aggregate exports are a luxury in developed countries. More surprisingly, we will show that even if preferences over goods are homothetic, extensive margin considerations can systematically give rise to non-homothetic preferences over aggregate imports and exports, i.e. non-linear Engel curves in (M, X) space.

General Armington Economy. Consider a general Armington economy. Preferences are CES; the supply of each good is fixed; and international trade is subject to iceberg trade costs. We do not restrict the number of goods, nor the distribution of taste shifters, endowments, and international prices. So this environment nests the standard Armington

[12]For the interested reader, Appendix E discusses how our results generalize to an economy with infinite horizon. In this case, one can show that a permanent tariff necessarily reduces trade deficits in the period with the maximum deficit, consistent with Corollary 1 in a two-period economy.

model in which there is one domestic good and one foreign good, as in Obstfeld and Rogoff (2000).[13]

Our next result shows that whether or not the extensive margin of trade is active—in the sense that changes in aggregate consumption, from C_1 to C_2, affects the sets of goods being exported, imported, or non-traded—is critical for the shape of the Engel curve.

Proposition 5 (Convex Engel Curves)**.** *In a general Armington economy, the Engel curves of C_t are convex for all $(M, X) > 0$ and strictly convex if the extensive margin of trade is active.*

The broad intuition is as follows. By definition, both imports and exports must be non-negative. When aggregate consumption C goes up, we move along an Engel curve increasing imports M and decreasing exports X. At the micro level, various imports and exports adjust. Due to the homotheticity of preferences, absent non-negativity constraints, these adjustments are linear, implying a linear Engel curve.[14] However, this is no longer the case when non-negativity constraints bind. In particular, as consumption C rises, a greater number of goods that were not previous imported start being imported, going from $m_i = 0$ to $m_i > 0$, facilitating the adjustment along the aggregate import M margin. Likewise, as consumption C rises, the non-negativity constraint for exports becomes binding for a greater number of goods, going from $x_i > 0$ to $x_i = 0$, blunting the adjustment on the aggregate export X margin. Both extensive margin considerations tend to make Engel curves convex.

In a standard Armington model, with one domestic good and one foreign good, the extensive margin is inactive for all $M, X > 0$. As a result, Engel curves are linear, as described in Figure 1b. In contrast, if there is a continuum of domestic goods, the extensive margin is generically active, and Engel curves are smooth and convex, as described in Figure 1a. Intermediate cases with a finite number of domestic goods give rise to Engel curves with finitely many kinks. Since strict convexity implies that tariffs reduce deficits, as discussed in Corollary 1, this highlights that the absence of extensive margin considerations in Proposition 2 was critical for neutrality to hold.[15]

[13]Note, though, that even if foreigners have no endowment of the domestic good, we treat the domestic economy as a small open economy. We will introduce terms-of-trade considerations in Section 3.4.

[14]More formally, the fact that we start from an endowment economy with homothetic preferences implies that Meade preferences absent non-negativity constraints and defined over the entire vector of goods, are quasi-homothetic in $m - x$. Absent non-negativity constraints, Engel curves would therefore be linear. The CES assumption then further guarantees that quasi-homotheticity is preserved when non-negativity constraints are binding for a subset of non-traded goods.

[15]For the interested reader, Appendix G offers a graphical way to relate, and distinguish, our results from the original analysis of Obstfeld and Rogoff (2000).

3.4 Interest Rate Channel and Terms-of-Trade Effects

So far, we have assumed fixed international prices. This implicitly restricts how the variation in aggregate consumption between today and tomorrow may affect domestic prices and, as a result, shape the incidence of a tariff. In general, a change in C may also affect a country's terms of trade. This is at the heart of the transfer problem debated by Keynes and Ohlin and studied by Dornbusch et al. (1977). To conclude this section, we show how terms-of-trade considerations, in addition to extensive margin considerations, can create a systematic relationship between a permanent tariff and the domestic real interest rate.

Adding terms-of-trade effects. Consider a generalized version of the environment in Section 2 in which import prices $p^*_{mt}(\tau)$ and export prices $p^*_{xt}(\tau)$ may vary with τ. R^* is still fixed, without loss of generality. Our aggregate approach remains valid in this case. We just need to keep track of the fact that $\mathcal{C}_t(M, X; p^*_{mt}(\tau), p^*_{xt}(\tau))$ and the associated expenditure function $e_t(C, \tau; p^*_{mt}(\tau), p^*_{xt}(\tau))$ now also depend on τ through its impact on import and export prices. Accordingly, the response of the domestic real interest rate to a change in the tariff τ is given by

$$\frac{d\ln R}{d\ln(1+\tau)} = \frac{d\ln e_{1C}}{d\ln(1+\tau)} - \frac{d\ln e_{2C}}{d\ln(1+\tau)}.$$

This is identical to (5), except for the fact that $d\ln e_{tC}/d\ln(1+\tau)$ is now a total derivative,

$$\frac{d\ln e_{tC}}{d\ln(1+\tau)} = \frac{\partial\ln e_{tC}}{\partial\ln(1+\tau)} + \frac{\partial\ln e_{tC}}{\partial\ln p^*_{mt}} \cdot \frac{\partial\ln p^*_{mt}}{\partial\ln(1+\tau)} + \frac{\partial\ln e_{tC}}{\partial\ln p^*_{xt}} \cdot \frac{\partial\ln p^*_{xt}}{\partial\ln(1+\tau)}. \tag{8}$$

The partial effect of a tariff, $\partial\ln e_{tC}/\partial\ln(1+\tau)$, is still given by equation (6). As before, it only depends on the slope of the Engel curve M_C/X_C. This slope, however, may now vary with the endogenous import and export prices, since $\mathcal{C}_t(M, X; p^*_{mt}(\tau), p^*_{xt}(\tau))$ depend on these. The extra terms in (8) further capture the direct effect of tariffs on prices, $\partial\ln p^*_{mt}/\partial\ln(1+\tau)$ and $\partial\ln p^*_{xt}/\partial\ln(1+\tau)$. Conveniently, their incidence on e_{tC} is itself a function of the slope of the Engel curve M_C/X_C, as we discuss below. In sum, the slope of the Engel curve continues to be a main determinant in this extension.

Ricardian economy. An illuminating special case is the "small economy" limit of a Ricardian model, as considered in Alvarez and Lucas (2007).[16] In this environment, import prices and the world interest rate are fixed, but export prices are proportional to the do-

[16]Details can be found in Appendix H. This limit obtains when the rest of the world is infinitely large, but its share of expenditure on domestic goods is infinitely small, as discussed in Demidova et al. (2024).

mestic wage $w_t(C_t, \tau)$, which is an endogenous function of aggregate consumption C_t and the import tariff τ:

$$p^*_{i,xt} = \bar{p}^*_{i,xt} \times w_t(C_t, \tau). \tag{9}$$

Combining equations (6), (8), and (9), and dropping the subscript t for notational convenience, one can then show

$$\frac{d \ln e_C}{d \ln(1+\tau)} = \frac{(1+\tau)M_C(C,\tau; w(C,\tau)) - X_C(C,\tau; w(C,\tau))\epsilon_w(C,\tau)}{(1+\tau)M_C(C,\tau; w(C,\tau)) - X_C(C,\tau; w(C,\tau))}, \tag{10}$$

where $\epsilon_w(C,\tau) \equiv \partial \ln w(C,\tau)/\partial \ln(1+\tau)$ denotes the elasticity of the domestic wage with respect to the tariff and $M_C(C,\tau; w(C,\tau))$ and $X_C(C,\tau; w(C,\tau))$ now also depend on C through its impact on the wage $w(C,\tau)$, which enter the aggregate preferences $\mathcal{C}$.

For the purposes of signing the impact of the tariff on the interest rate, there are now two sufficient statistics: M_C/X_C and ϵ_w. Another statistic ϵ_w is needed because export prices also respond to the tariff, $\partial \ln p^*_{xt}/\partial \ln(1+\tau) \neq 0$. No other statistic is needed because the incidence of changes in export prices also depends on the marginal spending on aggregate imports and exports, as reflected in M_C/X_C.

In this Ricardian economy, terms-of-trade effects create another reason for an increase in τ to reduce trade deficits via the interest rate channel, as we now demonstrate.

Proposition 6 (Terms-of-trade Effects)**.** *Consider the small economy limit of a stationary Ricardian economy as in Alvarez and Lucas (2007). Suppose that it runs a trade deficit in period 1. Then an increase in τ raises the incentives to save in this period, $\partial R/\partial \tau > 0$.*

The formal proof can be found in Appendix H. As mentioned above, there are now two forces that shape the interest rate channel, reflected in two sufficient statistics in (10). Both forces, however, reinforce each other. First, because of home-bias, an increase in C improves the country's terms of trade by raising w. This increase in the relative price of exports raises the marginal spending on imports relative to exports M_C/X_C, which makes the cost of living more sensitive to tariffs in periods of deficits. This is the counterpart to the mechanism at play in Proposition 5 via extensive margin considerations. Second, one can check that the new statistic ϵ_w is also increasing in C. So an increase in τ must disproportionately raise export prices in periods of deficits, magnifying the direct effect of tariffs on the cost of living.

Note that unlike our previous results, Proposition 6 does not restrict tariffs to be around zero. This reflects the fact that it focuses purely on the interest rate channel and leaves open the question of how wealth effects may affect the trade balance condition. Next we conclude our analysis with a discussion of wealth effects for large tariffs.

4 What if Tariffs Are Large?

4.1 The Distortion Channel

We return to the general environment of Section 2, without terms of trade effects. But in contrast to our analysis in Section 3, we now study the case where the distortion channel is active, which creates wealth effects.

Distortion Channel. Formally, we no longer restrict the tariff τ to be around zero so that

$$\mathcal{D}_{t\tau}(C,\tau) = e_{t\tau}(C,\tau) - M_t(C,\tau) - \tau M_{t\tau}(C,\tau) = -\tau M_{t\tau}(C,\tau) > 0,$$

if tariffs distorts aggregate imports, $M_{t\tau}(C,\tau) < 0$. This implies that the trade balance condition (4) (displayed in solid blue in Figure 2) may shift in response to a change in τ. Are there reasons to expect a systematic effect of a permanent tariff on deficits through this second channel? The answer again is yes, as we will demonstrate.

Consider a small change in tariff from τ to $\tau + d\tau$. Let $(\delta C_t)_{D_t=cst}$ denote the change in aggregate consumption that would be necessary to hold the trade deficit in period t fixed at its original level, $D_t = \mathcal{D}_t(C_t,\tau)$, under the new tariff, $\tau + d\tau$. It is given by

$$\mathcal{D}_C(C_t(\tau),\tau) \times (\delta C_t)_{D_t=cst} + \mathcal{D}_\tau(C_t(\tau),\tau) \times d\tau = 0.$$

By definition, an increase in tariffs from τ to $\tau + d\tau$ reduces the deficit in period t if and only if the change in aggregate consumption δC_t observed along the equilibrium is less than $(\delta C_t)_{D_t=cst}$. Note that the sum of the trade deficits across the two periods must remain equal to the country's NFA, which are fixed. So, trade deficits cannot move in the same direction in both periods. It follows that a necessary and sufficient condition for the trade deficit to go down in period 1 is

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_1=cst}} < \frac{\delta C_1}{\delta C_2}.$$

No Interest Rate Channel. To isolate the role of the distortion channel, we go back to the same economy as in Proposition 2. Static preferences are homothetic; the environment is stationary; and there are no extensive margin considerations. This guarantees that

$$\partial R(C_1,C_2,\tau)/\partial\tau = 0.$$

In this case, the Euler condition (3) (displayed in solid black in Figure 2) is fixed. So, we

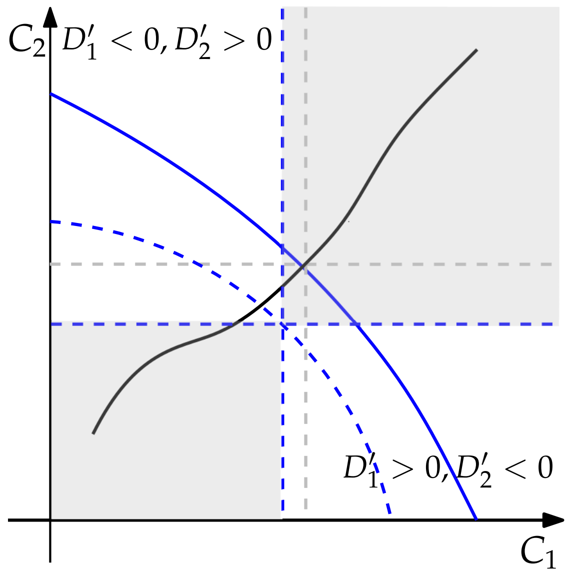


Figure 4: The Distortion Channel.

Notes: This figure illustrates how a shift in the deficit functions, $\partial \mathcal{D}_t / \partial \tau > 0$, causes a shift in the iso-deficit curves (in dashed blue) as well as the trade balance condition (4) (also in dashed blue) and, in turn, affects the equilibrium size of the trade deficits. In this example, an increase in τ lowers the deficit in period 1.

can rearrange the condition for the trade deficit to go down in period 1 as

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} < \frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}}, \tag{11}$$

where $\delta C_t = (\delta C_t)_{MRS=R}$ denotes the equilibrium change in C_t obtained from moving along the *dynamic* Engel curve in (C_1, C_2). This is the situation represented in Figure 4 where the intersection between the Euler condition (3) (in solid black) and the new trade balance condition (4) (in dashed blue) lies above the intersection of the new iso-deficit curves (in dashed blue). Intuitively, what matters for trade deficits is not only the difference between the magnitude of the income shocks in the two periods, reflected in $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst}$, but how this difference relates to the difference in the propensity to consume out of wealth in the two periods, as reflected in $(\delta C_1)_{MRS=R}/(\delta C_2)_{MRS=R}$.

Condition (11) is related to previous work by Svensson and Razin (1983). They derive a version of this condition in the context of an endowment economy subject to a permanent terms-of-trade shock. In contrast, we allow for a general neoclassical economy and endogenize income fluctuations across periods through the distortionary effects of tariffs. These two departures are critical to go beyond (11) and establish the following result.

Proposition 7 (Distortions)**.** *Consider the same economy as in Proposition 2, but with intertemporal preferences that are homothetic: U is homogeneous of degree one. Suppose that the economy runs a trade deficit in period 1. Then away from free trade ($\tau > 0$), an increase in τ lowers the* *deficit in this period:* $D_1'(\tau) \leq 0$*, with strict inequality if and only if tariffs distort production.*

The logic is the following. Since intertemporal preferences are homothetic and there is no interest rate channel, dynamic Engel curves are linear and $C_1(\tau)/C_2(\tau)$ is fixed. It follows that $(\delta C_1)_{MRS=R}/(\delta C_2)_{MRS=R}$ must be equal to $C_1(\tau)/C_2(\tau)$ as well. Given condition 11, the question then is whether $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst}$ is lower than this ratio. Proposition 7 states that it must be.

By distorting aggregate imports, $M_{t\tau}(C,\tau) < 0$, an increase in τ makes the economy poorer: $\mathcal{D}_{t\tau}(C,\tau) > 0$, which implies that a larger deficit is necessary to sustain the same level of aggregate consumption. In the case of an endowment economy, the import distortion is equal to the consumption distortion and is therefore proportional to C. This implies $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} = C_1(\tau)/C_2(\tau)$ and, in turn, the neutrality of tariffs, like in Proposition 2, even away from $\tau = 0$. In the general case, however, the import distortion also includes a production distortion, which is the same in the two periods, by stationarity. This implies $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} = [C_1(\tau) + a]/[C_2(\tau) + a]$, with $a > 0$ if and only if production is distorted, in which case tariffs reduce the deficit in period 1, since $(\delta C_1)_{D_1=cst}/(\delta C_2)_{D_2=cst} < C_1(\tau)/C_2(\tau)$.[17]

Increasing a permanent tariff makes the economy poorer in both periods. But it "succeeds" in reducing the trade deficit in period 1 because, as a fraction of aggregate consumption, it induces a disproportionately smaller income loss in this period.

4.2 A Quantitative Exploration

We have provided formal propositions about the impact of a permanent tariff on trade deficits when either the interest rate channel or the distortion channel are active. In either of these two polar cases, we have highlighted reasons why an increase in τ may reduce trade deficits, either because its impact on the cost of living systematically varies with C, as in Section 3, or because its impact on income does, as in 4.1. There is little more that can be said analytically when both channels are active and interact.[18] To conclude our analysis, we therefore turn to simulations. The goal is not to offer a definitive answer about what we expect the impact of tariffs on trade deficits to be in practice, but rather to explore the range of possible answers that might emerge under typical calibrations.

[17] No production distortion may also occur, locally, in an economy with production. The Ricardian model is one such example. In the absence of terms-of-trade effects, the pattern of specialization is independent of the level of the tariff as long as aggregate exports are non-zero.

[18] For the interested reader, Appendix J offers one such result in the special case of a general Armington economy in which both U and G_t are CES, with identical elasticity of substitution.

Parametric assumptions. In line with our previous analysis, we consider a general Armington economy, as in Section 3.3, as well as the "small economy" limit of a Ricardian economy, as in Section 3.4. So the interest rate channel may either derive from extensive margin considerations or from terms-of-trade effects. The environment is stationary and the world interest rate is normalized to unity: $R^* = 1$. Intertemporal preferences are CES with elasticity of substitution μ and static preferences are CES with elasticity of substitution σ. All goods enter preferences symmetrically. Iceberg trade costs are uniform: $\delta_{i,x} = \delta_{i,m} = \delta$.

In the general Armington economy, there is an exogenous endowment $y_i = [1 + e^{\frac{1}{\gamma}\left(i-\frac{1}{2}\right)}]^{-1}$ of each good $i \in [0,1]$. The parameter γ captures the importance of extensive margin considerations. As γ goes to zero, the model converges to the standard Armington model with one domestic good (with positive endowment) and one foreign good (with zero endowment). We normalize all international prices to unity in this economy: $p^*_m = p^*_x = 1$. Instead in the Ricardian economy, there is an exogenous endowment of labor L and a distribution of productivity that gives rise to a gravity equation with trade elasticity $\varepsilon > 0$. The domestic wage w is endogenously determined by the domestic labor market clearing condition, with ε determining the importance of terms-of-trade considerations. The expressions for aggregate imports and exports as well as the labor market clearing condition can be found in equations (H.1)-(H.3) of Appendix H.

Calibration. We set the intertemporal elasticity of substitution to $\mu = 1$. We consider two versions of the general Armington economy, one with $\gamma = 0.1$ and another with $\gamma = 0.005$. For each version, we choose σ to target a trade elasticity $\varepsilon = 5$. In the case of the Ricardian economy, we directly set $\varepsilon = 5$. Finally, in all three economies, we set the discount factor β and the iceberg trade cost δ to match a share of imports to GDP equal to 15% and a ratio of deficit to GDP equal to 6% in the free trade equilibrium ($\tau = 0$).

Simulation results. Figure 5a illustrates the relationship between tariffs and deficit in these three economies: general Armington with $\gamma = 0.1$, with an active extensive-margin around free trade; general Armington with $\gamma = 0.005$, with (almost) no extensive margin; and Ricardian. In all three cases, we see that an increase in τ lowers the trade deficit, in line with our analytical results. The extent to which it does, however, varies significantly across models, despite that the fact that all models are consistent with the same levels of openness to trade, both intra and inter-temporally, as well as the same trade elasticity.

Figure 5b explains why. As emphasized in our earlier analysis, the shape of static Engel curves is a key determinant of the structural relationship between tariffs and deficits.

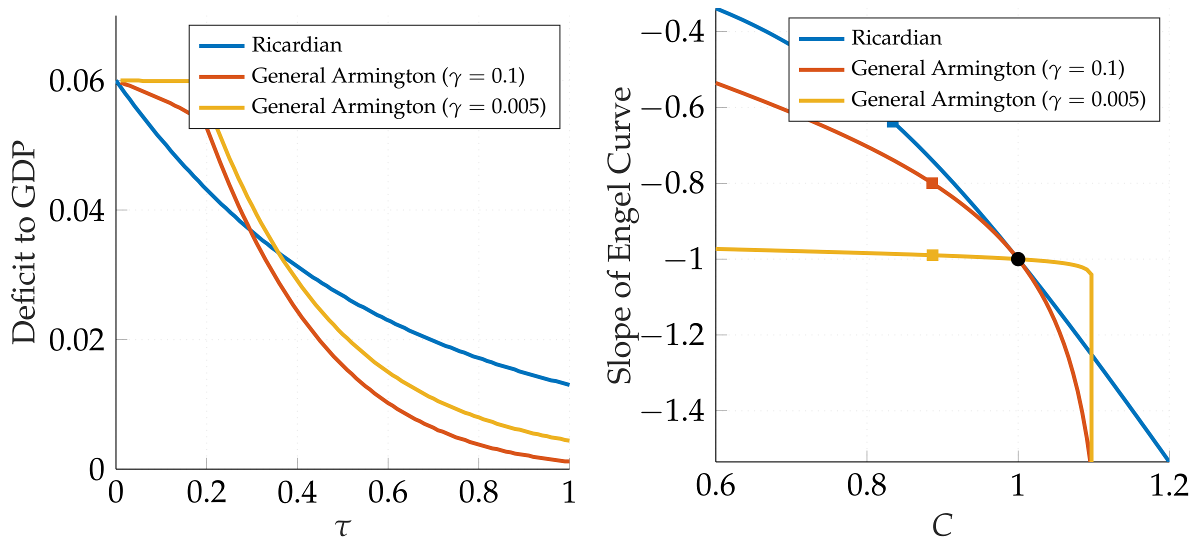


Figure 5: Quantitative Examples

Notes: Figure 5a plots the deficit D_t, expressed as a share of GDP, as a function of the tariff τ, in two Armington economies (with $\gamma = 0.1$ and $\gamma = 0.005$) and a Ricardian economy. All three economies are calibrated to match a share of imports to GDP equal to 15%, a ratio of deficit to GDP equal to 6%, and a trade elasticity $\varepsilon = 5$ in the free trade equilibrium ($\tau = 0$). Figure 5b plots the slopes of the Engel curves M_C/X_C. All three economies are normalized so that $C_1 = 1$ and the slope of the Engel curve is -1 at $C = 1$. Squares represent the slopes at C_2.

Matching the previous moments, however, imposes little discipline on the slope of the Engel curves across the three models. The trade elasticity is important in the Ricardian case, because it shapes terms-of-trade effects, but it is irrelevant in the Armington cases, where the parameter γ instead controls extensive margin considerations.[19] Different slopes, in turn, lead to different predictions. The contrast is particularly stark around free trade where we may be very close to neutrality.

To bring additional credibility to these (and other) simulations, one could estimate the slope of the Engel curve directly and use it as a new target in calibration. Empirically, one could proceed as follows. For a given tariff τ and a given level of aggregate consumption C and trade deficit D, consider an exogenous change in the world interest rate R^*. Everything else being equal, this will cause a change in consumption ΔC and, in turn, a change in trade deficit, ΔD, with associated changes in imports and exports, ΔM and ΔX. The slope of the Engel curve, evaluated around the original deficit level D, is simply the ratio of ΔM over ΔX. We are not aware of empirical work that has already estimated the previous responses of aggregate imports and exports. We hope that our theoretical

[19]Similar differences across these three models can be observed for alternative values of the trade elasticity. Appendix K reports the counterpart of Figure 5a for $\varepsilon = 2$ and 8.

analysis may provide further motivation.

5 Concluding Remarks

We have used a flexible trade model to study the effect of tariffs on the trade balance and to isolate the relevant sufficient statistics. We have shown that the response to this dynamic question is controlled by the static Engel curve for aggregate imports and exports.

Except in special cases, economic theory predicts that tariffs do affect trade imbalances and are likely to reduce them. Our results provide a step forward in understanding the underlying mechanism and determining the magnitudes. One central idea is that even if economic conditions are unchanged between today and tomorrow, the fact that a country is currently running a trade deficit implies that aggregate consumption will, in general, be different today and tomorrow. As a result, a permanent tariff can affect the real domestic interest rate and, in turn, the incentives to borrow and save. This can occur both through extensive margin and terms-of-trade considerations.

We conclude with a caveat and a reminder. First, intertemporal substitution requires that households and firms anticipate the endogenous effects of policy on future prices and income. Our analysis relied on the benchmark assumption of rational expectations. Alternatives with more backward-looking specifications of expectations would likely mitigate the effects of tariffs on deficits. Second, we started this paper by noting that our focus was a positive rather than a normative one. We are interested here in whether tariffs can affect trade deficits, not whether tariffs should be used to affect them. The answer to the second question is well-known. Regardless of whether or not tariffs affect deficits, a country may gain from unilaterally imposing tariffs, if it has the ability to affect international prices, and it may lose from foreign retaliation, if the rest of the world does as well.

References

Adao, Rodrigo, Arnaud Costinot, and Dave Donaldson, "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade," *American Economic Review*, 2017, *107* (3), 633–689.

Aguiar, Mark A., Manuel Amador, and Doireann Fitzgerald, "Tariff Wars and Net Foreign Assets," *NBER Working Paper 33743*, 2025.

Alessandria, George and Horag Choi, "The Dynamics of the U.S. Trade Balance and Real Exchange Rate: The J Curve and Trade Costs?," *Journal of International Economics*, 2021, *132.*

Alvarez, Fernando and Robert E. Lucas, "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, 2007, *54* (6), 1726–1768.

Anderson, James E. and Eric Van Wincoop, "Gravity with Gravitas: A Solution to the Border Puzzle," *The American Economic Review*, 2003, *93* (1), 170–192.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "The Macroeconomics of Tariff Shocks," *NBER Working Paper 33726*, 2025.

Baldwin, Richard, "The Tariffs and Balance of Trade Paradox: Why Macro Always Wins (in Pictures)," *Peterson Institute for International Economics Op-Ed*, 2024.

Borusyak, Kirill and Xavier Jaravel, "The Distributional Effects of Trade: Theory and Evidence from the United States," *NBER Working Paper 28957*, 2021.

Costinot, Arnaud and Iván Werning, "Lerner Symmetry: A Modern Treatment," *American Economic Review: Insights*, June 2019, *1* (1), 13–26.

Deaton, Angus and John Muellbauer, *Economics and consumer behavior*, Cambridge University Press, 1980.

Demidova, Svetlana, Konstantin Kucheryavyy, Takumi Naito, and Andres Rodríguez-Clare, "The Small-Open Economy in a Generalized Gravity Model," *Journal of International Economics*, 2024, *152.*

Dixit, Avinash K. and Victor Norman, *Theory of International Trade*, Cambridge University Press, 1980.

Dornbusch, Rudiger, "Real Interest Rates, Home Goods, and Optimal External Borrowing," *Journal of Political Economy*, 1983, *91* (1), 141–153.

Dornbusch, Rudiger S., Stanley Fischer, and Paul A. Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 1977, *67* (5), 823–839.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

_ , _ , and Brent Neiman, "Obstfeld and Rogoff's International Macro Puzzles: A Quantitative Assessment," *Journal of Economic Dynamics and Control*, 2016, *72*, 5–23.

Fitzgerald, Doireann, "Trade Costs, Asset Market Frictions, and Risk Sharing," *American Economic Review*, 2012, *102* (6), 2700–2733.

Harberger, Arnold C., "Currency Depreciation, Income, and the Balance of Trade," *Journal of Political Economy*, 1950, *58* (1), 47–60.

Hicks, J. R., *Value and Capital*, Oxford University Press, 1936.

Ignatenko, Anna, Ahmad Lashkaripour, Luca Macedoni, and Ina Simonovska, "Making America Great Again? The Economic Impacts of Liberation Day Tariffs," *NBER Working Paper 33771*, 2025.

Irwin, Douglas A., *Trade Policy Disaster: Lessons from the 1930s*, MIT Press, 2011.

Itskhoki, Oleg and Dmitry Mukhin, "The Optimal Macro Tariff," *NBER Working Paper 33839*, 2025.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *The American Economic Review*, 1980, *70* (5), 950–959.

_ , "Can Trump Reduce the Trade Deficit?," *mimeo Stone Center on Socio-Economic Inequality*, 2024.

Laursen, Svend and LLoyd A. Mezler, "Flexible Exchange Rates and the Theory of Employment," *Review of Economics and Statistics*, 1950, *32* (281-99).

Meade, J.E., *A Geometry of International Trade*, London: George Allen and Unwin, 1952.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Obstfeld, Maurice, "Aggregate Spending and the Terms of Trade: Is There a Laursen-Mezler Effect?," *Quarterly Journal of Economics*, 1982, *97* (251-70).

_ **and Kenneth Rogoff**, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," *The NBER Macroeconomics Annual*, 2000, *15.*

Panel, Clark Center Economic Expert, "US Poll: Tariffs, Reciprocal and Retaliatory," March 2025.

Razin, Assaf and Lars E. O. Svensson, "Trade Taxes and the Current Account," *Economics Letters*, 1983, *13*, 55–57.

Reyes-Heroles, Ricardo, "The Role of Trade Costs in the Surge of Trade Imbalances," *mimeo*, 2016.

Rodríguez-Clare, Andres, Mauricio Ulate, and Jose P. Vasquez, "The 2025 Trade War: Dynamic Impacts Across U.S. States and the Global Economy," *NBER Working Paper 33792*, 2025.

Svensson, Lars E. O. and Assaf Razin, "The Terms of the Trade and the Current Account: The Harberger-Laursen-Mezler Effect," *Journal of Political Economy*, 1983, *91* (1), 97–123.

A Definition of a Competitive Equilibrium

This appendix lays out the full set of equilibrium conditions.

The representative household faces a domestic budget constraint

$$p_{c1} \cdot c_1 + \frac{1}{R^*} p_{c2} \cdot c_2 = \Pi_1 + T_1 + \frac{1}{R^*}(\Pi_2 + T_2),$$

where p_{ct} are the domestic prices of consumption goods in period t; Π_t are total profits from firms in the same period; and T_t is the transfer from the government.

Firm profits in period t are equal to

$$\Pi_t = p_{ct} \cdot c_t + p^*_{xt} \cdot x_t - (1 + \tau) p^*_{mt} \cdot m_t.$$

Given an import tariff τ as well as international prices $\{p^*_{mt}, p^*_{xt}\}_{t=1,2}$ and R^*, a competitive equilibrium is $\{(c_t, m_t, x_t), p_{ct}\}_{t=1,2}$ such that

1. consumption (c_1, c_2) maximizes utility subject to the budget constraint taking p_{ct}, Π_t, and T_t as given;
2. each period, (c_t, m_t, x_t) maximizes profits Π_t taking p_{ct}, p^*_{mt}, p^*_{xt} and τ as given;
3. transfers satisfy

$$T_t = \tau \times (p^*_{mt} \cdot m_t).$$

B Proof of Proposition 1

We draw on the full equilibrium definition from Appendix A. We proceed in two steps. First, we establish that for any given C_t, the equilibrium in period t can be found using the aggregates and $\mathcal{C}_t$, as argued in Section 2.3. Second, we establish that we can find the aggregates C_1 and C_2 by focusing on the intertemporal problem in Section (2.4). To save notation, we fix τ, p^*_{mt} and p^*_{xt} and drop these variables from the arguments of all functions.

Static equilibrium conditions. For a given price p_c, the firm problem in period t is

$$\begin{aligned}\tilde{e}_t(p_c) \equiv & \min_{c,m,x} (1 + \tau) p^*_{mt} \cdot m - p^*_{xt} \cdot x - p_c \cdot c \\ & s.t. : (c, m, x) \in Y_t.\end{aligned}$$

The dual of the household problem can be written as

$$\hat{e}_t(p_c, C) \equiv \min_c p_c \cdot c$$
$$s.t : G_t(c) = C.$$

For a given C_t, a static equilibrium at t corresponds to (p_{ct}, c_t, m_t, x_t) that solve both the firm and household problems. A static equilibrium thus also solves

$$e_t(C) \equiv \min_{c,m,x} (1+\tau) p_{mt} \cdot m - p_{xt} \cdot x$$
$$s.t : (c, m, x) \in Y_t,$$
$$G_t(c) = C.$$

At the equilibrium price p_{ct}, note that

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

The minimization problem $e_t(C)$ is identical to minimizing $(1+\tau)M - X$ subject to $\mathcal{C}_t(M, X) = C$. We can then find m, x from the maximization problem defining the aggregate preferences $\mathcal{C}_t$. This establishes that for any given C_t the equilibrium in period t can be found using the aggregates and $\mathcal{C}_t$.

Dynamic equilibrium conditions. Since preferences are separable over time, we can use two-stage budgeting to express the intertemporal household problem as

$$\max_{C_1, C_2} U(C_1, C_2)$$

$$\hat{e}_1(p_{c1}, C_1) + \frac{1}{R^*} \hat{e}_2(p_{c2}, C_2) = NFA + T,$$

where $\hat{e}_t(p_c, C)$ denotes the static expenditure function of the household and the transfer T is taken as given. This gives the first-order condition

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R^* \frac{\hat{e}_{1C}(C_1, p_{c1})}{\hat{e}_{2C}(C_2, p_{c2})}.$$

To establish that this matches the first-order condition (3) in Section 2.4, i.e. equation it is sufficient to show that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t).$$

We already argued above that for $C = C_t$,

$$e_t(C_t) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C_t).$$

Now we want to show that for $C \neq C_t$,

$$e_t(C) \geq \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C). \tag{B.1}$$

This then implies that $e_t(\cdot)$ is an upper envelope for $\hat{e}_t(p_{ct}, \cdot)$ and, in turn, that

$$e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t),$$

as desired.

To establish (B.1) we first rewrite

$$\begin{aligned} e_t(C) = \min_{c_d, c_s, m, x} & (1+\tau) p^*_{mt} \cdot m - p^*_{xt} \cdot x \\ s.t\ : & (c_s, m, x) \in Y_t, \\ & G_t(c_d) = C, \\ & c_d = c_s. \end{aligned}$$

Then for any p_{ct}, we note that

$$\begin{aligned} e_t(C) \geq \min_{c_d, c_s, m, x} & (1+\tau) p_{mt} \cdot m - p_{xt} \cdot x + p_{ct} \cdot (c_d - c_s) = \tilde{e}_t(p_{ct}) + \hat{e}_t(p_{ct}, C) \\ s.t.\ : & (c_s, m, x) \in Y_t, \\ & G_t(c_d) = C, \end{aligned}$$

where the inequality derives from the fact the the right-hand side is a relaxed version of the original expenditure problem and the equality derives from the fact that c_d and c_s only appear in the household's and firm's problems, respectively. This completes the proof.

C Proof of Proposition 2

Let us first demonstrate that under the stated assumptions, the derivative of the expenditure function $e_{tC}(C_t, \tau)$ is independent of C_t. Start from the firm problem, as described in the proof of Proposition 1. Since the environment is stationary and trade is subject to iceberg trade cots, we have

$$\begin{aligned}\tilde{e}_t(p_{ct}) \equiv \min_{c,m,x,y} & (1+\tau)p_m^* \cdot m - p_x^* \cdot x - p_{ct} \cdot c \\ s.t :& y \in Y, \\ & y_i = c_i + (1+\delta_{i,x})x_i - \frac{m_i}{(1+\delta_{i,m})},\end{aligned}$$

where p_{ct} denotes the equilibrium price vector for consumption in period t. Since the equilibrium has each good i either strictly imported ($m_{it} > 0$) or strictly exported ($x_{it} > 0$) subject to iceberg trade costs ($\delta_{i,m}, \delta_{i,x} \geq 0$), the first-order conditions of the firm problem requires $p_{c1} = p_{c2} = p_c(\tau)$, with

$$p_{i,c}(\tau) = \begin{cases} (1+\tau)(1+\delta_{i,m})p_{i,m}^* & \text{if } m_{i1}, m_{i2} > 0, \\ p_{i,xt}^*/(1+\delta_{i,x}) & \text{if } x_{i1}, x_{i2} > 0. \end{cases}$$

Next, consider the dual of the household problem, as also described in the proof of Proposition 1. In this stationary environment, it can be written as

$$\begin{aligned}\hat{e}_t(p_{ct}, C_t) = & \min_c p_c(\tau) \cdot c \\ s.t :& G_t(c) = C_t.\end{aligned}$$

Since G_t is homogenous of degree one, we therefore have

$$\hat{e}(p_{ct}, C_t) = \beta(\tau)C_t.$$

In the proof of Proposition 1, we have already shown that $e_{tC}(C_t) = \hat{e}_{tC}(p_{ct}, C_t)$. We therefore obtain, as desired,

$$e_{tC}(C_t) = \beta(\tau). \tag{C.1}$$

To complete the argument, note that we can rearrange the equilibrium conditions (3) and (4) using (C.1) as

$$\begin{aligned}MRS(C_1, C_2) &= R^*, \\ \mathcal{D}_1(C_1, \tau) + \frac{1}{R^*}\mathcal{D}_2(C_2, \tau) &= NFA.\end{aligned}$$

So the Euler condition is independent of τ. Around $\tau = 0$, we have

$$\mathcal{D}_{t\tau}(C_t, \tau) = -\tau M_{t\tau}(C, \tau) = 0.$$

So the trade balance condition is also independent of τ. By the Implicit Function Theorem, we therefore have the neutrality of (C_1, C_2) with respect to the tariff τ. Since $D_t(\tau) = \mathcal{D}_t(C_t(\tau), \tau)$, we get $D_t'(\tau) = 0$.

D Proof of Proposition 3

We first formally state our normality assumptions for $\mathcal{C}$ and U. We assume that M and $-X$ are normal goods.

Assumption (Normality of M and $-X$)**.** *The utility function $\mathcal{C}(M, X)$ is concave in $(M, -X)$ and has decreasing interior Engel curves:*

$$\frac{\mathcal{C}_M(M, X)}{\mathcal{C}_X(M, X)} = 1 + \tau$$

defines a downward sloping relation between M and X for $M, X > 0$.

We also introduce the required normality assumptions for aggregate consumption over time:

Assumption (Normality of C_1 and C_2)**.** *The utility function U is concave and has increasing Engel curves, so that for any $R > 0$ the condition*

$$\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = R$$

defines a strictly upward sloping relation between C_1 and C_2 for $C_1, C_2 > 0$.

Let $\phi(e, \tau)$ denotes the aggregate consumption level associated with expenditure e given a tariff τ,

$$\phi(e, \tau) = \max_{m,x \geq 0} \mathcal{C}(M, X) \qquad (1+\tau)M - X = e,$$

and let $V(e_1, e_2)$ denote the indirect utility function,

$$V(e_1, e_2) \equiv U(\phi(e_1, \tau), \phi(e_2, \tau)).$$

The next lemma shows that normality for (C_1, C_2) in U implies normality for (e_1, e_2) in V.

Lemma. *If C_1 and C_2 are normal and $\phi(e, \tau)$ is concave in e then the solution to*

$$\max_{e_1, e_2} V(e_1, e_2)$$
$$s.t.{:}e + \frac{1}{R^*} e' = T$$

has both e_1 and e_2 strictly increasing in T.

We are now ready to derive our autarky result.

Proof of autarky result ($NFA = 0$). Choose τ large enough so that

$$\frac{\mathcal{C}_M(0,0)}{1+\tau} + \mathcal{C}_X(0,0) \le 0. \tag{D.1}$$

Since M and $-X$ are normal, the static optimum as a function of expenditure e,

$$\phi(e,\tau) = \max_{M,X\ge 0} \mathcal{C}(M,X) \qquad (1+\tau)M - X = e$$

is obtained at the corners: $X = 0$ for $e \ge 0$ or $M = 0$ for $e \le 0$. Thus,

$$\phi(e,\tau) = \begin{cases} \mathcal{C}(\frac{e}{1+\tau},0) & e \ge 0 \\ \mathcal{C}(0,-e) & e \le 0 \end{cases}$$

Furthermore, one can verify that $\phi(e,\tau)$ is a concave function of e. We denote the right derivative by $\phi_{e+}(e,\tau)$ and the left derivative by $\phi_{e-}(e,\tau)$. For $e \neq 0$ they coincide, but at $e = 0$ we have $\phi_{e+}(e,\tau) = \frac{1}{1+\tau}\mathcal{C}_M(\frac{e}{1+\tau},0) > 0$ and $\phi_{e-}(e,\tau) = -\mathcal{C}_X(0,-e) > 0$. Note that $\phi_{e-}(0,\tau) = -\mathcal{C}_X(0,0) \ge \frac{1}{1+\tau}\mathcal{C}_M(0,0) = \phi_{e-}(0,\tau)$ so there is a concave kink at $e = 0$.

Households solve the intertemporal problem

$$\max_{e_1,e_2} U(\phi(e_1,\tau),\phi(e_2,\tau))$$
$$\text{s.t.:} e_1 + \frac{1}{R^*}e_2 = T$$

taking T as given. In equilibrium

$$T = \left(M_1 + \frac{1}{R^*}M_2\right)\tau.$$

For any value of T the problem is strictly convex so the optimum is unique.

The first order conditions are

$$U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_{e+}(e_1,\tau) - U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_{e-}(e_2,\tau) \le 0,$$

$$-U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_{e-}(e_1,\tau) + U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_{e+}(e_2,\tau) \le 0.$$

The first inequality insures that it is not optimal to increase e_1; the second condition ensures it is not optimal to lower e_1. Note that when $e_1 \neq 0$ we have $\phi_{e-}(e_1,\tau) = \phi_{e+}(e_1,\tau)$

so the two conditions are equivalent to

$$U_1(\phi(e_1,\tau),\phi(e_2,\tau))\phi_e(e_1,\tau) - U_2(\phi(e_1,\tau),\phi(e_2,\tau))R^*\phi_e(e_2,\tau) = 0.$$

We first verify that $e_1 = e_2 = 0$ is an equilibrium with $T = 0$.

$$U_1(\phi(0,\tau),\phi(0,\tau))\phi_{e+}(0,\tau) - U_2(\phi(0,\tau),\phi(0,\tau))R^*\phi_{e-}(0,\tau) \leq 0$$

$$-U_1(\phi(0,\tau),\phi(0,\tau))\phi_{e-}(0,\tau) + U_2(\phi(0,\tau),\phi(0,\tau))R^*\phi_{e+}(0,\tau) \leq 0$$

Using that

$$\phi_{e+}(0,\tau) = \frac{1}{1+\tau}\mathcal{C}_M(0,0),$$
$$\phi_{e-}(0,\tau) = -\mathcal{C}_X(0,0),$$

this is equivalent to

$$\frac{1}{1+\tau}\mathcal{C}_M(0,0) + \beta R^*\mathcal{C}_X(0,0) \leq 0, \tag{D.2}$$
$$\beta R^*\frac{1}{1+\tau}\mathcal{C}_M(0,0) + C_X(0,0) \leq 0, \tag{D.3}$$

where we have defined

$$\beta \equiv \frac{U_2(\phi(0,\tau),\phi(0,\tau))}{U_1(\phi(0,\tau),\phi(0,\tau))}.$$

There are two cases to consider. If $\beta R^* \leq 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (D.2) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (D.1)-(D.3) are verified. If $\beta R^* > 1$ let $\hat{\tau}$ denote the lowest value of τ for which condition (D.3) is satisfied. Then for any $\tau \geq \hat{\tau}$ conditions (D.1)-(D.3) are verified. We conclude that $e_1 = e_2 = 0$ is a solution to the household's intertemporal problem for $T = 0$. It follows that $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$ is an equilibrium.

To establish that this is the unique equilibrium, we need to rule out $T > 0$. We proceed by contradiction. Suppose $T > 0$. Since C_1 and C_2 are normal, the previous lemma implies $e_1 \geq 0$ and $e_2 \geq 0$. But then

$$T = e_1 + \frac{1}{R^*}e_2 = (1+\tau)M_1 + \frac{1}{R^*}(1+\tau)M_2 > \tau M_1 + \frac{1}{R^*}\tau M_2 = T.$$

This concludes our proof for $NFA = 0$.

Proof of autarky result ($NFA \neq 0$). The argument is similar. The only difference is that for τ large the economy will not converge to $M_1 = X_1 = 0$ and $M_2 = X_2 = 0$, but instead to whatever level of aggregate imports or exports is consistent with the level of NFA.

Formally, if $NFA > 0$, define (M_1^a, M_2^a) that solves

$$\max_{M_1,M_2} U(\mathcal{C}(M_1,0),\mathcal{C}(M_2,0))$$
$$\text{s.t.:} M_1 + \frac{1}{R^*}M_2 = NFA.$$

One can then check the first-order conditions (D.1)-(D.3) around $M_1 = M_1^a$, $X_1 = 0$, $M_2 = M_2^a$, and $X_2 = 0$ to establish that for τ large enough this is the unique equilibrium.

Likewise, if $NFA < 0$, define (X_1^a, X_2^a) that solves

$$\max_{X_1,X_2} U(\mathcal{C}(0,X_1),\mathcal{C}(0,X_2))$$
$$\text{s.t.:} -X_1 - \frac{1}{R^*}X_2 = NFA.$$

The same approach as before shows that for τ large enough, the unique equilibrium is $M_1 = 0$, $X_1 = X_1^a$, $M_2 = 0$, and $X_2 = X_2^a$.

E Infinite Horizon Extension

The goal of this appendix is to extend Corollary 1 to an economy with infinite horizon.

Consider the same general environment as in Section 2, except that there is an infinite number of periods indexed by t and that intertemporal preferences are CES,

$$U(\{C_t\}) = \sum_{t=1} \beta^{t-1}\frac{(C_t)^{1-1/\mu}}{1-1/\mu}.$$

We can then express the household problem as

$$\max_{\{C_t\}} \sum_{t=1} \beta^{t-1}\frac{(C_t)^{1-1/\mu}}{1-1/\mu}$$

$$\sum_{t=1} \frac{1}{(R^*)^{t-1}} e_t(C_t,\tau) = NFA + T,$$

where the lump-sum transfer $T = \sum_t \tau M_t(C_t,\tau)$ is taken as given.

Suppose, in line with the assumptions in Corollary 1, that the economy is stationary.

The dynamic equilibrium conditions are then

$$\left(\frac{C_t}{C_s}\right)^{-1/\mu} = \beta^{s-t} R^* \frac{e_C(C_t,\tau)}{e_C(C_s,\tau)} \equiv \beta^{s-t} R(C_t, C_s, \tau), \tag{E.1}$$

$$\sum_{t=1} \frac{1}{(R^*)^{t-1}} \mathcal{D}(C_t,\tau) = NFA.$$

Pick t_0 such that $C_{t_0}(\tau) = \max_t\{C_t(\tau)\}$. Let $\tilde{C}_t(C_{t_0},\tau)$ denote the solution C to

$$\left(\frac{C_{t_0}}{C}\right)^{-1/\mu} = \beta^{t-t_0} R(C_{t_0}, C, \tau),$$

By the same argument as in Section 3.2, if the Engel curve for $\mathcal{C}$ is strictly convex, then

$$\frac{\partial \ln R(C_{t_0}, C_t, \tau)}{\partial \ln \tau} > 0.$$

From the second-order condition of the household problem, we must therefore have

$$\frac{\partial \ln \tilde{C}_t(C_{t_0},\tau)}{\partial \ln \tau} > 0 \text{ and } \frac{\partial \tilde{C}_t(C_{t_0},\tau)}{\partial C_{t_0}} > 0.$$

Now substituting $\tilde{C}_t(C_{t_0},\tau)$ in the intertemporal budget constraint and differentiating with respect to τ, we get

$$\sum_{t=1} \frac{1}{(R^*)^{t-1}} \mathcal{D}'_{Ct}[\frac{\partial \tilde{C}_t(C_{t_0},\tau)}{\partial \tau} + \frac{\partial \tilde{C}_t(C_{t_0},\tau)}{\partial C_{t_0}} \frac{dC_{t_0}}{d\tau}] = 0.$$

It follows that $dC_{t_0}/d\tau < 0$ and, in turn, that $\mathcal{D}'_{t_0}(\tau) = \mathcal{D}_{Ct_0} \times (dC_{t_0}/d\tau) < 0$.

F Proof of Proposition 5

Consider a general Armington economy. For notational convenience, we drop the subscript t. The representative household has CES preferences,

$$G(c) = u^{-1}\left(\int \theta_i u(c_i) di\right), \text{ with } u(c) = c^{1-\sigma}/(1-\sigma).$$

The domestic output of each good i is fixed and equal to $\omega_i \geq 0$. Imports and exports of good i are subject to iceberg trade costs $\delta_{i,m}$ and $\delta_{i,x}$. Define the "domestic dock" prices,

$$p_{i,m} = p_i^*(1+\delta_{i,m}),$$
$$p_{i,x} = p_i^*/(1+\delta_{i,x}),$$

with p_i^* the exogenous price of good i at the "foreign dock". We let F denote the cdf taken over the exogenous characteristics $(\omega, \theta, p_m, p_x)$ of individuals goods.

In a general Armington economy, preferences over aggregate imports and exports are given by

$$\mathcal{C}(M, X) = u^{-1}\left(\max_{m,x} \int \theta u(\omega + \frac{m(\omega, \theta, p_m, p_x)}{p_m} - \frac{x(\omega, \theta, p_m, p_x)}{p_x})dF\right)$$

subject to

$$\int x(\omega, \theta, p_m, p_x)dF = X,$$
$$\int m(\omega, \theta, p_m, p_x)dF = M.$$

To establish the convexity of the Engel curves of $\mathcal{C}(M, X)$, we first need to characterize the solution to the previous maximization problem.

Since $p_x \leq p_m$ we can ignore the possibility that both $m(\omega, \theta, p_m, p_x) > 0$ and $x(\omega, \theta, p_m, p_x) > 0$. The associated first-order conditions are then

$$\frac{\theta}{p_m} u'\left(\omega + \frac{m(\omega, \theta, p_m, p_x)}{p_m}\right) \leq \mu_m,$$
$$\frac{\theta}{p_x} u'\left(\omega - \frac{x(\omega, \theta, p_m, p_x)}{p_x}\right) \geq \mu_x,$$

with the usual complementary slackness, where μ_m and μ_x denote the Lagrange multipliers associated with the aggregate import and export constraints, respectively.

Together with $u(c) = c^{1-\sigma}/(1-\sigma)$, the two previous conditions imply

$$m(\omega, \theta, p_m, p_x) = \max\left\{0, \mu_m^{-1/\sigma}\left(\frac{p_m}{\theta}\right)^{-1/\sigma} - \omega\right\},$$
$$x(\omega, \theta, p_m, p_x) = \max\left\{0, \omega - \mu^{-1/\sigma}\left(\frac{p_x}{\theta}\right)^{-1/\sigma}\right\}.$$

For $k = m, x$, define

$$\tilde{\mu}_k \equiv \mu_k^{-1/\sigma},$$
$$\tilde{z}_k \equiv (p_k)^{1-1/\sigma}\theta^{1/\sigma},$$
$$\tilde{\omega}_k \equiv p_k \omega_k.$$

Using the previous notation, and summing across all goods, we can then express aggregate imports and exports as

$$M(\tilde{\mu}_m) = \int_{\tilde{\omega}_m/\tilde{z}_m \leq \tilde{\mu}_m} (\tilde{z}_m \tilde{\mu}_m - \tilde{\omega}_m) dF, \tag{F.1}$$

$$X(\tilde{\mu}_x) = \int_{\tilde{\omega}_x/\tilde{z}_x \geq \tilde{\mu}_x} (\tilde{\omega}_x - \tilde{z}_x \mu_x) dF. \tag{F.2}$$

Note that

$$M'(\tilde{\mu}_m) = \int_{\tilde{\omega}_m/\tilde{z}_m \leq \tilde{\mu}_m} \tilde{z}_m dF \equiv \bar{z}_m(\tilde{\mu}_m) \geq 0, \tag{F.3}$$

$$X'(\tilde{\mu}_x) = -\int_{\tilde{\omega}_x/\tilde{z}_x \geq \tilde{\mu}_x} \tilde{z}_x dF \equiv \bar{z}_x(\tilde{\mu}_x) \leq 0. \tag{F.4}$$

This further implies that $M''(\tilde{\mu}_m) \geq 0$ and $X''(\tilde{\mu}_x) \geq 0$, with strict inequality if there is an active extensive margin.

To see that the Engel curves of $\mathcal{C}(M, X)$ are convex, recall that that they correspond, for a given τ, to the locus of points (M, X) such that

$$\frac{\mathcal{C}_M(M, X)}{\mathcal{C}_X(M, X)} = 1 + \tau.$$

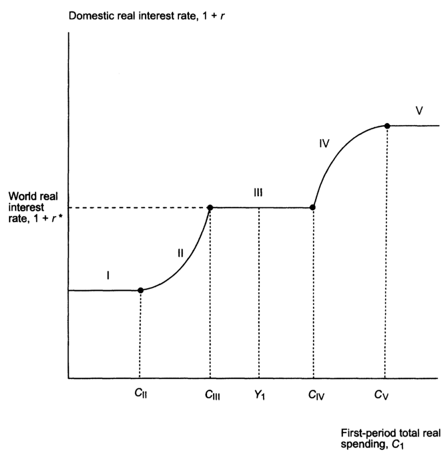
By the Envelope Theorem, the previous condition requires $\mu_m = \mu_x(1+\tau)$ so that $\tilde{\mu}_m = \tilde{\mu}_x(1+\tau)^{-1/\sigma}$. Thus the slope of the Engel curve is equal to

$$\frac{dM}{dX} = \frac{M'(\tilde{\mu}_x(1+\tau)^{-1/\sigma})}{X'(\tilde{\mu}_x)}.$$

Then it follows from our previous calculations that this ratio is negative and decreasing in $\tilde{\mu}_x$. Since aggregate consumption C must be decreasing in the Lagrange multiplier μ_x and $\tilde{\mu}_x = \mu_x^{-1/\sigma}$, it follows that this ratio is decreasing in C as well.

G Relationship to Obstfeld and Rogoff (2000)

Obstfeld and Rogoff (2000) (OR) consider a standard Armington model with two goods, H and F, and trade costs. They analyze the relationship between the real domestic interest rate and aggregate consumption in period 1. Figure 1 of their original paper offers a graphical representation. We reproduce it here in Figure G.1a. OR emphasize five different regions, depending on whether the goods are exported (X), imported (M), or non-traded (NT). Figure G.1b describes the pattern of trade associated with each region at $t = 1$ and 2.

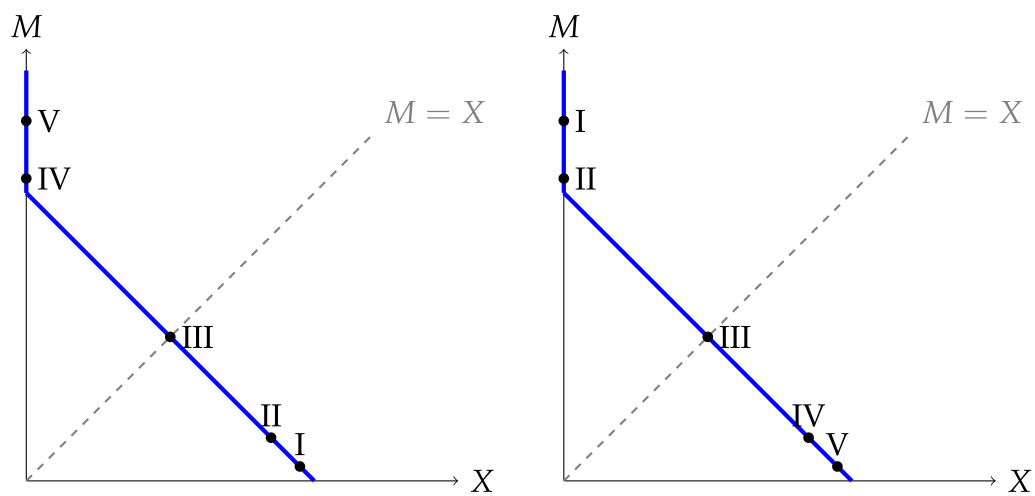


(a) Domestic Real Interest Rate

	H t=1	F t=1	H t=2	F t=2
I	X	M	M	M
II	X	M	NT	M
III	X	M	X	M
IV	NT	M	X	M
V	M	M	X	M

(b) Pattern of Exports and Imports

Figure G.1: Original Analysis from Obstfeld and Rogoff (2000)
Notes: Figure G.1a is the original Figure 1 from OR, which describes the domestic real interest rate ($1 + r$ in their notation, $R(C_1, C_2, \tau)$ in ours) as a function of C_1. Figure G.1b describes whether each good (H or F) is exported (X), imported (M) or non-traded (NT) in a given period t.



(a) Exports and Imports at $t = 1$ (b) Exports and Imports at $t = 1$

Figure G.2: Engel Curve in Obstfeld and Rogoff (2000)
Notes: Each panel plots the Engel curve in the standard Armington model considered by OR. In Figure G.2a, the five dots correspond to the pattern of exports and imports at $t = 1$ in the five regions of OR's original analysis. In Figure G.2a, the five dots correspond to the same five regions at $t = 2$.

In order to relate OR's original analysis to ours, Figure G.2 plots the Engel curve for aggregate imports and exports in their Armington model. This is the same as in Figure 1b. For each period $t = 1$ or 2, we report the pattern of aggregate imports and exports that correspond to each of the 5 regions in OR's analysis. Figure G.2 illustrates the distinction between our sufficient statistic, the slope of the Engel curve, and the real domestic interest rate emphasized in OR. From Figure G.1a, we see that the interest rate varies between regions I and II and as well as between regions IV and V. This reflects variation in the price of non-tradables across these regions. From Figure G.2a, however, we see that the slope of the Engel curve is the same in regions I and II and the same in regions IV and V. This slope, rather than the price of non-tradables, is what is relevant for the comparative static exercise that we are interested in, and that OR abstract from, namely the impact of changes in tariffs on the trade deficits.

H Proof of Proposition 6

We first derive equation (10). We then present the assumptions imposed in the small economy limit of a Ricardian economy as considered in Alvarez and Lucas (2007). Finally, we use equation (10) and the previous assumptions to establish Proposition 6.

Derivation of Equation (10). We drop t for notational convenience. Define

$$\mathcal{C}(M, X; w) \equiv \max_{(c,m,x)\in Y} G(c),$$

subject to

$$p_m^* \cdot m = M,$$
$$w(\bar{p}_x^* \cdot x) = X.$$

The associated expenditure function is

$$e(C, \tau; w) \equiv \min_{M,X\geq 0} \{(1+\tau)M - X\}$$
$$s.t : \mathcal{C}(M, X; w) \geq C.$$

We let $M(C, \tau; w)$ and $X(C, \tau; w)$ denote the solution.

Note that $\mathcal{C}(M, X; w) = \mathcal{C}(M, X/w; 1)$. Thus we can change variable, $\tilde{X} = X/w$, to rearrange the expenditure function as

$$e(C, \tau; w) = \min_{M,\tilde{X}\geq 0} \{(1+\tau)M - w\tilde{X}\}$$
$$s.t : \mathcal{C}(M, \tilde{X}; 1) \geq C.$$

Its derivative and cross-derivative then satisfy

$$e_w(C,\tau;w) = -\tilde{X}(C,\tau;w) = -X(C,\tau;w)/w \le 0,$$
$$e_{Cw}(C,\tau;w) = -\tilde{X}_C(C,\tau;w) = -X_C(C,\tau;w)/w \le 0.$$

This leads to

$$\frac{\partial \ln e_C}{\partial \ln w} = \frac{w e_{Cw}(C,\tau;w)}{e_C(C,\tau;w)} = -\frac{X_C(C,\tau;w)}{(1+\tau)M_C(C,\tau;w) - X_C(C,\tau;w)}.$$

From equation (6), we know that

$$\frac{\partial \ln e_C}{\partial \ln(1+\tau)} = \frac{(1+\tau)M_C(C,\tau;w)}{(1+\tau)M_C(C,\tau;w) - X_C(C,\tau;w)}.$$

Combining the two previous expressions and using the definition of ϵ_w, we finally get

$$\begin{aligned}\frac{d\ln e_C}{d\ln(1+\tau)} &= \frac{\partial \ln e_C}{\partial \ln(1+\tau)} + \frac{\partial \ln e_C}{\partial \ln w}\frac{\partial \ln w}{\partial \ln(1+\tau)} \\ &= \frac{(1+\tau)M_C(C,\tau;w)}{(1+\tau)M_C(C,\tau;w) - X_C(C,\tau;w)} - \frac{\epsilon_w X_C(C,\tau;w)}{(1+\tau)M_C(C,\tau;w) - X_C(C,\tau;w)},\end{aligned}$$

which can be rearranged into (10).

Assumptions for a Small Ricardian Economy à la Alvarez and Lucas (2007). There is a fixed endowment of labor L_t. Preferences are CES and the distribution of labor productivity is such that aggregate exports and imports follow a gravity equation with trade elasticity $\varepsilon > 0$. This may derive either from a Fréchet distribution, as in Eaton and Kortum (2002), or a degenerate distribution, as in Anderson and Van Wincoop (2003). Under these assumptions, aggregate imports and exports satisfy

$$(1+\tau)M_t = \frac{[(1+\delta_t)(1+\tau)]^{-\varepsilon}}{(w_t)^{-\varepsilon} + [(1+\delta_t)(1+\tau)]^{-\varepsilon}} \times P_t(w_t,\tau)C_t, \tag{H.1}$$

$$X_t = w_t L - \frac{(w_t)^{-\varepsilon}}{(w_t)^{-\varepsilon} + [(1+\delta_t)(1+\tau)]^{-\varepsilon}} \times P_t(w_t,\tau)C_t, \tag{H.2}$$

with the price index such that

$$P_t(w_t,\tau) \equiv [(w_t)^{-\varepsilon} + [(1+\delta_t)(1+\tau)]^{-\varepsilon}]^{-\frac{1}{\varepsilon}}.$$

Each period, the domestic wage w_t equalizes the global demand for domestic labor with its supply. In the limit when the rest of the world is infinitely large, but its share of expenditure on domestic goods is infinitely small, this labor market clearing condition takes the form,

$$\frac{(w_t)^{-\varepsilon} \times P_t(w_t, \tau) C_t}{(w_t)^{-\varepsilon} + [(1+\delta_t)(1+\tau)]^{-\varepsilon}} + [(1+\delta_t) w_t]^{-\varepsilon} = w_t L_t. \tag{H.3}$$

The equilibrium wage function $w_t(C_t, \tau)$ is the solution to (H.3).

Proof of Proposition 6. If the environment is stationary, the expenditure function is the same in the two periods: $e_t(C, \tau; w) = e(C, \tau; w)$. So is the equilibrium wage function: $w_t(C, \tau) = w(C, \tau)$. Accordingly, we can write the domestic real interest rate

$$R(C_1, C_2, \tau) = R^* \frac{e_C(C_1, \tau; w(C_1, \tau))}{e_C(C_2, \tau; w(C_2, \tau))}.$$

It follows that

$$\frac{d \ln R}{d \ln(1+\tau)} > 0 \text{ if and only if } \frac{d \ln e_C(C_1, \tau; w(C_1, \tau))}{d \ln(1+\tau)} > \frac{d \ln e_C(C_2, \tau; w(C_2, \tau))}{d \ln(1+\tau)}.$$

To establish Proposition 6, it is therefore sufficient to show that $d \ln e_C(C, \tau; w(C, \tau))/d \ln(1+\tau)$ is strictly increasing in C. We do so in three steps.

First, we establish that $\epsilon_w(c, \tau) \equiv \partial \ln w(C, \tau)/\partial \ln(1+\tau) \in (0,1)$ and is strictly increasing in C. Second, we show that $X_C(C, \tau; w(C, \tau))/M_C(C, \tau; w(C, \tau))$ is strictly increasing in C as well. Third, we combine these two results with equation (10) to establish the monotonicity of $d \ln e_C(C, \tau; w(C, \tau))/d \ln(1+\tau)$.

Step 1: $\partial \ln w(C, \tau)/\partial \ln(1+\tau) \in (0,1)$ ***and is strictly increasing in*** C**.** Define

$$\mathcal{L}(w) \equiv \left[\frac{w}{[(w)^{-\varepsilon} + (1+\delta)^{-\varepsilon}]^{-\frac{1}{\varepsilon}}} \right]^{-(1+\varepsilon)},$$
$$\mathcal{L}^*(w) \equiv (1+\delta)^{-\varepsilon} w^{-(1+\varepsilon)}.$$

Next, differentiate (H.3) with respect to τ. This implies

$$\frac{\partial \ln w(C, \tau)}{\partial \ln(1+\tau)} = \frac{\alpha(C, \tau) \epsilon_L(w(C, \tau)/(1+\tau))}{\alpha(C, \tau) \epsilon_L(w(C, \tau)/(1+\tau)) + (1-\alpha(C, \tau)) \epsilon_{L^*}(w(C, \tau))}, \tag{H.4}$$

with

$$\alpha(C,\tau) \equiv \frac{\mathcal{L}(w(C,\tau)/(1+\tau))C}{\mathcal{L}(w(C,\tau)/(1+\tau))C + \mathcal{L}^*(w(C,\tau))},$$
$$\epsilon_L(w) \equiv -\frac{d\ln \mathcal{L}(w)}{d\ln w},$$
$$\epsilon_{L^*}(w) \equiv -\frac{d\ln \mathcal{L}^*(w)}{d\ln w}.$$

Note that

$$\epsilon_L(w) = (1+\varepsilon)\times\left[1 - \frac{w^{-\varepsilon}}{w^{-\varepsilon}+(1+\delta)^{-\varepsilon}}\right] > 0, \tag{H.5}$$
$$\epsilon_{L^*}(w) = 1+\varepsilon > \epsilon_L(w). \tag{H.6}$$

From H.4-H.6, it follows that $\partial \ln w(C,\tau)/\partial \ln(1+\tau) \in (0,1)$.

Let us now show that $\partial \ln w(C,\tau)/\partial \ln(1+\tau)$ is strictly increasing in C. Define

$$\omega(\alpha,\epsilon_L,\epsilon_{L^*}) \equiv \frac{\alpha\epsilon_L}{\alpha\epsilon_L + (1-\alpha)\epsilon_{L^*}}.$$

It satisfies $\omega_\alpha > 0$, $\omega_{\epsilon_{L^*}} < 0$, and $\omega_{\epsilon_L} > 0$. Second, note that

$$\frac{d\alpha}{dC} = \frac{\mathcal{L}\mathcal{L}^*}{(\mathcal{L}C+\mathcal{L}^*)^2} + \alpha(1-\alpha)(\epsilon_{L^*}-\epsilon_L)(w_C/w) > 0,$$
$$\frac{d\epsilon_L}{dC} = \frac{\varepsilon(1+\varepsilon)}{(1+\tau)}\left[\frac{(w/(1+\tau))^{-\varepsilon-1}}{((w/(1+\tau))^{-\varepsilon}+(1+\delta)^{-\varepsilon})^2}\right]w_C > 0,$$
$$\frac{d\epsilon_{L^*}}{dC} = 0,$$

where we have used the fact that $\epsilon_{L^*} > \epsilon_L$, $\varepsilon > 0$, and $w_C > 0$, since $w(C,\tau)$ is the solution to (H.3). Totally differentiating (H.4) with respect to C and using the previous observations, we get $\partial \ln w(C,\tau)/\partial \ln(1+\tau)$ strictly increasing in C.

Step 2: $X_C(C,\tau;w(C,\tau))/M_C(C,\tau;w(C,\tau)) < 0$ ***is strictly increasing in*** C**.** From (H.1) and (H.2), we know that

$$M_C(C,\tau;w) = \frac{[(1+\delta)(1+\tau)]^{-\varepsilon}}{(w)^{-\varepsilon}+[(1+\delta)(1+\tau)]^{-\varepsilon}}\times P(w,\tau)C,$$
$$X_C(C,\tau;w) = -\frac{(w)^{-\varepsilon}}{(w)^{-\varepsilon}+[(1+\delta_t)(1+\tau)]^{-\varepsilon}}\times P(w,\tau)C.$$

This implies
$$\frac{X_C(C,\tau;w(C,\tau))}{M_C(C,\tau;w(C,\tau))} = -\frac{(w(C,\tau))^{-\varepsilon}}{[(1+\delta)(1+\tau)]^{-\varepsilon}} < 0.$$
Since $w_C > 0$ and $\varepsilon > 0$, we get that $X_C(C,\tau;w(C,\tau))/M_C(C,\tau;w(C,\tau))$ is strictly increasing in C.

Step 3: $d\ln e_C(C,\tau;w(C,\tau))/d\ln(1+\tau)$ is strictly increasing in C. Write (10) as
$$\frac{d\ln e_C(C,\tau;w(C,\tau))}{d\ln(1+\tau)} = \frac{1-g(C)h(C)}{1-h(C)},$$

with

$$g(C) \equiv \frac{\partial \ln w(C,\tau)}{\partial \ln(1+\tau)},$$
$$h(C) \equiv \frac{X_C(C,\tau;w(C,\tau))}{(1+\tau)M_C(C,\tau;w(C,\tau))}.$$

Taking log and totally differentiating with respect to C, we then get
$$\frac{d\ln}{d\ln C}\left[\frac{d\ln e_C(C,\tau;w(C,\tau))}{d\ln(1+\tau)}\right] = -\frac{g'(C)h(C)}{1-g(C)h(C)} + h'(C) \times \left[\frac{1-g(C)}{(1-h(C))(1-g(C)h(C))}\right].$$
Step 1 implies $g(C) \in (0,1)$ and $g'(C) > 0$. Step 2 implies $h(C) < 0$ and $h'(C) > 0$. It follows that $d\ln e_C(C,\tau;w(C,\tau))/d\ln(1+\tau)$ is strictly increasing in C.

I Proof of Proposition 7

Let us first compute aggregate imports and exports as a function of aggregate consumption C_t and the permanent tariff τ. Since the economy satisfies the same assumptions as in Proposition 2, we can follow same steps as our previous proof to show that the value function of the firm problem is

$$\tilde{e}_t(p_{ct}) = \min_{y\in Y} -(p_c(\tau)\cdot y) \equiv \alpha(\tau),$$

with
$$p_{i,c}(\tau) = \begin{cases} (1+\tau)(1+\delta_{i,m})p^*_{i,m} & \text{if } m_{i1}, m_{i2} > 0, \\ p^*_{i,xt}/(1+\delta_{i,x}) & \text{if } x_{i1}, x_{i2} > 0. \end{cases}$$
For future reference, we let $y(\tau)$ denote the associated solution to the firm problem. Likewise, since the economy satisfies the same assumptions as in Proposition 2, the value

function of the household problem is

$$\hat{e}(p_{ct}, C_t) = \min_c \{p_c(\tau) \cdot c | G_t(c) = C_t\} \equiv \beta(\tau) C_t.$$

We let $c(\tau)$ denote the solution of the previous problem for $C_t = 1$. The values of aggregate imports and exports can then be expressed as

$$M(C_t, \tau) = \sum_{i \in \mathcal{M}} p^*_{i,m}(1 + \delta_{i,m})[c_i(\tau)C_t - y_i(\tau)] \equiv -\alpha_M(\tau) + \beta_M(\tau)C_t,$$
$$X(C_t, \tau) = \sum_{i \in \mathcal{X}} p^*_{i,x}[y_i(\tau) - c_i(\tau)C_t]/(1 + \delta_{i,x}) \equiv \alpha_X(\tau) - \beta_X(\tau)C_t,$$

where $\mathcal{M}$ and $\mathcal{X}$ denote the set of imported and exported goods, respectively,

$$\alpha_M(\tau) \equiv \sum_{i \in \mathcal{M}} p^*_{i,m}(1 + \delta_{i,m}) y_i(\tau),$$
$$\alpha_X(\tau) \equiv \sum_{i \in \mathcal{X}} p^*_{i,x} y_i(\tau)/(1 + \delta_{i,x}),$$
$$\beta_M(\tau) \equiv \sum_{i \in \mathcal{M}} p^*_{i,m}(1 + \delta_{i,m}) c_i(\tau),$$
$$\beta_X(\tau) \equiv \sum_{i \in \mathcal{X}} p^*_{i,x} c_i(\tau)/(1 + \delta_{i,x}).$$

For future reference, note that $\alpha'_M(\tau) \geq 0$ and $\beta'_M(\tau) \leq 0$. For output choices, this follows from the fact the value of output produced among $\mathcal{M}$ and $\mathcal{X}$ in the import and export sectors, $\alpha_M(\tau)$ and $\alpha_X(\tau)$, must solve

$$\max_{\alpha_M, \alpha_X} (1 + \tau)\alpha_M + \alpha_X$$
$$F(\alpha_M, \alpha_X) \leq 0,$$

with $F(\alpha_M, \alpha_X)$ the aggregate production possibility frontier between the import and export sectors,

$$F(\alpha_M, \alpha_X) \equiv \min_y H(y)$$
$$\sum_{i \in \mathcal{M}} p^*_{i,m}(1 + \delta_{i,m}) y_i = \alpha_M,$$
$$\sum_{i \in \mathcal{X}} p^*_{i,x} y_i(\tau)/(1 + \delta_{i,x}) = \alpha_X,$$

and $H(y)$ the production possibility frontier associated with the domestic net-production set Ω, i.e. $y \in \Omega$ if and only if $H(y) \leq 0$. The argument for consumption choices is similar.

Next, let us compute $(\delta C_t)_{D_t=cst}$ in this environment. Recall that it is defined by

$$\mathcal{D}_C(C_t(\tau),\tau) \times (\delta C_t)_{D_t=cst} + \mathcal{D}_\tau(C_t(\tau),\tau) \times d\tau = 0.$$

Using the fact that

$$\mathcal{D}_\tau(C,\tau) = -\tau M_\tau(C,\tau) = -\tau[\beta'_M(\tau)C - \alpha'_M(\tau)],$$
$$\mathcal{D}_C(C,\tau) = M_C(C,\tau) - X_C(C,\tau) = \beta_M(\tau) + \beta_X(\tau),$$

we therefore get

$$(\delta C_t)_{D_t=cst} = -\frac{\beta'_M(\tau)C_t(\tau) - \alpha'_M(\tau)}{\beta_M(\tau) + \beta_X(\tau)}$$

and, in turn,

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} = \frac{\beta'_M(\tau)C_1(\tau) - \alpha'_M(\tau)}{\beta'_M(\tau)C_2(\tau) - \alpha'_M(\tau)}.$$

Since the economy runs a trade deficit in period 1, we know that $C_1(\tau) > C_2(\tau)$. It follows that

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} \leq \frac{C_1(\tau)}{C_2(\tau)}, \tag{I.1}$$

with strict inequality if and only $\alpha'_M(\tau) > 0$.

Finally, recall that since the economy satisfies the same assumptions as in Proposition 2, we have

$$\partial R(C_1, C_2, \tau)/\partial\tau = 0.$$

In addition, intertemporal preferences are homothetic. Hence the Euler condition (3) implies that the ratio of consumption between the two periods must remain constant,

$$\frac{C_1(\tau + d\tau)}{C_2(\tau + d\tau)} = \frac{C_1(\tau)}{C_2(\tau)},$$

and, in turn, that

$$\frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}} = \frac{C_1(\tau + d\tau) - C_1(\tau)}{C_2(\tau + d\tau) - C_2(\tau)} = \frac{C_1(\tau)}{C_2(\tau)}. \tag{I.2}$$

Combining (I.1) and (I.2), we get

$$\frac{(\delta C_1)_{D_1=cst}}{(\delta C_2)_{D_2=cst}} \leq \frac{(\delta C_1)_{MRS=R}}{(\delta C_2)_{MRS=R}},$$

with strict inequality if and only if $\alpha'_M(\tau) > 0$. Following the same argument as in Section 4.1, the trade deficit must go down in period 1, $D'_1(\tau) \leq 0$, with strict inequality if and only if $\alpha'_M(\tau) > 0$.

J Example with Interest Rate and Distortion Channels

Consider a general Armington economy, as described in Section 3.3, but impose additionally that both G_t and $U(C_1, C_2)$ are CES with identical elasticity of substitution. Then utility is additive

$$\int \theta u(c_i) dF + \beta \int \theta u(c_i) dF,$$

with $u(c) = c^{1-\sigma}/(1-\sigma)$ and $\sigma > 0$. We refer to this environment as an additive economy.

We then have the following result.

Proposition 8. *Consider an additive economy. If preferences and prices are stationary, but (i) $\beta R^* < 1$ and/or (ii) $y_1 \le y_2$ then $D_1(\tau) > D_2(\tau)$ and $D_1'(\tau) < 0$ for all $\tau < \hat{\tau}$ with $\hat{\tau}$ defined as in Proposition 3.*

Proof. We draw on the the characterization from F. In particular, using (F.1)-(F.2) we can write the intertemporal trade balance in as a function of the two multipliers $(\tilde{\mu}_m, \tilde{\mu}_x)$:

$$M_1(\tilde{\mu}_m) - X_1(\tilde{\mu}_x) + M_2(\tilde{\mu}_m) - X_2(\tilde{\mu}_x) = NFA$$

Since $\tilde{\mu}_m = \tilde{\mu}_x (1+\tau)^{-1/\sigma}$. An increase in the tariff amounts to a decrease in $\tilde{\mu}_m$ and an increase in $\tilde{\mu}_x$ with $\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} < 0$ solving

$$M_1'(\tilde{\mu}_m) - X_1'(\tilde{\mu}_x)\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} + M_2'(\tilde{\mu}_m) - X_2'(\tilde{\mu}_x)\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} = 0$$

Then

$$\frac{d\tilde{\mu}_x}{d\tilde{\mu}_m} = -\frac{\bar{z}_{1m} + \bar{z}_{2m}}{\bar{z}_{1x} + \bar{z}_{2x}}$$

It then follows that

$$dD = \bar{z}_{1m} - \bar{z}_{1x}\frac{\bar{z}_{1m} + \bar{z}_{2m}}{\bar{z}_{1x} + \bar{z}_{2x}} < 0$$

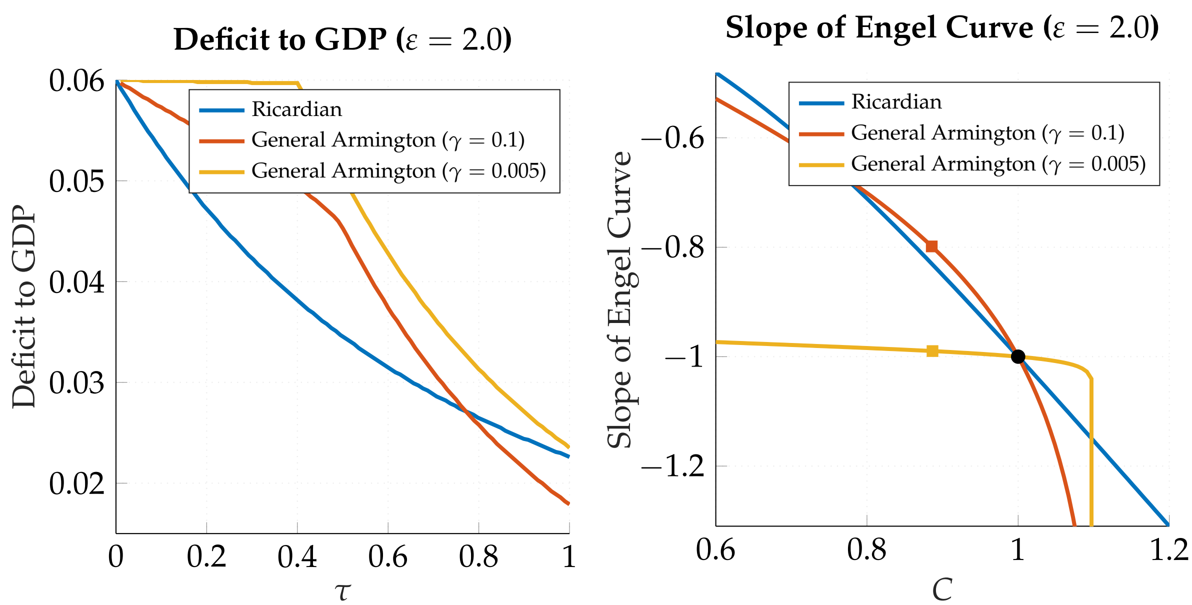
Since $\bar{z}_{ti} > 0$ this holds if and only if

$$\frac{\bar{z}_{2m}}{\bar{z}_{2x}} > \frac{\bar{z}_{1m}}{\bar{z}_{1x}}.$$

□

K Alternative Calibrations

In this Appendix, we report the counterpart of Figure 5 for alternative calibrations of the trade elasticity, namely for $\varepsilon = 2$ and $\varepsilon = 8$.

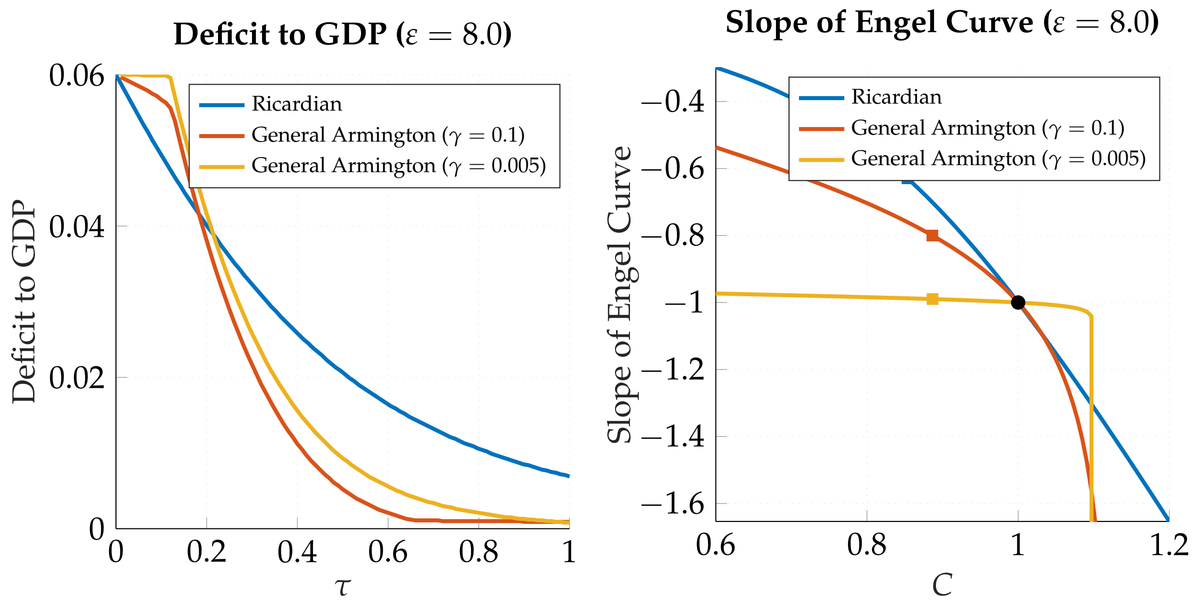


(a) Deficit and Tariff

(b) Slope of Engel Curves

Figure K.1: Quantitative Examples

Notes: Figure K.1a plots the deficit D_t, expressed as a share of GDP, as a function of the tariff τ, in two general Armington economies (with $\gamma = 0.1$ and $\gamma = 0.005$) and a Ricardian economy. All three economies are calibrated to match a share of imports to GDP equal to 15%, a ratio of deficit to GDP equal to 6%, and a trade elasticity $\varepsilon = 2$ in the free trade equilibrium ($\tau = 0$). Figure K.1b plots the slopes of the Engel curves: M_C/X_C in the Armington case and $M_C/(wX_C)$ in the Ricardian case. All three economies are normalized so that $C_1 = 1$ and the slope of the Engel curve is -1 at $C = 1$. Squares represent the slopes at C_2.



(a) Deficit and Tariff

(b) Slope of Engel Curves

Figure K.2: Quantitative Examples

Notes: Figure K.2a plots the deficit D_t, expressed as a share of GDP, as a function of the tariff τ, in two general Armington economies (with $\gamma = 0.1$ and $\gamma = 0.005$) and a Ricardian economy. All three economies are calibrated to match a share of imports to GDP equal to 15%, a ratio of deficit to GDP equal to 6%, and a trade elasticity $\varepsilon = 8$ in the free trade equilibrium ($\tau = 0$). Figure K.2b plots the slopes of the Engel curves: M_C/X_C in the Armington case and $M_C/(wX_C)$ in the Ricardian case. All three economies are normalized so that $C_1 = 1$ and the slope of the Engel curve is -1 at $C = 1$. Squares represent the slopes at C_2.