

Divide or Confer: Aggregating Information without Verification

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Abstract

We examine receiver-optimal mechanisms for aggregating information divided across many biased senders. Each sender privately observes an unconditionally independent signal about an unknown state, so no sender can verify another’s report. A receiver makes a binary accept/reject decision that determines the players’ payoffs via the state. When information is divided across a small population, and bias is low, the receiver-optimal mechanism coincides with the sender-preferred allocation, and can be implemented by letting senders *confer* privately before reporting. However, for larger populations, the receiver can benefit from the informational *divide*. We introduce a novel *incentive-compatibility-in-the-large* approach to solve the high-dimensional mechanism design problem for the large-population limit. Using this, we show that optimal mechanisms converge to one that depends only on the accept payoff and punishes excessive consensus in the direction of the common bias. These surplus burning punishments lead to payoffs that are bounded away from the first-best.

1 Introduction

Decision makers often have to rely on information that is divided across individuals with whom they have a conflict of interest. For instance, a CEO considering a new product needs to understand both the demand and the costs associated with it. However, this information is divided

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between the marketing manager (demand) and the production manager (costs) who have different incentives to the CEO. In other settings, the information is divided across much larger populations, which may fundamentally alter the decision maker’s approach to eliciting it. For example, governments have to assess the effectiveness of policies that may depend on local knowledge spread across large populations. Similarly, social media and e-commerce platforms often need to aggregate information about consumer products, political news, and opinions that is divided among a large number of individuals whose interests differ from those of the platform or society at large.¹

A natural solution to this problem is verification. In particular, it might be possible to compare a source’s report to that of the others and punish them collectively in case of disagreement. This strategy is effective when agents’ information sets overlap to such an extent that each individual’s information is largely dispensable.² However, when information is divided among individuals as in the aforementioned examples, cross-verification is not possible: no individual has information that can be used to verify the claims of another.³ A natural question then arises: Even though verification is impossible, is there some other way for a decision maker to benefit from information being divided across many agents? Specifically, how does dealing with a large number of senders affect optimal aggregation?

We examine this problem by studying optimal aggregation mechanisms in a multi-sender cheap talk environment. A receiver faces a binary decision: “accept” or “reject”. While the payoff to reject is known, all players’ payoffs from accept increase in an aggregate state variable. We consider a population of N biased senders, each receiving a private signal about the state. The senders share a common bias toward acceptance. Crucially, the senders’ signals are unconditionally independent, meaning no sender can verify another’s report.⁴ We take a mechanism design approach to find aggregation rules that maximize the *receiver’s* payoff in this environment.

Of particular interest is the mechanism’s ability to improve the payoffs when the same amount of information is divided among a larger number of agents. To examine this question, we adjust

¹In the context of product ratings, a constant problem that e-commerce platforms and regulators have to deal with is fake reviews and other forms of ratings manipulation by merchants and customers. A sizeable literature in computer science and marketing discusses methods for detecting fake reviews; see, for example, [Mohawesh et al. \(2021\)](#) or [He et al. \(2022\)](#).

²For instance, [Krishna and Morgan \(2001\)](#) show how to extract information perfectly when two senders have the same information. [Gerardi et al. \(2009\)](#) show a similar result with large populations. In these settings, in addition to having information that is valuable to an uninformed receiver, the senders’ information can be used as a verification tool to incentivize truth-telling.

³Verification mechanisms often rely on fact checking, which is known to be costly to implement. Fact checking is often noisy and, despite social media companies’ efforts, fairly ineffective. For example, since February 2025, Meta stopped fact checking to a great extent. For an example of ineffectiveness of fact checking in France, see [Barrera et al. \(2020\)](#).

⁴This structure of private information also satisfies the statistical properties of privacy developed in [Strack and Yang \(2024\)](#) and [He et al. \(2024\)](#). Hence, privacy of information and verifiability are intimately related.

the informativeness of each sender’s signal so that as the number of senders changes, their collective private information (i.e., the total variance of the state variable) stays constant relative to their conflict of interest with the receiver. Formally, in our baseline model, we assume that signals are i.i.d. across senders and that the state is the sum of the signals divided by $\sqrt{N}$.⁵

Our first result shows that when the number of senders is low and bias is moderate, the *receiver-optimal* aggregation mechanism coincides with the mechanism preferred by *senders*. The sender-preferred mechanism collects all signals and recommends accept whenever doing so serves the senders’ common interest. While senders have clear incentives to report truthfully under this mechanism, it imposes costs on the receiver: there exists an intermediate region of state values—the disagreement region—where the receiver would prefer reject but the mechanism recommends accept. Despite this cost, Proposition 1 establishes that the sender-preferred allocation remains constrained optimal for the receiver when N is small and bias is moderate. The key insight is that any attempt to tilt the mechanism toward the receiver’s preferences in the disagreement region requires costly surplus-burning punishments in the agreement region to maintain incentive compatibility. When the number of senders and bias are low, these punishment costs outweigh the benefits. This result suggests that allowing senders to *confer* privately and communicate as a unified entity can be optimal.

Our second and more important set of results concerns large populations, where each sender possesses relatively little information. We show that in this case, we can improve upon the sender-preferred allocation. While this may require complex mechanisms that depend on the entire profile of senders’ reports for any fixed N , we can characterize optimal aggregation mechanisms as the number of senders approaches infinity. Theorem 2 shows that a simple mechanism depending *only* on the aggregate state characterizes the optimum in the limit. This mechanism, depicted in Figure 1, recommends accept for an intermediate range of state values while rejecting both low and extremely high values.

Even though the limit mechanism is simple, deriving it requires new methodology. A challenge of working with mechanisms in the large economy is that the report of a single sender does not affect the distribution of reports, i.e., the probability of being pivotal is zero. To overcome this challenge, we introduce the concept of mechanisms that are *Incentive Compatible in the Large* (ICL). A mechanism defined for $N = \infty$ is ICL if there exists a sequence of incentive compatible mechanisms that converge to it as $N \rightarrow \infty$. Our key technical innovation (Theorem 1) fully characterizes these ICL mechanisms, showing that the incentive constraints collapse to simple conditions akin to standard envelope and monotonicity constraints.

⁵As we describe in Section 4, the large economy is equivalent to one in which the *state* is the realization path of a standard Brownian motion where each sender observes the incremental change of its path. In Section 5.2, we allow for general dependence of payoffs on the realization of signals.

The core insight is that as the number of senders grows, a sender's impact on the aggregate state vanishes at the same rate as their information rents. Information rents—the utility advantage one sender type can guarantee over another—depend on the probability of acceptance conditional on the sender's signal. By the law of large numbers, this probability becomes independent of any individual sender's report. While this intuition provides the necessary conditions for Theorem 1, establishing sufficiency requires sophisticated tools from functional analysis such as Banach's open mapping theorem and some inequalities related to the concentration of probability measures. Nevertheless, Theorem 1 enables a complete characterization of ICL mechanisms, allowing us to analyze the large economy (i.e., with an infinite number of senders) directly.

The intuition behind this interval mechanism centers on how it maintains incentive compatibility. The mechanism trades off two scenarios where a sender's upward lie matters: at the lower threshold, lying induces acceptance when payoffs are low; at the upper threshold, lying induces rejection when payoffs are high. To understand why this structure is optimal, consider starting from the sender-preferred mechanism and reducing acceptance probability at the bottom of the disagreement region, where the receiver gains most from rejection. Since senders are nearly indifferent at this boundary, the change minimally affects their over-reporting incentives. However, this perturbation still violates incentive compatibility. Surprisingly, the cheapest way to restore incentives is to punish senders by rejecting when the payoff to acceptance is highest for all players. While this punishment is costly when it occurs, such high-state reports have very low likelihood under truthful reporting relative to upward lies, due to the declining tail of the state distribution (as depicted in Figure 1). Hence, punishing at the top has the smallest ex-ante cost to the receiver relative to the incentive effect on the sender. Nonetheless, the use of such surplus-burning punishments imply that we are strictly bound away from first best even in the large-economy limit.

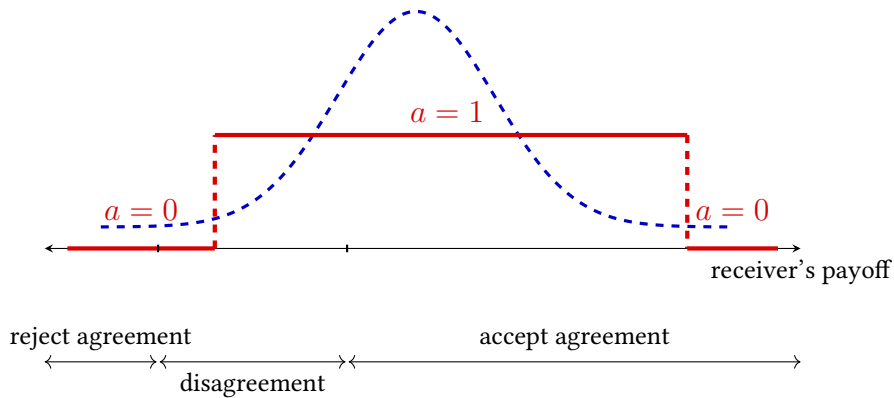


Figure 1: Optimal mechanism in the large economy. In the disagreement region, the preferred action of the senders is accept, and the receiver prefers reject. The dashed blue line represents the distribution of the receiver's payoff from accept as the number of senders tend to infinity.

Our results speak to the conditions under which aggregation mechanisms can benefit from keeping agents informationally divided in order to carefully manage the discourse. Proposition 1 states that there can be no benefit when groups are smaller and the conflict of interest is not too large. Indeed, the decision maker would like the informed parties to privately confer with one another and make a single joint recommendation. This implication is consistent with the common practice in firms, where the CEO canvasses the views of division managers before presenting a summary of the firm’s strategy to its board of directors. While private discussions are a simple way of implementing the sender preferred allocation, recent work by Antic et al. (2025) shows in a setting similar to ours that the senders can have “subversive conversations” where they are able to ensure the sender-preferred allocation despite being observed by the receiver. Thus, in combination with their results, Proposition 1 also provides conditions under which a completely laissez-faire attitude to managing the discourse can be optimal.

By contrast, Theorem 2 implies that when dealing with large societies, we can benefit by keeping agents informationally divided so that more sophisticated mechanisms can be applied. Increasingly, online platforms have the power to control how participants can share information. Moreover, as deep fakes, paid disinformation operatives, and fake reviews, make it ever harder to directly verify information, the need for more sophisticated tools becomes imperative. One tool in this fight, as suggested by Theorem 2, may be to punish suspiciously high levels of consensus by pooling these cases with low reports. Indeed, this may already happen in practice. For example, Facebook has a fact-checking policy that fits this insight to some extent. When there is a plethora of particular stories or reports, recommender algorithms flag them as possible misinformation. While this does not lead to removal of content, it can lead to Facebook reducing the visibility of these stories.⁶ Similarly, Amazon’s systems for deleting “fake” reviews appears very similar to a protocol that deletes just some proportion of the 5-star reviews whenever there are “too many” (He et al. (2022)).

The possibility of fake or paid reviews, deep fakes, and disinformation operatives suggests a model where some agents have different and extreme biases. To this end, we show that the simplicity of the optimal mechanisms identified using our concept of ICL for the baseline environment of the first part indeed generalizes beyond our environment to other aggregation problems. First, we consider a model where senders have heterogeneous biases but are otherwise the same as in the first part. In this setting, optimal mechanisms depend only on the aggregate state for each bias class of senders. Additionally, and when there are two groups of senders, the region of accept at the optimum is the area between two hyperbolic curves and relies more heavily on

⁶As of January 9th 2025, this is how Meta describes its fact-checking program, see <https://www.facebook.com/formedia/blog/third-party-fact-checking-how-it-works>. However, Meta announced at the beginning of 2025 that they plan to roll back this practice.

the report of low-bias senders. Second, we consider a model where the payoffs of senders are arbitrary functions of the distribution of signals. In some linear examples, optimal mechanisms are simple in that they depend only on the payoff realizations of the senders and of the receiver.

The rest of the paper is structured as follows. First, we review the relevant literature. Section 2 introduces the model. Sections 3 and 4 characterize the optimal mechanisms for small and large economies, respectively. Section 5 extends the model. Section 6 concludes the paper with a discussion. Proofs are relegated to the Appendix.

1.1 Related Literature

There is a long literature in information economics showing how information can be fully extracted via a cross-checking approach when agents have correlated information. [Cr mer and McLean \(1988\)](#) show that transfers can be used to achieve full surplus extraction when each agent’s information statistically identifies the others’. In a standard cheap talk setting ([Crawford and Sobel, 1982](#)) with multiple senders, [Krishna and Morgan \(2001\)](#), [Battaglini \(2002\)](#), and [Meyer et al. \(2019\)](#) show how punishing disagreement can support a fully revealing equilibrium when both senders have almost identical information. [Battaglini \(2004\)](#) and [Gerardi et al. \(2009\)](#) extend this to imperfectly correlated information, showing similar methods can still be used to implement the receiver-preferred allocation if the population is arbitrarily large. These results rely on being able to partially verify one sender’s claim by comparing it to the claims of the others. However, this comparison is not possible in our setting, so we cannot obtain the receiver-preferred allocation. Moreover, our optimal mechanisms may punish agreement rather than disagreement.

A notable exception to this literature in cheap talk is [Levy and Razin \(2007\)](#).⁷ In a setting with multiple senders with (multidimensional) imperfect signals, they show that the fully revealing equilibrium of [Battaglini \(2002\)](#) often fails to be an equilibrium. In their model, the senders and receivers fail to perfectly communicate, because information about a dimension with little conflict reveals too much about dimensions with too high of a conflict. While our models differ on details of payoffs and signals, both models share the feature that verification is impeded with perfect communication. To the extent that this is the case, a natural question is how can communication be improved by mediation or aggregation mechanisms. Our paper is one of the few that take a mechanism design approach to this question to study the optimal structure of communication.

The closest papers to ours are [Wolinsky \(2002\)](#) and, more recently, [Kattwinkel and Winter \(2024\)](#). [Wolinsky \(2002\)](#) examines a multiple-sender model with unconditionally independent binary types and a binary action. In his environment, senders can verifiably disclose their signals. He shows that optimal mechanisms are non-monotone and can consist of several intervals. In

⁷In addition to the multiple-sender case, and similar to [Chakraborty and Harbaugh \(2010\)](#), they also consider a one sender environment with multiple dimensions.

contrast, our paper allows for a general distribution of uncorrelated signals as well as a complete lack of commitment from the senders. This makes our model more suitable for analysis of information aggregation in environments where anonymity makes commitment difficult, such as social media. [Kattwinkel and Winter \(2024\)](#) show that, fixing population size, an interval mechanism is optimal for a sufficiently large bias in a binary-type binary-action setting with correlated types. The binary-type structure of both of these papers rules out the possibility of higher-dimensional mechanisms. We examine a complex multidimensional mechanism design problem. Nonetheless, we provide sharp conditions for the optimality of a very simple mechanism: the sender-preferred allocation. Moreover, we look at the comparative statics of population size, not just the conflict of interest. Finally, we show how the high-dimensional problem can be simplified for large populations. This insight extends beyond our particular interval mechanism structure, providing a novel approach for thinking about transfer-free mechanism design with large populations.

Another notable literature relevant to our paper is the one that studies communication by a single sender with mediation. [Goltsman et al. \(2009\)](#), [Salamanca \(2021\)](#), [Corrao and Dai \(2023\)](#), and [Best and Quigley \(2024\)](#) examine mediated communication with a single sender. One can interpret our results as providing conditions under which a mediator can (or cannot) improve on the single-sender setting by dividing information across many senders. [Carroll and Egorov \(2019\)](#) show that full information extraction is possible from a single expert who has multidimensional information when the designer can verify one dimension of the expert’s information. Instead, we examine a setting with multiple experts, each observing a single dimension of information. Critically, we have no source of verification, not even in equilibrium. Additionally, our techniques for dealing with large economies without transfers can be further employed to study different problems in this literature.

Additionally, our paper is related to a strand of the political economy literature that studies the problem of aggregation of voters’ preferences via elections. In a seminal paper, [Feddersen and Pesendorfer \(1997\)](#) show that voting can perfectly aggregate the information of voters with correlated information if the population is arbitrarily large. They show that in the large-population limit, an infinitesimal fraction of the population votes based on their private information about the candidate’s competence. However, due to the law of large numbers, this fraction has so much information about the candidate that the information of all the remaining voters is rendered irrelevant. Our results imply that the manner in which the economy becomes larger matters. In [Feddersen and Pesendorfer \(1997\)](#), as well as in [Battaglini \(2004\)](#) and [Gerardi et al. \(2009\)](#), increasing the size of the population increases the information held by any randomly chosen small set of the informed parties. In contrast, in our exercise, as we increase the size of the population, information is divided so that a small fraction of the population has little information and

the information of the remainder is indispensable. Thus, if an ever-shrinking proportion of the population provides information, then aggregation necessarily gets worse.

Our paper also relates to a rich literature on dynamic multi-sender cheap talk, such as [Aumann and Hart \(2003\)](#), [Krishna and Morgan \(2004\)](#), [Ambrus et al. \(2013\)](#), [Goloso et al. \(2014\)](#), [Migrow \(2021\)](#), and [Antic et al. \(2025\)](#). While these papers examine specific communication protocols via extensive form games, we take a mechanism design approach to find the receiver’s best possible communication outcome under any protocol.

Finally, our work is related to several recent papers that examine the limiting properties of optimal mechanisms. [Battaglini and Palfrey \(2024\)](#) study collective action problems where agents hold private information about their cost of contribution and optimal mechanisms to mitigate free-riding. [Frick et al. \(2023\)](#) and [Frick et al. \(2024\)](#) analyze mechanism design as data availability grows. Similar to our paper, they use properties from concentration of probability measures to describe limiting properties of various mechanisms. However, in contrast to our work, they apply mechanism design in environments where uncertainty vanishes as the economy becomes large. This approach implies that using mechanism design in the limit becomes trivial, and the critical step is to study the convergence properties of mechanisms for finite economies. In contrast, in our work, because uncertainty does not vanish in the limit, optimal mechanisms in the limit are nontrivial, and our characterization result in [Theorem 1](#) greatly simplifies the problem of finding them.

2 Model

We consider a model of strategic information transmission involving multiple senders (each “he”) and a single receiver (“she”).

There are N senders, denoted by $i \in \{1, 2, \dots, N\}$, who privately observe a random signal s_i (their “type”). This signal is distributed according to a common cumulative distribution function $F(s)$, with density $f(s)$ on an interval $S = [\underline{s}, \bar{s}] \subset \mathbb{R}$, that satisfies $\int_S s f(s) ds = 0$.⁸ Given the flexibility in the payoffs, as we will show, this normalization is without loss of generality. We assume s_i ’s are independent from each other. As a result, in our setting, each sender’s private signal contains no information about other senders’ information; therefore, it cannot be used to evaluate the truthfulness of the other agents’ reports. We use $\mathbf{s} = (s_1, s_2, \dots, s_N)$ to denote the senders’ type profile. Additionally, with a slight abuse of notation, we let $F(\mathbf{s}) = \prod_j F(s_j)$ and $F_{-i}(\mathbf{s}_{-i}) = \prod_{j \neq i} F(s_j)$ be the distribution functions of $\mathbf{s}$ and $\mathbf{s}_{-i}$, respectively, with the obvious corresponding density functions.

⁸In [Sections 4](#) and [5](#), we allow the distribution to be discrete, in which case, the density is taken with respect to a counting measure.

The receiver faces a binary decision problem, choosing an action $a \in \{0, 1\}$. We refer to action $a = 0$ as “reject”, and to action $a = 1$ as “accept”. The payoffs of the receiver and of the senders are, respectively,

$$\begin{aligned} u_R(a, \omega) &= a(\omega + r) \\ u_S(a, \omega) &= a(\omega + b) \end{aligned}$$

where

$$\omega = \sum_{i=1}^N \frac{s_i}{\sqrt{N}}, \tag{1}$$

with $b > 0$ and $r < b$ being the parameters determining the senders’ and the receiver’s preference toward $a = 1$, respectively. Note that since $b > r$, senders are “biased” toward $a = 1$. In other words, a conflict of interest exists between the senders and the receiver for realizations of the “state”, ω , between $-b$ and $-r$, where the senders prefer $a = 1$ while the receiver prefers $a = 0$.⁹ With a slight abuse of notation, we sometimes refer to ω as $\omega(s)$; we refer to the set of s such that $\omega(s) \in [-b, -r]$ as the *disagreement region*.

Our definition of ω scales each sender’s contribution by $1/\sqrt{N}$, so that the variance of the aggregate private information is constant in N , while the information held by each individual sender shrinks. This approach allows us to think of increasing N as dividing information into smaller increments. Indeed, even as $N \rightarrow \infty$, the distribution of ω remains non-degenerate, and individual senders may continue to influence outcomes at the margin. This perspective aligns with an interpretation in which each sender observes a disjoint increment of a Brownian motion, with its terminal value determining the payoff to action $a = 1$; we return to this formulation in Section 4.

We take a transfer-free mechanism design approach to identify the communication protocols that maximize the *receiver’s* payoff. In other words, we consider the set of mediated games between the senders and the receiver whose equilibrium is the standard perfect Bayesian equilibrium. The *revelation principle* of Myerson (1982) can be applied in our setting, and thus it is without loss to focus on direct recommendation mechanisms. Formally, a direct recommendation mechanism (henceforth, mechanism) collects the reports of the senders’ signals, the vector s , and recommends an action to the receiver. Hence, we can view a direct recommendation mechanism as a mapping $\sigma : S^N \rightarrow [0, 1]$, where $\sigma(s) = \Pr(a = 1|s)$ is the probability of recommending $a = 1$.

The revelation principle then requires that the mechanism σ satisfy (Bayesian) incentive com-

⁹This paper focuses on the case where senders are united in their disagreement with the receiver. Nonetheless, in Section 5.1, we show how our results extend to the case of heterogeneous bias.

patibility for the senders and obedience for the receiver:

1. *Incentive compatibility*: For each sender $i \in \{1, \dots, N\}$, and for all $s_i, \hat{s}_i \in S$,

$$\mathbb{E}[\sigma(\mathbf{s})(\omega(\mathbf{s}) + b) | s_i] \geq \mathbb{E}[\sigma(\hat{s}_i; \mathbf{s}_{-i})(\omega(\mathbf{s}) + b) | s_i]. \quad (2)$$

2. *Obedience*:

$$\mathbb{E}[\sigma(\mathbf{s})(\omega(\mathbf{s}) + r)] \geq 0 \geq \mathbb{E}[(1 - \sigma(\mathbf{s}))(\omega(\mathbf{s}) + r)]. \quad (3)$$

Inequality (2) requires that each sender prefer to truthfully report his own type to the mediator, given that the other senders do the same and the receiver obeys the mediator's recommendation. The obedience constraints in (3) state that the receiver should be willing to follow the recommendation of the mechanism. Since we are interested in mechanisms that maximize the receiver's payoff and the receiver can take only two actions, it is sufficient to focus on incentive compatibility and ignore obedience. This is because if for a mechanism σ , either of the inequalities in (3) is violated, the default action absent any information would dominate σ and satisfy incentive compatibility. We thus have the following lemma:¹⁰

Lemma 1. *If $\hat{\sigma}$ achieves the highest payoff for the receiver among all mechanisms that satisfy incentive compatibility (2), then $\hat{\sigma}$ satisfies obedience (3).*

Lemma 1 implies that the problem of finding the receiver's best mechanism boils down to the following:

$$\max_{\sigma: S^N \rightarrow [0,1]} \mathbb{E}[(\omega(\mathbf{s}) + r) \sigma(\mathbf{s})] \quad (\text{P})$$

subject to incentive compatibility (2).

Remark. In light of Lemma 1, the problem of finding the best mediated mechanism has an alternative interpretation. It can be directly considered a design problem in which the receiver can commit to her actions as a function of the senders' reports. For instance, we might equivalently consider an organization that commits to a voting rule that maps votes (reports) into a decision.

Before analyzing problem (P), it is useful to consider a few examples of mechanisms. Consider first the receiver's optimal *allocation*, which involves choosing $a = 1$ if and only if the receiver's payoff from accept is positive: $\sigma^R(\mathbf{s}) = \mathbf{1}[\omega(\mathbf{s}) + r \geq 0]$. Clearly, this mechanism can never be incentive compatible: Facing σ^R , a sender wishes to misreport his signal upward and increase the probability of $a = 1$. However, incentive compatibility is trivially satisfied by what we call the

¹⁰The formal proof of Lemma 1 is omitted for brevity and is available on request. When there are more than two actions, dropping obedience may not be without loss. See [Whitmeyer \(2024\)](#) for a counterexample. A similar result holds in [Ball \(2024\)](#). We thank Ian Ball for raising this point.

sender-preferred mechanism: $\sigma^S(s) = \mathbf{1}[\omega(s) + b \geq 0]$. Moreover, $\sigma^S(s)$ is obedient whenever it has a weakly positive payoff for the receiver: $\mathbb{E}[(\omega(s) + r)\sigma^S(s)] \geq 0$. When $N = 2$ and $r = 0 < b$, these allocations are depicted in Figure 2a. As illustrated, the sender-preferred mechanism has a cost to the receiver relative to σ^R . It implies that $a = 1$ whenever the state is in the disagreement region given by $\{s \mid -b < \omega(s) < -r\}$.

In addition to the sender-best mechanism, there are potentially other mechanisms that could satisfy incentive compatibility and obedience. Examples include equilibria of the cheap talk game where each sender sends a separate message. Such mechanisms could potentially be better for the receiver, but they are also costly, because the knowledge of the senders' total information could be beneficial for the receiver and for the senders.¹¹ The next section illustrates that indeed this logic proves right for small economies.

3 Optimal Aggregation in Small Economies

In this section, we characterize the optimal recommendation mechanism when the number of senders is small. For some intuition, we begin with the extreme case of $N = 1$, i.e., when a single sender wishes to communicate with the receiver. Because the sender fully knows the state, the incentive compatibility condition (2) becomes

$$\sigma(s_1)(s_1 + b) \geq \sigma(s'_1)(s_1 + b), \forall s_1, s'_1 \in S.$$

This condition implies that either $\sigma(s)$ is constant everywhere (i.e., is an uninformative mechanism) or it is constant and positive when $s_1 + b \geq 0$, and 0 otherwise. Hence, if the information that $s_1 \geq -b$ is valuable for the receiver (i.e., σ^S is obedient), the sender-preferred mechanism $\sigma^S(s_1)$ is also the best mechanism for the receiver.

The reason we cannot do better than σ^S with one sender is because he controls the entire report and can condition his deviation on perfect information about the state. This constraint implies that to reduce the probability of accept in the disagreement region, we also have to pay the high cost of reducing the probability of accept whenever the receiver agrees, i.e., when $\omega \geq -r$. When $N > 1$, each sender has fewer deviations available and less information; hence, there are more complicated mechanisms that can be used to incentivize agents more cheaply. Yet, if N is not too large, each sender still has too much information and control. Specifically, satisfying incentive compatibility while reducing $\sigma(s)$ requires costly distortions when the receiver agrees on the optimal action that outweigh the benefit. The following proposition shows that when

¹¹In the Online Appendix, Section C.2, we provide a few examples of mechanisms arising from independent cheap talk equilibria for the model with two senders as uniform signals.

$N > 1$ is below an explicit bound, we cannot improve upon the sender-preferred mechanism.

Proposition 1. *Let $f(\cdot)$ be a C^1 function with a finite derivative and full support, and define $\ell = \inf_{s_i \in S} f'(s_i) (\bar{s} - s_i) / f(s_i)$. Then, for any sender and receiver payoff parameters, (b, r) , there exists a positive finite value $\underline{N}(b, r, \ell, \bar{s})$ such that the sender-preferred mechanism $\sigma^S(s) = \mathbf{1}[\omega(s) + b \geq 0]$ is a solution to (P) whenever*

$$N \leq \underline{N}(b, r, \ell, \bar{s}).$$

Proposition 1 states that if the number of senders is low enough, then the best incentive compatible mechanism from the receiver's perspective is the same as that of the senders. It can be shown that the finite bound on the number of senders for the optimality of the sender-preferred mechanism, $\underline{N}(b, r, \ell, \bar{s})$, is decreasing in the bias of the sender, $b - r > 0$. Of course, for large levels of bias, even the uninformative allocation may be preferable to σ^S . Perhaps also unsurprising, when σ^S tends to the receiver first best, i.e., $b - r \rightarrow 0$, then this bound tends to infinity.¹² As we discuss in more detail later, one way to interpret $N \leq \underline{N}(b, r, \ell, \bar{s})$ is as a condition under which unrestricted private communication among the senders is indeed the best outcome from the receiver's perspective. Conversely, it identifies a necessary condition for the commitment of the mediator to be able to create value for the receiver by exploiting the informational division between the senders.

The proof of Proposition 1 has two parts. The first part establishes a weak duality result for the optimization (P). This implies that it is sufficient to come up with Lagrange multipliers—associated with the envelope formulation of the incentive compatibility and monotonicity of $\mathbb{E}[\sigma(s) | s_i]$ with respect to s_i —for which the senders' first-best allocation maximizes the Lagrangian associated with (P). We then prove the optimality of the sender-preferred mechanism by constructing the associated Lagrange multipliers.

To improve on the sender-preferred mechanism, it is necessary for N to be larger than the upper bound in Proposition 1. This condition is also sufficient when F is uniform.

Corollary 1. *Suppose that $f(s) = \frac{1}{|S|}$, i.e., s_i 's are uniformly distributed over the interval S . Then, the sender-preferred mechanism σ^S is the best mechanism for the receiver if and only if $N \leq \underline{N}(b, r, \ell, \bar{s})$.*

To understand these results better, we consider the case with uniform signals, $s_i \sim U[-1, 1]$,

¹²The bound is decreasing in $0 \geq \ell > -\infty$. The value of ℓ is the upper bound on the negative curvature of F . Thus, for highly negative values of ℓ , there may be a very large probability mass in the disagreement region relative to the probability of being in the agreement region. By considering the $N = 1$ case, one can see how this could easily imply that the cost of disagreement is just too high relative to the value of accepting when all parties agree.

and $r = 0$.¹³ Specifically, we consider a perturbation of the sender-preferred mechanism and its effect on the receiver's payoff. Obviously, any mechanism that benefits the receiver relative to the sender-preferred mechanism must reduce the probability of acceptance somewhere in the disagreement region (the red-dotted area in Figure 2a). Consider first reducing $\sigma(s)$ for some small amount ε_1 in the hypercube $B_1 = \left(-\frac{b}{\sqrt{N}}, -\frac{b}{\sqrt{N}} + dx\right]^N$. It is easy to see from the $N = 2$ case in 2b that B_1 lies within the disagreement region (for sufficiently small dx); therefore, such a reduction would benefit the receiver. However, it also violates incentive compatibility: Type $x_1 = -\frac{b}{\sqrt{N}} + dx$ now has a strict incentive to make a marginal upward lie. This, in turn, can be resolved by introducing a further reduction of σ by ε_2 in the hypercube $B_2 = (x_1, x_1 + dx]^N$ so this lie would now cause a loss when the other senders' types are in B_2 . In turn, to preserve incentive compatibility for $x_1 + dx$, we can reduce $\sigma(s)$ by some $\varepsilon_3 < \varepsilon_2$ in $B_3 = (x_2, x_2 + dx]^N$, and so on, as seen in Figure 2b.

Critically, in this perturbation, the reduction in σ is decreasing, i.e., $\varepsilon_{i+1} < \varepsilon_i$. This is because for any adjacent B_i and B_{i+1} , the reduction in σ is applied when the other senders' types are higher; thus, a lie for type x_i costs more per unit reduction in σ on B_{i+1} . Moreover, this per unit cost is increasing in N : When there are more senders, their aggregate contribution to the state is large conditional on being in B_{i+1} . Therefore, the rate at which ε_i decreases is increasing in N , and the cost of this perturbation to the receiver where $\omega \geq 0$ is lower when N is large. Hence, the net value of the perturbation is increasing in N . Indeed, in the proof of Corollary 1 (Online Appendix, Section C.3), we show that as dx becomes small, the gain for the receiver from such a perturbation is proportional to $b\sqrt{N} - 1$. That is, this perturbation increases the payoffs of the receiver if either the population or conflict of interest is large enough.

We briefly remark on the implications of our results for observed examples of information aggregation. As we have mentioned, an interpretation of Proposition 1 is that optimal mechanisms involve unrestricted private communication among the senders wherein the senders confer with each other and make a joint recommendation. In reality, expert information is often privately collated, before being disseminated via a single representative. Monetary policy committees announce their collated views to markets, spokespeople are frequently central in other political communications, and CEOs communicate an overall outlook for the firm in financial reporting periods. Our results indicate an advantage to such communication: When the number of senders or the senders' bias is not too large, it can maximize the expected payoffs of senders and receivers alike.¹⁴

¹³In this case, the necessary and sufficient condition for the optimality of σ^S is simply $N \leq \underline{N}(b, 0, 0, 1) = \frac{1}{b^2}$.

¹⁴While it is obvious that private communication among senders can implement the optimal mechanism in this case, it should be noted that Antic et al. (2025) illustrate that even in public this can be implemented via *subversive conversations* among the senders.

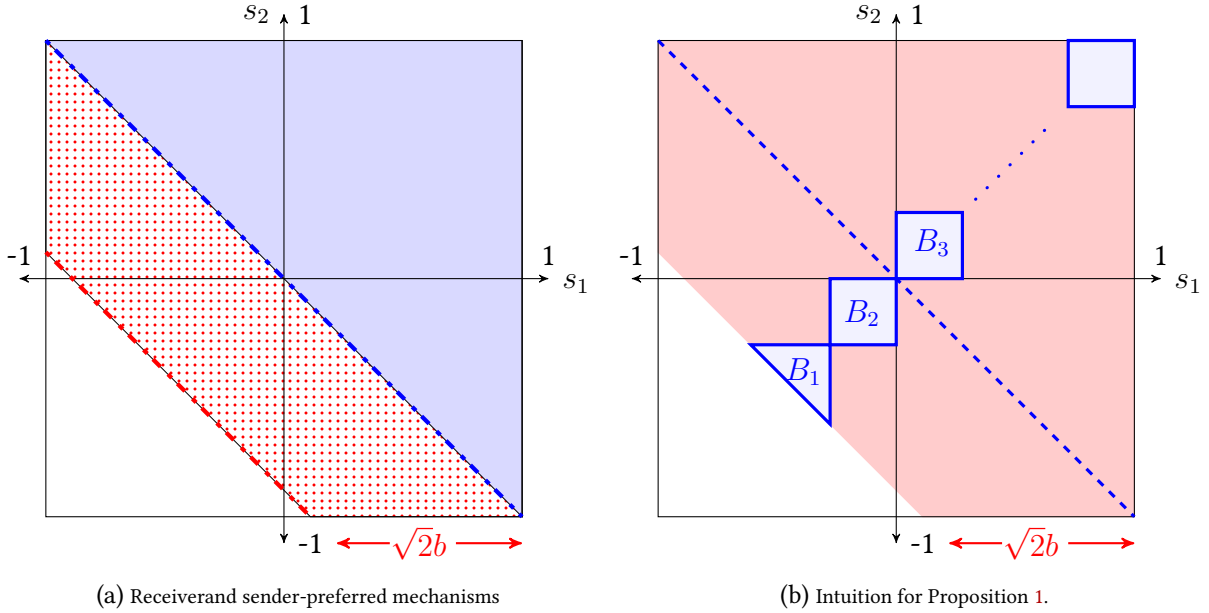


Figure 2: Two sender example: preferred mechanisms and intuition. In (a), the colored areas are associated with $\sigma = 1$. In (b), the red region represents the sender-preferred region with $\sigma = 1$. In the areas $B_1, B_2, B_3, \dots$, σ is reduced by $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots$, respectively.

Finally, the aforementioned discussion suggests two features of optimal mechanisms when the number of sellers or bias is large: First, the optimal mechanism may punish consensus among the senders in a non-monotone fashion. Second, the optimal mechanism is a complex high-dimensional function of the entire vector of reports $\mathbf{s} \in S^N$. Indeed, in the Online Appendix, Section D, we numerically show both of these to be true in the two-sender uniform example with a large bias. However, in the next section, we show that as the population becomes increasingly large, only the first feature is true, and that the optimal mechanism in the large-population limit takes on a simple non-monotonic interval structure.

4 Optimal Aggregation in Large Economies

In this section, we provide a tractable method to solve for optimal mechanisms in large economies. First, our main technical result (Theorem 1) provides a simple characterization for the limit set of incentive compatible mechanisms. Then, we use this result to provide a simple characterization of the optimal mechanism at infinity (Theorem 3), which we refer to as optimal aggregation in the large. This mechanism can be used to characterize the optimal recommendation mechanisms for large populations, because it is the limit point to which all optimal mechanisms converge.

As a first step, to deal with the exploding domain space S^N that accompanies large popula-

tions, we reformulate the design problem as a choice over mechanisms that map frequencies of signal realizations into the probability of recommending accept.

4.1 Preliminaries: Frequency Domain

For exposition, we assume a finite type space throughout the rest of the section.¹⁵ Let each sender's type s_i take on a finite number of values in the set $S = \{t_1, \dots, t_K\}$ with probability $f_k = \Pr(s_i = t_k)$, where t_k is increasing in k . We denote the vector of types by $\mathbf{t} = (t_1, \dots, t_K) \in \mathbb{R}^K$, and the probability distribution by $\mathbf{f} = (f_1, \dots, f_K)$. As before, we normalize the mean of s_i to zero; therefore, $\mathbf{f} \cdot \mathbf{t} = 0$.

As $N \rightarrow \infty$, by the central limit theorem, the state $\omega(\mathbf{s})$ becomes a normally distributed random variable. Moreover, the deviation of sample frequencies from distribution frequencies converges to the Normal distribution. We define the sample distribution, h_k , that represents this deviation as

$$h_k(\mathbf{s}) = \sqrt{N} \left(\frac{|\{i | s_i = t_k\}|}{N} - f_k \right), \quad (4)$$

with $\mathbf{h}^N(\mathbf{s}) = (h_1(\mathbf{s}), \dots, h_K(\mathbf{s}))$. We refer to $\mathbf{h}^N(\mathbf{s})$ as *normalized empirical frequencies (NEF)*, and to h_k as *k-NEF*. For completeness, we present the multidimensional central limit theorem for NEF in the following lemma:

Lemma 2. (Multidimensional Central Limit Theorem) *Let $\mathbf{h}^N(\mathbf{s})$ be the normalized empirical frequencies of $\mathbf{s}$ as defined in (4). Then, as $N \rightarrow \infty$,*

$$\mathbf{h}^N(\mathbf{s}) \rightarrow_d N(\mathbf{0}, \Sigma), \quad (5)$$

with $\Sigma_{kl} = f_k(\mathbf{1}(k = l) - f_l)$.

In what follows, we will use Lemma 2 to calculate the limits of the senders' and receiver's payoffs as $N \rightarrow \infty$.

Without loss of generality, we focus on symmetric recommendation mechanisms for which $\sigma(\mathbf{s}) = \sigma(\pi(\mathbf{s}))$ for any π , a permutation of $\{1, \dots, N\}$.¹⁶ This implies we may consider σ as a function in the frequency domain (i.e., NEF) for the realized s_i 's, which is given by the normalized $\mathbf{h}^N(\mathbf{s})$ into $[0, 1]$. With a slight abuse of notation, we denote this function by $\sigma(\mathbf{h}^N(\mathbf{s}))$. The state is defined as before, and we can write it in terms of $\mathbf{h}^N(\mathbf{s})$:

¹⁵While this assumption simplifies the technical analysis, it is not a substantive restriction. In Section 4.4, we discuss how our results still apply for a continuous type space.

¹⁶For any incentive compatible mechanism σ , the mechanism given by $\hat{\sigma}(\mathbf{s}) = \sum_{\pi} \sigma(\pi(\mathbf{s})) / N!$ is incentive compatible and symmetric and delivers the same payoff to the receiver.

$$\omega = \sum \frac{s_i}{\sqrt{N}} = \mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t}.$$

We further have that $\sum_{k=1}^K h_k^N(\mathbf{s}) = 0$. Finally, let $\mathbb{R}_0^K$ be the subset of $\mathbb{R}^K$ whose elements sum to zero.

4.1.1 Divided Information in the Large Economy

As we have emphasized, as $N \rightarrow \infty$, our model can be interpreted as one in which information is divided among a large group of senders. To see this, let $X_t^{(N)} = \sum_{i=1}^{\lfloor Nt \rfloor} \frac{s_i}{\sqrt{N}}$. The standard argument shows that as $N \rightarrow \infty$ $\left\{ X_t^{(N)} \right\}_{t \in [0,1]}$ converges to the sample paths of a Brownian motion with drift 0 and diffusion $\sqrt{\text{var}(s_i)}$; see, for example, Theorem 4.20 in [Karatzas and Shreve \(1988\)](#). In other words, $X^1 = \{X_t\}_{t \in [0,1]}$ is the state of the large economy, and $\omega = \int_0^1 dX_t = X_1$ is the payoff-relevant statistic. Figure 3 depicts two sample path versions of the state (i.e., X_t) and their associated NEFs. In subfigure (a), each sender's information is an increment of the accumulated signals. Subfigure (b) is constructed by calculating the frequency of the increments associated with the sample paths, subtracting the distribution of the signals, and multiplying the result by $\sqrt{N}$.¹⁷

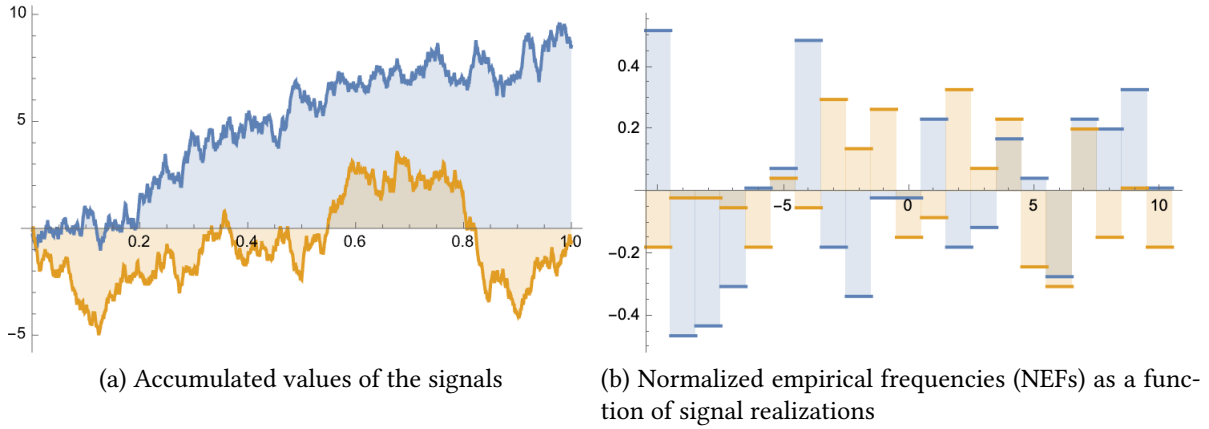


Figure 3: Simulated state of an economy with $N = 1000$ and type s_i uniformly distributed over the set $\{-10, -9, \dots, 10\}$. Subfigure (b) depicts the associated normalized empirical frequencies (NEFs).

¹⁷[Aybas and Callander \(2024\)](#) is another example of a cheap talk game with a Brownian state. In their model, however, a single sender observes the entire realization of the Brownian motion.

4.2 Optimality in the Large

We are interested in the limiting properties of optimal mechanisms as N grows large. To characterize this limit, we study a design problem in which feasible mechanisms must satisfy a notion of *incentive compatibility in the large*, which we define next. We show that this exercise is the correct way to think about large economies, by showing that the solution to this problem is indeed the unique limit point for all optimal, finite- N mechanisms (Theorem 1).

Definition 1. A recommendation mechanism $\sigma : \mathbb{R}_0^K \rightarrow [0, 1]$ is *incentive compatible in the large (ICL)* if a sequence of recommendation mechanisms $\sigma^N(\mathbf{h}^N(\mathbf{s}))$ exist that satisfy incentive compatibility, $\forall t_k, t_l \in S$,

$$\mathbb{E}^N [\sigma^N(\mathbf{h}^N(\mathbf{s})) (\mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t} + b) | s_i = t_k] \geq \mathbb{E}^N [\sigma^N(\mathbf{h}^N(t_l; s_{-i})) (\mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t} + b) | s_i = t_k],$$

and $\sigma^N(\mathbf{h})$ converges to σ in $L_\infty(\mathbb{R}_0^K)$, which is the set of bounded real functions over $\mathbb{R}_0^K$.

Our notion of convergence requires some discussion. Note that for any finite N , the set of feasible NEFs is a finite subset of $\mathbb{R}_0^K$. Therefore, for any finite N , a mechanism $\sigma^N(\mathbf{h})$ specifies the probability of $a = 1$ only for a finite number of points in $\mathbb{R}_0^K$. To embed σ^N in $L_\infty(\mathbb{R}_0^K)$, we consider a partition of $\mathbb{R}_0^K$ into closed convex subsets, each of which contains exactly one point of the form $\mathbf{h}^N(\mathbf{s})$ given by $A_{n_1, \dots, n_K}$, where $h_k^N(\mathbf{s}) = n_k/\sqrt{N} - f_k\sqrt{N}$.¹⁸ We then embed σ^N into $L_\infty(\mathbb{R}_0^K)$ by considering the function $\hat{\sigma}^N$ defined by

$$\hat{\sigma}^N(\mathbf{h}) = \sigma^N(\mathbf{h}^N(\mathbf{s})), \mathbf{h}, \mathbf{h}^N(\mathbf{s}) \in A_{n_1, \dots, n_K}.$$

Thus, according to our definition, σ is incentive compatible in the large if there exists an incentive compatible mechanism $\sigma^N(\mathbf{h})$ so that its equivalent $\hat{\sigma}^N(\mathbf{h})$ satisfies $\|\hat{\sigma}^N - \sigma\|_\infty \rightarrow 0$.

The following theorem fully describes the set of ICL mechanisms:

Theorem 1. A recommendation mechanism $\sigma : \mathbb{R}_0^K \rightarrow [0, 1]$ is ICL if and only if it satisfies

$$\mathbb{E}[\sigma(\mathbf{h}) (\mathbf{h} \cdot \mathbf{t} + b) h_k/f_k] = \mathbb{E}[\sigma(\mathbf{h})] t_k, 1 \leq k \leq K \quad (6)$$

$$\mathbb{E} \left[\sigma(\mathbf{h}) \frac{h_k}{f_k} \right] \text{ increasing in } k, \quad (7)$$

with $\mathbf{h} \sim \mathcal{N}(\mathbf{0}, \Sigma)$.

Theorem 1 is the main technical result of our paper and significantly simplifies the analysis of optimal mechanisms. As we illustrate next, the proof of the “only if” direction is straightforward

¹⁸There are $\binom{N+K-1}{K-1}$ such subsets.

and uses a simple application of the central limit theorem to calculate the incentive constraints in the limit as $N \rightarrow \infty$. In contrast, the proof of the “if” direction is much more technically challenging. Specifically, we use a multidimensional version of the Berry–Esseen theorem (see, for example, [Feller \(1991\)](#) for a one-dimensional version of it) that establishes convergence of the distribution of $\mathbf{h}^N(\mathbf{s})$ to that of the normal distribution $\mathcal{N}(0, \Sigma)$ according to the Kolmogorov–Smirnov metric (supremum difference between their measures), and the convergence is at rate $1/\sqrt{N}$. We can then use classic results from functional analysis, the *open mapping theorem* of Banach (see, for example, [Brezis \(2011\)](#)) to show existence of incentive compatible mechanisms with finite N that converge to a mechanism that satisfies (6) and (7).

To see the “only if” direction, note that with finitely many senders, incentive compatibility requires that type t_k prefer not to mimic t_l . Letting U_k^N denote t_k ’s truth-telling utility, we have

$$U_k^N - U_l^N \geq \frac{t_k - t_l}{\sqrt{N}} \mathbb{E} [\sigma^N(\mathbf{h}^N(\mathbf{s})) \mid t_l]. \quad (8)$$

If t_k mimics $t_l < t_k$, she can induce t_l ’s allocation and earn the right-hand side of (8) in marginal information rent. Because type realizations are independent across senders, the information rent of t_k beyond the payoff of t_l is t_l ’s allocation, i.e., the probability of $a = 1$ given t_l , when the state $\omega(\mathbf{s})$ is shifted by $(t_k - t_l)/\sqrt{N}$. By Bayes rule,

$$\Pr(\mathbf{h}^N(\mathbf{s}) \mid t_l) = \Pr(\mathbf{h}^N(\mathbf{s})) \frac{|\{i : s_i = t_l\}|}{f_l N} = \Pr(\mathbf{h}^N(\mathbf{s})) \left(\frac{h_l}{\sqrt{N} f_l} + 1 \right). \quad (9)$$

Because (8) must also hold for type t_l and a deviation to t_k , incentive compatibility bounds the rate at which utility varies in the sender’s private value, as follows:

$$\mathbb{E} [\sigma^N(\mathbf{h}^N(\mathbf{s})) \mid t_k] \geq \sqrt{N} \frac{U_k^N - U_l^N}{t_k - t_l} \geq \mathbb{E} [\sigma^N(\mathbf{h}^N(\mathbf{s})) \mid t_l]. \quad (10)$$

As $N \rightarrow \infty$, the aforementioned bounds converge to the unconditional expectation $\mathbb{E}[\sigma(\mathbf{h})]$. As (9) shows, when N becomes very large and each sender is a small component of the aggregate, his private information becomes uninformative for $\mathbf{h}$. Hence, in the limit, incentive compatibility pins the marginal utility of a unit increase in his private value (equivalently, a unit increase in ω due to an increase in his type) down to $\mathbb{E}[\sigma(\mathbf{h})]$. On the other hand, in any mechanism, the sender’s marginal utility depends only on the *rate* at which the sender’s conditional beliefs about $\mathbf{h}$ diverge from the prior. Indeed, a direct calculation of $\sqrt{N}(U_k^N - U_l^N)$ using (9) shows that $\sqrt{N}(U_k^N - U_l^N) \rightarrow \mathbb{E}[(\mathbf{h} \cdot \mathbf{t} + b) \sigma(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right)]$, and hence that in the limit, $\mathbb{E}[(\mathbf{h} \cdot \mathbf{t} + b) \sigma(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right)] = (t_k - t_l) \mathbb{E}[\sigma(\mathbf{h})]$.

Note that the incentive compatibility in the large is described by Theorem 1 in terms of condi-

tions known in the mechanism design literature. As we will see, Equation (6) can be interpreted as an envelope formulation of incentive compatibility; Equation (7) expresses a limiting version of the familiar interim monotonicity constraint.

Intuitively, one can view incentive compatibility in the large as balancing the cost and benefits of being pivotal. Consider adding sender i , whose signal realization is t_k . Sender i increases the payoff relevant state ω by $t_k/\sqrt{N}$ no matter what he reports. At the same time, by reporting t_k , he changes the distribution of reports by $\Pr(\mathbf{h}) \frac{h_k}{f_k \sqrt{N}}$. Hence, the pivotal effect of this sender is given by $\mathbb{E}[\sigma(\mathbf{h})((\mathbf{h} \cdot \mathbf{t} + b) h_k / f_k - t_k)]$.

The value of Theorem 1 is that it significantly simplifies the problem of finding the optimal mechanism as the number of senders converges to infinity. Specifically, an optimal mechanism in the limiting economy as $N \rightarrow \infty$ solves

$$\max_{\sigma: \mathbb{R}_0^K \rightarrow [0,1]} \mathbb{E}[\sigma(\mathbf{h})(\mathbf{h} \cdot \mathbf{t} + r)] \quad (\text{P1})$$

subject to

$$\mathbb{E}[\sigma(\mathbf{h})((\mathbf{h} \cdot \mathbf{t} + b) h_k / f_k - t_k)] = 0, \mathbb{E}\left[\sigma(\mathbf{h}) \frac{h_k}{f_k}\right] \geq \mathbb{E}\left[\sigma(\mathbf{h}) \frac{h_{k-1}}{f_{k-1}}\right], \quad (11)$$

where $\mathbf{h} \sim \mathcal{N}(\mathbf{0}, \Sigma)$.

The following theorem contains our main result on optimal mechanisms for large economies:

Theorem 2. *The optimum in (P1) is achieved by a recommendation mechanism that is a function of only the sample mean, $\omega = \mathbf{h} \cdot \mathbf{t}$, and satisfies*

$$\sigma^*(\mathbf{h}) = \begin{cases} 1 & \omega = \mathbf{h} \cdot \mathbf{t} \in [\underline{\omega}, \bar{\omega}], \\ 0 & \omega \notin [\underline{\omega}, \bar{\omega}]. \end{cases}$$

Furthermore, when $r \in \left(\frac{b-\sqrt{b^2+4}}{2}, b\right)$, then the cutoffs $\underline{\omega}, \bar{\omega}$ satisfy $\underline{\omega} \in (-b, -r)$, $\bar{\omega} \in (-r, \infty)$. If $r < \frac{b-\sqrt{b^2+4}}{2}$, then $\underline{\omega} = \bar{\omega}$ and thus $\sigma^*(\mathbf{h}) = 0$ almost surely.

Moreover, σ^* is the unique recommendation mechanism (up to measure zero changes) that achieves the optimum in (P1).

Theorem 2 establishes that there is a unique optimal ICL mechanism. Therefore, this theorem also establishes that for large populations, the optimal mechanism can be approximated by a simple one-dimensional mechanism that recommends accept when the aggregate reported state lies within a bounded interval. In particular, this mechanism recommends reject for a positive-measure set of states $[\bar{\omega}, \infty)$ at which the mediator commits to recommend reject, despite all

parties agreeing that accept would be better. This surplus burning at the top acts as a punishment for over-reporting that allows us to take the receiver's preferred action, reject, when $\omega \in [-b, \underline{\omega})$. In addition, Theorem 2 establishes that when r is negative, the bias $b - r$ may be so large that the uninformative mechanism is optimal.

The uniqueness result in Theorem 2 establishes that finding the optimal ICL mechanism was the correct exercise for thinking about large economies. Indeed, since σ^* is the unique ICL mechanism, applying Berge's theorem of the maximum implies that any convergent sequence of σ_N^* 's optimal mechanisms for N -sender economies converge to σ^* .

We need the notion of incentive compatibility in the large to solve this problem for the following reason. Had we expressed incentive constraints as simple differences in utilities, then the payoff consequences of any report would have been zero in the limit, as each sender's report would have had no impact on $\mathbf{h}(\mathbf{s})$. By scaling the signals of each sender appropriately, ICL ensures that incentive constraints continue to discipline the problem.

Beyond its technical implications, the scaling guarantees that senders remain informationally impactful despite being small in the limit. In other words, as the number of senders grows large, the benefits and costs of being pivotal decline together, and thus they have to balance each other even in the limit. Hence, when we consider division of information over large economies, we do not achieve the first best, not even in the limit. This result contrasts with the results in Feddersen and Pesendorfer (1997) and Gerardi et al. (2009), for example, because in their limiting exercise, they consider informational addition rather than division. They can thus achieve arbitrarily close to first best if they can manage to extract a small fraction of the population's information. In contrast, in our setup, no small group of senders can observe the state, and thus the mechanism has to discard meaningful information in order to provide incentives.

4.3 The Intuition Behind Optimality of Interval Mechanisms

Here, we provide an intuitive reasoning for the optimality of the interval mechanism of Theorem 2. Our intuitive argument can be broken into two parts: First, why is it that we can focus on simple mechanisms that depend only on ω . Second, why are interval mechanisms optimal?

4.3.1 Payoff-Dependent Mechanisms for Large N

Consider a mechanism σ^N that is incentive compatible. Then, consider an alternative mechanism $\hat{\sigma}^N(\hat{\omega}) = \mathbb{E}^N[\sigma^N(\mathbf{h}) | \hat{\omega} = \omega(\mathbf{h})]$ that replaces the probability of accept for each realization $\mathbf{h}$ with the probability of accept conditional on its associated state value $\omega(\mathbf{h})$. Since the receiver only cares about ω , this mechanism provides her with the same payoffs. However, this modification of the mechanism may no longer satisfy sender incentive compatibility. Nonetheless, it

has a notable property, specifically, that the effect of an individual sender's information, s_i , on the mechanism or the payoffs is an order of magnitude lower than it would be if we were simply controlling for ω .

To see this, consider the payoff of sender i , whose signal realization s_i takes a value $t_k \in S$ when he reports this signal truthfully. This is given by

$$U_k = \mathbb{E}^N [\sigma^N(\mathbf{h}) (\omega + b) | s_i = t_k].$$

Using the law of iterated expectations, we can write the above as

$$U_k = \mathbb{E}^N [\mathbb{E}^N [\sigma^N(\mathbf{h}) | \omega, s_i = t_k] (\omega + b) | s_i = t_k].$$

We can also write

$$\begin{aligned} \mathbb{E}^N [\sigma^N(\mathbf{h}) | \omega, s_i = t_k] &= \frac{\sum_{\mathbf{h}: \mathbf{h} \cdot \mathbf{t} = \omega} \Pr^N(\mathbf{h} | s_i = t_k) \sigma^N(\mathbf{h})}{\sum_{\mathbf{h}: \mathbf{h} \cdot \mathbf{t} = \omega} \Pr^N(\mathbf{h} | s_i = t_k)} \\ &= \frac{\sum_{\mathbf{h}: \mathbf{h} \cdot \mathbf{t} = \omega} \left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \Pr^N(\mathbf{h}) \sigma^N(\mathbf{h})}{\sum_{\mathbf{h}: \mathbf{h} \cdot \mathbf{t} = \omega} \left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \Pr^N(\mathbf{h})}. \end{aligned}$$

We can view the above as a function of $\frac{1}{\sqrt{N}}$ (without considering the effect of changes of N on the probabilities, $\Pr^N$), and a Taylor approximation around $1/\sqrt{N} = 0$ implies that

$$\mathbb{E}^N [\sigma^N(\mathbf{h}) | \omega, s_i = t_k] = \hat{\sigma}^N(\omega) + \frac{1}{\sqrt{N}} \text{cov}^N \left(\frac{h_k}{f_k}, \sigma^N(\mathbf{h}) | \omega \right) + O \left(\frac{1}{N} \right).$$

The above states that controlling for ω , the additional information contained in $s_i = t_k$ affects the expected probability of recommending $a = 1$ only by an order of $1/\sqrt{N}$, and thus this impact shrinks at that rate as N converges to infinity. A similar property holds for the payoff of the senders of a given type under truth-telling:

$$\begin{aligned} U_k &= \mathbb{E}^N [\mathbb{E}^N [\sigma^N(\mathbf{h}) | \omega, s_i = t_k] (\omega + b) | s_i = t_k] \\ &= \mathbb{E}^N \left[\left(\hat{\sigma}^N(\omega) + \frac{\text{cov}^N \left(\frac{h_k}{f_k}, \sigma^N(\mathbf{h}) | \omega \right) + O(1/\sqrt{N})}{\sqrt{N}} \right) (\omega + b) | s_i = t_k \right] \\ &= \mathbb{E}^N \left[\left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \hat{\sigma}^N(\omega) (\omega + b) \right] + O(1/N) \end{aligned}$$

In arriving at the last expression, we have used the typical property of the multinomial distribution in (9). Recall that we can cast the incentive compatibility using its envelope form and

write

$$U_k - U_l \geq \frac{t_k - t_l}{\sqrt{N}} \mathbb{E}^N [\sigma^N(\mathbf{h}) | s_i = t_l].$$

The right-hand side represents the extra surplus that a sender of type k is able to guarantee by pretending to be of type l . Therefore, it determines the marginal information rent captured by that sender. Applying the aforementioned logic to this incentive compatibility has a few implications as N becomes large. First, the right-hand side of the above can be replaced by $\frac{t_k - t_l}{\sqrt{N}} \mathbb{E}^N [\hat{\sigma}^N(\omega)] + O(1/N)$. Second, the left-hand side can be replaced by $\mathbb{E}^N \left[\left(\frac{h_k}{f_k \sqrt{N}} - \frac{h_l}{f_l \sqrt{N}} \right) \hat{\sigma}^N(\omega) (\omega + b) \right] + O(1/N)$. In other words, the conditional mean of σ^N is the main determinant of incentive provision up to a first order, and all of its higher moments—which fully determine the distribution of σ^N —affect the incentives with a lower order of magnitude. Since as we take limit, only the first-order effects become relevant, we can simply replace σ^N with $\hat{\sigma}^N$, and the resulting limit (as $N \rightarrow \infty$) will satisfy the conditions in Theorem 1.

4.3.2 Optimality of Interval Mechanism

As we have argued, it is without loss to focus on payoff-dependent mechanisms as N becomes large. Given this observation, the optimality of the interval mechanisms follows from the fact that ω is distributed normally (as $N \rightarrow \infty$) and that the normal distribution has thin tails.

To see this, consider the impact of a change in $\sigma(\omega)$ on k 's truth-telling payoff

$$\sum_{\mathbf{h}: \mathbf{h} \cdot \mathbf{t} = \omega} \left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \Pr^N(\mathbf{h} | \omega) (\omega + b) = \Pr^N(\omega) (\omega + b) \mathbb{E}^N \left[\frac{h_k}{f_k \sqrt{N}} + 1 | \omega \right].$$

As $N \rightarrow \infty$, $\mathbf{h}$ and ω are normally distributed, implying that $\mathbb{E}^N[h_k | \omega]$ converges to $\frac{f_k t_k}{\text{var}(s_i)} \omega$. As a result, the effect of a change in $\sigma(\omega)$ on the senders' truth-telling incentive is higher for higher values of ω . Because the relevant incentive constraints are those where $l > k$ (i.e., senders lie upward), information rents are more sensitive to changes in σ when ω is highest. As a result, it is optimal to reduce σ for high enough values of ω and increase them for intermediate values. Thus, the optimal mechanism is an interval mechanism.

4.4 The Case of Continuous Distributions

For ease of analysis, we have assumed that the distribution of the signals s_i had a finite support. This assumption allowed us to view the empirical distribution of the signals $\mathbf{h}(\mathbf{s})$ defined in (4) as a member of the Euclidean space $\mathbb{R}^K$ and thus the mechanism as a function that maps a subset of the Euclidean space—indeed a hyperplane—into $[0, 1]$. In this case, we were able to use a standard central limit theorem, Lemma 2, and show the convergence of mechanisms by restricting

attention to $L_\infty(\mathbb{R}_0^K)$, the set of bounded functions over $\mathbb{R}_0^K$, because each mechanism belongs to this space.

When s_i does not have a finite support and has an arbitrary distribution with support, say, in $\mathbb{R}$, then Lemma 2 becomes significantly more complex. This is because NEFs are cumulative distribution functions, and hence a mechanism maps each such function to $[0, 1]$. Indeed, as proved by Donsker—see, for example, Theorem 4.20 in Karatzas and Shreve (1988)—the NEFs converge to the set of sample realizations of a Brownian bridge. As a result, all expectations that describe the ICL have to be taken with respect to the Wiener measure on the Skorohod space of cadlag functions—representing NEFs. However, while this approach is feasible, it is an unnecessary difficulty. Given our results apply for arbitrarily large finite type spaces, we avoid these technical issues without losing any economic relevance.

5 Extensions

In this section, we extend our analysis in two directions. In Section 5.1, we study optimal mechanisms in the large when bias is heterogeneous across senders. In Section 5.2, we study the problem under more general preferences.

5.1 Heterogeneous Bias

Here, we consider an extension where the senders' bias is heterogeneous but observable. We show that the general structure of the optimal mechanisms in the large does not change.

Formally, there are M bias classes, where a sender in class $m \in \{1, \dots, M\}$ has bias b_m . There are N_m senders in bias class m , and $N = \sum_m N_m$. The relative size of class m is $\nu_m = N_m/N$, with $\nu_1 + \dots + \nu_M = 1$. We assume that the senders' signals are independent and distributed according to a discrete distribution. That is, for a sender i of bias class m , $s_{i,m} \in S_m = \{t_{1,m} < \dots < t_{K,m}\}$, with $f_{k,m} = \Pr(s_{i,m} = t_{k,m})$ such that

$$\sum_{k=1}^K f_{k,m} t_{k,m} = 0, \sum_{k=1}^K f_{k,m} t_{k,m}^2 = \text{var}(s_{i,m}) = \eta.$$

Moreover, the payoff of the sender is given by

$$\left(\frac{\sum_{m=1}^M \sum_{i=1}^{N_m} s_{i,m}}{\sqrt{N}} + b_m \right) a = (\omega + b_m) a,$$

and the payoff of the receiver is $(\omega + r) a$. As in the main model, we can define $\mathbf{h}_m \in \mathbb{R}_0^K$ as

the deviations of the sample distribution of signals among the senders of type m from the true distribution $\mathbf{f}_m$ multiplied by $\sqrt{N_m}$. By the central limit theorem, $\mathbf{h}_m \rightarrow_d \mathcal{N}(\mathbf{0}, \Sigma_m)$, where the k, k' element of Σ_m is $f_{k,m}(\mathbf{1}[k = k'] - f_{k',m})$. We will refer to $\omega_m = \mathbf{h}_m \cdot \mathbf{t}_m$ as the mean of group m . Again, we focus on symmetric mechanisms, which are of the form $\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \in [0, 1]$. We can write

$$\omega = \frac{\sum_{m=1}^M \mathbf{h}_m \cdot \mathbf{t}_m \sqrt{N_m}}{\sqrt{N}} = \sum_{m=1}^M \omega_m \sqrt{\nu_m}.$$

The central limit theorem implies that $\omega_m \rightarrow \mathcal{N}(0, \eta)$, and by assumption, ω_m 's are independent across groups. In this environment, an argument akin to that of Theorem 1 implies that the ICL holds if and only if, for all m :

$$\begin{aligned} \mathbb{E} \left[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \left((\omega + b_m) \frac{h_{k,m}}{f_{k,m}} - t_{k,m} \right) \right] &= 0, \\ \mathbb{E} \left[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \frac{h_{k,m}}{f_{k,m}} \right] &\text{ is increasing in } k. \end{aligned} \quad (12)$$

With this result in hand, the rest of the characterization follows that of Theorem 2. The following proposition states the main result for this extension:

Proposition 2. *The receiver-optimal ICL mechanism is a function of the sample mean for each class $\omega_m = \mathbf{h}_m \cdot \mathbf{b}_m$. Moreover, two vectors, $\lambda = (\lambda_1, \dots, \lambda_M) \in \mathbb{R}^M$ and $\zeta = (\zeta_1, \dots, \zeta_M) \in \mathbb{R}^M$, exist such that the optimal mechanism σ^* satisfies*

$$\sigma^*(\omega_1, \dots, \omega_M) = 1 \Leftrightarrow \omega + r + \sum_m \lambda_m [\omega \omega_m + b_m \omega_m - \eta] + \sum_m \zeta_m \omega_m \geq 0, \quad (13)$$

$$\mathbb{E}[\sigma^* \cdot ((\omega + b_m) \omega_m - \eta)] = 0, \quad (14)$$

$$\zeta_m \mathbb{E}[\sigma^* \cdot \omega_m] = 0, \zeta_m \geq 0, \mathbb{E}[\sigma^* \cdot \omega_m] \geq 0. \quad (15)$$

In Proposition 2, the λ_m 's are the Lagrange multipliers associated with the aggregated version of the incentive compatibility, (12), in each bias class. Additionally, the ζ_m 's are the Lagrange multipliers on the monotonicity constraints, which become $\mathbb{E}[\sigma^* \cdot \omega_m] \geq 0$. Thus, (15) is the complementary slackness associated with this constraint. The proof of Proposition 2 closely follows that of Theorems 1 and 2, and is relegated to the Appendix.

The variables λ, ζ that determine the region for which $\sigma = 1$ can be found by solving the system of equations defined by the incentive constraints, (14) and (15). Their existence is guaranteed by the existence of the solution of the mechanism design problem, as we show in the proof of Proposition 2.

Example 1. There are two equally sized groups with biases $b_1 = 0.1, b_2 = 0.3$, with $r = 0$. The payoff types have variance $\text{Var}(s_{i,m}) = 1$ for all m , which in turn implies that $\mathbb{E}[(\omega_m)^2] = 1$. In this case, the sufficient statistics are ω_1, ω_2 , which are the means in each group. We can numerically solve the system of equations in (14) and check that $\mathbb{E}[\sigma^* \cdot \omega_m] > 0$ for both types. The results are depicted in Figure 4. As can be seen, the area where $\sigma = 1$ falls between two parabolas. This mechanism is a natural extension of the interval mechanism in Section 4. When $b_1 = b_2$, the optimal mechanism is an interval, and thus it is the area between two lines that represent the cutoffs identified in Theorem 3. Here, since $b_2 > b_1$, the optimal mechanism relies more on the report of the senders of type 1, i.e., the set of ω_2 's for which $a = 1$ gets larger for high and low values of ω_1 . Similarly to the homogeneous case, when both values are high or low, $a = 0$ is recommended. A few observations are worth noting: The mediator relies more on the signal of the group with lower bias. In this case, the punishment is in the area where both the receiver and the two sender types prefer action 0, but action 1 is being recommended; however, the probability of landing on that area is very low, so the punishment still occurs at extreme values.

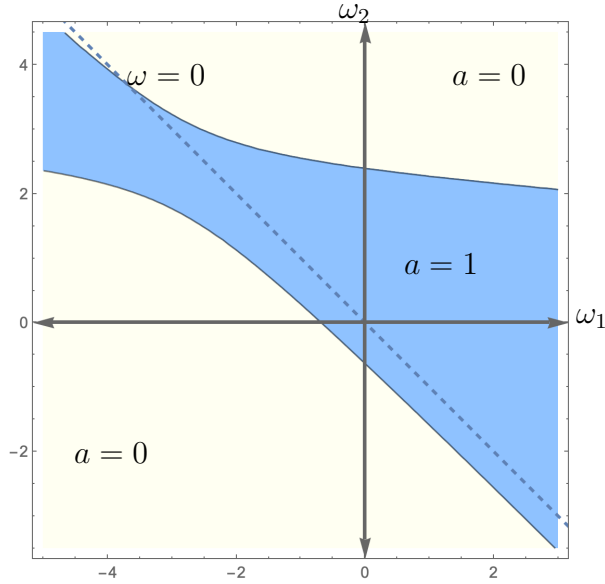


Figure 4: Regions of action with heterogeneous bias; in the white region, $a = 0$, while in the blue region, $a = 1$. The variance of the signals in each group is 1, and the biases are $b_1 = 0.1, b_2 = 0.3$.

5.2 General Preferences

In this section, we show how our main results on optimal aggregation in the large extend to more general preferences.

As before, there are N senders, and each has type s_i independently drawn from $\{t_1, \dots, t_K\}$ with probability f_k . Again, let h_k be the k -NEF. Focusing on symmetric mechanisms, we can assume that a mechanism is a function of NEF. Hence, we let the payoffs of the senders and of the receiver satisfy

$$u_R(a, \mathbf{h}) = \begin{cases} u_r(\mathbf{h}) & a = 1 \\ 0 & a = 0 \end{cases}, u_S(a, \mathbf{h}) = \begin{cases} u_s(\mathbf{h}) & a = 1 \\ 0 & a = 0 \end{cases}. \quad (16)$$

We make the following assumption on u_s, u_r :

Assumption 1. *The payoff functions $u_r, u_s : \mathbb{R}_0^K \rightarrow \mathbb{R}$ satisfy the following properties:*

1. *They are continuous and differentiable.*
2. *The marginal value of h_k for the sender, $\frac{\partial u_s(\mathbf{h})}{\partial h_k}$, is higher for higher values of k .*
3. *There exists $p > 1$ such that $u_s(\mathbf{h}) = O(\|\mathbf{h}\|_K^p)$ and $\|\nabla u_s(\mathbf{h})\|_K = O(\|\mathbf{h}\|_K^p)$.*
4. *The functions $\left\{ u_s(\mathbf{h}) \frac{h_k}{f_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_k} \right\}_{k=1}^K$ are linearly independent.*

In our model so far, the payoff functions are linear and thus satisfy the aforementioned assumption. The monotonicity assumption on the senders' marginal value of an additional type k sender is akin to the standard single crossing assumption used in mechanism design. The last parts of the assumption are technical ones that allow us to show the equivalent of Theorem 1 in this general setting.

The following proposition is the equivalent of Theorem 1 for arbitrary payoffs:

Proposition 3. *Suppose payoffs are given by (16) and satisfy Assumption 1. A recommendation mechanism $\sigma(\mathbf{h})$ is incentive compatible in the large if and only if there exists U such that it satisfies*

$$\mathbb{E} \left[\sigma(\mathbf{h}) \left(u_s(\mathbf{h}) \frac{h_k}{f_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_k} \right) \right] = U, \forall k \quad (17)$$

$$\mathbb{E} \left[\sigma(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right) \left(\frac{\partial u_s(\mathbf{h})}{\partial h_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_l} \right) \right] \geq \mathbb{E} \left[\sigma(\mathbf{h}) \left(\frac{\partial^2 u_s(\mathbf{h})}{\partial h_k^2} + \frac{\partial^2 u_s(\mathbf{h})}{\partial h_l^2} - 2 \frac{\partial^2 u_s(\mathbf{h})}{\partial h_l \partial h_k} \right) \right] \quad (18)$$

for all $k > l$.

Proposition 3 describes how incentive compatibility in the large is affected by having arbitrary payoffs. The first equality is the equivalent of the standard envelope condition as in (6). The second inequality is the standard monotonicity condition. Notably, with general payoffs, the

second derivative or curvature of the payoff function is also relevant to the standard monotonicity condition.

The complications arising from arbitrary payoffs imply that a general characterization of optimal mechanisms is not so straightforward. However, once the assumption of monotonicity is not binding, the characterization of the resulting optimal mechanism is simple:

Proposition 4. *Let $\sigma^* : \mathbb{R}_0^K \rightarrow \{0, 1\}$ be a mechanism that is ICL, i.e., satisfies the conditions in Proposition 3. If $\{\lambda_k\}_{k=1}^K$ exist such that*

$$\sigma^*(\mathbf{h}) = 1 \Leftrightarrow u_r(\mathbf{h}) + \sum_{k=1}^K \lambda_k \left(\frac{h_k}{f_k} u_s(\mathbf{h}) - \frac{\partial u_s}{\partial h_k}(\mathbf{h}) \right) \geq 0; \quad (19)$$

then σ^ is an optimal mechanism.*

This condition is equivalent to the interval condition in the case of our initial payoffs.

A particularly tractable specification is when u_r and u_s are linear and satisfy the specification in Proposition 5. Notably, this model is different from our baseline example, because the senders and the receiver evaluate different signal realizations differentially. For example, in the context of voting, the receiver's preferences can represent how an uninformed voter cares about different issues, while the senders' preferences represent the interests of the differentially informed political elite in terms of the same issues. In this case, optimal aggregation can be described by the following Proposition:

Proposition 5. *Suppose that $u_r(\mathbf{h}) = \mathbf{t}_r \cdot \mathbf{h}$ and that $u_s(\mathbf{h}) = \mathbf{t}_s \cdot \mathbf{h}$ with $\mathbf{t}_s, \mathbf{t}_r \in \mathbb{R}^K$ such that*

$$\begin{aligned} \sum_k f_k t_{s,k} &= \sum_k f_k t_{r,k} = 0, \\ \sum_k f_k t_{s,k}^2 &= \sum_k f_k t_{r,k}^2 = 1, \sum_k f_k t_{r,k} t_{s,k} = \gamma \end{aligned}$$

and $t_{s,k}, t_{r,k}$ are increasing in k . Then, the optimal mechanism σ^ is a function of $\omega_s = \mathbf{t}_s \cdot \mathbf{h}$, $\omega_r = \mathbf{t}_r \cdot \mathbf{h}$ and satisfies*

$$\begin{aligned} \sigma^*(\omega_r, \omega_s) = 1 \Leftrightarrow & \omega_r + (\lambda_r \omega_r + \lambda_s \omega_s) \omega_s - \gamma \lambda_r - \lambda_s \\ & + \omega_r \sum_k (t_{r,k} - \gamma t_{s,k}) (\zeta_k - \zeta_{k+1}) \\ & + \omega_s \sum_k (t_{s,k} - \gamma t_{r,k}) (\zeta_k - \zeta_{k+1}) \geq 0 \end{aligned}$$

for some α_r, α_s and $\zeta_k \geq 0$ with $\zeta_1 = \zeta_{K+1} = 0$ such that

$$\mathbb{E} [\sigma^* \cdot (\omega_s^2 - 1)] = 0 \quad (20)$$

$$\mathbb{E} [\sigma^* \cdot (\omega_s \omega_r - \gamma)] = 0 \quad (21)$$

and

$$\zeta_k [(t_{s,k} - t_{s,k-1}) \mathbb{E} \sigma^* \cdot (\omega_s - \gamma \omega_r) + (t_{r,k} - t_{r,k-1}) \mathbb{E} \sigma^* \cdot (\omega_r - \gamma \omega_s)] = 0. \quad (22)$$

Proposition 5 illustrates that the optimal mechanism takes a tractable form and is only a function of the ex post payoffs of the senders and of the receiver. Moreover, (22) is the usual complementary slackness associated with the monotonicity constraint (18). Hence, much in line with standard models of mechanism design, Proposition 5 illustrates a procedure for finding the optimal mechanism. That is, one can conjecture that the monotonicity constraints are slack, i.e., $\zeta_k = 0$, and find λ_r, λ_s so that ICL constraints (20) and (21) are satisfied. If the resulting mechanism σ^* satisfies the monotonicity constraint, it is the optimum. Otherwise, a procedure akin to ironing is required to find which monotonicity constraints are binding.

The next example illustrates this result.

Example 2. Consider an example in which $K = 3$ and $u_r(\mathbf{h}) = \rho(-2h_1 - h_2 + 3h_3)$, $u_s(\mathbf{h}) = \rho(-\sqrt{7}h_1 + \sqrt{7}h_3)$, where $\rho = \sqrt{3/14}$ and $f_1 = f_2 = f_3 = 1/3$. In this case, $\gamma = \frac{5}{2\sqrt{7}}$. Using Proposition 5, we can numerically find the values of λ_r, λ_s that satisfy the conditions in Proposition 5. Figure 5 depicts the set of ω_s, ω_r 's for which $a = 1$. In this example, the agreement regions are the positive and negative quadrants, while the disagreement regions are the remaining regions. The optimal mechanism creates value for the receiver by recommending $a = 1$ when $\omega_r > 0$ and $\omega_s < 0$ —the disagreement quadrant preferred by the receiver. Surplus burning is occurring in the positive and negative quadrants. In the positive quadrant, high reports of ω_s and low reports of ω_r are punished by recommending $a = 0$. In the negative quadrant, surplus burning is happening by recommending $a = 1$ when both the senders and the receiver prefer $a = 0$.

6 Conclusion and Discussion

We studied the optimal design of communication protocols when multiple biased senders possess indispensable private information that cannot be verified through cross-checking. Our results show that when information is concentrated among sufficiently few people, and conflicts of interest are moderate, then there can be no benefit from keeping people informationally divided. In this case, it would be optimal for a single person to hold all the information or, alternatively,

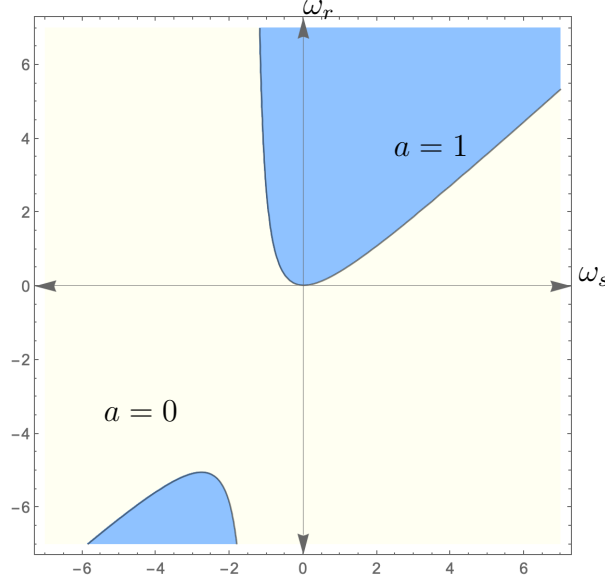


Figure 5: Optimal mechanism in Example 2. The sender's payoff ω_s is on the x-axis, and the receiver's payoff ω_r is on the y-axis.

to let all the agents confer with each other privately and make a joint recommendation. Such a communication protocol is consistent with the practice in firms of the CEO canvassing the views of division managers before presenting a summary of the firm's strategy to its board of directors. Given the results of [Antic et al. \(2025\)](#) on subversive conversations, this result might also be interpreted as conditions under which a completely laissez-faire attitude to managing discourse is optimal.

However, the picture changes when information is divided among larger groups. We demonstrate that keeping agents informationally divided enables more sophisticated mechanisms that can improve information elicitation. Crucially, and in contrast to settings where cross-verification is possible because of the senders' observing correlated information, these mechanisms work by punishing agreement. Of course, if agents could verify one another's information, then punishing disagreement would be more effective. Moreover, the results of [Gerardi et al. \(2009\)](#) and [Feddersen and Pesendorfer \(1997\)](#) suggest it could be possible to perfectly identify and extract information in large populations. Instead, in our setting, the designer falls short of the first best: While it may be optimal to punish suspiciously high levels of consensus, doing so still burns significant degrees of surplus.

Nonetheless, it appears that platforms have practices that bear some resemblance to our mechanism. For instance, Facebook has a fact-checking policy that fits this insight to some extent. When there is a plethora of particular stories or reports, Facebook flags them as possible misinformation. While this does not lead to removal of content, it can lead to Facebook reducing

the visibility of these stories. Our interval mechanism also has some similarities to Amazon’s practice of predominantly deleting 5-star reviews, discussed in [He et al. \(2022\)](#). The occurrence of such deletions, even when it appears the platform cannot reliably distinguish genuine from paid reviews, aligns with our findings.

In recent years, verification has become increasingly difficult because of deep fakes, fake reviews, sock-puppet accounts, and disinformation campaigns. The mechanisms we identify offer some guidance: Specifically, punishing consensus may substitute for verification, although imperfectly. The practical implications and implementation of such mechanisms remain important areas for future research.

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A Proofs

A.1 Proof of Proposition 1

Proof. We prove the claim in two steps: First, we show that one can use standard Lagrangian techniques to characterize the solution of problem (P). Second, we construct the Lagrange multipliers – members of an appropriate dual space to be precisely defined below – so that the sender-preferred allocation, $\sigma_S^*(s)$, maximizes the appropriate Lagrangian.

We begin by rewriting the problem (P) as follows:

$$\max_{\sigma: S^N \rightarrow [0,1], \underline{v}} \int_{S^N} (\omega(s) + r) \sigma(s) f(s) ds$$

subject to

$$\mathbb{E}[(\omega(s) + b) \sigma(s) | s_i] = \underline{v} + \frac{1}{\sqrt{N}} \int_{-1}^{s_i} \mathbb{E}[\sigma(s) | \tilde{s}_i] d\tilde{s}_i, \forall s_i \in S, i \in \{1, \dots, N\}, \quad (23)$$

$$\mathbb{E}[\sigma(s) | s_i] \leq \mathbb{E}[\sigma(s) | s'_i], \forall s_i \leq s'_i \quad (24)$$

$$0 \leq \sigma(s) \leq 1, s \in S^N. \quad (25)$$

The first constraint is the integral form of the envelope version of incentive compatibility (2), while the second constraint is the standard monotonicity of the interim allocations $\mathbb{E}[\sigma(s) | s_i]$. Finally, the last condition is the upper and lower bounds on $\sigma(s)$.¹⁹ We can thus frame the problem in a convenient way for establishing weak duality or standard sufficiency conditions for optimality—see Theorem 1 in Section 8.4 in [Luenberger \(1997\)](#).

More specifically, let $x = (\sigma, \underline{v}) \in X = L_2(S^N) \times \mathbb{R}$ and $Z = L_2(S)^{3N} \times L_2(S^N)^2$. The above program is a linear programming problem whose constraint set is of the form $G(x) \leq 0$ for some $G : X \rightarrow Z$, where $\leq$ is defined below. The function $G(x)$ is given by the quintuple

¹⁹That incentive compatibility is equivalent to the envelope condition (23), and the monotonicity of $\mathbb{E}[\sigma(s) | s_i]$ is standard. See, for example, [Myerson \(1981\)](#).

$(G_1(x), \dots, G_5(x))$. In this quintuple, $G_1, G_2 : X \rightarrow L_2(S)^N$ and are given by

$$G_{1,i}(x)(s_i) = \mathbb{E}[(\omega(\mathbf{s}) + b)\sigma(\mathbf{s}) | s_i] - \underline{v} - \frac{1}{\sqrt{N}} \int_{-1}^{s_i} \mathbb{E}[\sigma(\mathbf{s}) | \tilde{s}_i] d\tilde{s}_i \leq 0$$

$$G_{2,i}(x)(s_i) = -G_{1,i}(x)(s_i) \leq 0.$$

That is, instead of using an equality constraint in (23), we use two inequality constraints that have the same implication; $G_{3,i} : X \rightarrow L_2(S)$ is given by $G_{3,i}(x)(s_i) = -\mathbb{E}[\sigma(\mathbf{s}) | s_i]$ (associated with (25)), and $G_4, G_5 : X \rightarrow L_2(S^N)$ are simply $G_4(x)(\mathbf{s}) = -\sigma(\mathbf{s})$ and $G_5(x)(\mathbf{s}) = 1 - \sigma(\mathbf{s})$ (associated with (24)). The relation $\leq$ is associated with the cone of members of Z whose first, second, fourth, and fifth elements are nonnegative while its third element is a non-decreasing function.

Given this formulation, we can use Theorem 1 in Section 8.4 in Luenberger (1997) (switched to maximization), which states that if there exist $x_0 \in X$ and $z_0^* \in Z^*$ such that the Lagrangian $L(x, z^*) = w(x) + \langle z^*, G(x) \rangle$ has a saddle point at x_0, z_0^* ,

$$L(x, z_0^*) \leq L(x_0, z_0^*) \leq L(x_0, z^*), \forall x \in X, z^* \geq 0.$$

Then $x_0 \in \arg \max_{x \in X, G(x) \leq 0} w(x)$. In our setting, G is as defined before, while $w(x) = \mathbb{E}[\sigma(\mathbf{s}) \omega(\mathbf{s})]$. If x_0 satisfies $\langle z_0^*, G(x_0) \rangle = 0$, then the second inequality is satisfied by the definition of the dual cone in Z^* . Thus, we have to construct the Lagrange multiplier z_0^* such that $\langle z_0^*, G(x_0) \rangle = 0$, and the first of the aforementioned inequalities holds.

Using the Riesz representation theorem—see Theorem 14.12 in Aliprantis and Border (2006)—we can write the Lagrangian as

$$L(x, z^*) = \int (\omega(\mathbf{s}) + r) \sigma(\mathbf{s}) f(\mathbf{s}) d\mathbf{s}$$

$$+ \sum_i \int_S G_{1,i}(x)(s_i) d\Lambda_i^1(s_i) + \sum_i \int_S G_{2,i}(x)(s_i) d\Lambda_i^2(s_i)$$

$$+ \sum_i \int_S G_{3,i}(s_i) d\Omega_i(s_i) + \int \sigma(\mathbf{s}) d\zeta(\mathbf{s}) + \int (1 - \sigma(\mathbf{s})) d\bar{\zeta}.$$

Here, Λ_i^1, Λ_i^2 's are positive Borel measures over S , and $\zeta, \bar{\zeta}$ are positive Borel measures over S^N , since z_0^* should be a member of the dual cone of $\geq$ in Z^* . Additionally, Ω_i is a signed measure that satisfies $\Omega_i([s, \bar{s}]) \geq 0, \forall s \in S$.²⁰ Let $\Delta\Lambda_i = \Lambda_i^1 - \Lambda_i^2$. We can use integration by parts to

²⁰Recall that the dual cone of increasing functions is the set of signed Borel measures Ω so that $\int f(s_i) d\Omega \geq 0$ for all increasing f . This coincides with the set of measures that satisfy $\Omega([s, \bar{s}]) \geq 0$.

write the Lagrangian as

$$\begin{aligned}
L(x, z^*) &= \int (\omega(\mathbf{s}) + r) \sigma(\mathbf{s}) f(\mathbf{s}) d\mathbf{s} \\
&+ \sum_i \int_S \int_{S^{N-1}} [(\omega(s_i; s_{-i}) + b) \sigma(s_i; s_{-i}) f_{-i}(s_{-i}) ds_{-i} - \underline{v}] d\Delta\Lambda_i \\
&- \frac{1}{\sqrt{N}} \sum_i \int_S \int_{S^{N-1}} \sigma(s_i; s_{-i}) f_{-i}(s_{-i}) ds_{-i} \Delta\Lambda_i([s_i, \bar{s}]) ds_i \\
&+ \int \sigma(\mathbf{s}) d\underline{\zeta}(\mathbf{s}) + \int (1 - \sigma(\mathbf{s})) d\bar{\zeta}(\mathbf{s}) + \sum_i \int_S \int_{S^{N-1}} \sigma(s_i; s_{-i}) f_{-i}(s_{-i}) ds_{-i} d\Omega_i(s_i).
\end{aligned}$$

We are now ready to identify the relevant Lagrange multipliers and verify that—under the upper bound on N —they support the sender-preferred allocation $x_0 = (\sigma^S(\mathbf{s}), v^S(\underline{s}))$ as a saddle point of L . To that end, set

$$\begin{aligned}
\Delta\Lambda_i([s_i, \bar{s}]) &= \begin{cases} 0 & s_i = \underline{s} \\ f(s_i) \frac{b-r}{b+\sqrt{N}\bar{s}} (\bar{s} - s_i) & s_i > \underline{s} \end{cases} \\
\Omega_i([s_i, \bar{s}]) &= 0.
\end{aligned}$$

By the Jordan decomposition theorem, any signed measure can be written as the difference of two positive measures. Therefore, consider two positive measures Λ_i^1, Λ_i^2 such that $\Delta\Lambda_i = \Lambda_i^1 - \Lambda_i^2$. Given the aforementioned Lagrangian, let us define the “first order condition” as follows.²¹

$$\begin{aligned}
\forall i, s_i > \underline{s} : \lambda(\mathbf{s}) &= (\omega(\mathbf{s}) + r) f(\mathbf{s}) \\
&+ \sum_{i=1}^N (\omega(\mathbf{s}) + b) \frac{f(\mathbf{s})}{f(s_i)} \frac{b-r}{b+\sqrt{N}\bar{s}} \frac{d}{ds_i} (f(s_i) (\bar{s} - s_i)) \\
&+ \frac{1}{\sqrt{N}} \sum_{i=1}^N f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} (\bar{s} - s_i).
\end{aligned}$$

Moreover, we can write

$$\mathbf{s}, \exists i, s_i = \underline{s} : \frac{\lambda(\mathbf{s})}{(\omega(\mathbf{s}) + b) f(\mathbf{s})} = \frac{b-r}{b+\sqrt{N}\bar{s}} (\bar{s} - \underline{s}) \sum_{j=1}^N \mathbf{1}[s_j = \underline{s}].$$

This holds since Λ_i has a mass point at $\underline{s}$, and when calculating the Fréchet–Gateaux derivative of the Lagrangian, the effect of changes in the direction of the Dirac’s delta at $\mathbf{s}$, only mass points

²¹Formally, $\lambda(\mathbf{s})$ is the Fréchet derivative of $L(x, z)$ evaluated at x^* and in the direction of the dirac delta function at $\mathbf{s}$.

are relevant.

In order to show optimality of sender-preferred allocation, it is sufficient to show that

$$\lambda(\mathbf{s}) \geq 0 \iff \omega(\mathbf{s}) + b \geq 0.$$

When $\forall i, s_i > \underline{s}$, we have

$$\begin{aligned} s_i > \underline{s} : \lambda(\mathbf{s}) &= (\omega(\mathbf{s}) + r) f(\mathbf{s}) \\ &+ \frac{b-r}{b+\sqrt{N}\bar{s}} \sum_{i=1}^N (\omega(\mathbf{s}) + b) f(\mathbf{s}) \left(-1 + \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} \right) \\ &+ \frac{1}{\sqrt{N}} \sum_{i=1}^N f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} (\bar{s} - s_i) \\ &= (\omega(\mathbf{s}) + b) f(\mathbf{s}) + (r-b) f(\mathbf{s}) \\ &+ (\omega(\mathbf{s}) + b) f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} \sum_{i=1}^N \left(-1 + \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} \right) \\ &+ f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} \sum_{i=1}^N \frac{(\bar{s} - s_i)}{\sqrt{N}} \\ &= (\omega(\mathbf{s}) + b) f(\mathbf{s}) \left[1 + \frac{b-r}{b+\sqrt{N}\bar{s}} \sum_{i=1}^N \left(-1 + \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} \right) \right] \\ &+ (r-b) f(\mathbf{s}) + f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} \left(\sqrt{N}\bar{s} - \omega(\mathbf{s}) \right) \\ &= \left[1 - \frac{b-r}{b+\sqrt{N}\bar{s}} + \frac{b-r}{b+\sqrt{N}\bar{s}} \times \sum_{i=1}^N \left(-1 + \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} \right) \right] \times \\ &\quad (\omega(\mathbf{s}) + b) f(\mathbf{s}) + (r-b) f(\mathbf{s}) + f(\mathbf{s}) \frac{b-r}{b+\sqrt{N}\bar{s}} \left(\sqrt{N}\bar{s} + b \right) \\ &= \frac{b-r}{b+\sqrt{N}\bar{s}} \sum_{i=1}^N \left(-1 + \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} + \frac{1}{N} \frac{r + \sqrt{N}\bar{s}}{b-r} \right) \times \\ &\quad (\omega(\mathbf{s}) + b) f(\mathbf{s}) \end{aligned}$$

This implies that if the term in the last brackets is always positive, $\lambda(\mathbf{s}) \geq 0$ if and only $\omega(\mathbf{s}) + b \geq 0$. Note that the term in the brackets is always positive if and only if

$$\frac{1}{N} \frac{r + \sqrt{N}\bar{s}}{b-r} \geq 1 - \inf_{s_i \in S} \frac{f'(s_i)(\bar{s} - s_i)}{f(s_i)} = 1 - \ell \rightarrow r + \sqrt{N}\bar{s} \geq N(b-r)(1-\ell)$$

The above is a quadratic equation in $\sqrt{N}$ and since $1-\ell > 0$, it should hold when $\sqrt{N}$ is below its

higher root. The squared value of this higher root is given by $\underline{N}(b, r, \ell, \bar{s}) = \left(\frac{\sqrt{\bar{s}^2 + 4r(b-r)(1-\ell) + \bar{s}}}{2(b-r)(1-\ell)} \right)^2$ which gives us the (sufficient) bound on N .

When for some i , $s_i = \underline{s}$, then from the calculation of L , it is clear that $\lambda(\mathbf{s}) \geq 0$ if and only if $\omega(\mathbf{s}) + b \geq 0$. This concludes the proof. $\square$

A.2 Proof of Theorem 1

Proof. “Only If” Direction. The only if direction of the proof is essentially provided in Section 4.2. Here we provide the details. Let $\sigma : \mathbb{R}_0^K \rightarrow [0, 1]$ be an ICL mechanism. Then there must exist a sequence of σ^N that are incentive compatible and $\|\sigma^N - \sigma\|_{L_\infty} \rightarrow 0$.

The key property that we use in the derivations is the formulation of the conditional probability for the multinomial distribution, (9). An implication of (9) together with the independence of s_i 's, is that if $\mathbf{s}$ contains at least one t_k , then

$$\Pr_N(\mathbf{h}^N(\mathbf{s})) \frac{h_k^N(\mathbf{s}) \sqrt{N} + f_k N}{f_k N} = \Pr_N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k)) \frac{h_l^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k) \sqrt{N} + f_l N}{f_l N},$$

where $\mathbf{e}_k$ is a K -dimensional vector whose elements are 0 except for its k -th element, whose value is 1. This in turn implies that the value for t_k to pretend to be t_l is

$$\begin{aligned} \mathbb{E}_N[\sigma^N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k))(\mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t} + b) | t_k] &= \\ \sum_{\mathbf{s}} \sigma^N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k))(\mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t} + b) \frac{h_k^N(\mathbf{s}) \sqrt{N} + f_k N}{f_k N} \Pr_N(\mathbf{h}^N(\mathbf{s})) &= \\ \sum_{\mathbf{s}} \sigma^N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k))(\mathbf{h}^N(\mathbf{s}) \cdot \mathbf{t} + b) \times & \\ \frac{h_l^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k) \sqrt{N} + f_l N}{f_l N} \Pr_N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k)) &= \\ \sum_{\mathbf{s}} \sigma^N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k)) \left(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k) \cdot \mathbf{t} + \frac{t_k - t_l}{\sqrt{N}} + b \right) \times & \\ \frac{h_l^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k) \sqrt{N} + f_l N}{f_l N} \Pr_N(\mathbf{h}^N(\mathbf{s} + \mathbf{e}_l - \mathbf{e}_k)) &= \\ \sum_{\mathbf{s}'} \sigma^N(\mathbf{h}^N(\mathbf{s}')) \left(\mathbf{h}^N(\mathbf{s}') \cdot \mathbf{t} + \frac{t_k - t_l}{\sqrt{N}} + b \right) \frac{h_l^N(\mathbf{s}') \sqrt{N} + f_l N}{f_l N} \Pr_N(\mathbf{h}^N(\mathbf{s}')) &= \\ \mathbb{E} \left[\sigma^N(\mathbf{h}^N) \left(\mathbf{h}^N \cdot \mathbf{t} + \frac{t_k - t_l}{\sqrt{N}} + b \right) | t_l \right] \end{aligned}$$

Hence, we can write the incentive constraints as

$$\mathbb{E}^N [\sigma^N(\mathbf{h}) | t_k] \geq \frac{U_{k,k}^N - U_{l,l}^N}{\frac{t_k - t_l}{\sqrt{N}}} \geq \mathbb{E}^N [\sigma^N(\mathbf{h}) | t_l], t_k > t_l, \quad (26)$$

where $U_{k,k}^N$ is the utility of type k under truth-telling. Since by ICL, $\sigma^N \rightarrow_{L_\infty} \sigma$, taking the limit in (26) implies the only if direction—as explained in Section 4.2.

“If” Direction. For the if direction, the proof is much more involved. We prove it in steps and describe the basic structure of each step here. The required results that are less substantial are proven in the online Appendix. Consider a mechanism σ that satisfies (6) and (7). We prove that this is an ICL mechanism in the following steps:

Step 1. *The constraint $\sigma(\mathbf{h}) \in [0, 1]$ can be dropped and replaced by $\sigma(\mathbf{h}) \geq 0$ and $\sup_{\mathbf{h}} \sigma(\mathbf{h}) < \infty$, i.e., $\sigma \in L_\infty(\mathbb{R}_0^K)$.*

To see this, suppose that we have proven the only if direction under the above assumption $\sigma \geq 0, \sigma \in L_\infty(\mathbb{R}_0^K)$. Note that with finite N , any nonzero σ_N with $0 \leq \sigma_N$ that satisfies the incentive compatibility constraints (2) can be divided by $\max_{\mathbf{s} \in S^N} \sigma_N(\mathbf{h}^N(\mathbf{s}))$. The resulting mechanism satisfies incentive compatibility, and each σ_N is between 0 and 1.

Additionally, if we establish the result for arbitrary nonnegative mechanisms, then for any σ that takes values between 0 and 1 and satisfies (6), consider the mechanism $(1 - 1/M)\sigma$ for $M > 1$. Since $(1 - 1/M)\sigma$ satisfies the hypothesis, there must exist a sequence $\sigma_{M,N} \rightarrow (1 - 1/M)\sigma$ in $L_\infty(\mathbb{R}_0^K)$ where $\sigma_{M,N}$ is incentive compatible with N senders. Since $\sigma_{M,N}$ converges to $(1 - 1/M)\sigma$ and $\|(1 - 1/M)\sigma\|_{L_\infty} \leq 1 - 1/M$, there must exist n_M such that for all $N \geq n_M$, $\|\sigma_{M,N} - (1 - \frac{1}{M})\sigma\|_{L_\infty} < \frac{1}{2M}$ and $\|\sigma_{M,N}\|_{L_\infty} < 1$. Therefore,

$$\begin{aligned} \|\sigma_{M,N}\|_{L_\infty} - \left\| \left(1 - \frac{1}{M}\right) \sigma \right\|_{L_\infty} &\leq \left\| \sigma_{M,N} - \left(1 - \frac{1}{M}\right) \sigma \right\|_{L_\infty} < \frac{1}{2M} \\ \Rightarrow \|\sigma_{M,N}\|_{L_\infty} &< \left\| \left(1 - \frac{1}{M}\right) \sigma \right\|_{L_\infty} + \frac{1}{2M} \leq 1 - \frac{1}{M} \\ &\Rightarrow \|\sigma_{M,N} - \sigma\|_{L_\infty} \leq \left\| \sigma_{M,N} - \left(1 - \frac{1}{M}\right) \sigma \right\|_{L_\infty} + \frac{1}{M} \|\sigma\|_{L_\infty} \leq \frac{3}{2M}. \end{aligned}$$

By choosing the sequence $\left\{ \{\sigma_{M,N}\}_{N=n_M, \dots, n_{M+1}-1} \right\}_{M=2}^\infty$, we have a sequence of feasible incentive compatible mechanisms that converge to σ , which is the desired result.

Step 2. *It is sufficient to focus on functions $\sigma \in L_\infty(\mathbb{R}_0^K)$ for which $\mathbb{E} \left[\sigma(\mathbf{h}) \frac{h_k}{f_k} \right]$ is strictly increasing.*

To see this, suppose the only if direction is true for such σ 's. Now let σ be a function that satisfies (6), and $\mathbb{E} \left[\sigma(\mathbf{h}) \frac{h_k}{f_k} \right]$ be the same for two values of k . We can take another function $\hat{\sigma}$ for

which $\mathbb{E} \left[\hat{\sigma}(\mathbf{h}) \frac{h_k}{f_k} \right]$ is strictly increasing (this exists, obviously; for example, $\hat{\sigma} = \mathbf{1}[\mathbf{h} \cdot \mathbf{t} + b \geq 0]$ satisfies this condition). We then consider $\tilde{\sigma}_M = (1 - \frac{1}{M})\sigma + \frac{1}{M}\hat{\sigma}$. All such mechanisms satisfy (6), and $\mathbb{E} \left[\tilde{\sigma}_M \frac{h_k}{f_k} \right]$ is strictly increasing. Thus, they are ICL. Since as $M \rightarrow \infty$, $\tilde{\sigma}_M \rightarrow_{L_\infty} \sigma$, one can easily construct incentive compatible mechanisms that converge to σ .

Step 3. For finite N , local incentive compatibility implies incentive compatibility.

The argument here is standard.²² Note that the local incentive compatibility constraints are

$$\begin{aligned} \frac{t_k - t_{k-1}}{\sqrt{N}} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \frac{h_k \sqrt{N} + f_k N}{f_k N} \right] &\geq U_{k,k}^N - U_{k-1,k-1}^N \\ U_{k,k}^N - U_{k-1,k-1}^N &\geq \frac{t_k - t_{k-1}}{\sqrt{N}} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \frac{h_{k-1} \sqrt{N} + f_{k-1} N}{f_{k-1} N} \right], k \geq 2. \end{aligned} \quad (27)$$

Step 4. Consider $\sigma \in L_\infty(\mathbb{R}_0^K)$ that satisfies (6) and for which $\mathbb{E}[\sigma h_k / f_k]$ is strictly increasing and, as a result, $\sigma \neq 0$. Based on the local incentive constraints, we define linear operators T_N, T that map $L_\infty(\mathbb{R}_0^K)$ into $\mathbb{R}^{K-1}$.

For all $k \geq 2$, let us define the following functions:

$$\begin{aligned} w_k(\mathbf{h}) &= (\mathbf{h} \cdot \mathbf{t} + b) \left(\frac{h_k}{f_k} - \frac{h_{k-1}}{f_{k-1}} \right) - (t_k - t_{k-1}) \\ w_{k,N}(\mathbf{h}) &= (\mathbf{h} \cdot \mathbf{t} + b) \left(\frac{h_k}{f_k} - \frac{h_{k-1}}{f_{k-1}} \right) - (t_k - t_{k-1}) \left(1 + \frac{1}{\sqrt{N}} \frac{h_{k-1}}{f_{k-1}} \right). \end{aligned}$$

By the local incentive compatibility in (27), if a function σ^N satisfies incentive compatibility, then it must be that for all $k \geq 2$,

$$\frac{t_k - t_{k-1}}{\sqrt{N}} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_{k-1}}{f_{k-1}} \right) \right] \geq \mathbb{E}^N [\sigma^N(\mathbf{h}) w_{k,N}(\mathbf{h})] = \int \sigma^N(\mathbf{h}) w_{k,N}(\mathbf{h}) d\mu^N \geq 0,$$

where μ^N is the probability measure associated with $\mathbf{h}$, as we have defined. Moreover, since σ satisfies (6), we have that

$$\mathbb{E}[\sigma(\mathbf{h}) w_k(\mathbf{h})] = \int \sigma(\mathbf{h}) w_k(\mathbf{h}) d\mu = 0.$$

Let us also define $w(\mathbf{h}) = (w_2(\mathbf{h}), \dots, w_K(\mathbf{h}))$ and $w^N(\mathbf{h}) = (w_2^N(\mathbf{h}), \dots, w_K^N(\mathbf{h}))$. Then,

²²See, for example, Myerson (1981).

the linear operators T, T_N are defined by

$$T_N : L_\infty(\mathbb{R}_0^K) \rightarrow \mathbb{R}^{K-1}, T_N x = \int \sigma(\mathbf{h}) x(\mathbf{h}) w_N(\mathbf{h}) d\mu^N$$

$$T : L_\infty(\mathbb{R}_0^K) \rightarrow \mathbb{R}^{K-1}, Tx = \int \sigma(\mathbf{h}) x(\mathbf{h}) w(\mathbf{h}) d\mu.$$

Obviously, by assumption, σ satisfies $T\mathbf{1} = \int \sigma(\mathbf{h}) w(\mathbf{h}) d\mu = 0 \in \mathbb{R}^{K-1}$, where $\mathbf{1}$ is a function whose value is always 1.

Step 5. For large enough N , the linear operators T_N and T satisfy the following properties:

1. They map $L_\infty(\mathbb{R}_0^K)$ **onto** $\mathbb{R}^{K-1}$, i.e., for all $\mathbf{y} \in \mathbb{R}^{K-1}$, there exists $x \in L_\infty(\mathbb{R}_0^K)$ s.t. $Tx = \mathbf{y}$ ($T_N x = \mathbf{y}$).
2. There exists $c, c_N > 0$ such that

$$B_{K-1}(0, c) \subset T(B_\infty(0, 1)), B_{K-1}(0, c_N) \subset T_N(B_\infty(0, 1)),$$

where $B_{K-1}(0, c)$ is the ball of radius c around 0 in $\mathbb{R}^{K-1}$, and $B_\infty(0, 1)$ is the ball of radius 1 around 0 in $L_\infty(\mathbb{R}_0^K)$.

The first property follows from the fact that T is linear and maps a linear vector space into $\mathbb{R}^{K-1}$. This implies that the image of T is a subspace of $\mathbb{R}^{K-1}$ and if T is not onto, then $\text{Im}T$ must be isomorphic to a Euclidean space with dimension less than $K-1$. Hence, a basis $\mathbf{z}_1, \dots, \mathbf{z}_l \in \mathbb{R}^{K-1}$ with $l < K-1$ must exist such that

$$\forall x \in L_\infty(\mathbb{R}_0^{K-1}), Tx = \alpha_1 \mathbf{z}_1 + \dots + \alpha_l \mathbf{z}_l.$$

If we view the matrix $\mathbf{z} = (\mathbf{z}_1^{\text{tr}}, \dots, \mathbf{z}_l^{\text{tr}})$ as a linear operator that maps $\mathbb{R}^l$ to $\mathbb{R}^{K-1}$, then its rank cannot be higher than l . This implies that there exists $\mathbf{y} \neq 0 \in \mathbb{R}^{K-1}$ such that $\mathbf{y}\mathbf{z} = 0 \in \mathbb{R}^{K-1}$ or $\mathbf{y}\mathbf{z}_1^{\text{tr}} = \mathbf{y}\mathbf{z}_2^{\text{tr}} = \dots = \mathbf{y}\mathbf{z}_l^{\text{tr}} = 0 \in \mathbb{R}$. Therefore,

$$\mathbf{y}(Tx)^{\text{tr}} = \mathbf{y}\mathbf{z}_1^{\text{tr}}\alpha_1 + \dots + \mathbf{y}\mathbf{z}_l^{\text{tr}}\alpha_l = 0.$$

Since $\sigma \neq 0$ and measurable, there should be an open set U in $\mathbb{R}_0^K$ such that $\sigma \neq 0$. Choosing $x(\mathbf{h})$ to be 1 in an arbitrarily small open ball around any $\hat{\mathbf{h}} \in U$ and taking limit implies that $\mathbf{y}w(\hat{\mathbf{h}})^{\text{tr}} = 0$. We easily rule out this possibility because $w_k(\mathbf{h})$'s are quadratic with different coefficients; we establish this formally in the Online Appendix, Section C. To show the same property for T_N , one can argue that when N is large enough, $\mathbf{y}w_N(\hat{\mathbf{h}})^{\text{tr}} = 0$ for enough of $\hat{\mathbf{h}} \in U$, and thus by continuity, $\mathbf{y}w_N(\hat{\mathbf{h}})^{\text{tr}} = 0$ for an open set of $\hat{\mathbf{h}}$'s, leading to a similar

contradiction.

The second property is a restatement of the open mapping theorem applied to T and T_N ; see, for example, [Brezis \(2011\)](#)'s Theorem 2.6 on page 35.

Step 6. For all $x \in L_\infty(\mathbb{R}_0^K)$, $T_N x \rightarrow_{\mathbb{R}^{K-1}} Tx$, and this convergence is uniform. That is, there exists a function $\kappa(N)$ that is decreasing for N high enough and $\lim_{N \rightarrow \infty} \kappa(N) = 0$ such that for all x , $\|T_N x - Tx\|_{\mathbb{R}^{K-1}} \leq \kappa(N) \|x\|_\infty$.

The aforementioned property is an extension of the central limit theorem. To prove this, we use a version of the Berry–Esseen theorem. While in general, variants of the Berry–Esseen theorem provide uniform bounds on deviation of the probability distribution μ^N from the standard normal, our setting has two properties that require a stronger version: First, the function $w(\mathbf{h})$ is potentially unbounded; second, we have many dimensions. The version that we use is established in [Bhattacharya \(1975\)](#) (Theorem on page 818); we relax it to focus only on bounded random variables, strengthening its statement:

Theorem 3. Let $\mathbf{Y}_1, \dots, \mathbf{Y}_n$ be a sequence of i.i.d r.v.'s in $\mathbb{R}^d$, ($d \geq 2$) such that

1. $\mathbb{E}\mathbf{Y}_i = \mathbf{0} \in \mathbb{R}^d$ and $\text{Var}\mathbf{Y}_i = \mathbf{I}_d$, where $\mathbf{I}_d$ is the d -dimensional identity matrix; and
2. if $\mathbf{Y}_i = (y_{i,1}, \dots, y_{i,d})$, then $|y_{i,j}| \leq \bar{y} < \infty$.

Let ϕ_n be the probability measure associated with $\mathbf{S}_n = \frac{\mathbf{Y}_1 + \dots + \mathbf{Y}_n}{\sqrt{n}} \in \mathbb{R}^d$, and ϕ be the d -dimensional standard multivariate normal probability measure. Moreover, suppose that $f : \mathbb{R}^d \rightarrow \mathbb{R}$ is a Borel measurable function, and r , a natural number that satisfies

$$M_r(f) = \sup_{\mathbf{x} \in \mathbb{R}^d} \frac{|f(\mathbf{x})|}{1 + \|\mathbf{x}\|_d^r} < \infty, \quad (28)$$

$$\omega(f; \varepsilon) = \sup_{\mathbf{y} \in \mathbb{R}^d} \int \sup_{\|\mathbf{z} - \mathbf{x}\|_d \leq \varepsilon} |f(\mathbf{z} + \mathbf{y}) - f(\mathbf{x} + \mathbf{y})| d\phi(\mathbf{x}) < \infty. \quad (29)$$

Then, there exist constants c_1, c_2 , and c_3 that depend only on d, r such that

$$\left| \int_{\mathbb{R}^n} f(\mathbf{x}) d(\phi_n - \phi) \right| \leq c_1 M_r(f) \frac{\bar{y}}{\sqrt{n}} + c_2 \omega\left(f; c_3 \bar{y} \frac{\log n}{\sqrt{n}}\right).$$

The requirements of Theorem 3 are trivially satisfied for our setup. In our setting, each random variable is an adjusted vector-valued categorical or multinomial distribution

$$\mathbf{Y}_i = (\Sigma_{2:K})^{-1/2} \left(\mathbf{e}_k^{K-1} - \begin{bmatrix} f_2 \\ \vdots \\ f_K \end{bmatrix} \right) \text{ with } \Pr = \frac{f_k}{1 - f_1}, k \geq 2,$$

where $\Sigma_{2:K}$ is the matrix formed by the last $K - 1$ rows and columns of Σ as defined in Lemma 2, and $(\Sigma_{2:K})^{-1/2}$ is the square root of its inverse, which exists and is symmetric. We focus only on the last $K - 1$ element of the multinomial random variable because the matrix Σ is singular and thus not invertible. Given this, we have

$$\mathbf{S}_n = (\Sigma_{2:K})^{-1/2} \begin{bmatrix} h_2 \\ \vdots \\ h_K \end{bmatrix}.$$

The other requirements of Theorem 3 are more stringent. Specifically, if f is a function with unbounded derivatives, $\omega(f; \varepsilon)$ is not bounded. In Section C of the Online Appendix, we show how to modify this result to apply it to a function with unbounded derivatives. Specifically, for any function f that satisfies (28), we define $\tilde{f}(\mathbf{x}) = \mathbf{1} [\|\mathbf{x}\|_d^p + 1 \leq n^{1/3}] f(\mathbf{x})$. We can then use Hoeffding's inequality to bound the deviation of $\int \tilde{f} d\mu^N$ from $\int f d\mu^N$ and apply Theorem 3 to $\tilde{f}$. Given this modification, it can be shown that

$$\omega\left(\tilde{f}; c_3 \bar{y} \frac{\log n}{\sqrt{n}}\right) \leq \hat{C} M_r(f) n^{1/3-1/2} \log n = \hat{C} M_r(f) n^{-1/6} \log n.$$

Since $n^{-1/6} \log n$ eventually becomes decreasing and converges to 0, we can use this to show that the operator $T_N - T$ is bounded and linear, and its norm can be bounded by $\kappa(N)$, where $\kappa(N)$ has the form

$$\kappa(N) = \bar{C} (\hat{c}_1 N^{-1/3} + (\hat{c}_2 N^{-1/2} + \hat{c}_3 N^{-1/6}) \log N).$$

Step 7. *There exist $\bar{c} > 0$ and $\bar{N}$ such that for all $N \geq \bar{N}$,*

$$\begin{aligned} B_{K-1}(0, \varepsilon) &\subset T \left(\mathbf{1} + B_\infty \left(0, \frac{\varepsilon}{\bar{c}} \right) \right), \forall \varepsilon > 0, \\ T_N \mathbf{1} + B_{K-1}(0, \varepsilon) &\subset T_N \left(\mathbf{1} + B_\infty \left(0, \frac{\varepsilon}{\bar{c}} \right) \right), \forall \varepsilon > 0. \end{aligned}$$

The result from Step 7 is a direct implication of the results in Steps 5 and 6. Specifically, uniform convergence of T_N 's to T implies that we can choose c_N and c in Step 6 to be above a value $\bar{c} > 0$. To see this, consider $c > 0$ given by $B_{K-1}(0, c) \subset T(B_\infty(0, 1))$. Let $x \in B_\infty(0, 1)$ such that $\mathbf{z} = Tx$, with $\mathbf{z} \in B_{K-1}(0, 1)$. By uniform convergence, we know that for all N , $\|T_N x - Tx\|_{K-1} \leq \kappa(N) \|x\|_{L_\infty} \leq \kappa(N)$, where $\kappa(N)$ is decreasing in N and converges to 0 as N goes to infinity. We can thus choose $\bar{N}$ so that $\kappa(N) > c/3$ for all $N \geq \bar{N}$. Therefore, for all $N \geq \bar{N}$

$$\|\mathbf{z}\|_{K-1} - c/3 \leq \|Tx\|_{K-1} - \|T_N x - Tx\|_{K-1} \leq \|T_N x\|_{K-1}.$$

Since $\|\mathbf{z}\|_{K-1}$ can get arbitrarily close to c , the above means that for $N \geq \bar{N}$, $\|T_N x\|_{K-1}$ can get arbitrarily close to $\bar{c} = 2c/3$, and hence we must have that

$$B_{K-1}(0, \bar{c}) \subset T_N(B_\infty(0, 1)), \forall N \geq \bar{N}.$$

The statement in Step 7 then follows from the fact that T and T_N are linear and that $T\mathbf{1} = 0$ —take $B_{K-1}(0, \bar{c}) \subset T_N(B_\infty(0, 1))$ and multiply both sides by $\varepsilon/\bar{c}$ and shift $B_\infty(0, 1)$ by $\mathbf{1} \in L_\infty(\mathbb{R}_0^K)$.

Let us now show that Step 7 implies the existence of a sequence of $\sigma_N \in L_\infty(\mathbb{R}_0^K)$ that are positive and satisfy incentive compatibility, and $\sigma_N \rightarrow \sigma$. Since we know that $\mathbb{E}[\sigma(\mathbf{h}) h_k/f_k] - \mathbb{E}[\sigma(\mathbf{h}) h_{k-1}/f_{k-1}] > 0$, and that $\mathbb{E}^N[\sigma(\mathbf{h}) h_k/f_k] \rightarrow \mathbb{E}[\sigma(\mathbf{h}) h_k/f_k]$, there must exist N_0 such that for $N \geq N_0$, we can guarantee that

$$\mathbb{E}^N[\sigma(\mathbf{h}) h_k/f_k] - \mathbb{E}^N[\sigma(\mathbf{h}) h_{k-1}/f_{k-1}] \geq \frac{1}{2} \min_{k \geq 2} \mathbb{E}[\sigma(\mathbf{h}) h_k/f_k] - \mathbb{E}[\sigma(\mathbf{h}) h_{k-1}/f_{k-1}] > 0.$$

Additionally, for any $\bar{c} > \varepsilon > 0$, let N_ε be such that for all $N \geq N_\varepsilon$, $\|T_N \mathbf{1}\|_{K-1} \leq \varepsilon/2$. This implies that $T_N \mathbf{1} \in B_{K-1}(0, \varepsilon)$, and since $0 \in B_{K-1}(0, \varepsilon)$, there must exist $x_{N,\varepsilon} \in \mathbf{1} + B_\infty(0, \varepsilon/\bar{c})$ such that $T_N x_{N,\varepsilon} = 0$. In other words,

$$\int x_{N,\varepsilon}(\mathbf{h}) \sigma(\mathbf{h}) w_{k,N}(\mathbf{h}) d\mu^N = 0.$$

Since $x_{N,\varepsilon} \in \mathbf{1} + B_\infty(0, \varepsilon/\bar{c})$, we must have that $|x_{N,\varepsilon}(\mathbf{h}) - 1| \leq \varepsilon/\bar{c} < 1 \rightarrow x_{N,\varepsilon}(\mathbf{h}) > 0$. Now let us set $\sigma_{N,\varepsilon}(\mathbf{h}) = \sigma(\mathbf{h}) x_{N,\varepsilon}(\mathbf{h})$. If we choose $N \geq \max\{N_\varepsilon, N_0\}$, $\sigma_{N,\varepsilon}(\mathbf{h})$ should satisfy

$$\begin{aligned} \mathbb{E}^N[\sigma_{N,\varepsilon}(\mathbf{h}) w_{N,k}(\mathbf{h})] &= 0 \\ \mathbb{E}^N\left[\sigma_{N,\varepsilon}(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_{k-1}}{f_{k-1}}\right)\right] &> 0, \forall k \geq 2 \\ \|\sigma_{N,\varepsilon}(\mathbf{h}) - \sigma(\mathbf{h})\|_{L_\infty} &= \|\sigma(\mathbf{h}) (x_{N,\varepsilon}(\mathbf{h}) - 1)\|_{L_\infty} \leq \frac{\varepsilon}{\bar{c}} \|\sigma\|_{L_\infty}. \end{aligned}$$

Therefore, $\sigma_{N,\varepsilon}(\mathbf{h})$ is a positive function that satisfies the local incentive compatibility and thus is incentive compatible. Since the choice of ε can be arbitrarily small, we can choose a sequence of $\sigma_{N,\varepsilon_N}(\mathbf{h}) \rightarrow \sigma$ with $\varepsilon_N \rightarrow 0$. This concludes the proof. $\square$

A.3 Proof of Theorem 2

Proof. Note that any $\sigma(\mathbf{h})$ that is ICL satisfies

$$\mathbb{E} \left[\sigma(\mathbf{h}) \left((\mathbf{h} \cdot \mathbf{t} + b) \sum_k h_k t_k - \sum_k f_k t_k^2 \right) \right] = 0 \Rightarrow \mathbb{E} [\sigma(\mathbf{h}) ((\omega(\mathbf{h}) + b) \omega(\mathbf{h}) - \text{Var}(s))] = 0. \quad (30)$$

Now, consider the relaxed optimization where the ICL requirement, (6), is replaced with the above. Optimality implies that any optimal mechanism σ^* should satisfy

$$\sigma^*(\mathbf{h}) = 1 \Leftrightarrow \omega(\mathbf{h}) + r - \alpha((\omega(\mathbf{h}) + b) \omega(\mathbf{h}) - \text{Var}(s)) \geq 0$$

for some Lagrange multiplier associated with the relaxed constraint (30). This condition implies that $\sigma^*(\mathbf{h})$ depends only on $\omega(\mathbf{h})$.

Note that if $\alpha > 0$, the above condition defines two cutoffs for $\underline{\omega} < \bar{\omega}$ that satisfy the following quadratic equation:

$$\omega + r + \alpha \text{Var}(s) - \alpha(\omega + b)\omega = 0. \quad (31)$$

For the above to have two roots, we need to have $(1 - \alpha b)^2 + 4\alpha(r + \alpha \text{Var}(s)) \geq 0$. As we will show in the Online Appendix, Section B.1.1, when $b > r > \frac{b - \sqrt{b^2 + 4}}{2}$, for any $\alpha > 0$, this is the case. Moreover, it has to be that $-b < \underline{\omega} < 0 < -\underline{\omega} < \bar{\omega}$. Finally, there is a unique α such that the relaxed version of ICL, (30), holds.

This implies that the optimal mechanism in the relaxed problem must satisfy $\sigma^*(\mathbf{h}) = 1$ when $\omega = \mathbf{h} \cdot \mathbf{t} \in [\underline{\omega}, \bar{\omega}]$, and $\sigma^*(\mathbf{h}) = 0$, otherwise. Note that when $\sigma(\mathbf{h})$ is only a function of the sample mean, basic properties of the normal distribution implies that $\mathbb{E}[h_k | \omega] = \frac{f_k t_k}{\text{Var}(s)} \omega$. We can use this to show that

$$\begin{aligned} \mathbb{E} \left[\sigma^*(\mathbf{h}) \left((\omega(\mathbf{h}) + b) \frac{h_k}{f_k} - t_k \right) \right] &= \mathbb{E} \left[\mathbb{E} \left[\sigma^*(\mathbf{h}) \left((\omega(\mathbf{h}) + b) \frac{h_k}{f_k} - t_k \right) \mid \omega(\mathbf{h}) = \omega \right] \right] \\ &= \mathbb{E} \left[\sigma^*(\mathbf{h}) \left((\omega(\mathbf{h}) + b) \mathbb{E} \left[\frac{h_k}{f_k} \mid \omega(\mathbf{h}) = \omega \right] - t_k \right) \right] \\ &= \mathbb{E} \left[\sigma^*(\mathbf{h}) \left((\omega(\mathbf{h}) + b) \frac{t_k \omega(\mathbf{h})}{\text{Var}(s)} - t_k \right) \right] \\ &= \frac{t_k}{\text{Var}(s)} \mathbb{E} [\sigma^*(\mathbf{h}) ((\omega(\mathbf{h}) + b) \omega(\mathbf{h}) - \text{Var}(s))] = 0, \end{aligned}$$

where we have used the law of iterated expectations and the fact that $\sigma^*(\mathbf{h})$ depends only on

$\omega(\mathbf{h})$. Similarly, we can write

$$\mathbb{E} \left[\sigma^*(\mathbf{h}) \frac{h_k}{f_k} \right] = \mathbb{E} \left[\sigma^*(\mathbf{h}) \mathbb{E} \left[\frac{h_k}{f_k} | \omega \right] \right] = \frac{t_k}{\text{Var}(s_i)} \mathbb{E} [\sigma^*(\mathbf{h}) \omega].$$

Since $\bar{\omega} + \underline{\omega} > 0$, $\mathbb{E} \sigma^*(\mathbf{h}) \omega(\mathbf{h}) > 0$ and thus the above expression is increasing in k . Therefore, when $r > \frac{b - \sqrt{b^2 + 4}}{2}$, the solution of the relaxed problem is also the optimal mechanism.

In the Online Appendix, Section **B.1.1**, we show that $\alpha < 0$ is not a possibility. Moreover, whenever $r \leq \frac{b - \sqrt{b^2 + 4}}{2}$, we must have $\underline{\omega} = \bar{\omega}$, and thus the solution of the relaxed problem as well as the optimal mechanism is $\sigma^* \equiv 0$. $\square$

B Online Appendix

B.1 Other Proofs

B.1.1 Existence of the Multiplier in Theorem 2

Proof. Here, we show that when $r > \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$ there is a unique α for which

$$\int_{D_\alpha} \left(1 - \frac{\omega(\omega + b)}{\text{Var}(s)}\right) \phi\left(\frac{\omega}{\sqrt{\text{Var}(s)}}\right) d\omega = 0$$

$$\{\omega | \psi(\omega, \alpha) = \omega + r + \alpha\text{Var}(s) - \alpha(\omega + b)\omega \geq 0\} = D_\alpha$$

Moreover $\alpha \in (0, 1/b)$ which implies that $D_\alpha = [\underline{\omega}, \bar{\omega}]$ where

$$-b < \underline{\omega} < -r < -\underline{\omega} < \bar{\omega}.$$

Additionally, we show that if $r \leq \frac{b - \sqrt{b^2 + 4}}{2}$, then whenever the above holds, $\underline{\omega} = \bar{\omega}$. We do so via a series of claims.

Claim 1. For any value of α , $\int_{D_\alpha} \left(1 - \frac{\omega(\omega + b)}{\text{Var}(s)}\right) \phi\left(\frac{\omega}{\sqrt{\text{Var}(s)}}\right) d\omega$ is increasing in α .

To prove this claim, note that if $\alpha < 0$, then either $\psi(\omega, \alpha)$ is always positive or at most two values of $\omega, \bar{\omega}$ exist such that $\psi(\underline{\omega}, \alpha) = \psi(\bar{\omega}, \alpha) = 0$ in which case $D_\alpha = \mathbb{R} \setminus (\underline{\omega}, \bar{\omega})$. On the other hand, if $\alpha > 0$, either $D_\alpha = [\underline{\omega}, \bar{\omega}]$ with $\psi(\underline{\omega}, \alpha) = \psi(\bar{\omega}, \alpha) = 0$ or $D_\alpha = \emptyset$. Evidently, if $\psi(\omega, \alpha)$ does not have a zero, then the integral is 0.

Note that when the zeros exist, they are given by

$$\underline{\omega} = \frac{1 - b\alpha}{2\alpha} - \frac{1}{2\alpha} \sqrt{(1 - b\alpha)^2 + 4r\alpha + 4\alpha^2\text{Var}(s)} = \frac{1 - \alpha b - \sqrt{\Delta}}{2\alpha} \quad (32)$$

$$\bar{\omega} = \frac{1 - \alpha b}{2\alpha} + \frac{1}{2\alpha} \sqrt{(1 - b\alpha)^2 + 4r\alpha + 4\alpha^2\text{Var}(s)} = \frac{1 - \alpha b + \sqrt{\Delta}}{2\alpha} \quad (33)$$

We thus have that

$$\frac{d\bar{\omega}}{d\alpha} = -\frac{\psi_\alpha(\bar{\omega}, \alpha)}{\psi_\omega(\bar{\omega}, \alpha)} = -\frac{\text{Var}(s) - \bar{\omega}(\bar{\omega} + b)}{1 - \alpha(2\bar{\omega} + b)} = -\frac{-\frac{\bar{\omega} + r}{\alpha}}{-\sqrt{\Delta}} = -\frac{\bar{\omega} + r}{\alpha\sqrt{\Delta}}$$

$$\frac{d\underline{\omega}}{d\alpha} = -\frac{\psi_\alpha(\underline{\omega}, \alpha)}{\psi_\omega(\underline{\omega}, \alpha)} = -\frac{\text{Var}(s) - \underline{\omega}(\underline{\omega} + b)}{1 - \alpha(2\underline{\omega} + b)} = -\frac{-\frac{\underline{\omega} + r}{\alpha}}{\sqrt{\Delta}} = \frac{\underline{\omega} + r}{\alpha\sqrt{\Delta}}$$

When $\alpha > 0$, $\bar{\omega} \geq \underline{\omega}$ and we have

$$\begin{aligned}
& \frac{d}{d\alpha} \int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& \frac{d}{d\alpha} \int_{\underline{\omega}}^{\bar{\omega}} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& \phi \left(\frac{\bar{\omega}}{\sqrt{\text{Var}(s)}} \right) \frac{d\bar{\omega}}{d\alpha} \left[1 - \frac{\bar{\omega}(\bar{\omega}+b)}{\text{Var}(s)} \right] - \phi \left(\frac{\underline{\omega}}{\sqrt{\text{Var}(s)}} \right) \frac{d\underline{\omega}}{d\alpha} \left[1 - \frac{\underline{\omega}(\underline{\omega}+b)}{\text{Var}(s)} \right] = \\
& \phi \left(\frac{\bar{\omega}}{\sqrt{\text{Var}(s)}} \right) \left(-\frac{\bar{\omega}+r}{\alpha\sqrt{\Delta}} \right) \left(-\frac{\bar{\omega}+r}{\alpha\text{Var}(s)} \right) - \phi \left(\frac{\underline{\omega}}{\sqrt{\text{Var}(s)}} \right) \left(\frac{\underline{\omega}+r}{\alpha\sqrt{\Delta}} \right) \left(-\frac{\underline{\omega}+r}{\alpha\text{Var}(s)} \right) = \\
& \frac{1}{\sqrt{\Delta}\text{Var}(s)} \left[\phi \left(\frac{\bar{\omega}}{\sqrt{\text{Var}(s)}} \right) \left(\frac{\bar{\omega}+r}{\alpha} \right)^2 + \phi \left(\frac{\underline{\omega}}{\sqrt{\text{Var}(s)}} \right) \left(\frac{\underline{\omega}+r}{\alpha} \right)^2 \right] > 0
\end{aligned}$$

where the above holds since both terms are squares multiplied by a density. Similarly when $\alpha < 0$, $\bar{\omega} < \underline{\omega}$ and thus

$$\begin{aligned}
& \int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& \int_{\mathbb{R} \setminus (\bar{\omega}, \underline{\omega})} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& \int_{\underline{\omega}}^{\bar{\omega}} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega
\end{aligned}$$

where in the above we have used the fact that the integral over the entire real line is 0. Then a logic similar to the case of $\alpha > 0$ implies that the above is increasing.

Claim 2. For all values $\alpha < 0$, $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega < 0$.

To show this, it is sufficient to show that it holds as $\alpha \rightarrow 0$ from below. Note that as $\alpha \nearrow 0$, $\underline{\omega} \rightarrow -r$ and $\bar{\omega} \rightarrow -\infty$. Hence,

$$\begin{aligned}
& \int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& \int_{-r}^{-\infty} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \phi \left(\frac{\omega}{\sqrt{\text{Var}(s)}} \right) d\omega = \\
& - \int_{-\infty}^{-r} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)} \right] \frac{e^{-\frac{\omega^2}{2\text{Var}(s)}}}{\sqrt{2\pi}} d\omega = -(\omega+b) \frac{e^{-\frac{\omega^2}{2\text{Var}(s)}}}{\sqrt{2\pi}} \Big|_{-\infty}^{-r} = -(b-r) \frac{e^{-\frac{r^2}{2\text{Var}(s)}}}{\sqrt{2\pi}} < 0
\end{aligned}$$

which proves the claim.

Claim 3. If $r > \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, then $\exists! \alpha \in (0, 1/b)$ such that $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega = 0$. At this value of α , $-b < \underline{\omega} < -r < \bar{\omega}$ and $0 < \underline{\omega} + \bar{\omega}$.

Note that as $\alpha \searrow 0$, $\underline{\omega} \rightarrow -r$ and $\bar{\omega} \rightarrow \infty$. Hence and as above, $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega < 0$ holds for $\alpha = 0$. In contrast, at $\alpha = 1/b$,

$$\begin{aligned}\underline{\omega} &= -\frac{b}{2} \sqrt{4r/b + 4\text{Var}(s)/b^2} = -\sqrt{rb + \text{Var}(s)} \\ \bar{\omega} &= \sqrt{rb + \text{Var}(s)}\end{aligned}$$

At these values

$$\begin{aligned}\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega &= \int_{-\sqrt{rb+\text{Var}(s)}}^{\sqrt{rb+\text{Var}(s)}} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega \\ &= \left(\sqrt{rb + \text{Var}(s)} + b\right) \frac{e^{-\frac{rb+\text{Var}(s)}{2\text{Var}(s)}}}{\sqrt{2\pi}} \\ &\quad - \left(b - \sqrt{rb + \text{Var}(s)}\right) \frac{e^{-\frac{rb+\text{Var}(s)}{2\text{Var}(s)}}}{\sqrt{2\pi}} \\ &= 2\sqrt{rb + \text{Var}(s)} \frac{e^{-\frac{rb+\text{Var}(s)}{2\text{Var}(s)}}}{\sqrt{2\pi}} > 0\end{aligned}$$

Hence, there exists a unique $\alpha \in (0, 1/b)$ for which $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega = 0$. Since $\underline{\omega}, \bar{\omega}$ satisfy (32) and (33), their sum is $\frac{1-b\alpha}{\alpha} > 0$. Moreover, since $\Delta = (1 - b\alpha)^2 + 4r\alpha + 4\alpha^2\text{Var}(s)$, we can sign $\underline{\omega}, \bar{\omega}$ by signing Δ . The expression for Δ is a quadratic function of α whose discriminant is $(4r - 2b)^2 - 4(b^2 + 4\text{Var}(s)) = 16(r^2 - rb - \text{Var}(s))$. When $b > r > \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, this expression is always negative which means that for all values of α , $\Delta > 0$. Since $\alpha < 1/b$, we must have that $\bar{\omega} > 0 > \underline{\omega}$. Finally, $\psi(-r, \alpha) = \alpha(\text{Var}(s) + rb - r^2)$. When $b > r > \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, this expression is positive which means that $\underline{\omega} < -r < \bar{\omega}$.

Claim 4. If $r \leq \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, then $\exists \alpha > 0$ such that $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega = 0$. For all such values of α , $\underline{\omega} = \bar{\omega}$.

The proof of this claim follows from the same argument as before together with the fact that when $r < \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, there is an $1/b > \alpha > 0$ for which $\Delta = (1 - b\alpha)^2 + 4r\alpha + 4\alpha^2\text{Var}(s) = 0$. To see this, note that the values of α for which this holds are given by

$$\alpha = \frac{b - 2r \pm 2\sqrt{r^2 - rb - \text{Var}(s)}}{b^2 + 4\text{Var}(s)}$$

Since $r \leq \frac{b - \sqrt{b^2 + 4\text{Var}(s)}}{2}$, we must have that $2r \leq b - \sqrt{b^2 + 4\text{Var}(s)}$ which implies that the lower root above is positive. Since as we have argued before, $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega$ is strictly increasing in α whenever $\alpha > 0$ and $\underline{\omega}, \bar{\omega}$ are real and at the above values of α , $\underline{\omega} = \bar{\omega}$ and $\int_{D_\alpha} \left[1 - \frac{\omega(\omega+b)}{\text{Var}(s)}\right] \phi\left(\omega/\sqrt{\text{Var}(s)}\right) d\omega = 0$, this establishes the claim. $\square$

B.2 Extensions: Proofs

Heterogeneous Biases: Proof of Proposition 2

Proof. The fact that ICL is equivalent to 2 follows a similar existence proof as of that of Theorem 1. Such an extension is possible because the number of bias types and signal realizations is finite. Hence, we can apply the same technique – consider a linear operator for all signal realization in each bias group and show the existence of a mechanism for any finite number of senders.

Now, consider the problem of choosing $\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M)$ to maximize the receiver's payoff $\mathbb{E}[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M)(\omega + r)]$ subject to the ICL requirements:

$$\begin{aligned} \mathbb{E} \left[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \left(\frac{h_{k,m}}{f_{k,m}} (\omega + b_m) - t_{k,m} \right) \right] &= 0 \\ \mathbb{E} \left[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \frac{h_{k,m}}{f_{k,m}} \right] &\geq \mathbb{E} \left[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M) \frac{h_{k-1,m}}{f_{k-1,m}} \right], k > 1 \end{aligned}$$

By multiplying the top condition by $f_{k,m}t_{k,m}$ and summing over k for a fixed m and using the fact that $\sum_k f_{k,m}t_{k,m}^2 = \eta$, we arrive at

$$\mathbb{E}[\sigma(\mathbf{h}_1, \dots, \mathbf{h}_M)(\omega_m(\omega + b_m) - \eta)] = 0 \quad (34)$$

Moreover, since $t_{k,m}$ and $\mathbb{E} \left[\sigma \frac{h_{k,m}}{f_{k,m}} \right]$ are both increasing in k , their covariance has to be positive. In other words,

$$\sum_k f_{k,m}t_{k,m} \mathbb{E} \left[\sigma \frac{h_{k,m}}{f_{k,m}} \right] \geq \sum_k f_{k,m}t_{k,m} \sum_k f_{k,m} \mathbb{E} \left[\sigma \frac{h_{k,m}}{f_{k,m}} \right]$$

The left hand side of the above is $\mathbb{E}[\sigma\omega_m]$ while the right hand side is 0. Hence, we must have that for all m , $\mathbb{E}[\sigma\omega_m] \geq 0$.

In other words, if a mechanism is ICL, then it must satisfy (34) and $\mathbb{E}[\sigma\omega_m] \geq 0$. One can thus focus on the relaxed problem of maximizing $\mathbb{E}[\sigma \cdot (\omega + r)]$ subject to (34) and $\mathbb{E}[\sigma \cdot \omega_m] \geq 0$. In this relaxed problem, all constraint only depend on $(\mathbf{h}_1, \dots, \mathbf{h}_M)$ via $\omega_1, \dots, \omega_M$ which implies that the solution should only depend on $\omega_1, \dots, \omega_M$. Additionally, σ^* is a solution to this relaxed problem if and only if multipliers $\lambda_1, \dots, \lambda_M$ associated with (34) and $\zeta_m \geq 0$ associated with

$\mathbb{E}[\sigma \cdot \omega_m] \geq 0$ exists that satisfy the conditions provided in statement of the proposition.

It therefore remains to be shown that if σ^* solves the relaxed problem, then it is indeed ICL. To see this, Since $\mathbf{h}_m \sim \mathcal{N}(0, \Sigma_m)$ and $h_{k,m}$'s are independent across different bias groups, we must have that

$$\mathbb{E}[h_{k,m}|\omega_1, \dots, \omega_M] = \mathbb{E}[h_{k,m}|\omega_m] = \frac{\mathbb{E}[h_{k,m}\omega_m]}{\mathbb{E}[\omega_m^2]}\omega_m = \frac{f_{k,m}t_{k,m}}{\eta}\omega_m$$

Therefore,

$$\begin{aligned} \mathbb{E}\left[\sigma^* \cdot \left(\frac{h_{k,m}}{f_{k,m}}(\omega + b_m) - t_{k,m}\right)\right] &= \mathbb{E}\left[\mathbb{E}\left[\sigma^* \cdot \frac{h_{k,m}}{f_{k,m}}(\omega + b_m) \mid \omega_1, \dots, \omega_M\right] - \sigma^* \cdot t_{k,m}\right] \\ &= \mathbb{E}\left[\sigma^* \cdot (\omega + b_m) \mathbb{E}\left[\frac{h_{k,m}}{f_{k,m}} \mid \omega_1, \dots, \omega_M\right] - \sigma^* \cdot t_{k,m}\right] \\ &= \mathbb{E}\left[\sigma^* \cdot (\omega + b_m) \frac{t_{k,m}}{\eta}\omega_m - \sigma^* \cdot t_{k,m}\right] \\ &= \frac{t_{k,m}}{\eta}\mathbb{E}[\sigma^* \cdot (\omega + b_m)\omega_m - \eta\sigma^*] = 0 \end{aligned}$$

where in the above we have used the law of iterated expectations and that σ^* is the solution to the relaxed problem. Similarly,

$$\begin{aligned} \mathbb{E}\left[\sigma^* \cdot \frac{h_{k,m}}{f_{k,m}}\right] &= \mathbb{E}\left[\sigma^* \cdot \mathbb{E}\left[\frac{h_{k,m}}{f_{k,m}} \mid \omega_1, \dots, \omega_M\right]\right] \\ &= \frac{t_{k,m}}{\eta}\mathbb{E}[\sigma^* \cdot \omega_m] \end{aligned}$$

Since σ^* is the solution to the relaxed problem, it must satisfy $\mathbb{E}[\sigma^* \cdot \omega_m]$ which implies that the above is increasing in k . Therefore, σ^* is ICL. $\square$

B.2.1 General Preferences: Proof of Proposition 3

Proof. Similar to the first part of Theorem 1, we can write

$$U_{l,k}^N = \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) \quad (35)$$

where in the above $\mathbf{e}_k$ is a K -dimensional vector whose elements are 0 except for its k -th element which is 1

Consider a vector of realizations $\mathbf{s}_- \in S^{N-1}$ for which $\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k) = \mathbf{h}$. In this case, the count of s_i 's which are of type m is given by $n_m = \sqrt{N}h_m + Nf_m$ with $\sqrt{N}h_k + Nf_k \geq 1$. Then

probability of this occurring – using the multi-nomial distribution – is given by

$$\begin{aligned}
\Pr_{N-1}(\mathbf{s}_-) &= \binom{N-1}{n_1, \dots, n_k-1, \dots, n_K} f_1^{n_1} \dots f_k^{n_k-1} \dots f_K^{n_K} \\
&= \binom{N}{n_1, \dots, n_K} f_1^{n_1} \dots f_K^{n_K} \frac{n_k}{f_k N} \\
&= \Pr_N(\mathbf{s}_- + \mathbf{e}_k) \frac{h_k \sqrt{N} + N f_k}{f_k N}
\end{aligned}$$

Using this adjustment of the probabilities, we can write

$$\begin{aligned}
U_{l,k}^N &= \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) \\
&= \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) + \\
&\quad \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) [u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) - u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l))] \\
&= U_{l,l}^N + \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) [u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) - u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l))]
\end{aligned}$$

Let $\mu^N(\mathbf{h})$ be the probability of the adjusted frequencies being equal to $\mathbf{h}$. If $\mathbf{h} = \mathbf{h}^N(\mathbf{s})$, then it must be that $\mu^N(\mathbf{h}) = \Pr_N(\mathbf{s})$. Let us also define $H^N \subset \mathbb{R}_0^K$ to be the support of μ^N . We can thus write the above as

$$\begin{aligned}
U_{l,k}^N &= U_{l,l}^N + \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_{N-1}(\mathbf{s}_-) \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) [u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) - u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l))] \\
&= U_{l,l}^N + \sum_{\mathbf{s}_- \in S^{N-1}} \Pr_N(\mathbf{s}_- + \mathbf{e}_l) \frac{h_l^N(\mathbf{s}_- + \mathbf{e}_l) \sqrt{N} + N f_l}{f_l N} \sigma^N(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l)) \times \\
&\quad [u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_k)) - u_s(\mathbf{h}^N(\mathbf{s}_- + \mathbf{e}_l))] \\
&= U_{l,l}^N + \sum_{\mathbf{h} \in H^N, h_l \geq \frac{1-f_l\sqrt{N}}{N}} \mu^N(\mathbf{h}) \sigma^N(\mathbf{h}) \frac{h_l \sqrt{N} + N f_l}{f_l N} \left[u_s\left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}}\right) - u_s(\mathbf{h}) \right] \\
&= U_{l,l}^N + \sum_{\mathbf{h} \in H^N, h_l \geq \frac{1-f_l\sqrt{N}}{N}} \mu^N(\mathbf{h}) \sigma^N(\mathbf{h}) \frac{h_l \sqrt{N} + N f_l}{f_l N} \left[u_s\left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}}\right) - u_s(\mathbf{h}) \right] + \\
&\quad \sum_{\mathbf{h} \in H^N, h_l = -f_l \sqrt{N}} \mu^N(\mathbf{h}) \sigma^N(\mathbf{h}) \frac{h_l \sqrt{N} + N f_l}{f_l N} \left[u_s\left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}}\right) - u_s(\mathbf{h}) \right]
\end{aligned}$$

If we use $\mathbb{E}^N$ to refer to the expectation with respect to μ^N , we can write the incentive constraints as

$$U_{k,k}^N - U_{l,l}^N \geq \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \frac{h_l \sqrt{N} + f_l N}{f_l N} \left(u_s \left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \right) - u_s(\mathbf{h}) \right) \right] \quad (36)$$

As N tends to infinity, $\sqrt{N} \left(u_s \left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \right) - u_s(\mathbf{h}) \right)$ converges to $\frac{\partial u_s}{\partial h_k} - \frac{\partial u_s}{\partial h_l}$. Similar to the specific case of section 4, we have

$$\begin{aligned} \sqrt{N} (U_{k,k}^N - U_{l,l}^N) &= \sqrt{N} \mathbb{E}^N \left[\sigma(\mathbf{h}) \left(\frac{h_k \sqrt{N} + f_k N}{f_k N} - \frac{h_l \sqrt{N} + f_l N}{f_l N} \right) u_s(\mathbf{h}) \right] \\ &= \mathbb{E}^N \left[\sigma(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right) u_s(\mathbf{h}) \right] \end{aligned}$$

Now taking a limit in (36) gives us

$$\mathbb{E} \left[\sigma(\mathbf{h}) u_s(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right) \right] \geq \mathbb{E} \left[\sigma(\mathbf{h}) \left(\frac{\partial u_s(\mathbf{h})}{\partial h_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_l} \right) \right]$$

Since this has to hold for all k, l , it should hold with equality and thus

$$\mathbb{E} \left[\sigma(\mathbf{h}) \left(u_s(\mathbf{h}) \frac{h_k}{f_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_k} \right) \right] = \bar{U}$$

Similar to before, we also need that when $k > l$,

$$\begin{aligned} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \left(u_s(\mathbf{h}) - u_s \left(\mathbf{h} + \frac{\mathbf{e}_l - \mathbf{e}_k}{\sqrt{N}} \right) \right) \right] &\geq \\ \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_l}{f_l \sqrt{N}} + 1 \right) \left(u_s \left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \right) - u_s(\mathbf{h}) \right) \right] \end{aligned}$$

Using Taylor's formula, we can write

$$\begin{aligned} u_s \left(\mathbf{h} + \frac{\mathbf{e}_l - \mathbf{e}_k}{\sqrt{N}} \right) - u_s(\mathbf{h}) &= \frac{\mathbf{e}_l - \mathbf{e}_k}{\sqrt{N}} \nabla u_s(\mathbf{h})^{\text{tr}} + \frac{1}{2} \frac{\mathbf{e}_l - \mathbf{e}_k}{\sqrt{N}} \nabla^2 u_s(\mathbf{h}) \frac{\mathbf{e}_l^{\text{tr}} - \mathbf{e}_k^{\text{tr}}}{\sqrt{N}} + O \left(\frac{1}{N \sqrt{N}} \right) \\ u_s \left(\mathbf{h} + \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \right) - u_s(\mathbf{h}) &= \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla u_s(\mathbf{h})^{\text{tr}} + \frac{1}{2} \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla^2 u_s(\mathbf{h}) \frac{\mathbf{e}_k^{\text{tr}} - \mathbf{e}_l^{\text{tr}}}{\sqrt{N}} + O \left(\frac{1}{N \sqrt{N}} \right) \end{aligned}$$

Thus the above inequality becomes

$$\begin{aligned} & \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_k}{f_k \sqrt{N}} + 1 \right) \left(\frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla u_s(\mathbf{h})^{\text{tr}} - \frac{1}{2} \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla^2 u_s(\mathbf{h}) \frac{\mathbf{e}_k^{\text{tr}} - \mathbf{e}_l^{\text{tr}}}{\sqrt{N}} + O\left(\frac{1}{N\sqrt{N}}\right) \right) \right] \geq \\ & \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_l}{f_l \sqrt{N}} + 1 \right) \left(\frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla u_s(\mathbf{h})^{\text{tr}} + \frac{1}{2} \frac{\mathbf{e}_k - \mathbf{e}_l}{\sqrt{N}} \nabla^2 u_s(\mathbf{h}) \frac{\mathbf{e}_k^{\text{tr}} - \mathbf{e}_l^{\text{tr}}}{\sqrt{N}} + O\left(\frac{1}{N\sqrt{N}}\right) \right) \right] \end{aligned}$$

We thus have

$$\begin{aligned} & \frac{1}{N} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_k}{f_k} (\mathbf{e}_k - \mathbf{e}_l) \nabla u_s(\mathbf{h})^{\text{tr}} - \frac{1}{2} (\mathbf{e}_k - \mathbf{e}_l) \nabla^2 u_s(\mathbf{h}) (\mathbf{e}_k^{\text{tr}} - \mathbf{e}_l^{\text{tr}}) \right) \right] + O\left(\frac{1}{N\sqrt{N}}\right) \geq \\ & \frac{1}{N} \mathbb{E}^N \left[\sigma^N(\mathbf{h}) \left(\frac{h_l}{f_l} (\mathbf{e}_k - \mathbf{e}_l) \nabla u_s(\mathbf{h})^{\text{tr}} + \frac{1}{2} (\mathbf{e}_k - \mathbf{e}_l) \nabla^2 u_s(\mathbf{h}) (\mathbf{e}_k^{\text{tr}} - \mathbf{e}_l^{\text{tr}}) \right) \right] + O\left(\frac{1}{N\sqrt{N}}\right) \end{aligned}$$

Hence, as N converges to infinity, the above multiplied by N converges to

$$\begin{aligned} & \mathbb{E} \left[\sigma(\mathbf{h}) \left(\frac{h_k}{f_k} - \frac{h_l}{f_l} \right) \left(\frac{\partial u_s(\mathbf{h})}{\partial h_k} - \frac{\partial u_s(\mathbf{h})}{\partial h_l} \right) \right] \geq \\ & \mathbb{E} \left[\sigma(\mathbf{h}) \left(\frac{\partial^2 u_s(\mathbf{h})}{\partial h_k^2} + \frac{\partial^2 u_s(\mathbf{h})}{\partial h_l^2} - 2 \frac{\partial^2 u_s(\mathbf{h})}{\partial h_l \partial h_k} \right) \right] \end{aligned}$$

The only if proof follows closely that of Theorem 1. Specifically, we use the power bound from Assumption 1 and apply Lemma (3) to show the uniform convergence used in proof of Theorem 2. This concludes the proof. $\square$

B.3 Proof of Proposition 5

Proof. If we apply the characterization result of Proposition 3 to this setup, it implies that a mechanism $\sigma(\mathbf{h})$ is ICL if and only if it satisfies

$$\mathbb{E} \left[\sigma(\mathbf{h}) \left(\mathbf{t}_s \cdot \mathbf{h} \frac{h_k}{f_k} - t_{s,k} \right) \right] = U, \forall k \quad (37)$$

$$\mathbb{E} \left[\sigma(\mathbf{h}) \frac{h_k}{f_k} \right] \geq \mathbb{E} \left[\sigma(\mathbf{h}) \frac{h_l}{f_l} \right], \forall k > l \quad (38)$$

The fact that $\sum_k f_k t_{s,k} = 0$ implies that $U = 0$.

The rest of the proof closely follows that of Proposition 2. Namely, if σ is ICL, i.e., satisfies (37) and (38), then by multiplying (37) by $f_k t_{s,k}$ and summing over k , we arrive at

$$\mathbb{E} \left[\sigma(\mathbf{h}) (\omega_s^2 - 1) \right] = 0 \quad (39)$$

where $\omega_s = \mathbf{t}_s \cdot \mathbf{h}$. Similarly, by repeating this for $f_k t_{r,k}$, we arrive at

$$\mathbb{E} [\sigma(\mathbf{h}) (\omega_s \omega_r - \gamma)] = 0 \quad (40)$$

Finally, note that properties of the normal distribution implies that

$$\mathbb{E} [h_k | \omega_s, \omega_r] = \begin{bmatrix} \omega_s & \omega_r \end{bmatrix} \begin{bmatrix} \mathbb{E} [\omega_s^2] & \mathbb{E} [\omega_s \omega_r] \\ \mathbb{E} [\omega_s \omega_r] & \mathbb{E} [\omega_r^2] \end{bmatrix}^{-1} \begin{bmatrix} \mathbb{E} [\omega_s h_k] \\ \mathbb{E} [\omega_r h_k] \end{bmatrix}$$

where

$$\begin{aligned} \mathbb{E} [\omega_s^2] &= \sum_{k,l} \mathbb{E} [t_{s,k} t_{s,l} h_k h_l] = \sum_{k,l} t_{s,k} t_{s,l} f_k (\mathbf{1}[k=l] - f_l) \\ &= \sum_k f_k t_{s,k}^2 = 1 \\ \mathbb{E} [\omega_r^2] &= 1, \mathbb{E} [\omega_r \omega_s] = \sum_k f_k t_{r,k} t_{s,k} = \gamma \\ \mathbb{E} [\omega_s h_k] &= f_k t_{s,k}, \mathbb{E} [\omega_r h_k] = f_k t_{r,k} \end{aligned}$$

and hence

$$\begin{aligned} \mathbb{E} [h_k | \omega_s, \omega_r] &= f_k \begin{bmatrix} \omega_s & \omega_r \end{bmatrix} \begin{bmatrix} 1 & \gamma \\ \gamma & 1 \end{bmatrix}^{-1} \begin{bmatrix} t_{s,k} \\ t_{r,k} \end{bmatrix} \\ &= \frac{f_k}{1-\gamma^2} \begin{bmatrix} \omega_s & \omega_r \end{bmatrix} \begin{bmatrix} 1 & -\gamma \\ -\gamma & 1 \end{bmatrix} \begin{bmatrix} t_{s,k} \\ t_{r,k} \end{bmatrix} \\ &= \frac{f_k}{1-\gamma^2} [(t_{s,k} - \gamma t_{r,k}) \omega_s + (t_{r,k} - \gamma t_{s,k}) \omega_r] \end{aligned}$$

Replacing the above in the (38) leads to

$$\mathbb{E} [\sigma(\mathbf{h}) ((t_{s,k} - \gamma t_{r,k}) \omega_s + (t_{r,k} - \gamma t_{s,k}) \omega_r)] : \text{increasing in } k \quad (41)$$

Hence, similar to the proof of Proposition 2, we can focus on a relaxed problem where instead of ICL we impose (39), (40), and (41). The solution of the relaxed problem should then satisfy the conditions provided in proposition 5. Finally, we must show that the solution of the relaxed problem is indeed ICL. This follows steps similar to those in proof of Proposition 2. $\square$

C Proof of Uniform Convergence in Theorem 1

Proof. In this section, we provide a detailed proof of Step 6 in the proof of Theorem 1. Let us first show the following lemma:

Lemma 3. *Let $f : \mathbb{R}^d \rightarrow \mathbb{R}$ be a continuously differentiable function (except possibly for a measure zero set of values) and suppose that $p \geq 1$ exists such that $f(\mathbf{x}) = O(\|\mathbf{x}\|_d^p)$ and $\frac{\partial f}{\partial x_j}(\mathbf{x}) = O(\|\mathbf{x}\|_d^p)$ for all $j = 1, \dots, d$. Moreover, let ϕ_n and ϕ be constructed as in Theorem 3. Then, there exists a function $\kappa(n)$ which is strictly decreasing for high enough values of n , it is independent of f , and $\lim_{n \rightarrow \infty} \kappa(n) = 0$ such that*

$$\left| \int f(\mathbf{x}) d(\phi_n - \phi) \right| \leq \kappa(n) \max \left\{ M_p(f), \left\{ M_p \left(\frac{\partial f}{\partial x_j} \right) \right\}_{j=1}^d \right\} < \infty.$$

Proof. Note that f and ∇f are continuous functions (almost surely everywhere). Thus the functions $|f(\mathbf{x})| / (1 + \|\mathbf{x}\|_d^p)$ and $|\nabla f(\mathbf{x})| / (1 + \|\mathbf{x}\|_d^p)$ have the same property. This implies they are bounded in a ball around the origin. Moreover, since $f, \nabla f$ both satisfy $O(\|\mathbf{x}\|_d^p)$, there must exist $\bar{C} > 0$ such that

$$|f(\mathbf{x})|, \left| \frac{\partial}{\partial x_j} f(\mathbf{x}) \right| \leq \bar{C} (1 + \|\mathbf{x}\|_d^p), \forall \mathbf{x} \in \mathbb{R}^d$$

Obviously, we can set $\bar{C} = \max \left\{ M_p(f), \left\{ M_p \left(\frac{\partial f}{\partial x_j} \right) \right\}_{j=1}^d \right\}$.

Now, let us consider the following function

$$\tilde{f}(\mathbf{x}) = \begin{cases} f(\mathbf{x}) & \|\mathbf{x}\|_d \leq n^{\frac{1}{3p}} \\ 0 & \|\mathbf{x}\|_d > n^{\frac{1}{3p}} \end{cases}$$

We can use the triangle inequality and write

$$\begin{aligned} \left| \int f(\mathbf{x}) d(\phi_n - \phi) \right| &= \left| \int \tilde{f}(\mathbf{x}) d(\phi_n - \phi) + \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} f(\mathbf{x}) d(\phi_n - \phi) \right| \\ &\leq \left| \int \tilde{f}(\mathbf{x}) d(\phi_n - \phi) \right| + \left| \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} f(\mathbf{x}) d\phi_n \right| + \left| \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} f(\mathbf{x}) d\phi \right| \end{aligned} \tag{42}$$

The above expression is consisted of the three integrals which we will bound next. By applying

the intermediate value theorem to the function $g(t) = f(t(\mathbf{z} - \mathbf{x}) + \mathbf{x} + \mathbf{y})$ we have that

$$\exists t_1 \in [0, 1], f(\mathbf{z} + \mathbf{y}) - f(\mathbf{x} + \mathbf{y}) = (\mathbf{z} - \mathbf{x})^T \nabla f(\mathbf{y} + t_1 \mathbf{x} + (1 - t_1) \mathbf{z})$$

Now, if $\|\mathbf{x} - \mathbf{z}\|_d \leq \varepsilon$, we must have that

$$|f(\mathbf{x} + \mathbf{y}) - f(\mathbf{z} + \mathbf{y})| \leq \overline{C}\varepsilon (1 + (\|\mathbf{x} + \mathbf{y}\| + \varepsilon)^p), \forall \mathbf{x}, \mathbf{y}$$

Therefore, for $\tilde{f}$, we have

$$\begin{aligned} \omega(\tilde{f}; \varepsilon) &= \sup_{\mathbf{y} \in \mathbb{R}^d} \int_{\mathbb{R}^d} \sup_{\mathbf{z}: \|\mathbf{x} - \mathbf{z}\|_d \leq \varepsilon} \left| \tilde{f}(\mathbf{x} + \mathbf{y}) - \tilde{f}(\mathbf{z} + \mathbf{y}) \right| d\phi(\mathbf{x}) \\ &\leq \sup_{\|\mathbf{y}\|_d \leq n^{1/(3p)} + \varepsilon} \int \sup_{\|\mathbf{x} + \mathbf{y}\|_d \leq n^{1/3p} + \varepsilon, \|\mathbf{x} - \mathbf{z}\|_d \leq \varepsilon} |f(\mathbf{x} + \mathbf{y}) - f(\mathbf{z} + \mathbf{y})| d\phi \\ &\leq \sup_{\|\mathbf{y}\|_d \leq n^{1/(3p)} + \varepsilon} \int \overline{C}\varepsilon (1 + (\|\mathbf{x} + \mathbf{y}\| + \varepsilon)^p) d\phi \\ &\leq \overline{C}\varepsilon \left(1 + (n^{1/(3p)} + 2\varepsilon)^p \right) \end{aligned}$$

This implies that $\tilde{f}$ satisfies the requirements of Theorem 3 for f . Hence, we can write

$$\left| \int \tilde{f} d(\phi_n - \phi) \right| \leq c_1 M_r(\tilde{f}) \frac{\bar{y}}{\sqrt{n}} + c_2 \omega\left(\tilde{f}; c_3 \bar{y} \frac{\log n}{\sqrt{n}}\right)$$

Now, let $\bar{n}$ satisfy $c_3 \bar{y} \frac{\log \bar{n}}{\sqrt{\bar{n}}} = 1/2$ which is guaranteed to exist since $\log n / \sqrt{n}$ is decreasing for values of $n \geq e^2$. Therefore, for values of $n \geq \bar{n}$, we have

$$\begin{aligned} \omega\left(\tilde{f}; c_3 \bar{y} \frac{\log n}{\sqrt{n}}\right) &\leq \overline{C} c_3 \bar{y} \frac{\log n}{\sqrt{n}} \left(1 + (n^{1/(3p)} + 1)^p \right) \\ &\leq \overline{C} c_3 \bar{y} \frac{\log n}{\sqrt{n}} (1 + 2^p n^{1/3}) \\ &= \overline{C} c_3 \bar{y} (n^{-1/2} + 2^p n^{-1/6}) \log n \end{aligned}$$

As a result, we can write

$$\begin{aligned} \left| \int \tilde{f} d(\phi_n - \phi) \right| &\leq \overline{C} \bar{y} (c_1 n^{-1/2} + c_2 c_3 (n^{-1/2} + 2^p n^{-1/6}) \log n) \\ &= \overline{C} (\hat{c}_1 n^{-1/3} + \hat{c}_2 n^{-1/2} \log n + \hat{c}_3 n^{-1/6} \log n) \end{aligned}$$

where in the above $\hat{c}_1, \hat{c}_2, \hat{c}_3$ are only functions of $\bar{y}, d$ and p .

Additionally, since each $y_{i,j}$ is bounded above and below by $\bar{y}$ and $-\bar{y}$, we can apply a multi-

dimensional Hoeffding's inequality to it and write

$$\Pr \left(\frac{|\sum_i y_{i,j}|}{\sqrt{n}} \geq t \right) \leq 2e^{-\frac{t^2}{2\bar{y}^2}}$$

We can also write

$$\begin{aligned} \Pr (\|\mathbf{S}_n\|_d \geq t) &\leq \Pr \left(\max_j |S_{n,j}| \geq \frac{t}{\sqrt{d}} \right) \\ &\leq \sum_{j=1}^d \Pr \left(\frac{|\sum_i y_{i,j}|}{\sqrt{n}} \geq \frac{t}{\sqrt{d}} \right) \leq 2de^{-\frac{t^2}{2d\bar{y}^2}} \end{aligned} \quad (43)$$

where the first inequality follows from the fact that $\|\mathbf{x}\|_d \leq \sqrt{d} \max_j |x_j|$. We can then write

$$\begin{aligned} \left| \int_{\|\mathbf{x}\|_d \geq n^{\frac{1}{3p}}} f(\mathbf{x}) d\phi_n \right| &\leq \int_{\|\mathbf{x}\|_d \geq n^{\frac{1}{3p}}} |f(\mathbf{x})| d\phi_n \\ &\leq \bar{C} \int_{\|\mathbf{x}\|_d \geq n^{\frac{1}{3p}}} (1 + \|\mathbf{x}\|_d^p) d\phi_n \end{aligned}$$

By applying Fubini's theorem, we can write the last integral as

$$\int_{\|\mathbf{x}\|_d \geq n^{\frac{1}{3p}}} (1 + \|\mathbf{x}\|_d^p) d\phi_n = \left(n^{\frac{1}{3}} + 1 \right) \phi_n \left(\left\{ \mathbf{x} | 1 + \|\mathbf{x}\|_d^p \geq n^{\frac{1}{3}} + 1 \right\} \right) + \int_{n^{\frac{1}{3}}+1}^{\infty} \phi_n \left(\left\{ \mathbf{x} | 1 + \|\mathbf{x}\|_d^p \geq t \right\} \right) dt$$

We can apply the inequality (43) to the above and have

$$\begin{aligned} \left| \int_{\|\mathbf{x}\|_d \geq n^{\frac{1}{3p}}} f(\mathbf{x}) d\phi_n \right| &\leq \bar{C} \int_{n^{\frac{1}{3}}+1}^{\infty} \phi_n \left(\left\{ \mathbf{x} | 1 + \|\mathbf{x}\|_d^p \geq t \right\} \right) dt + \\ &\quad \bar{C} \left(n^{\frac{1}{3}} + 1 \right) \phi_n \left(\left\{ \mathbf{x} | 1 + \|\mathbf{x}\|_d^p \geq n^{\frac{1}{3}} + 1 \right\} \right) \\ &= \bar{C} \int_{n^{\frac{1}{3p}}}^{\infty} pz^{p-1} \phi_n \left(\left\{ \mathbf{x} | \|\mathbf{x}\|_d \geq z \right\} \right) dz + \\ &\quad \bar{C} \left(n^{\frac{1}{3}} + 1 \right) \phi_n \left(\left\{ \mathbf{x} | \|\mathbf{x}\|_d \geq n^{\frac{1}{3p}} \right\} \right) \\ &\leq 2d\bar{C} \int_{n^{\frac{1}{3p}}}^{\infty} pz^{p-1} e^{-\frac{z^2}{2d\bar{y}^2}} dz + 2d\bar{C} \left(n^{\frac{1}{3}} + 1 \right) e^{-\frac{n^{\frac{2}{3p}}}{2d\bar{y}^2}} \\ &\leq 2d\bar{C} (2pd\bar{y}^2)^p e^{-p} n^{-1/3} + 4d\bar{C} (2pd\bar{y}^2)^p e^{-p^2} n^{-\frac{1}{3}} \end{aligned}$$

Where the first term in the last inequality follows from realizing that the function $x^p e^{-x}$ is maximized at $x = p$ and has a maximum value of $p^p e^{-p}$.²³ The second term comes from a similar logic

²³This implies that $e^{-x} \leq p^p e^{-p} x^{-p}$ for all values of x . We can then write the last integral as $\int_{n^{1/3}}^{\infty} e^{-\frac{x^{2/p}}{2d\bar{y}^2}} dx \leq$

applied to $(n^{1/3} + 1) e^{-n^{2/(3p)}/(2d\bar{y}^2)}$ realizing that since $n \geq 1$, $n^{1/3} + 1 \leq 2n^{1/3}$. We can write the last term in the inequality above as $\bar{C}\hat{c}_4 n^{-1/3}$ where $\hat{c}_4$ only depends on d, p and $\bar{y}$.

Finally, the last term in (42) satisfies

$$\begin{aligned} \left| \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} f(\mathbf{x}) d\phi \right| &\leq \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} |f(\mathbf{x})| d\phi \\ &\leq \bar{C} \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} (1 + \|\mathbf{x}\|_d^p) d\phi \end{aligned}$$

We can use the spherical coordinates and rewrite the above integral as

$$\begin{aligned} \bar{C} \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} (1 + \|\mathbf{x}\|_d^p) d\phi &= \\ \bar{C} \int_{n^{\frac{1}{3p}}}^{\infty} (1 + r^p) r^{d-1} \frac{e^{-\frac{r^2}{2}}}{(2\pi)^{d/2}} dr \int_{[0, \pi]^{n-2} \times [0, 2\pi]} \sin^{n-2} \varphi_1 \cdots \sin \varphi_{n-2} d\varphi_1 \cdots d\varphi_{n-1} &= \\ \bar{C} \int_{n^{\frac{1}{3p}}}^{\infty} (1 + r^p) r^{d-1} \frac{e^{-\frac{r^2}{2}}}{(2\pi)^{d/2}} dr \frac{2\pi^{d/2}}{\Gamma(d/2)} &\leq \\ 2\bar{C} \int_{n^{\frac{1}{3p}}}^{\infty} r^{p+d-1} e^{-\frac{r^2}{2}} dr \frac{2^{1-d/2}}{\Gamma(d/2)} \end{aligned}$$

where in the above we have used the fact that if $r \geq 1$, then $r^p + 1 \leq 2r^p$ and $\Gamma(\cdot)$ is Euler's Gamma function. Using the fact that $x^{d+2p} e^{-x^2/2}$ is maximized at $\frac{d+2p}{2}$ and that the highest possible value it takes is $(d+2p)^{\frac{d+2p}{2}} e^{-(d+2p)^2/4}$, we can show that

$$\left| \int_{\|\mathbf{x}\|_d > n^{1/(3p)}} f(\mathbf{x}) d\phi \right| \leq 2\bar{C} (d+2p)^{\frac{d+2p}{2}} e^{-(d+2p)^2/4} n^{-1/3} \frac{2^{1-d/2}}{p\Gamma(d/2)} = \bar{C}\hat{c}_5 n^{-1/3}$$

where in the above $\hat{c}_5$ only depends on d and p . Replacing all of these bounds into (42) we have

$$\begin{aligned} \left| \int f(\mathbf{x}) d(\phi_n - \phi) \right| &\leq \bar{C} (\hat{c}_1 n^{-1/3} + (\hat{c}_2 n^{-1/2} + \hat{c}_3 n^{-1/6}) \log n) + \\ &\quad \bar{C} (\hat{c}_4 + \hat{c}_5) n^{-1/3} = \bar{C} \kappa(n) \end{aligned}$$

To see that $\kappa(n)$ is eventually decreasing, notice that in the above, $(\hat{c}_2 n^{-1/2} + \hat{c}_3 n^{-1/6}) \log n$ is the only term that can be decreasing. Since the function $n^{-\alpha} \log n$ is maximized at $n = e^{1/\alpha}$, for all values of $n \geq e^6$, $\kappa(n)$ is guaranteed to be decreasing. Moreover, using a similar reasoning we see that $\lim_{n \rightarrow \infty} \kappa(n) = 0$. This establishes the claim. $\square$

$p^p e^{-p} \int_{n^{1/3}}^{\infty} (2d\bar{y})^2 x^{-2} dx$ which implies the inequality.

To establish the claim that $T_N - T$ is a bounded linear operator, recall that $T_N - T$ is given by

$$\begin{aligned}
(T_N - T)x &= \int \sigma(\mathbf{h}) x(\mathbf{h}) w_N(\mathbf{h}) d\mu^N - \int \sigma(\mathbf{h}) x(\mathbf{h}) w(\mathbf{h}) d\mu \\
&= \int \sigma(\mathbf{h}) x(\mathbf{h}) \left(w(\mathbf{h}) - \frac{1}{\sqrt{N}} (\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1}) \odot \mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1} \right) d\mu^N \\
&\quad - \int \sigma(\mathbf{h}) x(\mathbf{h}) w(\mathbf{h}) d\mu \\
&= \int \sigma(\mathbf{h}) x(\mathbf{h}) w(\mathbf{h}) d(\mu^N - \mu) \\
&\quad - \frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int \sigma(\mathbf{h}) x(\mathbf{h}) \mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1} d\mu^N
\end{aligned}$$

where in the above $\odot$ is the Hadamard (pointwise) vector multiplication and $\oslash$ is the Hadamard division. Moreover, $\mathbf{h}_{1:K-1}$ is consisted of the first $K - 1$ entries of $\mathbf{h}$. Note that for the last integral above we have

$$\begin{aligned}
\left| \frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int \sigma(\mathbf{h}) x(\mathbf{h}) \mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1} d\mu^N \right| &\leq \\
\frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int |\sigma(\mathbf{h})| |x(\mathbf{h})| |\mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1}| d\mu^N &\leq \\
\frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int |\sigma(\mathbf{h})| |\mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1}| d\mu^N \|x\|_{L_\infty} &
\end{aligned}$$

where the inequalities above are element wise, i.e., each side is a member of $\mathbb{R}^{K-1}$. By the Central Limit Theorem,

$$\int |\sigma(\mathbf{h})| |\mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1}| d\mu^N \rightarrow \int |\sigma(\mathbf{h})| |\mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1}| d\mu$$

Thus for N large enough, we have that

$$\begin{aligned}
\left| \frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int \sigma(\mathbf{h}) x(\mathbf{h}) \mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1} d\mu^N \right| &\leq \\
(1 + \varepsilon) \frac{(\mathbf{t}_{2:K} - \mathbf{t}_{1:K-1})}{\sqrt{N}} \odot \int |\sigma(\mathbf{h})| |\mathbf{h}_{1:K-1} \oslash \mathbf{f}_{1:K-1}| d\mu \|x\|_{L_\infty} &= \mathbf{a}(\sigma) \frac{\|x\|_{L_\infty}}{\sqrt{N}}
\end{aligned} \tag{44}$$

Let us now focus on the first term $\int \sigma(\mathbf{h}) x(\mathbf{h}) w(\mathbf{h}) d(\mu^N - \mu)$ and refer to it as $\tilde{T}_N x$ where $\tilde{T}_N$ is a linear operator that maps $L_\infty(\mathbb{R}_0^K)$ to $\mathbb{R}^{K-1}$. Consider the definition of $\tilde{T}_N$'s norm as an

operator over L_∞ :

$$\begin{aligned}\left\|\tilde{T}_N\right\|_\infty &= \sup_{x \in L_\infty(\mathbb{R}_0^K), \|x\|_{L_\infty} \leq 1} \int x(\mathbf{h}) \sigma(\mathbf{h}) w(\mathbf{h}) d(\mu^N - \mu) \\ &= \sup_{z \in L_\infty(\mathbb{R}_0^K), \|z\|_{L_\infty} \leq \|\sigma\|_{L_\infty}} \int z(\mathbf{h}) w(\mathbf{h}) d(\mu^N - \mu)\end{aligned}$$

We can thus in the above view $w(\mathbf{h}) d(\mu^N - \mu)$ as a signed measure and thus, the optimum value in the above should assign $z(\mathbf{h}) = \|\sigma\|_{L_\infty}$ to a maximal set of $w(\mathbf{h}) d(\mu^N - \mu)$, i.e., sets $A \subset \mathbb{R}_0^K$ where $\int_A w(\mathbf{h}) d(\mu^N - \mu)$ is maximized (such a set and all of its subsets should have non-negative measure) and $z(\mathbf{h}) = -\|\sigma\|_{L_\infty}$ for its complement. Therefore,

$$\begin{aligned}\left\|\tilde{T}_N\right\|_\infty &= \sup_{A \subset \mathbb{R}_0^K, A: \text{Borel}} \|\sigma\|_{L_\infty} \int_A w(\mathbf{h}) d(\mu^N - \mu) - \|\sigma\|_{L_\infty} \int_{A^c} w(\mathbf{h}) d(\mu^N - \mu) \\ &= 2\|\sigma\|_{L_\infty} \sup_{A \subset \mathbb{R}_0^K, A: \text{Borel}} \int_A w(\mathbf{h}) d(\mu^N - \mu) + \|\sigma\|_{L_\infty} \int_{A^c} w(\mathbf{h}) d(\mu^N - \mu)\end{aligned}\quad (45)$$

Now consider the function $v(\mathbf{h}) = \mathbf{1}[\mathbf{h} \in A] w(\mathbf{h})$. Since $w(\mathbf{h})$ is a quadratic form, this function must satisfy $M_2(v) \leq M_2(w) < \infty$. Moreover, since $w(\mathbf{h})$ is continuously differentiable, $v(\mathbf{h})$ is also continuously differentiable except at the boundary of A . Since A is a Borel subset of $\mathbb{R}_0^K$, its boundary must have a zero measure. So $v(\mathbf{h})$ is almost surely continuously differentiable and its derivative also satisfies $M_2(\partial v / \partial h_k) \leq M_2(\partial w / \partial h_k) < \infty$ (since it is linear). This implies that we can use Lemma 3 (apply it to both terms in (45)) and thus have that

$$\left\|\tilde{T}_N\right\|_\infty \leq 3\|\sigma\|_\infty \kappa(N) \max \left\{ M_2(w), \{M_2(\partial w / \partial h_k)\}_{k=1}^{K-1} \right\}$$

and thus using, the Holder inequality implies that

$$\left\|\tilde{T}_N x\right\|_{K-1} \leq \left\|\tilde{T}_N\right\|_\infty \|x\|_\infty \leq 3\|\sigma\|_\infty \|x\|_\infty A \kappa(N)$$

for some $A > 0$. This together with inequality (44) implies that

$$\begin{aligned}\|(T_N - T)x\|_{K-1} &\leq \left\|\tilde{T}_N x\right\|_{K-1} + \|\mathbf{a}(\sigma)\|_{K-1} \frac{\|x\|_{L_\infty}}{\sqrt{N}} \\ &\leq \|x\|_\infty \left(3\|\sigma\|_\infty A \kappa(N) + \|\mathbf{a}(\sigma)\|_{K-1} / \sqrt{N} \right)\end{aligned}$$

which is the desired result. □

C.1 Surjectivity of the operator T

Proof. The only remaining part of the proof of Theorem 1 is to show surjectivity of T . As we have shown in the proof of Theorem 1, if T is not surjective, then there must exist an open set of values of $\mathbf{h}$, U , in $\mathbb{R}_0^K$ together with a non-zero vector $\mathbf{y} \in \mathbb{R}^{K-1}$ such that $\mathbf{y}w(\mathbf{h})^{\text{tr}} = 0$. In other words,

$$\sum_{k=2}^K y_k w_k(\mathbf{h}) = 0, \forall \mathbf{h} \in U$$

Recall that $w_k(\mathbf{h}) = (\mathbf{h} \cdot \mathbf{t} + b) \left(\frac{h_k}{f_k} - \frac{h_{k-1}}{f_{k-1}} \right) - (t_k - t_{k-1})$. Since the above has to hold in an open set, we should also have that

$$\sum_{k=2}^K y_k \frac{\partial^2}{\partial h_l^2} w_k(\mathbf{h}) = 0$$

where by taking derivatives, we take into account the fact that in $\mathbb{R}_0^K$ a change in h_k , $k \geq 2$, should decrease h_1 by the same amount. We have

$$\frac{\partial^2}{\partial h_l^2} w_k(\mathbf{h}) = \mathbf{1}[k=l] \frac{t_k}{f_k} - \mathbf{1}[k-1=l] \frac{t_{k-1}}{f_{k-1}} + \mathbf{1}[k=2] \frac{t_l}{f_1}$$

Therefore,

$$\begin{aligned} \sum_{k=2}^K y_k \frac{\partial^2}{\partial h_l^2} w_k(\mathbf{h}) &= 0 \\ \sum_{k=2}^K y_k \left(\mathbf{1}[k=l] \frac{t_k}{f_k} - \mathbf{1}[k-1=l] \frac{t_{k-1}}{f_{k-1}} + \mathbf{1}[k=2] \frac{t_l}{f_1} \right) &= \\ y_l \frac{t_l}{f_l} - y_{l+1} \frac{t_l}{f_l} + y_2 \frac{t_l}{f_1} &= 0, l \leq K-1 \\ y_K \frac{t_K}{f_K} + y_2 \frac{t_K}{f_1} &= 0 \end{aligned}$$

In other words,

$$\frac{y_l - y_{l+1}}{f_l} + \frac{y_2}{f_1} = 0, l \leq K-1, \frac{y_K}{f_K} + \frac{y_2}{f_1} = 0$$

Multiplying the l -th equation by f_l and summing over them we have

$$y_2 + \sum_l f_l \frac{y_2}{f_1} = 0 \rightarrow y_2 = 0 \rightarrow y_K = 0 \rightarrow y_l = 0, \forall l$$

which is a contradiction. □

C.2 A Cheap-Talk Two Sender Example

In this section we explore how the mechanisms change if we revert to traditional cheap-talk a la [Crawford and Sobel \(1982\)](#), where two senders get signals $s_i \sim U[-1, 1], i = 1, 2$ and in which they independently and simultaneously make one report each. In this case, senders cannot achieve the sender best allocation as it requires them to report their signal truthfully in which the receiver will not obey the sender-preferred mechanism. One can show that the pure strategy cheap talk equilibria must be monotone partitional. For instance, there are two types of partitional equilibria that divide $[-1, 1]$ into two sub-intervals. Their associated allocations are:

$$\sigma_1(\mathbf{s}) = \begin{cases} 1 & s_1, s_2 \geq -\frac{1+2\sqrt{2}b}{3} \\ 0 & \text{otherwise} \end{cases}, \sigma_2(\mathbf{s}) = \begin{cases} 0 & s_1, s_2 \leq \frac{1-2\sqrt{2}b}{3} \\ 1 & \text{otherwise} \end{cases}.$$

Considering σ_1 and σ_2 as direct mechanisms the cutoffs are chosen so that the cutoff type is indifferent between truth-telling and lying. This is depicted in Figure 6.

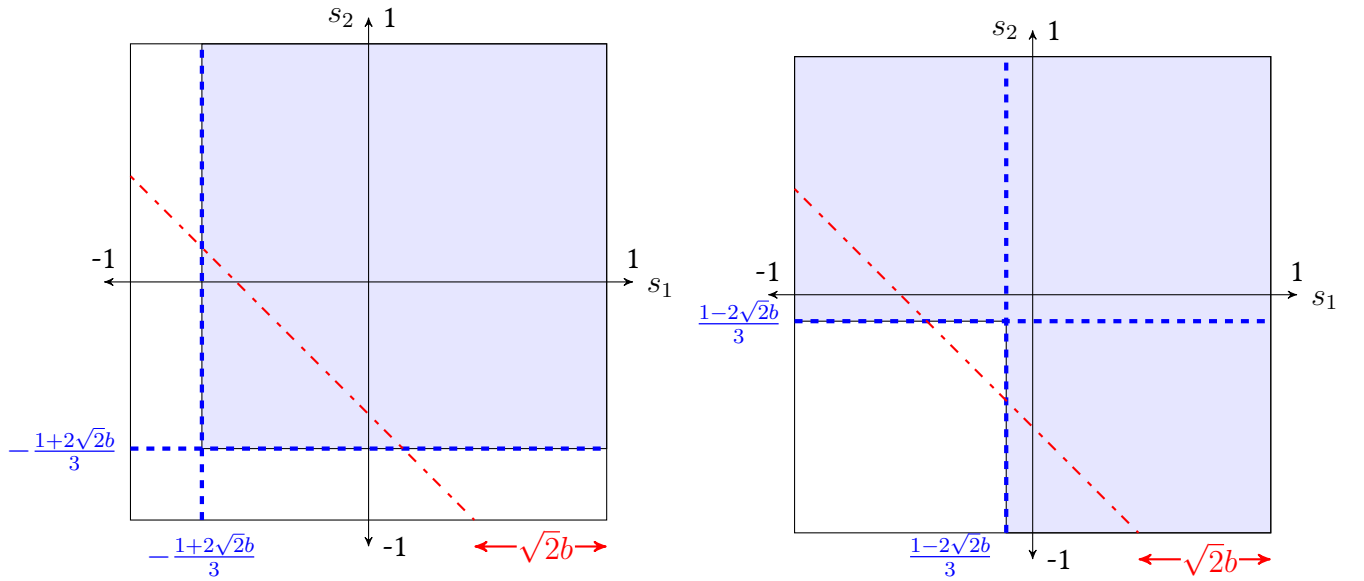


Figure 6: Recommendation mechanisms associated with simultaneous-move cheap talk. The mechanisms recommend $a = 1$ for high report by both sender (left) or either of the senders (right).

As can be observed from Figure 6, equilibria of the simultaneous-move cheap talk game add some areas (recommends $a = 1$) to the south-west of the sender-preferred area (depicted by the dashed red line in Figure 6) while removing some areas from this area.

C.3 Intuition of Proposition 1

In this section, we formalize the perturbation reported in section 3. Consider the hypercubes introduced in that section and take the limit when the length of hypercubes dx , goes to zero. We may evaluate the receiver's marginal payoff from such a perturbation. Indeed, for a type x_m at the border of hypercube $B_m^N = (x_{m-1}, x_m]^N$, incentive compatibility requires that the costs of the distortions ε_m and ε_{m+1} associated with reporting x_m and x_{m+1} , respectively, are equal:

$$\left(\frac{x_m}{\sqrt{N}} + b\right)(\varepsilon_m - \varepsilon_{m+1}) + \frac{(N-1)}{\sqrt{N}} \left(\frac{x_m + x_{m-1}}{2} \varepsilon_m - \frac{x_m + x_{m+1}}{2} \varepsilon_{m+1}\right) = 0.$$

Equivalently,

$$\left(\sqrt{N}x_m + b\right)(\varepsilon_m - \varepsilon_{m+1}) - dx \frac{(N-1)}{\sqrt{N}} \left(\frac{\varepsilon_m + \varepsilon_{m+1}}{2}\right) = 0. \quad (46)$$

The first term on the left side of (46) is a marginal benefit from over-reporting, and reducing the probability of distortion by $\varepsilon_m - \varepsilon_{m+1}$.²⁴ The second term is a marginal cost associated with suffering distortion when all other types are higher. For dx small, the difference equation (46) is approximated by the differential equation

$$\frac{d \ln \varepsilon}{d\omega} = -\frac{N-1}{N} \times \frac{1}{\omega + b}, \quad (47)$$

for $\omega > -b$, where we have used the substitution $\omega = \sqrt{N}x$ along the line $s_1 = \dots = s_N = x$. Equation (47) is solved by $\varepsilon(\omega) = \left(\frac{1}{\sqrt{N}(\omega+b)}\right)^{-\frac{N-1}{N}}$. Having characterized the distortions themselves, the marginal payoff to the receiver' is simply

$$-\sum \frac{\omega_{m-1} + \omega_m}{2} \varepsilon_m \frac{1}{2^N} dx^N \approx -dx^N \int_{-b}^{\sqrt{N}} \omega \left(\frac{1}{\sqrt{N}(\omega+b)}\right)^{-\frac{N-1}{N}} d\omega \propto \frac{\sqrt{N}}{N+1} (b\sqrt{N} - 1),$$

where $\omega_m = \sqrt{N}x_m$. Thus, this perturbation beats the sender-preferred allocation precisely when Assumption 1 is violated. That is, Assumption 1 is tight in the uniform case.

The above exercise also provides intuition for the role played by b and N in Proposition 1. As we have seen, reducing acceptance in the disagreement region must be accompanied by distortions in the agreement region to preserve incentives. When b increases, the distortions apply to a greater extent over a region for which the receiver benefits. When N increases, the cost

²⁴Indeed, it is easy to see from (46) that indifference requires $\varepsilon_m > \varepsilon_{m+1}$ – since ω is everywhere greater in B_{m+1}^N than in B_m^N , the probability of distortion must decrease in m .

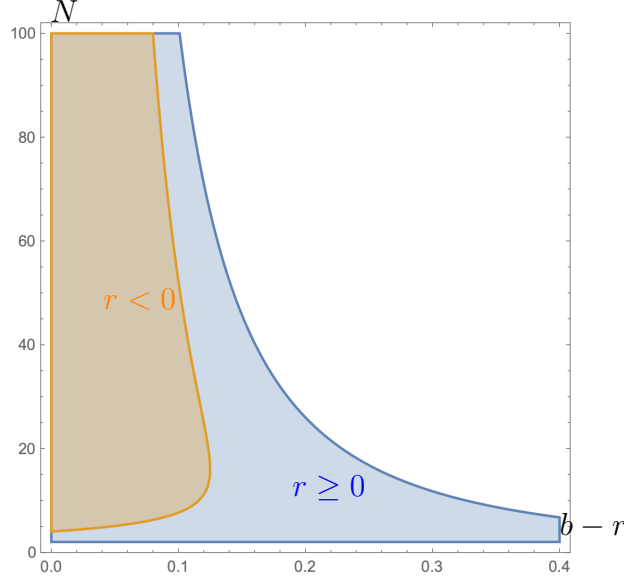


Figure 7: Set of N and $b - r$ that satisfy Assumption 1 when $s_i \sim U[-1, 1]$

of punishment becomes larger relative to a marginal upward report (see, for instance, equation (46)), because that punishment occurs when many other senders also have higher types. For this reason, the distortions diminish more rapidly in ω while preserving incentives (equation (47)). Since these distortions are actually good for the receiver at low values of ω , the benefits of the distortion become large relative to its costs. By contrast, small b and N drive towards the optimality of the sender-preferred allocation.

Generally speaking, one can fix the distribution $f(s)$ and preference parameter of the receiver, r , and consider the set of biases $b - r$ and the number of senders that satisfy Assumption 1. As depicted in Figure 7, as the degree of bias increases the maximum number of senders that satisfies Assumption 1 goes down. Moreover, the relevant statistic of the distribution is $\max_{s \in S} 1 - \frac{(\max S - s)f'(s)}{f(s)}$. A higher number for this statistic leads to a smaller set of possible values for $(b - r, N)$.

D Numerical Solution of Small Economy with Large Bias

The problem of finding the optimal mechanism, (P), is a linear program. Here we plot the numerical solution of this linear program when $N = 2$, $b = 0.6\sqrt{2}$, $s_i \sim U[-1, 1]$, and $r = 0$. For the numerical solution, we discretize $[-1, 1]$ into 200 subintervals and solve for optimal σ for each square generated by all such subintervals, i.e., we solve for a vector of 40000 values. We use the `linprog` function in MATLAB to solve this linear program. Figure 8 depicts the solution. The

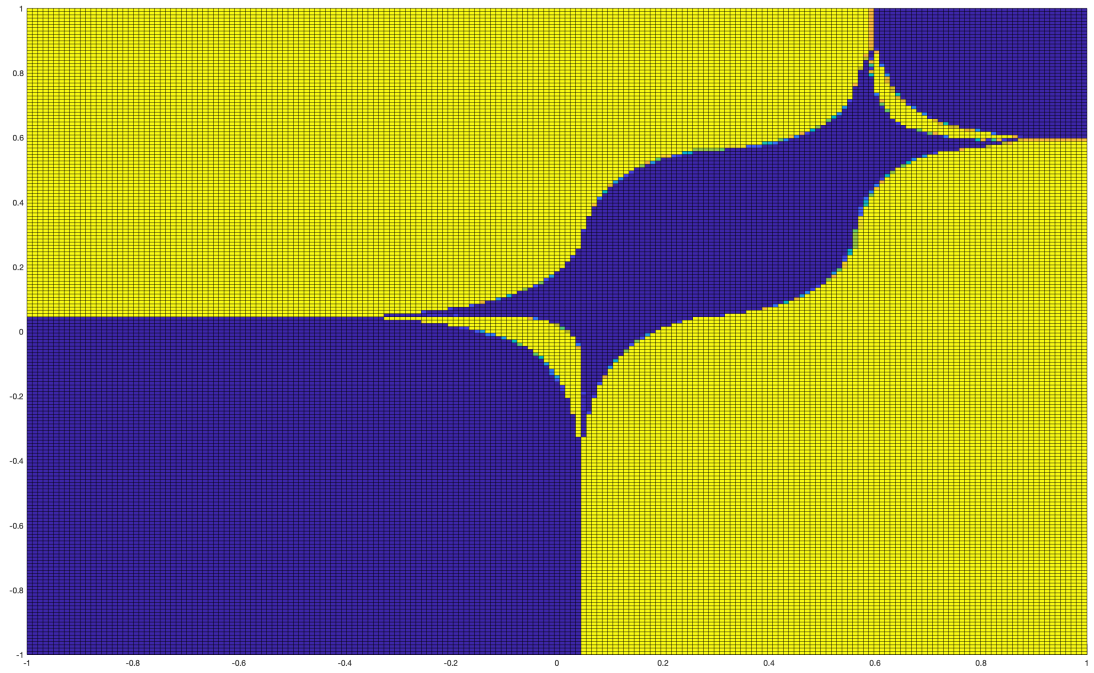


Figure 8: Numerical simulation of the optimal mechanism

yellow area represents the values for which $\sigma = 1$ and the blue area is associated with $\sigma = 0$.